Ptolemy's ALMAGEST

Translated and Annotated by

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Preface

A new English translation of the Almagest needs no apology. As one of the most influential scientific works in history, and a masterpiece of technical exposition in its own right, it deserves a much wider audience than can be found amongst those able to read it in the original. The existing English translation by R. Catesby Taliaferro,[1] besides being difficult to acquire, is such that silence is the kindest comment one can make. The French translation by N. Halma, virtually unobtainable, suffers from excessive literalness, particularly where the text is difficult. The other modern version, Karl Manitius' German translation, is on an entirely different level from these. It was done by a man who had studied the text and made a strenuous and on the whole successful effort to understand Ptolemy's meaning and methods. I have used it constantly for twenty years, and those to whom it is familiar will recognise how much I owe to it. Nevertheless, it is not free from mistakes, and, to my taste, errs in the direction of paraphrasing where it should be translating. Most important, one can no longer assume that those with a serious interest in history are able to read German with ease. I have been able to improve on Manitius' translation, in part because of work published since he made it, in part because I had independent access to much of the textual evidence, notably the mediaeval Arabic translations. I have drawn attention to a few passages where I have noticed that he is in error, but I have made no systematic comparison between my translation and his or any other version.

Every translator, and especially one dealing with an ancient language, is confronted with the dilemma of being faithful to the original and at the same time comprehensible to his readers. My intention was that this translation should serve both those who know no Greek, as a substitute for the text, and those who do, as an aid to reading it. This has inevitably led to compromises. On the whole, I have kept closely to the meaning and structure of the Greek, even, on occasion, where this entailed abandoning idiomatic English. But I have usually broken up Ptolemy's enormously long sentences (characteristic of Hellenistic scientific prose) into shorter units more suitable for English, and I have frequently substituted mathematical symbols (=, + etc.) and a symmetric presentation for the continuous rhetorical exposition of the ancient text. I have been liberal with explanatory additions, which are marked as such by enclosure within square brackets. Wherever the need to be intelligible forced me to a paraphrase, I give the literal translation in a footnote.

It would have made what is an already big book impossibly unwieldy if I had

[1] For full references here and elsewhere see the Bibliography.

provided a full technical and historical commentary on the Almagest. Fortunately two recent works, by Neugebauer (*HAMA*) and Pedersen, are excellent guides to the technical content, and the former is also of considerable help on the numerous historical problems which arise from it. I have therefore confined my own commentary to footnotes on points of detail (referring to the above works for expository treatments), and to an introduction giving the minimum of information necessary to understand and use the translation.

In the course of making the translation I recomputed all the numerical results in the text, and all the tables (the latter mostly by means of computer programs). The main purpose of this was to detect scribal errors (in which I have been moderately successful). But my calculations incidentally revealed a number of computing errors or distortions committed by Ptolemy himself. Where these are explicable as the result of rounding in the course of computation they are ignored, since to list some thousands of slightly more accurate results which I have found with modern mechanical aids would invite Ptolemy's own sardonic remark: 'Scrupulous accuracy about such a small amount is a sign of vain conceit rather than love of truth'. However, I have noted every computing error of a significant amount, and also those cases where the rounding errors are not random, but seem directed towards obtaining some 'neat' result. I hope that this will shed some light on the problem of Ptolemy's manipulation of his material (both computational and observational) in order to present an appearance of rigor in his theoretical treatment which he could never have found in his actual experience. The problem is an interesting one, which deserves an informed and critical discussion. Unfortunately, the recent book on this subject by R. R. Newton provides nothing of the kind, but rather tends to bring the whole topic into disrepute. The only detailed discussion which is useful is that by Britton [1].[2] This, however, is confined to certain classes of the observations. My own inferences from the computations tend to confirm Britton's conclusions about the nature and purpose of Ptolemy's manipulations of his data.

This book owes much to the help of numerous people and institutions, which I gratefully acknowledge here. The Bibliothèque Nationale, Paris, the Biblioteca Apostolica Vaticana and the Biblioteca de El Escorial provided me with microfilms of various Greek and Arabic manuscripts of the Almagest (detailed on pp. 3–4). I thank my colleague, David Pingree, Prof. Dr. Fuat Sezgin and Prof. Dr. Paul Kunitzsch for providing me with other microfilms and photocopies which I needed. Mr. Colin Haycraft not only gave me the necessary encouragement actually to embark on a project which I had been contemplating for a long time, but also bore patiently with the repeated delays until the book was ready for publication. When B. R. Goldstein, who was already engaged in preparing an English version of the Almagest, heard that I had decided to make this translation, he generously abandoned the project and turned over his materials to me. I owe to these and to him several ideas about format and notation. My pupil, Don Edwards, detected a number of slips and

[2] It is regrettable that this has never been formally published. It is available in Xerox copy from University Microfilms International, Ann Arbor, Michigan 48106.

typing errors in my preliminary version, and performed many useful services in comparing the translation with the Greek text. Michele Wilson drew Fig. F for me. Janet Sachs provided invaluable help in preparing the typescript for publication and eliminating numerous mistakes. Several of my footnotes on difficult problems have been influenced by my discussions with Noel Swerdlow. Rather than trying to disentangle his contribution at each place, I here record, with thanks, the stimulus he has given to my thinking. N. G. Wilson answered my questions on points of Greek palaeography and went out of his way to examine manuscripts at my request. My colleague, A. J. Sachs, gave me the benefit of his unrivalled expertise on several points of Babylonian astronomy and Mesopotamian history. To my colleague O. Neugebauer I owe more than I can express here. Let me say only that it was he who first introduced me to the Almagest more than twenty years ago, that his own investigations of it (only part of which have been published in his monumental *A History of Ancient Mathematical Astronomy*) have been invaluable to me as an aid and as a model, and that many will recognize his draughtsmanship in several of the supplementary diagrams. As an inadequate token I dedicate this book to him.

Providence, 1982 G.J.T.

Introduction

1. *Ptolemy*

For a detailed discussion of what little is known of the life of the author of the Almagest, and an account of his numerous other works, on astronomy, astrology, geography, optics and other mathematical subjects, I refer the reader to my article in the Dictionary of Scientific Biography (Toomer [5]). Here I mention only that his name was Claudius Ptolemaeus (Κλαύδιος Πτολεμαῖος), that he lived from approximately A.D. 100 to approximately A.D. 175, and that he worked in Alexandria, the principal city of Greco-Roman Egypt, which possessed, among other advantages, what was probably still the best library in the ancient world.

2. *The Almagest*

The Almagest is firmly dated to the reign of the Roman emperor Antoninus (A.D. 138–161). The latest observation used in it is from 141 February 2 (IX 7 p. 450), and Ptolemy takes the beginning of the reign of Antoninus as the epoch of his star catalogue (VII 4 p. 340). Although it is clear that Ptolemy had spent much time on it and that it is a work of his maturity (his own observations recorded in it range from A.D. 127 to 141), it has always been considered as his earliest extant work, because of the changes from it and references back to it in other works by him (for details see Toomer [5] p. 187). However, a recent discovery by Norman T. Hamilton (see IV n.51 p. 205) has shown that the 'Canobic Inscription' represents a stage in the development of Ptolemy's astronomical theory earlier than the Almagest. Since Ptolemy erected that dedication in the tenth year of Antoninus (A.D. 146/7), the Almagest can hardly have been published earlier than the year 150.

As is implied by its Greek name, μαθηματικὴ σύνταξις, 'mathematical systematic treatise', the Almagest is a complete exposition of mathematical astronomy as the Greeks understood the term. Whether there were any comparable works (i.e. *comprehensive* astronomical treatises) before it is not known. In any case, its success contributed to the loss of most of the work of Ptolemy's scientific predecessors, notably Hipparchus, by the end of antiquity, because, being obsolete, they ceased to be copied. Whereas Hipparchus' works are still used by Ptolemy's younger contemporaries, Galen and Vettius Valens,[1]

[1] E.g. Galen, *On Seven-month Children*, ed. Walzer 347, 350; Commentary on Hippocrates' *Airs Waters and Places* (see GAS VI 98). Vettius Valens, *Anthologiae* 354.

by the early fourth century (and probably much earlier),[2] when Pappus wrote his commentary on it, the Almagest had become the standard textbook on astronomy which it was to remain for more than a thousand years. Thus its importance for us lies not only in its value as a historical source for earlier theories and observations, but also, and perhaps chiefly, in its influence on all later astronomy in antiquity and the middle ages (in both Islamic and Christian areas) down to the sixteenth century. It was dominant to an extent and for a length of time which is unsurpassed by any scientific work except Euclid's *Elements*.

No attempt can be made here to sketch even an outline of the history of its influence.[3] I mention only some points to which I will make reference in the notes to the translation. The position of the Almagest as the standard textbook in astronomy for 'advanced students' in the schools at Alexandria (and no doubt at Athens and Antioch too) in late antiquity is amply demonstrated by the partially extant commentaries on it by Pappus (*c.* 320) and by Theon of Alexandria (*c.* 370). In the late eighth and ninth centuries, with the growth of interest in Greek science in the Islamic world, the Almagest was translated, first into Syriac, then, several times, into Arabic. In the middle of the twelfth century no less than five such versions were still available to the amateur ibn aṣ-Ṣalāḥ: a Syriac translation, two versions made under the Caliph al-Ma'mūn (an older one by al-Ḥasan ibn Quraysh, and one dated 827/8 by al-Ḥajjāj), a version by the famous translator Isḥāq ibn Ḥunayn (*c.* 879–90), and a revision of the latter by Thābit ibn Qurra (d. 901).[4] Two of these translations are still extant, those of al-Ḥajjāj and Isḥāq-Thābit. In them we find the title of Ptolemy's treatise given as 'al-mjsṭy' (consonantal skeleton only). This is undoubtedly derived (ultimately) from a Greek form μεγίστη (?sc. σύνταξις), meaning 'greatest [treatise]', but it is only later that it was incorrectly vocalised as al-majasṭī, whence are derived the mediaeval Latin 'almagesti', 'almagestum', the ancestors of the modern title 'Almagest'. The available evidence has been assembled and discussed by Kunitzsch, *Der Almagest* 115–25, where he makes a good case for supposing that the Arabic form was derived, not directly from the Greek, but from a middle Persian (Pahlavi) translation of the Almagest. There is independent evidence for the existence of the latter, but whether it was made as early as the reign of the Sassanid king Shahpuhr I (241–272), as later Persian accounts maintain, seems very dubious to me.

While Ptolemy's work in the original Greek continued to be copied and studied in the eastern (Byzantine) empire, all knowledge of it was lost to western

[2] The evidence for the practice of astronomy in the third century is pitifully small, but there exists a fragment of a text from about A.D. 213 which is closely related to the Almagest (see *HAMA* II 948–49), and there are several third-century papyri related to the Handy Tables (*ibid.* 974–75, 979–80). P. Ryl. 27 (written c. 260) quotes Ptolemy's solstice and equinox observations from Almagest III 1, and in the late third century Porphyry (*Comm. on Harmonica* 2, p. 24,15 ff.) quotes Almagest I 2 (H9, 11–16). The only evidence I have seen for knowledge of the Almagest in the second century, Galen, Commentary on Hippocrates' *Airs Waters and Places* III (ms. Cairo, Ṭal'at ṭibb 550, p. 73a), where Ptolemy is mentioned at the end of a list of authorities on astronomy, must be an interpolation in the Arabic tradition, since Ptolemy is there characterized as 'the king of Egypt'.

[3] I know of no satisfactory account of this. I gave a very brief sketch, Toomer[5] 202.

[4] For a full account of this see Kunitzsch, *Der Almagest*, especially 15–71. Kunitzsch has also published the work of ibn aṣ-Ṣalāḥ (see Bibliography).

Europe by the early middle ages. Although translations from the Greek text into Latin were made in mediaeval times,[5] the principal channel for the recovery of the Almagest in the west was the translation from the Arabic by Gerard of Cremona, made at Toledo and completed in 1175.[6] Manuscripts of the Greek text began to reach the west in the fifteenth century, but it was Gerard's text which underlay (often at several removes) books on astronomy as late as the Peurbach-Regiomontanus epitome of the Almagest (see Bibliography under Regiomontanus). It was also the version in which the Almagest was first printed (Venice, 1515). The sixteenth century saw the wide dissemination of the Greek text (printed at Basel by Hervagius, 1538), and also the obsolescence of Ptolemy's astronomical system, brought about not so much by the work of Copernicus (which in form and concepts is still dominated by the Almagest), as by that of Brahe and Kepler.

3. *The translation*

The basis of my translation is the Greek text established by Heiberg. I have, however, found it necessary to make several hundred corrections to that text. These are noted at the places in the translation where they occur,[7] and are also listed in Appendix B. In many cases (usually involving numerical computations), my correction consists of adopting the reading of the manuscript D, unjustly spurned by Heiberg as descended from an archetype due to an Alexandrian recension in late antiquity (Prolegomena, in Ptolemy, *Opera Minora* CXXVI–VII). Whatever the truth about that, and despite the fact that D itself is, as Heiberg says, 'most negligently written', I am convinced on grounds of internal consistency that it represents a sounder tradition than that of the mss. ABC, generally preferred by Heiberg. In many cases its obviously correct readings are shared by all or part of the Arabic tradition. Nevertheless, I have not deviated from Heiberg's text except where it seemed essential for sense or numerical consistency. In making corrections I have referred to photographs of the following manuscripts.

Greek (I use Heiberg's notation)

A Parisinus graecus 2389. Mainly uncial, ninth century

B Vaticanus graecus 1594. Minuscule, ninth century

D Vaticanus graecus 180. Several hands, but not, as Heiberg, *Almagest* I p. V, of the twelfth century, but rather of the tenth: see the Vatican Catalogue by Mercati and Franchi de' Cavalieri, I p. 206. N. G. Wilson has confirmed this dating for me by personal inspection. (Heiberg himself seems to have changed his opinion later: see Prolegomena LXXIX.)

Arabic (I have used the abbreviations 'Ar' to refer to the consensus of the

[5] See Haskins, *Studies* 103–112, 157–165.

[6] Kunitzsch, *Der Almagest* 83–112, gives a valuable account of the evidence for this, and of Gerard's method of work: evidently he used more than one of the Arabic translations.

[7] I have acknowledged there all cases known to me where my correction has been anticipated by others, notably Manitius.

Arabic tradition, and 'Is' to the consensus of the mss. containing the Isḥāq-Thābit version).

L Leiden, or. 680. Eleventh century according to Kunitzsch, *Der Almagest* 38. This is the only surviving manuscript of the version of al-Ḥajjāj.

T Tunis, Bibliothèque Nationale, 07116 (see Kunitzsch, *Der Almagest* 38–40). Completed October 1085. The Isḥāq-Thābit version, complete.

P Paris, B.N. ar. 2482. Completed December 1221. See Kunitzsch, *Der Almagest* 42–3. The Isḥaq-Thābit version, Books I–VI 13.

Q Paris, B.N. ar. 2483. Fifteenth century. See Kunitzsch, *Der Almagest* 43. The Isḥāq-Thābit version, Books I–VII.

E Escorial 914. See Kunitzsch, *Der Almagest* 43–4. The Isḥāq-Thābit version, Books V–IX.

F Escorial 915. Completed September 1276. See Kunitzsch, *Der Almagest* 44–5. The Isḥāq-Thābit version, allegedly containing Books VII–XIII, but in fact lacking large sections even of these, and bound in such disorder as to be almost useless.

Ger The Latin translation of Gerard of Cremona, for which I have used only the printed edition (Venice, Liechtenstein, 1515). For the complex dependence of this on the various Arabic versions see Kunitzsch, *Der Almagest* 97–104.

I did not undertake a complete collation of any of the above mss. For the Greek mss. that would have been largely useless, since Heiberg's reports are, as in all his editions, very accurate (to judge from my sporadic verifications; I remarked the rare exceptions in the notes to the translation). To collate the Arabic translation would have delayed this book for several years, with no commensurate gain. I have consulted the above mss. only in passages where I already considered Heiberg's text wrong or suspect. Therefore no conclusions should be drawn about the readings of the Arabic mss. where I do not explicitly report them.

There are a number of places where, if I were to establish a Greek text, it would differ from Heiberg's, but which I have not bothered to record in this book. Examples are:

mere orthography:

ηὑρίσκομεν	for εὑρίσκομεν (imperfect)	I 327,15
Κάλλιππος	for Κάλιππος	I 199,5
ἀμετάπειστον	for ἀμετάπιστον	I 6,18 (cf. Boll, *Studien* 74)
κρίκος	for κρῖκος	I 196,8

changes in form not affecting the sense: ἄν for ἐάν I 393,11
reversals of letters referring to figures: ZK for KZ I 243, 22
obvious misprints:

σελήνης	for σηλήνης	I 406,25
ἀνωμαλίας	for ἀμωμαλίας	I 462,19

(less obvious misprints, particularly those involving numbers, are recorded).

During the course of making the translation, I became convinced that the text contains quite a large number of interpolations, which must go back to antiquity, since they are in the whole manuscript tradition, both Greek and Arabic. I was first led to this conclusion by the discovery that there are places in the text, nonsensical as they stand, which can be made to yield perfect sense by the simple elimination of a clause or sentence, which must have been inserted as 'explanation' by someone who failed to understand Ptolemy's meaning. A notable example is V 1 (see p. 219 n.5). Cf also V 12, p. 245 with n.41. I later realised that there are whole classes of textual matter which must also be regarded as interpolations. One of these is the totals in the star catalogue (see pp. 16–17). The other is the chapter headings. Some of these (e.g. IX 2) are so inept as descriptions of the actual content of the chapter that it is impossible to attribute them to Ptolemy. In fact I do not believe that Ptolemy himself used any chapter divisions at all. It is obvious that he is responsible for the division into 13 books, both from the summaries that are found at the beginning of most books, and from explicit references such as 'in Book I' (ἐν τῷ πρώτῳ τῆς συντάξεως, II 1 p. 75) and 'in the preceding book' (ἐν τῷ πρὸ τούτων συντάγματι, VI 5 p. 283). But he never refers to a chapter division. Furthermore, there is some discrepancy in the manuscript tradition (especially between the branch represented by D and that represented by A) as to the points of division between chapters (e.g. at the beginning of Book III), and it is clear from Pappus' commentary that although a division into chapters already existed in his time, it was very different, at least in Book V, from the present division.[8] If the chapter division and headings are spurious, so too must be the table of contents preceding each book. Nevertheless, since this method of subdividing the text is useful for reference purposes, and appears in all editions, I have retained it, merely marking the character of the chapter headings by enclosing them in brackets thus: {   }.

4. *What is in the Almagest, and what is not*

The order of treatment of topics in the Almagest (outlined in I 2) is completely logical. In Book I, after a brief treatment of the nature of the universe (in so far as it concerns the astronomer), Ptolemy develops the trigonometrical theory necessary for the work as a whole. In Book II he discusses those aspects of spherical astronomy which are related to the observer's position on earth (rising-times, length of daylight, etc.). Book III is devoted to the theory of the sun. This is a necessary preliminary for the treatment of the moon in Book IV, since the use of lunar eclipses there depends on one's ability to *calculate* the solar position. Book V treats the advanced lunar theory, which is a refinement of that in Book IV, and also lunar and solar parallax. Book VI is on eclipses, and thus requires a knowledge of both solar and lunar theory, and also of parallax. Books VII and VIII treat the fixed stars: since the moon is used as a 'marker' to determine the position of some crucial fixed stars, lunar theory must precede this, and since some planetary observations are made with respect to fixed stars,

[8] See the note in Rome[1] I p. 106, and cf. (for Theon) II p. 448 n. (1).

the establishment of a star catalogue (VII 5 and VIII 1) must precede the planetary theory. The last five books are devoted to the planets. Books IX–XI develop the theory of their longitudinal motion, Book XII treats retrogradations and greatest elongations (which depend only on longitude), while Book XIII deals with planetary latitude and those phenomena (the 'phases') which are partially dependent on it. Ptolemy occasionally anticipates later results for the sake of convenience (see IV 3 p. 179 and IX 3 p. 423, where the mean motion tables of moon and planets incorporate some later corrections), but in general the order of presentation, within books as well as in the treatise as a whole, is dictated by the logic of the didactic method.

There are, however, certain topics which Ptolemy does not discuss either because he takes it for granted that they are already known to his readers, or because it seemed superfluous to go into details (here I am referring especially to chronological matters). He says specifically (I 1 p. 37 with n.13) that the work is for 'those who have already made some progress in the field'. This means, in practice, that he assumes a knowledge of elementary geometry ('Euclid') and 'logistic' (thus he does not consider it necessary to explain how to extract a square root), and also of 'spherics'. The latter is illustrated by the extant works of Autolycus, Euclid (*Phaenomena*) and Theodosius (*Sphaerica*), which deal with the phenomena arising from the rotation of stars and sun about a central, spherical earth, e.g. their risings, settings, first and last visibilities, periods of invisibility etc., using elementary geometry, but arriving mainly at qualitative rather than quantitative results.[9] These results are mostly irrelevant to Ptolemy's work, but he does use much of the terminology and concepts of spherics without explanation.

5. *What the reader of the Almagest needs to know*

The modern reader, too, is likely to be familiar with elementary geometry. So I have not burdened the translation with references to Euclid except where the theorems assumed are not immediately obvious. However, in what follows I give a brief explanation of methods, concepts and facts not explained by Ptolemy which the reader of the Almagest needs to know, but which may be less familiar. On Ptolemy's mathematical methods in general one may profitably consult Pedersen 47–56.

(*a*) *The sexagesimal system*

This was taken over by the Greeks (one may guess by the Hellenistic astronomers) from the Babylonians as a convenient way of expressing fractions and (to a lesser extent) large numbers, and of performing calculations with them. It is the first place-value system in history. In the translation and notes I use the convenient modern 'comma and semi-colon' notation, in which

[9] For more detail see *HAMA* II 755–71.

6,13;10,0,58 represents $6 \times 60 + 13 + 10 \times 60^{-1} + 0 \times 60^{-2} + 58 \times 60^{-3}$. Ptolemy uses the system only for fractions, and represents whole numbers, even when combined with sexagesimal fractions, by the standard Greek (alphabetic) notation. The translation follows this mixed notation (thus the above number would be written 373;10,0,58 in the translation, and $\overline{\tau o\gamma}$ $\bar{\iota}$ o $\overline{\nu\eta}$ in Greek).

(*b*) *Fractions*

Except where it is necessary to be precise, Ptolemy prefers the traditional Greek fractional system to the sexagesimal. In this, although it is possible to express proper fractions as e.g. '4 5ths', preference is given to unit fractions, so that, e.g. '$\frac{3}{4}$' is expressed as the sum of $\frac{1}{2}$ and $\frac{1}{4}$ (written ∠′δ′, i.e. '$\frac{1}{2}\frac{1}{4}$'). There is a special sign for $\frac{2}{3}$. In the translation I have usually converted these sums of unit fractions to proper fractions without comment. However, I have always retained the fractional form where Ptolemy has it, since it gives a misleading appearance of precision to convert to sexagesimals (as Manitius often does, putting an exact number of minutes instead of a fraction of a degree). This is particularly true of the star catalogue.

(*c*) *Trigonometry*

The sole trigonometrical function used by Ptolemy is the chord. The derivation and structure of his chord table are fully explained in I 10. However, Ptolemy does not give explicit instructions for its use in trigonometrical calculations, although his method is obvious enough from the worked examples. In what follows I give a literal translation, with commentary, of a typical calculation involving trigonometry.

See Fig. A, and, for my conventions, compare the translation pp. 163–4. In the given situation arc ΘH is 30°, AD is 60^p, AH is $2;30^p$, and it is required to find the angle ADH (the 'equation'). In modern trigonometry we would use the cosine formula. Ptolemy has no equivalent, so he drops the perpendicular HK, thus transforming the problem into one of solving only right triangles, which is his standard procedure.[10]

'Then since arc ΘH is again 30 degrees, angle ΘAH would be 30 of those [units] of which 4 right angles are 360, and 60 of those [units] of which 2 right angles are 360. So the arc on HK is 60 of the units of which the circle [circumscribed] about the right-angled [triangle] HKA is 360, and the arc on AK is 120, the supplement making up the semi-circle. And so, of the chords subtended by them, HK will be 60 of the units of which hypotenuse AH is 120, and AK 103;55 of the same [units].'

[10] He knows the equivalent of the sine formula, namely that in the general triangle the sides are proportional to the chords of the doubles of the opposite angles, but uses it surprisingly infrequently. An example is IX 10 p. 462 (cf. n.96 there).

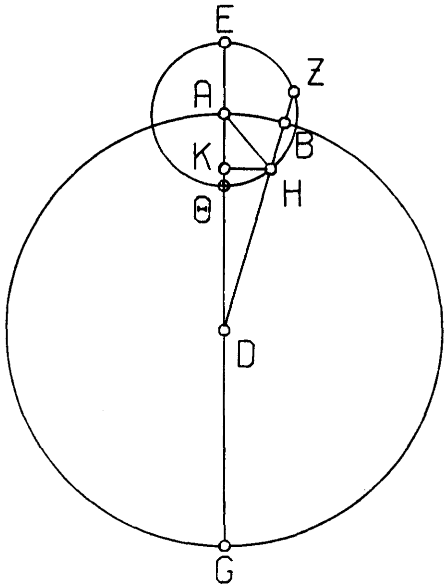

Fig. A

To solve a right-angled triangle (here HKA), Ptolemy imagines a circle circumscribed about it. Then the hypotenuse of the triangle is the diameter of the circle, and is taken (initially) as 120 parts (R = 60 being the standard on which Ptolemy's chord table is constructed). The two acute angles of the triangle being given, the other two sides can now be expressed in the same units: they are the chords of the arcs of the circumscribed circle, which are the doubles of the angles of the triangle (since they are equal to the angles at the centre). Instead of explicitly doubling these angles, Ptolemy always first expresses them in 'units of which 2 right angles are 360'. (Following the convention invented by B. R. Goldstein, I indicate these 'demi degrees' by the notation $^{\circ\circ}$, reserving $^{\circ}$ for the standard degree of which there are 90 in a right angle.) This enables him to switch smoothly from the triangle to the circle (and hence to the chord table, which gives him the actual numbers 60^p and $103;55^p$): an angle of size θ° is $2\theta^{\circ\circ}$, and hence the arc of the circumscribing circle which corresponds to that angle is $2\theta^{\circ}$.

'Therefore in those [units] of which line AH is 2;30, and the radius AD is 60, HK will be 1;15 and AK, likewise, 2;10, and KD, the remainder, 57;50.'

The sides of triangle AKH are converted to the norm representing their actual size (AH = $2;30^p$, hence they are multiplied by 2;30/120). This gives two sides of the next right triangle to be solved, DHK:HK and (by subtraction of AK from the given AD) KD.

'And since the squares on these added together make the square on DH, the latter will be, in length, approximately 57;51 of the units of which line KH was [found to be] 1;15.'

Since Ptolemy has no tangent function, he has to use 'Pythagoras' theorem' to find the hypotenuse of the right triangle in question. He uses the word μήκει, 'in length', to indicate that he is taking the square root (considered as the side of a square, hence a line length).

'And so of those [units] of which hypotenuse DH is 120, line HK will be 2;34 and the arc on it [HK, will be] 2;27 of those [units] of which the circle about DHK is 360. So that angle HDK is 2;27 of those [units] of which 2 right angles are 360, and about 1;14 of those of which 4 right angles are 360.'

The sides of triangle DHK are now converted to the standard in which the hypotenuse is 120^{p}, thus enabling Ptolemy to use the chord table to determine the size of the arc corresponding to the side opposite the angle to be determined, HDK. The latter, being at the circumference of the circumscribed circle, is half the arc. Ptolemy again expresses this relationship by saying that it is the same number of 'demi degrees' as the arc is 'single degrees', and then converting the 'demi degrees' to 'single degrees' by halving. Note that I frequently translate expressions like '30 degrees of the kind of which the great circle is 360' simply as '30°'.

(*d*) *Chronology and calendars*

Ptolemy's own chronological system is very simple. He uses the *Egyptian year* and the *era Nabonassar*. The Egyptian year is of unvarying length of 365 days, consisting of twelve 30-day months and 5 extra ('epagomenal') days at the end. Ptolemy uses the Greek transliterations of the Egyptian month names. For the reader's convenience, I usually add a Roman numeral indicating the number of the month. The order of the months is:

I	Thoth	VII	Phamenoth
II	Phaophi	VIII	Pharmouthi
III	Athyr	IX	Pachon
IV	Choiak	X	Payni
V	Tybi	XI	Epiphi
VI	Mechir	XII	Mesore.

The reason for choosing the era Nabonassar is given by Ptolemy at III 7 (p. 166: the earliest (Babylonian) observations available to him were from the reign of King Nabonassar. Ptolemy's epoch, Nabonassar 1, Thoth 1 corresponds to −746 February 26 in our reckoning.[11]

[11] Throughout this book I use the 'astronomical' system of dating according to the Christian era, since it is far simpler for calculating intervals than the 'B.C./A.D.' system. In this, year −1 corresponds to 2 B.C., year 0 to 1 B.C., year 1 to A.D. 1, etc.

Even when he refers to other calendars, Ptolemy usually gives the equivalent date in his own system, so there is no uncertainty. Sometimes, however, he gives, not the running date in the era Nabonassar, but only the regnal year of a king. It is clear that there already existed, in some form, a 'king-list' enabling one to relate the regnal year of a given king to a standard epoch.[12] Later, in his 'Handy Tables', Ptolemy published such a king-list (known as 'Canon Basileon'), and it survives, in a considerably augmented form, in Byzantine versions of Theon of Alexandria's revision of the Handy Tables. From these I have excerpted and 'reconstructed' the table on p. 11, which makes no historical pretensions, but is intended solely as an aid to readers of this book. The basis of the table is Usener's edition of the two versions in the manuscript Leidensis gr. 78, in *Monumenta Germaniae Historica, Auctores Antiquissimi* XIII (*Chronica Minora Saec. IV. V. VI. VII*, ed. Th. Mommsen), Vol. III, 447-53, supplemented by my own reading of the version in the ms. Vaticanus gr. 1291, 16^{v}-17^{r}. The names of the Babylonian and Assyrian kings are obviously very corrupt, and I have made no attempt to emend them, but have chosen those manuscript variants which seem closest to the forms now known from the cuneiform sources, which are listed in the second column (supplied to me by A. Sachs).

For the purposes of astronomical chronology, an integer number of years is assigned to each reign. As far as can be checked from independent sources, 'Year 1' of each reign was assumed to begin on the Thoth 1 *preceding* the historical date on which the king began to reign.[15] Thus, to use the table to go from a given regnal year to the era Nabonassar, one simply adds the number of the regnal year to the total listed (in the fourth column) for the previous king.[16] E.g. to find the second year of Mardokempad in the era Nabonassar (cf. IV 8 p. 204), we add 2 to the total of 26 given for his predecessor, Ilulai, and get the twenty-eighth year in the era Nabonassar.

Although I supply in the translation the modern equivalent of all dates in the Almagest, I have added, for the use of those readers who wish to check them, a fifth column listing the Julian equivalent of the first day of each king's reign. If one bears in mind that every Julian year divisible by 4 is a leap-year, while the Egyptian year is constant, this is a sufficient basis for the calculation. However, I recommend as an easier alternative the use of Schram's *Kalendariographische Tafeln*: from pp. 182-9 of that one can find the Julian day number of any date in

[12] Papyrus fragments of such king-lists are found in P. Oxy. 1.35 and Sattler, *Studien* 39-50. These are, however, later than Ptolemy. P. Oxy. 19.2222, a list of the Ptolemies of Egypt, is earlier than the Almagest, but is very different in format from Ptolemy's king-list.

[13] It is not known why these two kings are combined. In cuneiform sources (e g. the king-list translated in Pritchard, *Ancient Near Eastern Texts* 272 (iv), they appear consecutively, Ukīn-zēr being assigned 3 years and Pūlu 2.

[14] This must be a corruption in the Greek tradition of Arses ('Αρσης), the usual form of this king's name (also known as 'Οαρσης).

[15] This was recognised long ago. See Usener, *MGH* XIII.3 p. 441, with references to older literature in his n.5.

[16] In the Handy Tables Ptolemy adopted the 'era Philip' (which already occurs in the Almagest as 'death of Alexander'); hence in the mss. the totals for era Nabonassar go only as far as Alexander the Macedonian (no. 31), and a new totalling system begins with Philip (no. 32). I have converted all these later totals to the era Nabonassar by the addition of 424 to each. Cf. Schram p. 173.

	Ruler	Correct form	Years of reign	Total years to end of reign	Julian date of beginning of reign
	Kings [of Assyria and Babylonia]				
1	Nabonassar	Nabû-nāṣir	14	14	-746 Feb. 26
2	Nadi	Nādin	2	16	-732 Feb. 23
3	Chinzer and Por[13]	Ukīn-zēr; Pūlu	5	21	-730 Feb. 22
4	Ilulai	Elūlai	5	26	-725 Feb. 21
5	Mardokempad	Marduk-apla-iddin	12	38	-720 Feb. 20
6	Arkean	Šarru-ukīn	5	43	-708 Feb. 17
7	First interregnum		2	45	-703 Feb. 15
8	Belib	Bēl-ibni	3	48	-701 Feb. 15
9	Aparanad	Aššur-nādin-šumi	6	54	-698 Feb. 14
10	Regebel	Nergal-ušēzib	1	55	-692 Feb. 13
11	Mesesemordak	Mušēzib-Marduk	4	59	-691 Feb. 12
12	Second interregnum		8	67	-687 Feb. 11
13	Asaridin	Aššur-aḫa-iddina	13	80	-679 Feb. 9
14	Saosdouchin	Šamaš-šuma-ukīn	20	100	-666 Feb. 6
15	Kiniladan	Kandalanu	22	122	-646 Feb. 1
16	Nabopolassar	Nabû-apla-uṣur	21	143	-624 Jan. 27
17	Nabokolassar	Nabû-kudurra-uṣur	43	186	-603 Jan. 21
18	Illoaroudam	Amīl-Marduk	2	188	-560 Jan. 11
19	Nerigasolassar	Nergal-šarra-uṣur	4	192	-558 Jan. 10
20	Nabonadi	Nabû-na'id	17	209	-554 Jan. 9
	Kings of the Persians				
21	Cyrus	Kūruš	9	218	-537 Jan. 5
22	Kambyses	Kambužiya	8	226	-528 Jan. 3
23	Darius I	Dārayava^{h}u	36	262	-520 Jan. 1
24	Xerxes	χšayāršā	21	283	-485 Dec. 23
25	Artaxerxes I	Artaχšaθrā	41	324	-464 Dec. 17
26	Darius II	Dārayava^{h}u	19	343	-423 Dec. 7
27	Artaxerxes II	Artaχšaθra	46	389	-404 Dec. 2
28	Ochus	Vahauka	21	410	-358 Nov. 21
29	Arogos[14]	?Hawarša	2	412	-337 Nov. 16
30	Darius III	Darayava^{h}u	4	416	-335 Nov. 15
31	Alexander the Macedonian	Ἀλέξανδρος	8	424	-331 Nov. 14
	Kings of the Macedonians				
32	Philip who succeeded Alexander the founder	Φίλιππος	7	431	-323 Nov. 12
33	Alexander II	Ἀλέξανδρος ἕτερος	12	443	-316 Nov. 10
34	Ptolemy son of Lagos	Πτολεμαῖος Λάγου	20	463	-304 Nov. 7
35	Ptolemy Philadelphos	Φιλάδελφος	38	501	-284 Nov. 2
36	Ptolemy Euergetes	Εὐεργέτης	25	526	-246 Oct. 24
37	Ptolemy Philopator	Φιλοπάτωρ	17	543	-221 Oct. 18
38	Ptolemy Epiphanes	Ἐπιφανής	24	567	-204 Oct. 13
39	Ptolemy Philometor	Φιλομήτωρ	35	602	-180 Oct. 7
40	Ptolemy Euergetes II	Εὐεργέτης β′	29	631	-145 Sept. 29
41	Ptolemy Soter	Σωτήρ	36	667	-116 Sept. 21
42	Ptolemy Neos Dionysus	Διόνυσος νέος	29	696	-80 Sept. 12
43	Cleopatra	Κλεοπάτρα	22	718	-51 Sept. 5
	Kings of the Romans				
44	Augustus	Augustus	43	761	-29 Aug. 31
45	Tiberius	Tiberius	22	783	14 Aug. 20
46	Gaius	Gaius	4	787	36 Aug. 14
47	Claudius	Claudius	14	801	40 Aug. 13
48	Nero	Nero	14	815	54 Aug. 10
49	Vespasian	Vespasianus	10	825	68 Aug. 6
50	Titus	Titus	3	828	78 Aug. 4
51	Domitian	Domitianus	15	843	81 Aug. 3
52	Nerva	Nerva	1	844	96 July 30
53	Trajan	Traianus	19	863	97 July 30
54	Hadrian	Hadrianus	21	884	116 July 25
55	Antoninus	Aelius Antoninus	23	907	137 July 20

the era Nabonassar in a few seconds, and hence (from his other tables) the equivalent date in any standard calendar.

The only other aspect of Ptolemy's own chronology requiring remark is the 'double dates'. He frequently characterises the day of an observation by expressions like Παχὼν ιζ′ εἰς τὴν ιη′, translated 'Pachon 17/18', but literally 'Pachon, the seventeenth towards the eighteenth'. Modern commentators have made unnecessarily heavy weather of this. Ptolemy himself uses a noon epoch, but this is an artificial starting-point (the reason for which he explains at III 9 pp. 170–1), and has nothing to do with *numbering* the day. In antiquity the 'civil epoch' of the day was either dawn (as in Egypt) or sunset (as in Babylon). In either system, an event which took place in the daylight would be on the same 'day', but one which took place in the night would be on 'day n' for those using dawn epoch and 'day n+1' for those using sunset epoch. Hence ambiguity was possible. Ptolemy uses double dates (which are found *only* for night-time observations) to avoid this ambiguity. The form he uses implies the Egyptian, i.e. dawn epoch (cf. the long form III 1 p. 138,τῇ ια′ τοῦ Μεσορὴ μετὰ β̄ ὥρας ἐγγὺς τοῦ εἰς τὴν ιβ′ μεσονυκτίου (literally 'on the eleventh of Mesore, approximately two hours after the midnight towards the twelfth'), but it would be clear even to someone using sunset epoch (who would date the above event to 'Mesore 12') what day he means.

In using the observations of his predecessors Ptolemy often has occasion to refer to other systems of chronology and calendars. Although in such cases one can always readily derive the equivalent date in Ptolemy's own system (he almost always gives it explicitly), I shall describe them briefly here.

The most frequently mentioned is the *Kallippic Cycles*. To explain this, we must go back to Meton, who in −431 devised a 19-year 'cycle', i.e. a fixed scheme of intercalation of months containing 6940 days (thus the average length of a year was $365\frac{1}{4} + \frac{1}{76}$ days).[17] Since he was an Athenian, he used the month names of the Athenian civil calendar for the months of his artificial 'calendar'. A hundred years later an associate of Aristotle, Kallippos, produced a revision of this, based on the more accurate year-length of $365\frac{1}{4}$ days. In order to achieve this, he eliminated one day from 4 Metonic cycles, thus producing the 'Kallippic cycle' of 76 years and 27759 days. What was later known as the 'First Kallippic Cycle' began at the summer solstice (probably June 28th) of the year −329. In the Almagest we find references also to the Second and Third Kallippic Cycles, which began in −253 and −177 respectively. To judge from the Almagest, this chronological system was the one most used by earlier Hellenistic astronomers.[18] In VII 3 four observations by Timocharis (Alexandria, third century B.C.) are given according to the year of the First Kallippic Cycle and 'Athenian' month and day. On the basis of these, several attempts have been made to reconstruct the whole 'Kallippic calendar', with discrepant results. Since the above constitute the whole evidential basis, apart from the

[17] For a detailed discussion see Toomer[7]. I give there the arguments for supposing that Meton's purpose was not to reform the Athenian calendar, but to establish an 'astronomical chronology'.

[18] The dates of the three eclipses in IV 11 (p. 211, cf. n.63 there) which, though observed in Babylon, are given according to Athenian archon and Athenian month, are presumably in the Metonic calendar.

passage in Geminus, *Eisagoge* VIII, which I regard as fiction, and two dubious equivalences in the Milesian parapegma, any reconstruction is academic.[19] Here I note only that Kallippos evidently retained the peculiar Athenian method of counting the days of the month by decads, and in the last decad counting backwards, so that VII 3 p. 336 τῇ ς′ φθίνοντος, literally 'on the sixth [day] of the waning [moon]', means 'the sixth day from the end of the last decad', i.e. the twenty-fifth.[20]

Hipparchus too used the Kallippic cycles for astronomical dating, but combined them, not with Kallippos' 'Athenian' calendar, but with the Egyptian calendar (i.e. he used the cycles simply as a year count), at least as far as we can tell from the Almagest. This seems to have led to ambiguities, since the 'Kallippic' year began at or near the summer solstice, while the Egyptian year is a 'wandering year', which in Hipparchus' time began about the end of September. Thus there arose the possibility of a discrepancy of 1 in the year count, for certain stretches of the year (whether it is +1 or −1 depends on Hipparchus' choice). Such a discrepancy is firmly attested in Almagest IV 11 (see p. 214 n. 72), and cannot plausibly be removed by emendation, though this has been done (by Ideler and others) in the interest of consistency. In fact it is impossible to make all of Hipparchus' 'Kallippic cycle' dates in the Almagest consistent with one another (see p. 224 no. 13), and we must allow for the possibility that Hipparchus used different systems in different works.

Three planetary observations in the Almagest are dated κατὰ Χαλδαίους, 'according to the Chaldaeans', with a year number and a Macedonian month name and day number. The year numbers show that the era used is that known in modern times as the *Seleucid Era* (dating from the year which Seleucus I counted as the first of his reign, −311/10), which was common throughout the Seleucid empire. Since the observations are undoubtedly Babylonian, the particular epoch used in them is, as one would expect, that known from the surviving Babylonian astronomical texts, 1 Nisan (April) −310 (Greeks under the Seleucid empire commonly used an epoch of autumn −311). The use of Macedonian month names has rightly been taken to show that the Babylonian lunar months were simply called by the names of the Macedonian months by the Greeks under the Seleucid empire: if one computes the date of the first day of the 'Macedonian' month from the equivalent date in the era Nabonassar given by Ptolemy, it coincides (with an error of no more than one day) with the computed day of first visibility of the lunar crescent at Babylon.[21] There is other evidence for the assimilation of the month names,[22] but this is the strongest.

Unattested outside the Almagest is the *Calendar of Dionysius*. This had a

[19] Those who care to may consult Ginzel II 409–19 and Samuel, *Greek and Roman Chronology*, 42–9 for details and literature.

[20] For this system see Samuel, *Greek and Roman Chronology* 59–60. I do not know why it is not used for the other three 'Kallippic' dates in which the days are simply numbered consecutively.

[21] These are conveniently listed in Parker-Dubberstein.

[22] For details see Samuel, *Greek and Roman Chronology* 140–2. However, Samuel is wrong in saying that the Almagest evidence proves that the assimilation was made as early as the date of the earliest observation (Nov. −244). In the cuneiform record from which this was derived the Babylonian names must have been used. It was only when this was translated into Greek (which may have been as much as a century later) that the Macedonian names were substituted.

running year count and months named after the signs of the zodiac (corresponding, at least approximately, to the period of the year when the sun was in the sign in question). The months Tauron (♉), Didymon (♊), Leonton (♌), Parthenon (♍), Skorpion (♏), Aigon (♑) and Hydron (♒) are attested. From analysis of the Almagest evidence Böckh, *Sonnenkreise* 286–340, showed that the epoch of the calendar was the summer solstice of -284. Since Thoth 1 (Nov. 2) of -284 is the beginning of the first regnal year of Ptolemy Philadelphos, it is plausibly concluded that Dionysius observed in Egypt. Böckh's further conclusions, that the calendar was similar to the Egyptian one in having 12 months of 30 days, but was modified by introducing a sixth epagomenal day every four years, cannot be regarded as certain, especially since this requires 'emending' some of the Almagest dates. Here, as for the Kallippic calendar, 'reconstruction' seems pointless when the evidence is so scanty and the likelihood of verification utterly remote.[23]

One observation is dated in the *Bithynian calendar* of the imperial period. Like a number of other contemporary calendars in Asia Minor, this was simply the Julian calendar, with different month-names, and with the first day of the year Augustus' birthday, Sept. 23. For details and literature see Samuel, *Greek and Roman Chronology* 174–5.

(*e*) *Ptolemy's star catalogue*

The list of the coordinates and magnitudes of the principal fixed stars visible to Ptolemy poses special problems to the translator. In particular, there are numerous manuscript variants in the coordinates, and while one must put some number in the translation, it is often difficult to be certain about one's choice. The solution I have adopted is (in the star catalogue only) to append an asterisk to any element (longitude, latitude, magnitude, description or identification) where there is reason to suppose that it may be incorrect (i.e. not what Ptolemy wrote or intended),[24] either because there is a plausible ms. variant, or because of some gross inconsistency with the astronomical facts. In such cases I give all significant variants known to me in a footnote. I have made no effort to record all variants, since most are obviously wrong. The reader who wishes to go further must still consult Peters-Knobel, on which I have drawn heavily, and which is still the best treatment of the catalogue as a whole, though badly in need of updating and revision in certain respects.[25]

Ptolemy lists the stars under 48 constellations, and gives for each star (1) a description of its location on the 'figure' and (sometimes) of its brightness and colour; (2) its longitude; (3) its latitude and direction (north or south of the ecliptic); and (4) its magnitude. I have followed my predecessors (notably Manitius) in adding to these: (a) an initial column giving a running number to

[23] The interested reader may consult *HAMA* III 1067 n.2 and Samuel, *Greek and Roman Chronology* 50, n.6 for further literature.

[24] The lack of an asterisk does not imply that I regard the reading adopted as Ptolemy's beyond any question, but only that I have no good reason to doubt it.

[25] See the strictures of Kunitzsch, *Der Almagest* 46.

the star within its constellation (stars listed at the end of some constellations by Ptolemy as 'outside the constellation', i.e. not part of the imaginary figure, are numbered continuously with those preceding them); (b) a final column giving the modern identification of the star. For those stars which have them, this is the Bayer letter or Flamsteed number. Certain fainter stars have neither; for these I give the number in the Yale Bright Star Catalogue (abbreviated as 'BSC'). From that publication those interested can find the corresponding number in the Durchmusterung and the Henry Draper and Boss General Catalogues. I have abandoned all references to the antiquated Piazzi catalogue (still used by Peters-Knobel).

I have used Roman numerals to number the constellations, and refer to individual stars (throughout the translation) by the combination of Roman and Arabic numerals (thus 'catalogue XXXIX 2' refers to the second star in the thirty-ninth constellation (Canis Minor), namely Procyon).

The star descriptions pose numerous individual problems, only a few of which are touched on in the footnotes. Ideally one should provide a reconstruction of the outline of each constellation as it appears on Ptolemy's star-globe. Unfortunately no one has done the necessary work of assembling and comparing all the literary and iconographic evidence from antiquity and from the derivative Arabic tradition (notably aṣ-Ṣūfī). This would be an interesting and valuable enterprise. Meanwhile, for the reader who needs some visual illustration, I can recommend only the old work of Bayer, *Uranometria*, with the warning that in many cases his positioning of the stars on the figures, and the outlines of the figures themselves, are certainly different from Ptolemy's.[26] On the matter of the orientation of the figures, I have satisfied myself that Ptolemy describes them as if they were drawn on the *inside* of a globe, as seen by an observer at the centre of that globe, and facing towards him. This is in agreement with what Hipparchus says (*Comm. in Arat.* I 4 5): 'for all the stars are described in constellations (ἠστέρισται) from our point of view, and as if they were facing us, except for such of them as are drawn in profile' (κατάγραφον, as interpreted by Manitius, whom I follow dubiously). It is in this sense that we must interpret 'left hand', 'right leg', etc. This has to be said, since on the actual star globes the constellations were necessarily drawn on the *outside*. Hence the orientation of the figures was (at least in some cases) reversed, which could lead to confusion.[27] I have rendered the prepositions used by Ptolemy in indicating the positions of stars with respect to parts of the figures consistently, as follows:

in = ἐν
on = ἐπί
over = ὑπέρ

[26] The work of Thiele, *Antike Himmelsbilder*, is very little help, although I have referred to it to illustrate some particulars.

[27] Cf. the scholion on Aratus, Maass, *Comm. in Arat.* p. 384 no. 251: 'the signs look inward with respect to the heavens . . . but they have their backs to the globe, so that their faces may be seen. Hence, if he says "right hand" or "left hand" and we find the opposite on the globe, we should not be confounded.'

above = ἐπάνω
under = ὑπό
below = ὑποκάτω
just over = κατά + genitive
advance, in advance = προηγούμενος
rear, to the rear = ἑπόμενος

On the meaning of the last two terms see below p. 20. Note that 'rear' is never used in a sense other than directional. To indicate the back parts of an animal figure I use 'hind'.

Both longitudes and latitudes are given, not in degrees and minutes, but in degrees and fractions of a degree. I have retained this in the translation (see p. 7). With very few exceptions, the longitudes are not given more accurately than to $\frac{1}{6}°$. (This has been taken to imply that the ecliptic ring of Ptolemy's instrument was graduated only every 10′). However, one frequently finds the fractions $\frac{1}{4}°$ and $\frac{3}{4}°$ for the latitudes.

The latitudes in Ptolemy's list are preceded by the direction (βο = βόρειος, 'northern'; νο = νότιος, 'southern'). I have rendered these by + and − respectively.

The magnitudes range (according to a system which certainly precedes Ptolemy, but is only conjecturally attributed to Hipparchus) from 1 to 6. Ptolemy indicates intermediate magnitudes by adding (after the number) μείζων, 'greater' or ἐλάσσων, 'less' (abbreviated in the mss.). I have rendered these by > and < (before the number) respectively. One occasionally finds for the magnitude, instead of a number, the remark ἀμαυρός (rendered 'f.' for 'faint') or νεφελ. (for νεφελοειδής), 'nebulous', abbreviated as 'neb.'

For the identifications, wherever Peters-Knobel and Manitius are in agreement, I have usually been content to adopt their opinion. Where they differ (and even when they agree, in some special cases),[28] I have checked the possibilities as carefully as I could, using the large-scale *Atlas of the Heavens* by Bečvář, and transforming Ptolemy's coordinates to right ascension and declination at the modern epoch, where necessary. However, I have made no attempt to redo the work of Peters and Knobel, namely to compute the longitude and latitude of the relevant stars for Ptolemy's time from modern data (in particular using the most up-to-date values for the proper motions). This might be worth while, though I doubt whether the degree of improvement over Peters-Knobel would justify the large amount of computation. In any case, it is unlikely that it would eliminate the doubts that remain about the identification of many of the fainter stars.

At the end of each constellation in the mss. are listed the total number of stars in the constellation, and the sub-totals of each magnitude. These in turn are added up at various intermediate points (the northern segment, the zodiac, and the southern segment), and the grand totals are given at the end. I am

[28] Notably, where Ptolemy describes a star as a 'nebulous mass' (νεφελοειδής συστροφή), I have preferred to give the globular cluster (abbreviated 'CGlo') or galactic cluster (abbreviated 'CGal') rather than some particular star inside it.

convinced that this was not done by Ptolemy (who makes no mention of it in his description of the catalogue, VII 4 pp. 339–40). Another indication of the spuriousness of these passages is that no separate count is made in the totals of the stars which are greater (>) or less (<) than a certain magnitude: all are lumped in with the stars of that magnitude. I have translated the passages in question, but enclosed them in brackets thus: { }.

(*f*) *Explanations of special terms*

(i) *Geometrical*

by subtraction (λοιπός -ή -όν): literally 'the remaining [part]', 'remainder' (I have on occasion so rendered it).

by addition (ὅλος -η -ον): literally 'the total'.

Crd x: chord of the angle x° (R = 60^p). Greek has no word with the specific meaning 'chord', but uses the generic εὐθεῖα, 'straight line'. 'Crd x' renders ἡ τὰς x μοίρας ὑποτείνουσα εὐθεῖα, 'the straight line subtending x degrees'.

In connection with the Menelaus Theorem (see p. 18), an expression of the type 'Crd arc 2AB' represents ἡ ὑπὸ τὴν διπλῆν τῆς AB περιφερείας, literally 'the [line] subtended by the double of arc AB'.

supplement, supplementary arc (ἡ λείπουσα [λοιπὴ] εἰς τὸ ἡμικύκλιον περιφέρεια): literally 'the arc which is the remainder to the semi-circle'.

complement (λοιπὴ εἰς τὸ τεταρτημόριον): literally, 'the remainder to the quadrant'.

|| literally, 'is similar to'. Used of arcs of different-sized circles. Arc AB|| arc GD if each arc is the same fraction of its circle.

||| (ἰσογώνιόν ἐστι): literally, 'has [all] its angles equal to', i.e. is similar to (used only of triangles).

≡ (ἰσόπλευρόν ἐστι): literally 'has its sides equal to', i.e. is congruent to. Used only of spherical triangles. Sometimes ἰσογώνιον καὶ ἰσόπλευρόν ἐστι, 'has its angles and sides equal to'.

Q.E.D. (ὅπερ ἔδει δεῖξαι): literally 'which is what it was required to prove'.

componendo (συνθέντι). Expresses the operation of addition of ratios: if a : b = c : d, then (a + b):b = (c + d):d.

dividendo (διελόντι, κατὰ διαίρεσιν) (1) Usually expresses the operation of subtraction of ratios: if a : b = c : d, then (a − b) : b = (c − d) : d.

(2) Once, at XII 1 (see p. 558 n.4) διελόντι expresses *division* of members of ratios. If a : b = c : d, then $\frac{a}{n} : b = \frac{c}{n} : d$.

Menelaus Configuration and *Menelaus Theorem* (used only in the footnotes and explanatory additions). Cf. *HAMA* 26–9. Fig. B represents a Menelaus Configuration. m,n,r and s are four great circle arcs on the surface of the sphere, intersecting each other as shown, and divided by the intersections into the parts m_1, m_2 etc. (thus $m = m_1 + m_2$ etc.) In I 10 Ptolemy proves the theorems

$$\text{I} \qquad \frac{\text{Crd } 2m}{\text{Crd } 2m_1} = \frac{\text{Crd } 2r}{\text{Crd } 2r_1} \times \frac{\text{Crd } 2s_2}{\text{Crd } 2s}$$

$$\text{II} \qquad \frac{\text{Crd } 2r_2}{\text{Crd } 2r_1} = \frac{\text{Crd } 2m_2}{\text{Crd } 2m_1} \times \frac{\text{Crd } 2n}{\text{Crd } 2n_2} .$$

Since it is known that these were discovered by Menelaus, Neugebauer has named them 'Menelaus Theorem I' and 'Menelaus Theorem II' respectively, and I follow him, abbreviating to 'M.T.I.' and 'M.T.II'.

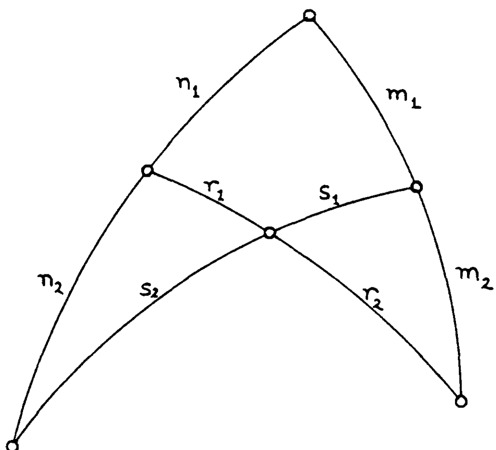

Fig. B

(ii) *Spherical astronomy*

(*at*) *sphaera recta* (ἐπ' ὀρθῆς τῆς σφαίρας) and (*at*) *sphaera obliqua* (ἐπ' ἐγκεκλιμένης τῆς σφαίρας). These mediaeval Latin terms are the literal translations of the Greek, meaning 'on the upright sphere' and 'on the inclined sphere' respectively. Probably taken from the use of celestial globes, they refer to the phenomena which occur when the celestial equator is perpendicular to the local horizon (*sphaera recta*) or inclined to it at an acute angle (*sphaera obliqua*). In particular, we use *rising-time at sphaera recta* or *right ascension*, and *rising-time at sphaera obliqua* or *oblique ascension* to designate the arc of the equator which crosses the horizon together with a given arc of the ecliptic (e.g. one zodiacal sign) at *sphaera recta* (i.e. at the terrestrial equator), and at *sphaera obliqua* (i.e. any other terrestrial latitude) respectively.

equator represents ἰσημερινὸς (κύκλος), literally 'circle of equal day', so called for the reason Ptolemy gives in I 8 (pp. 45–6).

meridian represents μεσημβρινὸς (κύκλος), literally 'midday circle' (defined and explained at I 8 p. 47). Meridian passage of a heavenly body is called *culmination*. The Greek terms for culminate and culmination, μεσουρανεῖν, μεσουράνησις, mean literally 'being in the middle of the heaven'. *upper* and *lower* culmination are expressed by ὑπὲρ γῆν and ὑπὸ γῆν, meaning 'above the earth' and 'below the earth' respectively, and sometimes so translated.

An *altitude circle* is any circle drawn through the zenith perpendicular to the horizon. Ptolemy has no special term for this in the Almagest, merely saying 'the (great) circle drawn through the zenith (through the poles of the horizon)', e.g. II 12, HI 166, 20–1.

colure. This term is used by Ptolemy only once, at II 6 p. 83. I translate part of Manitius' note on that passage: Two of the circles of declination through the poles of the equator are named 'colure' (κόλουρος): the solsticial colure, which goes through the solstices and hence carries the poles of the ecliptic, and the equinoctial colure. These two colures divide the sphere into four equal parts and divide both ecliptic and equator into four quadrants, so that one quadrant corresponds to each season of the year. Ptolemy counts the solsticial colure as boundary of the daily revolution [I 8 pp. 46–7, where however the term 'colure' is not used], but never explicitly mentions the equinoctial colure. Both colures were already defined by Eudoxus (Hipparchus, *Comm. in Arat.* 117 ff.) The term is explained by Achilles, *Isagoge* 27 (Maass, *Comm. in Arat.* 60) as follows: 'They are called colures because they appear to have their tails cut off as it were (κεκολοῦσθαι ὥσπερ τὰς οὐράς), since we cannot see the parts of them beginning at the antarctic, always invisible parallel'.

It is unfortunate that we have to use the same word *latitude* to refer both to the celestial coordinate (vertical to the ecliptic) and to the unrelated terrestrial coordinate. Ptolemy uses, for the former πλάτος, and for the latter κλίμα, literally 'inclination'. When necessary I gloss this e.g. as '[terrestrial] latitude'. κλίμα, however, does not refer to the coordinate as such (for which Ptolemy uses ἔγκλιμα, HI 68,9, ἔγκλισις, HI 101,23 or, once, πλάτος, HI 188,4), but to a specific 'band' of the earth where the same phenomena (e.g. length of longest daylight) are found. Hence in early Hellenistic times arose the notion of the division of the known world (the οἰκουμένη) into 7 standard *climata* (see *HAMA* 334 ff., II 727 ff. and Honigmann, *Die sieben Klimata*). This is reflected in several places in the Almagest, e.g. in Table II 13. I refer to these seven standard parallels by Roman numerals, e.g. Clima IV = the parallel through Rhodes, longest day 14½ hours.

(iii) *Referring to the heavenly bodies*

As Ptolemy explains in I 8, in his system the whole heavens are conceived as rotating from east to west, making one revolution daily. The direction defined by this motion, and the direction counter to it, are called εἰς τὰ προηγούμενα ('towards the leading [parts]') and εἰς τὰ ἑπόμενα ('towards the following [parts]') respectively. The corresponding adjectives προηγούμενος and ἑπόμενος are also found, particularly in the star catalogue, and Ptolemy frequently uses the phrases εἰς τὰ προηγούμενα (ἑπόμενα) τῶν ζῳδίων, 'towards the leading (following) [parts] of the zodiacal signs', to indicate the direction of motion in the ecliptic. A modern reader may find this confusing: since the normal motion of bodies in the ecliptic is from west to east, what we regard as forward motion, e.g. of a planet, is described as 'towards the following [parts]' ('towards the rear' in my translation). No version of these terms in a modern language is satisfactory. One cannot use 'west' and 'east' because these must be reserved for Ptolemy's δυσμαί and ἀνατολαί, which are confined to situations where a terrestrial observer is implied. It is a distortion to translate (with Manitius) 'in the reverse order of the signs' and 'in the order of the signs', since this implies that the terms define *ecliptic* coordinates, whereas they are in the *equatorial* system, and while it is usually true that a celestial object which προηγεῖται ('leads') another will have a lesser ecliptic longitude, if their latitudes differ greatly the reverse may be true, especially at very high ecliptic latitudes. Precisely this situation occurs in the star catalogue, despite Ptolemy's own statement at VII 4 p. 340 that the terms in the catalogue define ecliptic coordinates (see n.93 there). Although I am aware that my choice too has its drawbacks, I have settled on *in advance* for εἰς τὰ προηγούμενα, and *towards the rear* for εἰς τὰ ἑπόμενα. These always imply 'with respect to the daily motion from east to west', with the paradoxical consequence, as remarked above, that in the ecliptic a body which is 'in advance' of another has a lesser longitude. However, I have committed an inconsistency in translating the derived noun προήγησις as *retrogradation*. This is used only for the portion of the courses of the five planets in which they reverse their normal direction of motion, and it would be too confusing to render this by 'motion in advance'.

ecliptic. Ptolemy never refers to this circle by the term ἐκλειπτικός (which he confines strictly to the meaning 'having to do with eclipses'). His normal term is ὁ διὰ μέσων τῶν ζῳδίων (κύκλος), 'the (circle) through the middle of the zodiacal signs' (e.g. HI 18,23–4); more fully, ὁ λόξος καὶ διὰ μέσων τῶν ζῳδίων κύκλος, 'the inclined circle through the middle of the signs' (HI 64,4). Occasionally, when the context is clear, simply λόξος κύκλος, 'inclined circle' (HI 8,22). However, the latter can be used for other things, notably the moon's orbit (which is 'inclined' to the ecliptic). I normally use 'ecliptic' throughout.

[zodiacal] sign. The conventional subdivision of the ecliptic into twelve 30° stretches named Aries, Taurus, etc. For this Ptolemy uses, not ζῴδιον ('animal sign'), but δωδεκατημόριον ('twelfth'), presumably because he wishes to

distinguish the ecliptic, a notional circle, from the zodiac, a band of actual constellations.

star. The Greek term ἀστήρ really means 'heavenly body', and can be used indifferently for a star (in the modern sense), a planet, or even the sun and moon. When Ptolemy wishes to distinguish what we call stars, he says 'fixed stars'. I have normally translated ἀστήρ according to the context, as 'planet', 'star' or 'body'. However, in I 3–8, where Ptolemy uses the term to include all heavenly bodies, I too have used *star* in this special sense. When naming the five planets, Ptolemy almost always uses the periphrasis 'star of . . ', thus ὁ τοῦ Κρόνου [ἀστήρ], '[star] of Kronos'. I always translate simply 'Saturn' etc.

latitude (celestial). πλάτος (literally 'breadth') refers not only to 'the direction orthogonal to the ecliptic', but to any 'vertical' direction, e.g. that normal to the equator. In such cases I use, not 'latitude', but another appropriate term (see I 12 p. 63 with n. 74). In VII 3, however, I have been forced to use 'latitude' to express the more general meaning of the Greek (see p. 329 n.55).

Ptolemy uses ἔκκεντρος as both adjective and noun. It may be that in the latter case one has always to understand ἔκκεντρος κύκλος, 'eccentric circle'. However, to avoid ambiguity, I have (following mediaeval usage) consistently denoted the noun by *eccentre* and the adjective by *eccentric*. An 'eccentre' is simply an eccentric circle. Similarly for *concentre* and *concentric*.

I have occasionally used the convenient mediaeval term *deferent* to denote the circle on which an epicycle is 'carried'. Ptolemy has no one-word equivalent, but uses phrases like 'the concentric carrying the epicycle', 'the circle carrying it'.

anomaly. As noted e.g. by Pedersen (139 with n.9), ἀνωμαλία in the Almagest has a number of different meanings. Despite the ambiguity, I have generally rendered ἀνωμαλία and the adjective from which it is derived, ἀνώμαλος, by 'anomaly', 'anomalistic', although where necessary I have translated the latter literally as 'non-uniform'. Besides referring to non-uniform motion, 'anomaly' is also used for the mean (hence uniform) motion of the moon and planets on their epicycles (because the motion on the epicycle produces the appearance of 'non-uniformity'). For the planets Ptolemy distinguishes between the *synodic anomaly* (ἡ πρὸς τὸν ἥλιον ἀνωμαλία, 'the anomaly with respect to the sun', HII 255,8), which produces the phenomena of retrogradation and varies with the planet's elongation from the sun, and the *ecliptic anomaly* (ζῳδιακὴ ἀνωμαλία, HII 258,11), which varies according to the planet's position in the ecliptic.

equation. I use this convenient mediaeval term for the angle (or arc) to be applied to a mean motion to 'correct' it to account for a particular feature of the geometric model. Ptolemy uses the vaguer terms τὸ διάφορον 'difference' (which is also used for many other things) and προσθαφαίρεσις ('amount to be added or subtracted'). *equation of anomaly* refers to the correction for the varying position of a body on its epicycle, and *equation of centre* (only in the footnotes, not the text) to the correction due to the eccentricity of a planet's deferent.

centrum. I have occasionally used this mediaeval term in the footnotes to denote the angular distance from apogee (see below) to the centre of the epicycle.

elongation (ἀποχή) is the angular distance along the ecliptic between two bodies or points. It is used particularly, but not exclusively, for the ecliptic distance between sun and moon.

apogee and *perigee* are simply transcriptions of ἀπόγειον and περίγειον, literally '[point] far from earth' and '[point] near to earth'. These are the usual terms for the points on a body's orbit which are respectively farthest from and nearest to the terrestrial observer. Ptolemy also uses the superlative forms ἀπογειότατον (περιγειότατον) σημεῖον ('point farthest from (nearest to) earth'), with no obvious difference in meaning. However, in the case of Mercury, translation of both by 'perigee' generates an ambiguity. For all other bodies, in Ptolemy's models, the perigee is diametrically opposite the apogee, but for Mercury the point of closest approach is about 120° from apogee. Ptolemy still refers to the point 180° from apogee as the 'perigee' (περιγεῖον) for Mercury, and when he wants to refer to the point of that planet's closest approach uses the superlative (περιγειότατος). I have mitigated the ambiguity by translating the latter, not as 'perigee', but as 'closest to earth' (for Mercury alone).

phase. Used for the fixed stars and planets, this is simply a transcription of φάσις, and is a general term including all the significant 'configurations with respect to the sun' (listed by Ptolemy at VIII 4 pp. 409–10, and exemplified in his partially extant work φάσεις ἀπλανῶν ἀστέρων, 'Phases of the Fixed Stars'), such as first visibility at sunset, or last visibility just before dawn. But the literal meaning of φάσις is 'appearance', and Ptolemy also uses it to mean specifically 'first visibility' of a body after a period of invisibility. To avoid ambiguity, I have translated the latter case by 'first visibility', reserving 'phase' for the general term.

(iv) *Referring to sun and moon*

conjunction is a fairly literal rendering of σύνοδος ('meeting'), but *opposition* renders πανσέληνος (literally 'full moon', which occurs when sun and moon are in opposition). *syzygy* is a transcription of the convenient συζυγία (literally 'yoking together'), a general term to denote either or both conjunction and opposition. In eclipses the partial phases are denoted by *immersion* (ἔμπτωσις, 'falling in', the phase from the beginning of the eclipse to totality) and *emersion* (ἀναπλήρωσις, 'filling up again', the phase from the end of totality to the end of the eclipse). The total phase is denoted by μονή ('remaining') and rendered by *duration (of totality)*.

(v) *Time-reckoning*

Ptolemy often uses the term νυχθήμερον, which combines the Greek words for night and day, to mean the 'solar day' of 24 hours. There is no such convenient term in English. I have generally translated it *day* when no ambiguity is possible, but have occasionally resorted to periphrasis (e.g. II 3 p. 79 = HI 96, 7-9). Since we use clocks, we reckon time by the *mean solar day* of uniform length, the average time taken by the sun to go from one meridian crossing to the next. In antiquity, where the normal means of telling time was the sundial, it was usually reckoned by the *true solar day*, of varying length, the time taken by the sun to go from one meridian crossing to the next on a specific day. In III 9 Ptolemy explains why they are different, and how to transform one into the other. He uses the terms ὁμαλὰ νυχθήμερα ('uniform days') and ἀνώμαλα νυχθήμερα ('non-uniform days') for mean and true solar days respectively. When he is talking about intervals, he often refers to those measured in true solar days as 'reckoned simply', and those measured in mean solar days as 'reckoned accurately'.

The kind of hours normally used in the ancient world were *seasonal hours* (ὧραι καιρικαί), sometimes known as 'civil hours'. An hour was $\frac{1}{12}$th of the actual length of daylight or night-time at a given place, and hence the length of an hour varied according to terrestrial latitude and time of year, and a day-hour was of different length from a night-hour except at equinox. For astronomical purposes, however, the uniform $\frac{1}{24}$th of a day was used; these were known as *equinoctial hours* (ὧραι ἰσημεριναί), because they were the same length as the seasonal hour at equinox. If an ordinal number is attached to an hour, it indicates a seasonal hour, counted from dawn (or sunset, if specified by 'of night' or by the context). Thus 'the sixth hour' is the same as noon.

time-degrees. Another way of measuring time was by the amount of the celestial equator which had passed a bound (horizon or meridian). This was often connected with the rising-times of ecliptic arcs (see pp. 18–19). This measurement was in degrees. Since 360° of the equator cross the meridian in about one day, one 'time-degree' equals $\frac{1}{15}$th of an equinoctial hour or 4 minutes. The Greek term is χρόνοι ἰσημερινοί ('equatorial times'), sometimes abbreviated to χρόνοι ('times').

(vi) *Other*

mean (μέσος) can imply 'of average length' (as in 'mean synodic month') or 'uniform' (as in 'mean motion in longitude').

hypothesis. With some hesitation, I have used this to translate ὑπόθεσις, although the connotation in the Almagest never really coincides with the modern one. Whereas we use 'hypothesis' to denote a tentative theory which has still to be verified, Ptolemy usually means by ὑπόθεσις something more like 'model', 'system of explanation', often indeed referring to 'the hypotheses which we have demonstrated'. The word still retains much of the etymological meaning of 'basis on which something else is constructed'. The corresponding verbal forms are ὑποτίθεται, ὑποκεῖται, which I have frequently translated, not only as 'assume', but even as 'it is given'. They are standard terms of Greek geometry in this sense at least as early as Euclid.

6. *Editorial procedures*

Since the translation is based principally on the Teubner text of Heiberg (see p. 3), it is keyed to that edition by the addition of Heiberg's page numbers in the margin. There and elsewhere references to Heiberg are preceded by 'H'. Thus HI 236,15 means 'Heiberg's edition, Vol. I p. 236 line 15'. Where the context makes it unnecessary the volume number is omitted.

Brackets are used as follows. Square brackets [ ] enclose explanatory additions to or expansions of the Greek text by the translator. Curved brackets { } enclose passages which I believe to be later additions to Ptolemy's original text. Parentheses ( ) are used merely for clarity, better to express the author's sequence of thought.

As explained on p. 5, I believe the list of chapter headings preceding each book to be a later addition. Nevertheless, since these serve a useful purpose, I have grouped them together at the beginning (pp. 27–32) to serve as a table of contents.

I have made no effort to provide a continuous commentary, but refer the reader to the relevant sections in Olaf Pedersen's *A Survey of the Almagest* (abbreviated 'Pedersen') and O. Neugebauer's *A History of Ancient Mathematical Astronomy* (abbreviated *HAMA*). My footnotes are confined to particulars not treated by them, or requiring some elaboration, and to textual corrections. In Appendix A, however, I have provided worked examples of every type of problem (including, where it is not utterly trivial, the use of the tables) which arises in the Almagest, except where Ptolemy himself gives a worked example. Where possible, my example is taken from a calculation or observation actually occurring in the Almagest. Appendix B lists all my corrections to Heiberg's text. Appendix C discusses the problem of the derivation of Ptolemy's planetary mean motions, which has never been adequately treated.

The index includes all proper names occurring in the text, and certain selected topics (mostly taken from the Introduction and footnotes). It also contains all observations recorded in the Almagest, under the topic or body concerned (e.g. 'equinox', 'moon'). For a list of the observations in chronological order the reader is referred to Pedersen's Appendix A.

In drawing the diagrams I have tried to reproduce the manuscript tradition, while at the same time making the figures as clear as possible by marking the points unambiguously. Since there is often considerable variation in the manuscript representations, I have been forced to make many choices; but I have not 'modernized' the figures. Where a figure seemed inadequate, I have not changed it, but have added an explanatory one of my own. Such explanatory (and other supplementary) figures are distinguished by alphabetical numbering ('Fig. A' etc.), whereas figures reproduced from the manuscripts are numbered according to the book and the order within that book (thus 'Fig. 3.10' indicates that this is the tenth diagram in Book III; in the manuscripts they are not usually numbered, but where they are, they are numbered separately in each book). I have represented the Greek letters of the figures by the following system:

Text	Trans.	Text	Trans.	Text	Trans.
Α	A	Ι	J	Π	P
Β	B	Κ	K	Ρ	R
Γ	G	Λ	L	Σ	S
Δ	D	Μ	M	Τ	T
Ε	E	Ν	N	Υ	Y
Ζ	Z	Ξ	X	Φ	F
Η	H	Ο	O	Χ	Q
Θ	Θ			Ψ	V

7. *Other conventional symbols and abbreviations*

e	eccentricity
r	radius of epicycle or body
M	length of longest day in hours
m	length of shortest day in hours
R	radius of principal circle (e.g. of deferent)
α	(1) right ascension (see p. 18)
	(2) anomaly (see p. 21)
β	celestial latitude
δ	declination
ε	obliquity of ecliptic
η	elongation
θ	equation
ι	inclination of orbit (of moon or planet)
κ	'centrum', i.e. distance from apogee (see p. 22)
λ	longitude
ρ	(1) oblique ascension (see p. 18)
	(2) geocentric distance
φ	terrestrial latitude
ω	distance from northpoint on orbit

A bar over a letter denotes 'mean', thus $\bar{\lambda}$ = 'mean longitude'.

The following are used in a raised position (e.g. 2^p) to denote units:

d	days
h	equinoctial hours

m	months
y	years
p	'parts', i.e. the arbitrary units in trigonometrical calculations (see pp. 7–9)
°	degrees
°°	demi degrees ($2^{\circ\circ} = 1^\circ$, see p. 8)
°/d	degrees per day

In the star catalogue only, * indicates some doubt about the reading. For other abbreviations particular to the star catalogue see p. 341 n.95.

Zodiacal signs

♈	Aries	♈ 0° =	0° in longitude
♉	Taurus	♉ 0° =	30°
♊	Gemini	♊ 0° =	60°
♋	Cancer	♋ 0° =	90°
♌	Leo	♌ 0° =	120°
♍	Virgo	♍ 0° =	150°
♎	Libra	♎ 0° =	180°
♏	Scorpius	♏ 0° =	210°
♐	Sagittarius	♐ 0° =	240°
♑	Capricornus	♑ 0° =	270°
♒	Aquarius	♒ 0° =	300°
♓	Pisces	♓ 0° =	330°

Planetary symbols

♄ Saturn
♃ Jupiter
♂ Mars
♀ Venus
☿ Mercury
☉ Sun
☽ Moon

Other astronomical symbols

⊕ Earth
☊ ascending node
☋ descending node

On 'sexagesimal' representations such as 6,13;10,0,58 see pp. 6–7.

For the mathematical symbols || and ||| (both meaning 'is similar to') and ≡ ('is congruent to') see p. 17.

For 'M. T. I' and 'M. T. II' see p. 18.

For manuscript abbreviations see pp. 3–4.

Contents of the *Almagest*[1]

[1] These lists of the chapter headings are found in the mss. at the beginning of each book preceded by the words 'The following are the contents of Book *n* of Ptolemy's mathematical treatise'. I believe that the division into chapters and the chapter headings are later additions (see Introduction p. 5).

BOOK III

BOOK IV

[2] Most mss., followed by Heiberg, read at H86,20 κανονίων τῶν κατὰ δεκαμοιρίαν παράλληλον, which is untranslatable. I read with D κανονίων τῶν κατὰ παράλληλον. Someone who compared the text at II 8 (H 134,I), κανόνιον τῶν κατὰ δεκαμοιρίαν ἀναφορῶν, imported δεκαμοιρίαν here and tried to combine the two inconsistent descriptions.

BOOK V

BOOK VI

[3] In the text the 'method of calculation' is explained at the end of ch. 9, and ch. 10 consists solely of the table. This variation is perhaps a remnant of a different chapter division. Cf. Introduction p. 5.

BOOK VII

BOOK VIII

BOOK IX

page

BOOK X

BOOK XI

BOOK XII

BOOK XIII

Translation of the ALMAGEST

Book I

1. {*Preface*}[4]

The true philosophers, Syrus,[5] were, I think, quite right to distinguish the theoretical part of philosophy from the practical. For even if practical philosophy, before it *is* practical, turns out to be theoretical,[6] nevertheless one can see that there is a great difference between the two: in the first place, it is possible for many people to possess some of the moral virtues even without being taught, whereas it is impossible to achieve theoretical understanding of the universe without instruction; furthermore, one derives most benefit in the first case [practical philosophy] from continuous practice in actual affairs, but in the other [theoretical philosophy] from making progress in the theory. Hence we
thought it fitting to guide our actions (under the impulse of our actual ideas [of H5
what is to be done]) in such a way as never to forget, even in ordinary affairs, to strive for a noble and disciplined disposition, but to devote most of our time to intellectual matters, in order to teach theories, which are so many and beautiful, and especially those to which the epithet 'mathematical' is particularly applied. For Aristotle divides theoretical philosophy too, very fittingly, into three primary categories, physics, mathematics and theology.[7] For everything that exists is composed of matter, form and motion; none of these [three] can be observed in its substratum by itself, without the others: they can only be imagined. Now the first cause of the first motion of the universe, if one considers it simply, can be thought of as an invisible and motionless deity; the division [of theoretical philosophy] concerned with investigating this [can be called] 'theology', since this kind of activity, somewhere up in the highest reaches of the universe, can only be imagined, and is completely separated from

[4] This 'philosophical' preface and its relationship to Ptolemy's attitude to philosophy is discussed by Boll, *Studien* 68–76, to which the reader is referred for the relevant passages in ancient literature. The general standpoint is Aristotelian.

[5] Syrus is also the addressee of a number of other works by Ptolemy (see Toomer[5] 187). Nothing is known about him. The name is very common in (but not confined to) Greco-Roman Egypt. The statement in a scholion to the Tetrabiblos (quoted by Boll, *Studien* 67, n. 2) that some say he was a fictitious person, others that he was a doctor, merely reveals that he was equally unknown in late antiquity.

[6] Theon in his commentary (Rome II 320, 13–14) gives φησὶ . . . συμβεβηκέναι τῷ πρακτικῷ τό πρότερον αὐτοῦ τοῦ θεωρητικοῦ τυγχάνειν. This is a paraphrase rather than a different reading, but shows that he understood the text as I have translated it. By this obscure expression I take Ptolemy to mean that before actually practising virtues one must have some concept of them (even though this is innate rather than taught).

[7] E. g. Metaphysics E 1, 1026a 18 ff., ὥστε τρεῖς ἂν εἶεν φιλοσοφίαι θεωρητικαί, μαθηματική, φυσική, θεολογική.

perceptible reality. The division [of theoretical philosophy] which investigates material and ever-moving nature, and which concerns itself with 'white', 'hot', 'sweet', 'soft' and suchlike qualities one may call 'physics'; such an order of being is situated (for the most part) amongst corruptible bodies and below the lunar sphere. That division [of theoretical philosophy] which determines the
H6 nature involved in forms and motion from place to place, and which serves to investigate shape, number, size, and place, time and suchlike, one may define as 'mathematics'. Its subject-matter falls as it were in the middle between the other two, since, firstly, it can be conceived of both with and without the aid of the senses, and, secondly, it is an attribute of all existing things without exception, both mortal and immortal: for those things which are perpetually changing in their inseparable form, it changes with them, while for eternal things which have an aethereal[8] nature, it keeps their unchanging form unchanged.

From all this we concluded:[9] that the first two divisions of theoretical philosophy should rather be called guesswork than knowledge, theology because of its completely invisible and ungraspable nature, physics because of the unstable and unclear nature of matter; hence there is no hope that philosophers will ever be agreed about them; and that only mathematics can provide sure and unshakeable knowledge to its devotees, provided one approaches it rigorously. For its kind of proof proceeds by indisputable methods, namely arithmetic and geometry. Hence we were drawn to the investigation of that part of theoretical philosophy, as far as we were able to the whole of it, but especially to the theory concerning divine and heavenly things. For that alone is devoted to the investigation of the eternally
H7 unchanging. For that reason it too can be eternal and unchanging (which is a proper attribute of knowledge) in its own domain, which is neither unclear nor disorderly. Furthermore it can work in the domains of the other [two divisions of theoretical philosophy] no less than they do. For this is the best science to help theology along its way, since it is the only one which can make a good guess at [the nature of] that activity which is unmoved and separated; [it can do this because] it is familiar with the attributes of those beings[10] which are on the one hand perceptible, moving and being moved, but on the other hand eternal and unchanging, [I mean the attributes] having to do with motions and the arrangements of motions. As for physics, mathematics can make a significant contribution. For almost every peculiar attribute of material nature becomes apparent from the peculiarities of its motion from place to place. [Thus one can distinguish] the corruptible from the incorruptible by [whether it undergoes] motion in a straight line or in a circle, and heavy from light, and passive from active, by [whether it moves] towards the centre or away from the centre. With

[8] 'aethereal' (αἰθερώδης) has a precise meaning in Aristotelian physics: everything above the sphere of the moon is composed of an 'incorruptible' substance, unlike anything known on earth in its consistency (very thin) and in its natural motion (circular). See I 3 p. 40. One of the names for this substance is 'aether', another 'fifth essence'. See Campanus IV n. 56, pp. 394–5.

[9] In this exaltation of mathematics above the other two divisions of philosophy Ptolemy parts company with Aristotle, for whom theology was the most noble pursuit for the human mind.

[10] The heavenly bodies.

regard to virtuous conduct in practical actions and character, this science, above all things, could make men see clearly; from the constancy, order, symmetry and calm which are associated with the divine, it makes its followers lovers of this divine beauty, accustoming them and reforming their natures, as it were, to a similar spiritual state.

It is this love of the contemplation of the eternal and unchanging which we constantly strive to increase, by studying those parts of these sciences which H8
have already been mastered by those who approached them in a genuine spirit of enquiry, and by ourselves attempting to contribute as much advancement as has been made possible by the additional time between those people and ourselves.[11] We shall try to note down[12] everything which we think we have discovered up to the present time; we shall do this as concisely as possible and in a manner which can be followed by those who have already made some progress in the field.[13] For the sake of completeness in our treatment we shall set out everything useful for the theory of the heavens in the proper order, but to avoid undue length we shall merely recount what has been adequately established by the ancients. However, those topics which have not been dealt with [by our predecessors] at all, or not as usefully as they might have been, will be discussed at length, to the best of our ability.

2. {*On the order of the theorems*}

In the treatise which we propose, then, the first order of business is to grasp the relationship of the earth taken as a whole to the heavens taken as a whole.[14] In the treatment of the individual aspects which follows, we must first discuss the position of the ecliptic[15] and the regions of our part of the inhabited world and also the features differentiating each from the others due to the [varying] latitude at each horizon taken in order.[16] For if the theory of these matters is H9
treated first it will make examination of the rest easier. Secondly, we have to go through the motion of the sun and of the moon, and the phenomena accompanying these [motions];[17] for it would be impossible to examine the theory of the stars[18] thoroughly without first having a grasp of these matters. Our final task in this way of approach is the theory of the stars. Here too it would be appropriate to deal first with the sphere of the so-called 'fixed stars',[19]

[11] This notion of the advancement of science, and particularly astronomy, by the additional time available is one to which Ptolemy recurs in the epilogue (XIII 11 p. 647), and also, in a specifically astronomical context, at VII 1 p. 321 and VII 3 p. 329.

[12] ὑπομνηματίσασθαι. A ὑπόμνημα is a 'memoir', usually implying summary brevity. Ptolemy recurs to this too in the epilogue (XIII 11 p. 647).

[13] Ptolemy assumes that his readers will have a certain competence. See Introduction p. 6.

[14] I 3–8. On the logic of Ptolemy's order see Introduction pp. 5–6.

[15] I 12–16. The mathematical section I 10–11 is not specifically mentioned here.

[16] Book II.

[17] Books III–VI.

[18] 'Stars' here and throughout chs. 3–8 includes both fixed stars and planets (see Introduction p. 21) and also, sometimes, sun and moon.

[19] Books VII–VIII.

and follow that by treating the five 'planets', as they are called.[20] We shall try to provide proofs in all of these topics by using as starting-points and foundations, as it were, for our search the obvious phenomena, and those observations made by the ancients and in our own times which are reliable. We shall attach the subsequent structure of ideas to this [foundation] by means of proofs using geometrical methods.

The general preliminary discussion covers the following topics: the heaven is spherical in shape, and moves as a sphere; the earth too is sensibly spherical in shape, when taken as a whole; in position it lies in the middle of the heavens very much like its centre; in size and distance it has the ratio of a point to the sphere of
H10 the fixed stars; and it has no motion from place to place. We shall briefly discuss each of these points for the sake of reminder.

3. {*That the heavens move like a sphere*}[21]

It is plausible to suppose that the ancients got their first notions on these topics from the following kind of observations. They saw that the sun, moon and other stars were carried from east to west along circles which were always parallel to each other, that they began to rise up from below the earth itself, as it were, gradually got up high, then kept on going round in similar fashion and getting lower, until, falling to earth, so to speak, they vanished completely, then, after remaining invisible for some time, again rose afresh and set; and [they saw] that the periods of these [motions], and also the places of rising and setting, were, on the whole, fixed and the same.

What chiefly led them to the concept of a sphere was the revolution of the ever-visible stars, which was observed to be circular, and always taking place about one centre, the same [for all]. For by necessity that point became [for
H11 them] the pole of the heavenly sphere: those stars which were closer to it revolved on smaller circles, those that were farther away described circles ever greater in proportion to their distance, until one reaches the distance of the stars which become invisible. In the case of these, too, they saw that those near the ever-visible stars remained invisible for a short time, while those farther away remained invisible for a long time, again in proportion [to their distance]. The result was that in the beginning they got to the aforementioned notion solely from such considerations; but from then on, in their subsequent investigation, they found that everything else accorded with it, since absolutely all phenomena are in contradiction to the alternative notions which have been propounded.

For if one were to suppose that the stars' motion takes place in a straight line towards infinity, as some people have thought,[22] what device could one

[20] Books IX–XIII.
[21] See Pedersen 36–7.
[22] According to Theon's commentary (Rome II 338) this belief was Epicurean, but I know of no other evidence. The only other relevant passage appears to be Xenophanes, Diels-Kranz A41a (the sun really moves towards infinity).

conceive of which would cause each of them to appear to begin their motion from the same starting-point every day? How could the stars turn back if their motion is towards infinity? Of, if they did turn back, how could this not be obvious? [On such a hypothesis], they must gradually diminish in size until they disappear, whereas, on the contrary, they are seen to be greater at the very moment of their disappearance, at which time they are gradually obstructed and cut off, as it were, by the earth's surface.

But to suppose that they are kindled as they rise out of the earth and are extinguished again as they fall to earth is a completely absurd hypothesis.[23] For
even if we were to concede that the strict order in their size and number, their H12
intervals, positions and periods could be restored by such a random and chance process; that one whole area of the earth has a kindling nature, and another an extinguishing one, or rather that the same part [of the earth] kindles for one set of observers and extinguishes for another set; and that the same stars are already kindled or extinguished for some observers while they are not yet for others: even if, I say, we were to concede all these ridiculous consequences, what could we say about the ever-visible stars, which neither rise nor set? Those stars which are kindled and extinguished ought to rise and set for observers everywhere, while those which are not kindled and extinguished ought always to be visible for observers everywhere. What cause could we assign for the fact that this is not so? We will surely not say that stars which are kindled and extinguished for some observers never undergo this process for other observers. Yet it is utterly obvious that the same stars rise and set in certain regions [of the earth] and do neither at others.

To sum up, if one assumes any motion whatever, except spherical, for the heavenly bodies, it necessarily follows that their distances, measured from the earth upwards, must vary, wherever and however one supposes the earth itself to be situated. Hence the sizes and mutual distances of the stars must appear to
vary for the same observers during the course of each revolution, since at one H13
time they must be at a greater distance, at another at a lesser. Yet we see that no such variation occurs. For the apparent increase in their sizes at the horizons[24] is caused, not by a decrease in their distances, but by the exhalations of moisture surrounding the earth being interposed between the place from which we observe and the heavenly bodies, just as objects placed in water appear bigger than they are, and the lower they sink, the bigger they appear.

The following considerations also lead us to the concept of the sphericity of the heavens. No other hypothesis but this can explain how sundial constructions produce correct results; furthermore, the motion of the heavenly bodies is the most unhampered and free of all motions, and freest motion belongs among

[23] Theon (Rome II 340) ascribes this to Heraclitus. Otherwise it is attested for Xenophanes (Diels-Kranz A38), and was admitted as one possible explanation by Epicurus (e.g. *Letter to Pythocles* 92) and his followers.

[24] Ptolemy refers to the well-known phenomenon that the sun and moon appear larger when close to the horizon. He gives an incorrect physical and optical explanation here. In a later work (*Optics* III 60, ed. Lejeune p. 116) he correctly explains it as a purely psychological phenomenon. No doubt instrumental measurement of the apparent diameters had convinced him that the enlargement is entirely illusory.

plane figures to the circle and among solid shapes to the sphere; similarly, since of different shapes having an equal boundary those with more angles are greater [in area or volume], the circle is greater than [all other] surfaces, and the sphere greater than [all other] solids;[25] [likewise] the heavens are greater than all other bodies.

Furthermore, one can reach this kind of notion from certain physical
H14 considerations. E.g., the aether is, of all bodies, the one with constituent parts which are finest and most like each other; now bodies with parts like each other have surfaces with parts like each other; but the only surfaces with parts like each other are the circular, among planes, and the spherical, among three-dimensional surfaces. And since the aether is not plane, but three-dimensional, it follows that it is spherical in shape. Similarly, nature formed all earthly and corruptible bodies out of shapes which are round but of unlike parts, but all aethereal and divine bodies out of shapes which are of like parts and spherical. For if they were flat or shaped like a discus[26] they would not always display a circular shape to all those observing them simultaneously from different places on earth. For this reason it is plausible that the aether surrounding them, too, being of the same nature, is spherical, and because of the likeness of its parts moves in a circular and uniform fashion.

4. {*That the earth too, taken as a whole, is sensibly spherical*}[27]

That the earth, too, taken as a whole,[28] is sensibly spherical can best be grasped from the following considerations. We can see, again, that the sun, moon and
H15 other stars do not rise and set simultaneously for everyone on earth, but do so earlier for those more towards the east, later for those towards the west. For we find that the phenomena at eclipses, especially lunar eclipses,[29] which take place at the same time [for all observers], are nevertheless not recorded as occurring at the same hour (that is at an equal distance from noon) by all observers. Rather, the hour recorded by the more easterly observers is always later than that recorded by the more westerly. We find that the differences in the hour are proportional to the distances between the places [of observation]. Hence one can reasonably conclude that the earth's surface is spherical, because its evenly curving surface (for so it is when considered as a whole) cuts off [the heavenly bodies] for each set of observers in turn in a regular fashion.

If the earth's shape were any other, this would not happen, as one can see from the following arguments. If it were concave, the stars would be seen rising first by those more towards the west; if it were plane, they would rise and set

[25] These propositions were proved in a work by Zenodorus (early second century B.C., see Toomer[1]) from which extensive excerpts are given by (among others) Theon (Rome II 355–79). There is a good summary in Heath *HGM* II 207–13.

[26] The only relevant passage I know is Empedocles, Diels-Kranz A60, who maintained that the moon is disk-shaped.

[27] See Pedersen 37–9.

[28] 'taken as a whole': ignoring local irregularities such as mountains, which are negligible in comparison to the total mass.

[29] The timings for solar eclipses are complicated by parallax.

simultaneously for everyone on earth; if it were triangular or square or any other polygonal shape, by a similar argument, they would rise and set simultaneously for all those living on the same plane surface. Yet it is apparent that nothing like this takes place. Nor could it be cylindrical, with the curved surface in the east-west direction, and the flat sides towards the poles of the universe, H16
which some might suppose more plausible. This is clear from the following: for those living on the curved surface none of the stars would be ever-visible, but either all stars would rise and set for all observers, or the same stars, for an equal [celestial] distance from each of the poles, would always be invisible for all observers. In fact, the further we travel toward the north, the more[30] of the southern stars disappear and the more of the northern stars appear. Hence it is clear that here too the curvature of the earth cuts off [the heavenly bodies] in a regular fashion in a north-south direction, and proves the sphericity [of the earth] in all directions.

There is the further consideration that if we sail towards mountains or elevated places from and to any direction whatever, they are observed to increase gradually in size as if rising up from the sea itself in which they had previously been submerged: this is due to the curvature of the surface of the water.

5. {*That the earth is in the middle of the heavens*}[31]

Once one has grasped this, if one next considers the position of the earth, one will find that the phenomena associated with it could take place only if we H17
assume that it is in the middle of the heavens, like the centre of a sphere. For if this were not the case, the earth would have to be either

[a] not on the axis [of the universe] but equidistant from both poles, or
[b] on the axis but removed towards one of the poles, or
[c] neither on the axis nor equidistant from both poles.

Against the first of these three positions militate the following arguments. If we imagined [the earth] removed towards the zenith or the nadir of some observer, then, if he were at *sphaera recta*, he would never experience equinox, since the horizon would always divide the heavens into two unequal parts, one above and one below the earth; if he were at *sphaera obliqua*, either, again, equinox would never occur at all, or, [if it did occur,] it would not be at a position halfway between summer and winter solstices, since these intervals would necessarily be unequal, because the equator, which is the greatest of all parallel circles drawn about the poles of the [daily] motion, would no longer be bisected by the horizon; instead [the horizon would bisect] one of the circles parallel to the equator, either to the north or to the south of it. Yet absolutely everyone agrees that these intervals are equal everywhere on earth, since H18
[everywhere] the increment of the longest day over the equinoctial day at the

[30] Reading πλείονα (with D) for τὰ πλείονα at H16,9. Corrected by Manitius.
[31] See Pedersen 39–42.

summer solstice is equal to the decrement of the shortest day from the equinoctial day at the winter solstice. But if, on the other hand, we imagined the displacement to be towards the east or west of some observer, he would find that the sizes and distances of the stars would not remain constant and unchanged at eastern and western horizons, and that the time-interval from rising to culmination would not be equal to the interval from culmination to setting. This is obviously completely in disaccord with the phenomena.

Against the second position, in which the earth is imagined to lie on the axis removed towards one of the poles, one can make the following objections. If this were so, the plane of the horizon would divide the heavens into a part above the earth and a part below the earth which are unequal and always different for different latitudes,[32] whether one considers the relationship of the same part at two different latitudes or the two parts at the same latitude. Only at *sphaera recta* could the horizon bisect the sphere; at a *sphaera obliqua* situation such that the nearer pole were the ever-visible one, the horizon would always make the part above the earth lesser and the part below the earth greater; hence another phenomenon would be that the great circle of the ecliptic would be divided into
H19 unequal parts by the plane of the horizon. Yet it is apparent that this is by no means so. Instead, six zodiacal signs are visible above the earth at all times and places, while the remaining six are invisible; then again [at a later time] the latter are visible in their entirety above the earth, while at the same time the others are not visible. Hence it is obvious that the horizon bisects the zodiac, since the same semi-circles are cut off by it, so as to appear at one time completely above the earth, and at another [completely] below it.

And in general, if the earth were not situated exactly below the [celestial] equator, but were removed towards the north or south in the direction of one of the poles, the result would be that at the equinoxes the shadow of the gnomon at sunrise would no longer form a straight line with its shadow at sunset in a plane parallel to the horizon, not even sensibly.[33] Yet this is a phenomenon which is plainly observed everywhere.

It is immediately clear that the third position enumerated is likewise impossible, since the sorts of objection which we made to the first [two] will both arise in that case.

To sum up, if the earth did not lie in the middle [of the universe], the whole order of things which we observe in the increase and decrease of the length of daylight would be fundamentally upset. Furthermore, eclipses of the moon would not be restricted to situations where the moon is diametrically opposite the sun (whatever part of the heaven [the luminaries are in]), since the earth
H20 would often come between them when they were not diametrically opposite, but at intervals of less than a semi-circle.

[32] The word translated here and elsewhere as '[terrestrial] latitude' is κλίμα, for the meaning of which see Introduction p. 19.

[33] The *caveat* 'sensibly' is inserted because the equinox is not a date but an instant of time. Therefore on the day of equinox the sun does not rise due east and set due west (as is implied by the rising and setting shadows lying on the same straight line). However, the difference would be 'imperceptible to the senses'.

6. {*That the earth has the ratio of a point to the heavens*}[34]

Moreover, the earth has, to the senses, the ratio of a point to the distance of the sphere of the so-called fixed stars.[35] A strong indication of this is the fact that the sizes and distances of the stars, at any given time, appear equal and the same from all parts of the earth everywhere, as observations of the same [celestial] objects from different latitudes are found to have not the least discrepancy from each other. One must also consider the fact that gnomons set up in any part of the earth whatever, and likewise the centres of armillary spheres,[36] operate like the real centre of the earth; that is, the lines of sight [to heavenly bodies] and the paths of shadows caused by them agree as closely with the [mathematical] hypotheses explaining the phenomena as if they actually passed through the real centre-point of the earth.

Another clear indication that this is so is that the planes drawn through the observer's lines of sight at any point [on earth], which we call 'horizons', always bisect the whole heavenly sphere. This would not happen if the earth were of H21
perceptible size in relation to the distance of the heavenly bodies; in that case only the plane drawn through the centre of the earth could bisect the sphere, while a plane through any point on the surface of the earth would always make the section [of the heavens] below the earth greater than the section above it.

7. {*That the earth does not have any motion from place to place, either*}[37]

One can show by the same arguments as the preceding that the earth cannot have any motion in the aforementioned directions, or indeed ever move at all from its position at the centre. For the same phenomena would result as would if it had any position other than the central one. Hence I think it is idle to seek for causes for the motion of objects towards the centre, once it has been so clearly established from the actual phenomena that the earth occupies the middle place in the universe, and that all heavy objects are carried towards the earth. The following fact alone would most readily lead one to this notion [that all objects fall towards the centre]. In absolutely all parts of the earth, which, as we said, has been shown to be spherical and in the middle of the universe, the direction[38] and path of the motion (I mean the proper, [natural] motion) of all H22
bodies possessing weight is always and everywhere at right angles to the rigid plane drawn tangent to the point of impact. It is clear from this fact that, if

[34] See Pedersen 42-3.

[35] Ptolemy qualifies the traditional terminology for the fixed stars as 'so-called' (καλουμένων) because they do in fact, according to him, have a motion (the modern 'precession'). He develops the point further at VII 1 p. 321, q.v. In general, however, he uses the traditional terminology without qualification.

[36] An example of an armillary sphere (κρικωτὴ σφαῖρα) is the 'astrolabe' described in V 1. For references to the term in other works see LSJ s.v. κρικωτός.

[37] See Pedersen 43-4.

[38] πρόσνευσις, which I have translated 'the direction of motion' here, means basically 'direction in which something points' (for astronomical usages see V 5 p. 227 n. 19 and VI 11 p. 313 n. 77). Thus it would also include here the direction of a plumb-line (cf. I 12 p. 62).

[these falling objects] were not arrested by the surface of the earth, they would certainly reach the centre of the earth itself, since the straight line to the centre is also always at right angles to the plane tangent to the sphere at the point of intersection [of that radius] and the tangent.

Those who think it paradoxical that the earth, having such a great weight, is not supported by anything and yet does not move, seem to me to be making the mistake of judging on the basis of their own experience instead of taking into account the peculiar nature of the universe. They would not, I think, consider such a thing strange once they realised that this great bulk of the earth, when compared with the whole surrounding mass [of the universe], has the ratio of a point to it. For when one looks at it in that way, it will seem quite possible that that which is relatively smallest should be overpowered and pressed in equally from all directions to a position of equilibrium by that which is the greatest of all
H23 and of uniform nature. For there is no up and down in the universe with respect to itself,[39] any more than one could imagine such a thing in a sphere: instead the proper and natural motion of the compound bodies in it is as follows: light and rarefied bodies drift outwards towards the circumference, but seem to move in the direction which is 'up' for each observer, since the overhead direction for all of us, which is also called 'up', points towards the surrounding surface;[40] heavy and dense bodies, on the other hand, are carried towards the middle and the centre, but seem to fall downwards, because, again, the direction which is for all us towards our feet, called 'down', also points towards the centre of the earth. These heavy bodies, as one would expect, settle about the centre because of their mutual pressure and resistance, which is equal and uniform from all directions. Hence, too, one can see that it is plausible that the earth, since its total mass is so great compared with the bodies which fall towards it, can remain motionless under the impact of these very small weights (for they strike it from all sides), and receive, as it were, the objects falling on it. If the earth had a single motion in common with other heavy objects, it is obvious that it would be carried down faster than all of them because of its much greater size: living
H24 things and individual heavy objects would be left behind, riding on the air, and the earth itself would very soon have fallen completely out of the heavens. But such things are utterly ridiculous merely to think of.

But certain people,[41] [propounding] what they consider a more persuasive view, agree with the above, since they have no argument to bring against it, but think that there could be no evidence to oppose their view if, for instance, they supposed the heavens to remain motionless, and the earth to revolve from west to east about the same axis [as the heavens], making approximately one revolution each day;[42] or if they made both heaven and earth move by any amount whatever, provided, as we said, it is about the same axis, and in such a

[39] Reading αὐτόν (with D, Is) for αὐτήν at H23,1.

[40] It is not clear to me whether Ptolemy means the outmost boundary of the *universe* or merely the surface (of the 'aether') surrounding the *earth*.

[41] Heraclides of Pontos (late fourth century B.C.) is the earliest certain authority for the view that the earth rotates on its axis. See *HAMA* II 694–6. It was also adopted by Aristarchus as part of his more radical heliocentric hypothesis.

[42] 'approximately' because one revolution takes place in a sidereal, not a solar day.

way as to preserve the overtaking of one by the other. However, they do not realise that, although there is perhaps nothing in the celestial phenomena which would count against that hypothesis, at least from simpler considerations, nevertheless from what would occur here on earth and in the air, one can see that such a notion is quite ridiculous. Let us concede to them [for the sake of argument] that such an unnatural thing could happen as that the most rare and light of matter should either not move at all or should move in a way no different from that of matter with the opposite nature (although things in the air, which are less rare [than the heavens] so obviously move with a more rapid motion than any earthy object); [let us concede that] the densest and heaviest objects H25
have a proper motion of the quick and uniform kind which they suppose (although, again, as all agree, earthy objects are sometimes not readily moved even by an external force). Nevertheless, they would have to admit that the revolving motion of the earth must be the most violent of all motions associated with it, seeing that it makes one revolution in such a short time; the result would be that all objects not actually standing on the earth would appear to have the same motion, opposite to that of the earth: neither clouds nor other flying or thrown objects would ever be seen moving towards the east, since the earth's motion towards the east would always outrun and overtake them, so that all other objects would seem to move in the direction of the west and the rear. But if they said that the air is carried around in the same direction and with the same speed as the earth, the compound objects in the air would none the less always seem to be left behind by the motion of both [earth and air]; or if those objects too were carried around, fused, as it were, to the air, then they would never appear to have any motion either in advance or rearwards: they would always appear still, neither wandering about nor changing position, whether they were flying or thrown objects. Yet we quite plainly see that they do undergo all these H26
kinds of motion, in such a way that they are not even slowed down or speeded up at all by any motion of the earth.

8. {*That there are two different primary motions in the heavens*}[43]

It was necessary to treat the above hypotheses first as an introduction to the discussion of particular topics and what follows after. The above summary outline of them will suffice, since they will be completely confirmed and further proven by the agreement with the phenomena of the theories which we shall demonstrate in the following sections. In addition to these hypotheses, it is proper, as a further preliminary, to introduce the following general notion, that there are two different primary motions in the heavens. One of them is that which carries everything from east to west: it rotates them with an unchanging and uniform motion along circles parallel to each other, described, as is obvious, about the poles of this sphere which rotates everything uniformly. The greatest of these circles is called the 'equator',[44] because it is the only [such

[43] See Pedersen 45.

[44] 'equator': ἰσημερινός, literally 'of equal day' or 'equinoctial'. See Introduction p. 19.

parallel circle] which is always bisected by the horizon (which is a great circle), and because the revolution which the sun makes when located on it produces equinox everywhere, to the senses. The other motion is that by which the
H27 spheres of the stars perform movements in the opposite sense to the first motion, about another pair of poles, which are different from those of the first rotation. We suppose that this is so because of the following considerations. When we observe for the space of any given single day, all heavenly objects whatever are seen, as far as the senses can determine, to rise, culminate and set at places which are analogous and lie on circles parallel to the equator; this is characteristic of the first motion. But when we observe continuously without interruption over an interval of time, it is apparent that while the other stars retain their mutual distances and (for a long time) the particular characteristics arising from the positions they occupy as a result of the first motion,[45] the sun, the moon and the planets have certain special motions which are indeed complicated and different from each other, but are all, to characterise their general direction,[46] towards the east and opposite to [the motion of] those stars which preserve their mutual distances and are, as it were, revolving on one sphere.

Now if this motion of the planets too took place along circles parallel to the equator, that is, about the poles which produce the first kind of revolution, it
H28 would be sufficient to assign a single kind of revolution to all alike, analogous to the first. For in that case it would have seemed plausible that the movements which they undergo are caused by various retardations, and not by a motion in the opposite direction. But as it is, in addition to their movement towards the east, they are seen to deviate continuously to the north and south [of the equator]. Moreover the amount of this deviation cannot be explained as the result of a uniformly-acting force pushing them to the side: from that point of view it is irregular, but it is regular if considered as the result of [motion on] a circle inclined to the equator. Hence we get the concept of such a circle, which is one and the same for all planets, and particular to them. It is precisely defined and, so to speak, drawn by the motion of the sun, but it is also travelled by the moon and the planets, which always move in its vicinity, and do not randomly pass outside a zone on either side of it which is determined for each body. Now since this too is shown to be a great circle, since the sun goes to the north and south of the equator by an equal amount, and since, as we said, the eastward motion of all of the planets takes place on one and the same circle, it became necessary to suppose that this second, different motion of the whole takes place
H29 about the poles of the inclined circle we have defined [i.e. the ecliptic], in the opposite direction to the first motion.

If, then, we imagine a great circle drawn through the poles of both the above-mentioned circles, (which will necessarily bisect each of them, that is the equator and the circle inclined to it [the ecliptic], at right angles), we will have four points on the ecliptic: two will be produced by [the intersection of] the

[45] These characteristics of the fixed stars are e.g. dates of first and last visibility. They are unchanged 'for a long time' because the effect of precession is very slow.

[46] The qualification is inserted here to allow for the retrogradations of the planets.

equator, diametrically opposite each other; these are called 'equinoctial' points. The one at which the motion [of the planets] is from south to north is called the 'spring' equinox, the other the 'autumnal'. Two [other points] will be produced by [the intersection of] the circle drawn through both poles; these too, obviously, will be diametrically opposite each other; they are called 'tropical' [or 'solsticial'] points. The one south of the equator is called the 'winter' [solstice], the one north, the 'summer' [solstice].

We can imagine the first primary motion, which encompasses all the other motions, as described and as it were defined by the great circle drawn through both poles [of equator and ecliptic] revolving, and carrying everything else with it, from east to west about the poles of the equator. These poles are fixed, so to speak, on the 'meridian' circle, which differs from the aforementioned [great] H30
circle in the single respect that it is not drawn through the poles of the ecliptic too at all positions of the latter. Moreover, it is called 'meridian' because it is considered to be always orthogonal to the horizon.[47] For a circle in such a position divides both hemispheres, that above the earth and that below it, into two equal parts, and defines the midpoint of both day and night.

The second, multiple-part motion is encompassed by the first and encompasses the spheres of all the planets. As we said, it is carried around by the aforementioned [first motion], but itself goes in the opposite direction about the poles of the ecliptic, which are also fixed on the circle which produces the first motion, namely the circle through both poles [of ecliptic and equator]. Naturally they [the poles of the ecliptic] are carried around with it [the circle through both poles], and, throughout the period of the second motion in the opposite direction, they always keep the great circle of the ecliptic, which is described by that [second] motion, in the same position with respect to the equator.[48]

9. {*On the individual concepts*}

Such, then are the necessary preliminary concepts which must be summarily set out in our general introduction. We are now about to begin the individual demonstrations, the first of which, we think, should be to determine the size of H31
the arc between the aforementioned poles [of the ecliptic and equator] along the great circle drawn through them. But we see that it is first necessary to explain the method of determining chords:[49] we shall demonstrate the whole topic geometrically once and for all.

[47] See Introduction p.19.

[48] My translation follows the interpretation of Theon (Rome II 447). Manitius (p. 24 n. a) wrongly considers τοῦ γραφομένου δι' αὐτῆς μεγίστου κἀι λοξοῦ κύκλου interpolated, partly because he misinterprets συντηροῦσιν (which is used here in a way similar to συντηροῦσαν at HI 6,10).

[49] 'chords': literally 'straight lines in a circle'. On this term see Introduction p. 17.

10. {*On the size of chords*}[50]

For the user's convenience, then, we shall subsequently set out a table of their amounts, dividing the circumference into 360 parts, and tabulating the chords subtended by the arcs at intervals of half a degree, expressing each as a number of parts in a system where the diameter is divided into 120 parts. [We adopt this norm] because of its arithmetical convenience,[51] which will become apparent from the actual calculations. But first we shall show how one can undertake the calculation of their amounts by a simple and rapid method, using as few theorems as possible, the same set for all. We do this so that we may not merely
H32 have the amounts of the chords tabulated unchecked, but may also readily undertake to verify them by computing them by a strict geometrical method. In general we shall use the sexagesimal system for our arithmetical computations, because of the awkwardness of the [conventional] fractional system. Since we always aim at a good approximation, we will carry out multiplications and divisions only as far as to achieve a result which differs from the precision achievable by the senses by a negligible amount.

First, then, [see Fig. 1.1] let there be a semi-circle ABG about centre D and on diameter ADG. Draw DB perpendicular to AG at D. Let DG be bisected at E, join EB, and let EZ be made equal to EB. Join ZB.

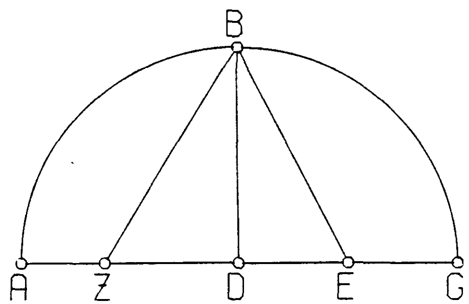

Fig. 1.1

I say that ZD is the side of the [regular] decagon, and BZ the side of the [regular] pentagon.

[Proof:] Since the straight line DG is bisected at E, and a straight line DZ is adjacent to it,

H33 $$GZ.ZD + ED^2 = EZ^2.$$ [52]

$$\text{But } EZ^2 = BE^2 \ (EB = ZE),$$

$$\text{and } EB^2 = ED^2 + DB^2.$$

$$\therefore GZ.ZD + ED^2 = ED^2 + DB^2.$$

[50] On Ptolemy's calculation of his chord table see *HAMA* 21–4, Pedersen 56–63.

[51] The principal convenience is that the radius is 60 parts, or 1,0 in the sexagesimal system. Hence in some ways this resembles a sine table with R = 1.

[52] Euclid II 6.

$$\therefore GZ.ZD = DB^2 \text{ (subtracting } ED^2\text{, common).}$$
$$\therefore GZ.ZD = DG^2.$$

So ZG has been cut in extreme and mean ratio at D.[53]

Now since the side of the hexagon and the side of the decagon, when both are inscribed in the same circle, make up the extreme and mean ratios of the same straight line,[54] and since GD, being a radius, represents the side of the hexagon,[55] DZ is equal to the side of the decagon.

Similarly, since the square on the side of the pentagon equals the sums of the squares on the sides of the hexagon and decagon when all are inscribed in the same circle,[56] and, in the right-angled triangle BDZ, the square on BZ equals H34 the sum of the squares on BD, which is the side of the hexagon, and on DZ, which is the side of the decagon, it follows that BZ equals the side of the pentagon.

Since, then, as I said, we set the diameter of the circle as 120 parts, it follows from the above that

$$DE = 30^p \text{ (DE half the radius)}$$
$$\text{and } DE^2 = 900^p;$$
$$BD = 60^p \text{ (BD a radius)}$$
$$\text{and } BD^2 = 3600^p.$$
$$\text{And } EZ^2 = EB^2 = 4500^p, \text{ the sum [of } DE^2 \text{ and } BD^2]$$
$$\therefore EZ \approx 67;4,55^p$$

and by subtraction [of DE from EZ], $DZ = 37;4,55^p$.
So the side of the decagon, which subtends 36°, has $37;4,55^p$ where the diameter has 120^p.

$$\text{Again, since } DZ = 37;4,55^p,$$
$$DZ^2 = 1375;4,15^p;\text{[57]}$$
$$\text{and } DB^2 = 3600^p,$$
$$\text{so } BZ^2 = DZ^2 + DB^2 = 4975;4,15^p.$$
$$\therefore BZ \approx 70;32,3^p.$$ H35

Therefore the side of the pentagon, which subtends 72°, contains $70;32,3^p$ where the diameter has 120^p.

It is immediately obvious that the side of the [inscribed] hexagon, which subtends 60° and is equal to the radius, contains 60^p.

Similarly, since the side of the [inscribed] square, which subtends 90°, is equal, when squared, to twice the square on the radius, and since the side of the [inscribed] triangle, which subtends 120°, is equal, when squared, to three times the square on the radius, and the square on the radius is 3600^p, we compute that

the square on the side of the square is 7200^p

and the square on the side of the triangle is 10800^p.

[53] Euclid VI Def. 3 states that 'a straight line has been cut in extreme and mean ratio when, as the whole line is to the greater segment, so is the greater to the less'; i.e. here ZG:DG = DG:ZD.

[54] Euclid XIII 9.

[55] Euclid IV 15 porism.

[56] Euclid XIII 10.

[57] The reading 14 (for 15) occurs as a marginal variant, in the Greek mss., here and at related places (see apparatus at H34,16; 34,18; 36,4 and 36,7), and, in the Arabic, in T, and was adopted in Ḥajjāj's translation. It is more accurate, but makes no difference to the final result.

$$\left.\begin{array}{r}\therefore \text{Crd } 90° \approx 84;51,10^p\\ \text{and Crd } 120° \approx 103;55,23^p\end{array}\right\}\text{ where the diameter is } 120^p.$$

We can, then, consider the above chords as established individually by the above straightforward procedures. It will immediately[58] be obvious that if any chord be given, the chord of the supplementary arc is given in a simple fashion,
H36 since the sum of their squares equals the square on the diameter. For instance, since the chord of 36° was shown to be $37;4,55^p$, and the square of this is $1375;4,15^p$, and the square on the diameter is 14400^p, the square on the chord of the supplementary arc (which is 144°) will be the difference, namely 13024;55,45, and so

$$\text{Crd } 144° \approx 114;7;37^p.$$

Similarly for the other chords [of the supplements].

We shall next show how the remaining individual chords can be derived from the above [chords], first of all setting out a theorem which is extremely useful for the matter at hand.

[See Fig. 1.2.] Let there be a circle with an arbitrary quadrilateral ABGD inscribed in it. Join AG and BD.

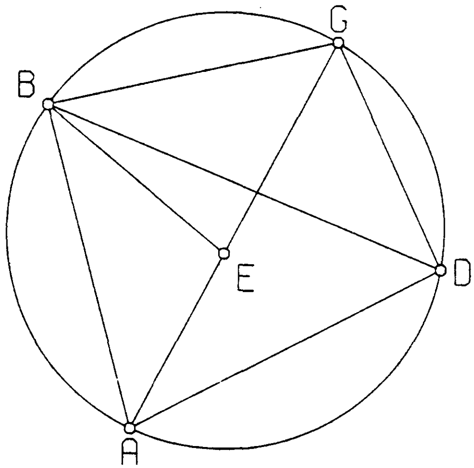

Fig. 1.2

We must prove that

$$AG.BD = AB.DG + AD.BG.$$ [59]

[Proof:] Make $\angle$ ABE = $\angle$ DBG.

Then, if we add $\angle$ EBD common,

$$\angle\, ABD = \angle\, EBG.$$

[58] Reading αὐτόθεν (with D) for ἐντεῦθεν at H35,18.

[59] This proposition, commonly known as 'Ptolemy's Theorem', is not in fact attested before him. It remains uncertain whether any of the earlier chord tables (e.g. Menelaus') used any geometrical basis beyond the half-angle theorem (see n. 60 and Toomer[2] 18–19).

But ∠ BDA = ∠ BGE also, since they subtend the same segment. H37

∴ triangle ABD ||| triangle BGE.

∴ BG:GE = BD:DA.

∴ BG.AD = BD.GE.

Again, since ∠ ABE = ∠ DBG,

and ∠ BAE = ∠ BDG,

triangle ABE ||| triangle BGD.

∴ BA:AE = BD:DG.

∴ BA.DG = BD.AE.

But it was shown that

BG.AD = BD.GE.

Therefore, by addition, AG.BD = AB.DG + AD.BG.

Q.E.D.

Having established this preliminary theorem, we draw [Fig. 1.3] semi-circle ABGD on diameter AD, and draw from A two chords, AB, AG, each given in H38 size in terms of a diameter of 120^p. Join BG.

I say that BG too is given.

[Proof:] Join BD,GD.

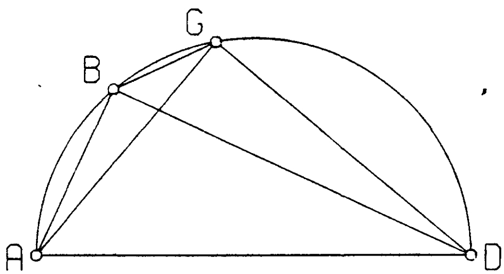

Fig. 1.3

Then, clearly, BD and GD too will be given, since they are chords of [arcs] supplementary [to the arcs of the given chords AB and AG].

Now since ABGD is a cyclic quadrilateral,

AB.GD + AD.BG = AG.BD.

But AG.BD and AB.GD are given.

∴ AD.BG is given by subtraction.

And AD is a diameter.

Therefore chord BG is given.

And we have shown that, if two arcs and the corresponding chords are given, the chord of the difference between the two arcs will also be given.

It is obvious that by means of this theorem we shall be able to enter [in the table] quite a few chords derived from the difference between the individually calculated chords, and notably the chord of 12°, since we have those of 60° and H39 72°.

Let us now consider the problem of finding the chord of the arc which is half that of some given chord.[60]

Let [Fig. 1.4] ABG be a semi-circle on diameter AG. Let GB be a given chord. Bisect arc GB at D, join AB, AD, BD, DG, and drop perpendicular DZ from D on to AG.

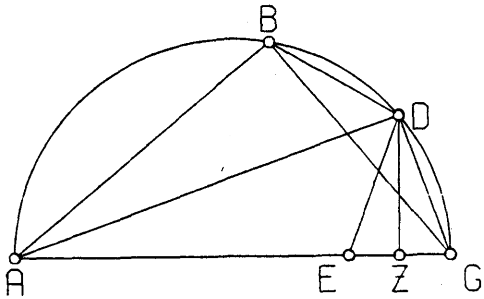

Fig. 1.4

I say that

$$ZG = \tfrac{1}{2}(AG - AB).$$

[Proof:] Let AE = AB, and join DE.

Then since [in the triangles ABD, ADE]

AB = AE, and AD is common,

the two pairs of sides AB, AD, and AE, AD are equal.

Furthermore ∠ BAD = ∠ EAD.

∴ base BD = base DE.

But BD = DG [by construction]

∴ DG = DE.

So, since, in the isosceles triangle DEG, perpendicular DZ has been drawn from apex to base

H40 EZ = ZG.

But EG = [AG − AE =] AG − AB.

∴ $ZG = \tfrac{1}{2}(AG - AB)$.

Now, if the chord of arc BG is given, the supplementary chord AB is immediately given.

Therefore ZG, which is $\tfrac{1}{2}(AG - AB)$, is also given.

But, since, in the right-angled triangle AGD, the perpendicular DZ has been drawn,

triangle ADG ||| triangle DGZ (both right-angled).[61]

∴ AG:GD = GD:GZ.

∴ $AG.GZ = GD^2$.

[60] Although Ptolemy's formula for the chord of the half-angle can easily be derived from his general theorem (see Toomer[2] 16–17), he introduces instead another theorem, which goes back to Archimedes (see *HAMA* 23–4). It is a plausible inference that this is because the latter theorem was the sole basis of earlier chord tables, notably Hipparchus', as I have argued, Toomer[2] 18–19.

[61] Euclid VI 8.

But AG.GZ is given.
Therefore GD^2 is given, and so chord GD, which subtends an arc half of [the arc of the given chord] BG, is also given.

By means of this theorem too a large number of chords will be derived by halving [the arcs of] the previously determined chords, and notably, from the chord of 12°, the chords of 6°, 3°, 1½° and ¾°. By calculation we find the chord of 1½° to be approximately $1;34,15^p$ where the diameter is 120^p, and the chord of ¾° H41
to be approximately $0;47,8^p$ in the same units.

Again, [see Fig. 1.5] let there be a circle ABGD on diameter AD, with centre Z. From A let there be cut off in succession two given arcs, AB, BG. Join the corresponding chords AB, BG; they too will be given.

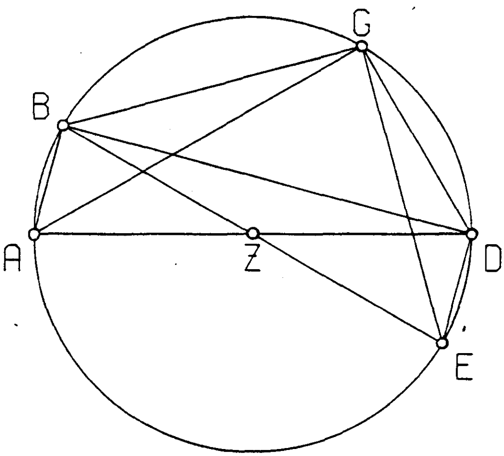

Fig. 1.5

I say, that if we join AG, that [chord] too will be given.

[Proof:] Draw through B diameter BZE, and join BD,DG,GE,DE. It is immediately clear that from BG one can derive GE, and from AB one can derive BD and DE [all as chords of the supplementary arc]. By an argument similar to the preceding [p. 51], since BGDE is a cyclic quadrilateral, in which BD and GE are diagonals, the product of the diagonals will be equal to the sum of the products of the opposite sides [i.e. BD.GE = BG.DE + BE.GD]. Therefore, since (BD.GE) and (BG.DE) are both given, (BE.GD) is also given. But BE also is given, being a diameter: therefore the remaining[62] part, GD, will also H42
be given, and hence GA, the [chord of the] supplement.

Therefore, if two arcs and the corresponding chords are given, the chord corresponding to the sum of these two arcs will be given by means of this theorem.

It is obvious that by combining [in this way] the chord of 1½° with all the chords we have already obtained, and then computing successive chords, we will be able to enter [in the table] all chords [of arcs] which when doubled are

[62] Reading ἡ λοιπή (with A) at H42,1 for λοιπῇ ('by subtraction').

divisible by three [i.e. multiples of $1\frac{1}{2}°$]. Then the only chords remaining to be determined will be those between the $1\frac{1}{2}°$ intervals, two in each interval, since our table is made at $\frac{1}{2}°$ intervals. If, therefore, we can find the chord of $\frac{1}{2}°$, this will enable us to complete [the table with] all the remaining intermediate chords, by finding the sum or difference [of $\frac{1}{2}°$] from the given chords at either end of the [$1\frac{1}{2}°$] intervals. Now, if a chord, e.g. the chord of $1\frac{1}{2}°$, is given, the chord corresponding to an arc which is one-third of the previous one cannot be found by geometrical methods.[63] (If this were possible, we should immediately
H43 have the chord of $\frac{1}{2}°$). Therefore we shall first derive the chord of $1°$ from those of $1\frac{1}{2}°$ and $\frac{3}{4}°$. [We shall do this] by establishing a lemma which, though it cannot in general exactly determine the sizes [of chords], in the case of such very small quantities can determine them with a negligibly small error.

I say, then, that if two unequal chords be given, the ratio of the greater to the lesser is less than the ratio of the arc on the greater to the arc on the lesser.

[See Fig. 1.6] Let there be a circle ABGD, in which there are drawn two unequal chords, the lesser AB and the greater BG.

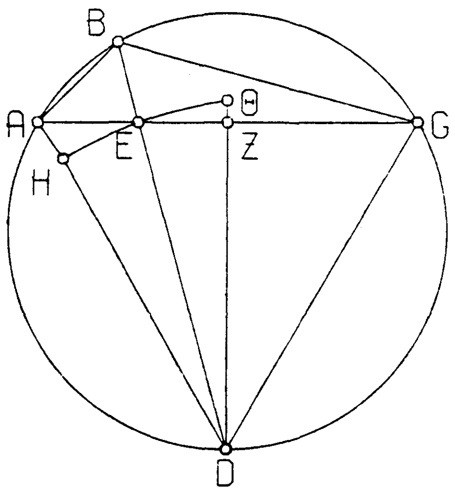

Fig. 1.6

I say that

$$GB:BA < \text{arc } BG: \text{arc } BA.$$

[Proof:] Let ∠ ABG be bisected by [chord] BD. Join AEG, AD and GD. Then, since ∠ ABG is bisected by chord BED,

$$GD = AD$$

H44 and $GE > EA$.[64]

[63] This is true: the problem of finding Crd α from given Crd 3α can be reduced to a cubic equation of the kind which cannot (except for a few particular values of α) be solved by Euclidean geometry (using straight line and circle). See Toomer[3] 138.

[64] Derivable from Euclid VI 3, which states that the bisector of the angle at the apex of a triangle divides the base in the ratio of the two sides enclosing the angle. Here, since $BG > BA$, $GE > EA$.

So drop perpendicular DZ from D on to AEG.

Then, since AD > ED and ED > DZ, a circle drawn on centre D with radius DE will cut AD and pass beyond DZ. Let it be drawn as HEΘ, and let DZ be produced to Θ. Now, since sector DEΘ is greater than triangle DEZ, and triangle DEA is greater than sector DEH,

triangle DEZ: triangle DEA < sector DEΘ: sector DEH.

But triangle DEZ: triangle DEA = EZ:EA,[65]

and sector DEΘ: sector DEH = $\angle$ ZDE:$\angle$ EDA.

$\therefore$ ZE:EA < $\angle$ ZDE:$\angle$ EDA.

So, *componendo*,

ZA:EA < $\angle$ ZDA:$\angle$ ADE.

And, doubling the first members [of the ratios],

GA:AE < $\angle$ GDA:$\angle$ EDA. H45

Then, *dividendo*,

GE:EA < $\angle$ GDE:$\angle$ EDA.

But GE:EA = GB:BA,[66]

and $\angle$ GDB:$\angle$ BDA = arc GB:arc BA.

$\therefore$ GB:BA < arc GB:arc BA.

Having established this, let us draw [Fig. 1.7] circle ABG, and in it two chords, AB and AG. Let us suppose, first, that AB is the chord of ¾° and AG the chord of 1°. Then, since

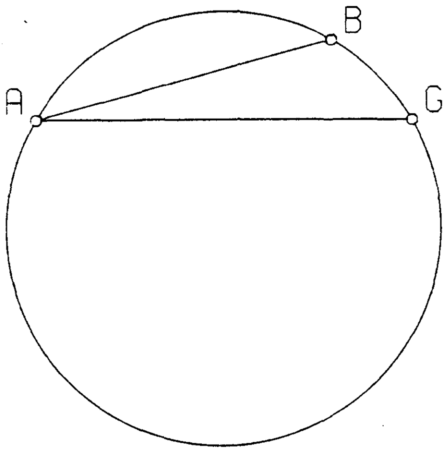

Fig. 1.7

AG:BA < arc AG:arc AB

$$\text{and arc AG} = \frac{4 \text{ arc AB}}{3},$$

$$\text{GA} < \frac{4\text{AB}}{3}.$$

[65] Euclid VI 1.
[66] Euclid VI 3.

But, in units of which the diameter contains 120, we showed that

$$AB = 0;47,8^p.$$

$$\therefore\ GA < 1;2,50^p \text{ (for } 1;2,50 \approx \tfrac{4}{3}.0;47,8).$$

H46 Again, using the same figure, let us set AB as the chord of 1° and AG as the chord of $1\frac{1}{2}$°. By the same argument, since

$$\text{arc } AG = \frac{3 \text{ arc } AB}{2},$$

$$GA < \frac{3BA}{2}.$$

But, in units of which the diameter contains 120, we showed that

$$AG = 1;34,15^p.$$

$$\therefore\ AB > 1;2,50^p \text{ (for } 1;34,15 = \tfrac{3}{2}.1;2,50).$$

Therefore, since the chord of 1° was shown to be both greater and less than the same amount, we can establish it as approximately $1;2,50^p$ where the diameter is 120^p. By the preceding propositions we can also establish the chord of $\frac{1}{2}$°, which we find to be approximately $0;31,25^p$. The remaining intervals can [now] be completed, as we said [p. 54]. For example, in the first [$1\frac{1}{2}$°] interval, we can calculate the chord of 2° by using the addition formula for the chord of $\frac{1}{2}$° applied to the chord of $1\frac{1}{2}$°, while the chord of $2\frac{1}{2}$° is given by using the difference formula for [the chord of $\frac{1}{2}$°] applied to the chord of 3°. Similarly for the remaining chords.

Such, then, I think, is the easiest way to undertake the calculation of the
H47 chords. But, as I said, in order that we may have the actual amounts of the chords readily available for every occasion, we shall set out tables [for that purpose] below. They will be arranged in sections of 45 lines[67] to achieve a symmetrical appearance. The first column [in each section] will contain the arcs tabulated at intervals of $\frac{1}{2}$°, the second the corresponding chords in units of which the diameter contains 120, and the third the thirtieth part of the increment in the chord for each interval. [This last] is so that we may have the average increment corresponding to one minute [of arc], which will not be sensibly different from the true increment [for each minute]. Thus we can easily calculate the amount of the chord corresponding to fractions which fall between the [tabulated] half-degree intervals.

It is easy to see that, if we suspect some scribal corruption in one of the values for the chord in the table, the same theorems which we have already set out will enable us to test and correct it easily, either by taking the chord of double the arc [of that] of the chord in question, or from the difference with some other given chord, or from the chord of the supplement.

The layout of the table is as follows.

H48—63

11. {*Table of Chords*}[68]

[See pp. 57-60.]

[67] 45 lines is the standard height of tables throughout the *Almagest*. It is presumably chosen to conform to some standard height of papyrus roll (on papyrus standards see Lewis, *Papyrus in Classical Antiquity*, 36-9, 56, on Pliny *NH* 13, 78). Various consequences flow from it, notably the 18-year interval in mean motion tables (see III 1 p. 140 with n. 28).

TABLE OF CHORDS

Arcs	Chords	Sixtieths	Arcs	Chords	Sixtieths
½	0 31 25	1 2 50	23	23 55 27	1 1 33
1	1 2 50	1 2 50	23½	24 26 13	1 1 30
1½	1 34 15	1 2 50	24	24 56 58	1 1 26
2	2 5 40	1 2 50	24½	25 27 41	1 1 22
2½	2 37 4	1 2 48	25	25 58 22	1 1 19
3	3 8 28	1 2 48	25½	26 29 1	1 1 15
3½	3 39 52	1 2 48	26	26 59 38	1 1 11
4	4 11 16	1 2 47	26½	27 30 14	1 1 8
4½	4 42 40	1 2 47	27	28 0 48	1 1 4
5	5 14 4	1 2 46	27½	28 31 20	1 1 0
5½	5 45 27	1 2 45	28	29 1 50	1 0 56
6	6 16 49	1 2 44	28½	29 32 18	1 0 52
6½	6 48 11	1 2 43	29	30 2 44	1 0 48
7	7 19 33	1 2 42	29½	30 33 8	1 0 44
7½	7 50 54	1 2 41	30	31 3 30	1 0 40
8	8 22 15	1 2 40	30½	31 33 50	1 0 35
8½	8 53 35	1 2 39	31	32 4 8	1 0 31
9	9 24 54	1 2 38	31½	32 34 22	1 0 27
9½	9 56 13	1 2 37	32	33 4 35	1 0 22
10	10 27 32	1 2 35	32½	33 34 46	1 0 17
10½	10 58 49	1 2 33	33	34 4 55	1 0 12
11	11 30 5	1 2 32	33½	34 35 1	1 0 8
11½	12 1 21	1 2 30	34	35 5 5	1 0 3
12	12 32 36	1 2 28	34½	35 35 6	0 59 57
12½	13 3 50	1 2 27	35	36 5 5	0 59 52
13	13 35 4	1 2 25	35½	36 35 1	0 59 48
13½	14 6 16	1 2 23	36	37 4 55	0 59 43
14	14 37 27	1 2 21	36½	37 34 47	0 59 38
14½	15 8 38	1 2 19	37	38 4 36	0 59 32
15	15 39 47	1 2 17	37½	38 34 22	0 59 27
15½	16 10 56	1 2 15	38	39 4 5	0 59 22
16	16 42 3	1 2 13	38½	39 33 46	0 59 16
16½	17 13 9	1 2 10	39	40 3 25	0 59 11
17	17 44 14	1 2 7	39½	40 33 0	0 59 5
17½	18 15 17	1 2 5	40	41 2 33	0 59 0
18	18 46 19	1 2 2	40½	41 32 3	0 58 54
18½	19 17 21	1 2 0	41	42 1 30	0 58 48
19	19 48 21	1 1 57	41½	42 30 54	0 58 42
19½	20 19 19	1 1 54	42	43 0 15	0 58 36
20	20 50 16	1 1 51	42½	43 29 33	0 58 31
20½	21 21 11	1 1 48	43	43 58 49	0 58 25
21	21 52 6	1 1 45	43½	44 28 1	0 58 18
21½	22 22 58	1 1 42	44	44 57 10	0 58 12
22	22 53 49	1 1 39	44½	45 26 16	0 58 6
22½	23 24 39	1 1 36	45	45 55 19	0 58 0

[68] Ptolemy's chord table has been recomputed, using a computer program which reproduces, as far as possible, Ptolemy's own methods of calculation, by Glowatzki and Göttsche. Although much of their book is superfluous (see my review, Toomer[4]), it contains some interesting results, notably that Ptolemy must have carried out his calculations to five sexagesimal places to achieve the

Arcs	Chords	Sixtieths	Arcs	Chords	Sixtieths
45½	46 24 19	0 57 54	68	67 6 12	0 52 1
46	46 53 16	0 57 47	68½	67 32 12	0 51 52
46½	47 22 9	0 57 41	69	67 58 8	0 51 43
47	47 51 0	0 57 34	69½	68 23 59	0 51 33
47½	48 19 47	0 57 27	70	68 49 45	0 51 23
48	48 48 30	0 57 21	70½	69 15 27	0 51 14
48½	49 17 11	0 57 14	71	69 41 4	0 51 4
49	49 45 48	0 57 7	71½	70 6 36	0 50 55
49½	50 14 21	0 57 0	72	70 32 3	0 50 45
50	50 42 51	0 56 53	72½	70 57 26	0 50 35
50½	51 11 18	0 56 46	73	71 22 44	0 50 26
51	51 39 42	0 56 39	73½	71 47 56	0 50 16
51½	52 8 0	0 56 32	74	72 13 4	0 50 6
52	52 36 16	0 56 25	74½	72 38 7	0 49 56
52½	53 4 29	0 56 18	75	73 3 5	0 49 46
53	53 32 38	0 56 10	75½	73 27 58	0 49 36
53½	54 0 43	0 56 3	76	73 52 46	0 49 26
54	54 28 44	0 55 55	76½	74 17 29	0 49 16
54½	54 56 42	0 55 48	77	74 42 7	0 49 6
55	55 24 36	0 55 40	77½	75 6 39	0 48 55
55½	55 52 26	0 55 33	78	75 31 7	0 48 45
56	56 20 12	0 55 25	78½	75 55 29	0 48 34
56½	56 47 54	0 55 17	79	76 19 46	0 48 24
57	57 15 33	0 55 9	79½	76 43 58	0 48 13
57½	57 43 7	0 55 1	80	77 8 5	0 48 3
58	58 10 38	0 54 53	80½	77 32 6	0 47 52
58½	58 38 5	0 54 45	81	77 56 2	0 47 41
59	59 5 27	0 54 37	81½	78 19 52	0 47 31
59½	59 32 45	0 54 29	82	78 43 38	0 47 20
60	60 0 0	0 54 21	82½	79 7 18	0 47 9
60½	60 27 11	0 54 12	83	79 30 52	0 46 58
61	60 54 17	0 54 4	83½	79 54 21	0 46 47
61½	61 21 19	0 53 56	84	80 17 45	0 46 36
62	61 48 17	0 53 47	84½	80 41 3	0 46 25
62½	62 15 10	0 53 39	85	81 4 15	0 46 14
63	62 42 0	0 53 30	85½	81 27 22	0 46 3
63½	63 8 45	0 53 22	86	81 50 24	0 45 52
64	63 35 25	0 53 13	86½	82 13 19	0 45 40
64½	64 2 2	0 53 4	87	82 36 9	0 45 29
65	64 28 34	0 52 55	87½	82 58 54	0 45 18
65½	64 55 1	0 52 46	88	83 21 33	0 45 6
66	65 21 24	0 52 37	88½	83 44 4	0 44 55
66½	65 47 43	0 52 28	89	84 6 32	0 44 43
67	66 13 57	0 52 19	89½	84 28 54	0 44 31
67½	66 40 7	0 52 10	90	84 51 10	0 44 20

accuracy he does in the third place. The book also enables one to make a number of corrections of scribal errors in the table. Before seeing it I had already made those given below. None of the other corrections (all of 1 in the last place) suggested by the authors seem likely to me, although some are possible.

Corrections to Heiberg's text:

Crd 9°, seconds, νδ (with D, Ar) for να (51) at H48,20 (corrected by Hultsch, *Sehnentafeln* 52)

Arcs	Chords	Sixtieths	Arcs	Chords	Sixtieths
90½	85 13 20	0 44 8	113	100 3 59	0 34 34
91	85 35 24	0 43 57	113½	100 21 16	0 34 20
91½	85 57 23	0 43 45	114	100 38 26	0 34 6
92	86 19 15	0 43 33	114½	100 55 28	0 33 52
92½	86 41 2	0 43 21	115	101 12 25	0 33 39
93	87 2 42	0 43 9	115½	101 29 15	0 33 25
93½	87 24 17	0 42 57	116	101 45 57	0 33 11
94	87 45 45	0 42 45	116½	102 2 33	0 32 57
94½	88 7 7	0 42 33	117	102 19 1	0 32 43
95	88 28 24	0 42 21	117½	102 35 22	0 32 29
95½	88 49 34	0 42 9	118	102 51 37	0 32 15
96	89 10 39	0 41 57	118½	103 7 44	0 32 0
96½	89 31 37	0 41 45	119	103 23 44	0 31 46
97	89 52 29	0 41 33	119½	103 39 37	0 31 32
97½	90 13 15	0 41 21	120	103 55 23	0 31 18
98	90 33 55	0 41 8	120½	104 11 2	0 31 4
98½	90 54 29	0 40 55	121	104 26 34	0 30 49
99	91 14 56	0 40 42	121½	104 41 59	0 30 35
99½	91 35 17	0 40 30	122	104 57 16	0 30 21
100	91 55 32	0 40 17	122½	105 12 26	0 30 7
100½	92 15 40	0 40 4	123	105 27 30	0 29 52
101	92 35 42	0 39 52	123½	105 42 26	0 29 37
101½	92 55 38	0 39 39	124	105 57 14	0 29 23
102	93 15 27	0 39 26	124½	106 11 55	0 29 8
102½	93 35 11	0 39 13	125	106 26 29	0 28 54
103	93 54 47	0 39 0	125½	106 40 56	0 28 39
103½	94 14 17	0 38 47	126	106 55 15	0 28 24
104	94 33 41	0 38 34	126½	107 9 27	0 28 10
104½	94 52 58	0 38 21	127	107 23 32	0 27 56
105	95 12 9	0 38 8	127½	107 37 30	0 27 40
105½	95 31 13	0 37 55	128	107 51 20	0 27 25
106	95 50 11	0 37 42	128½	108 5 2	0 27 10
106½	96 9 2	0 37 29	129	108 18 37	0 26 56
107	96 27 47	0 37 16	129½	108 32 5	0 26 41
107½	96 46 24	0 37 3	130	108 45 25	0 26 26
108	97 4 55	0 36 50	130½	108 58 38	0 26 11
108½	97 23 20	0 36 36	131	109 11 44	0 25 56
109	97 41 38	0 36 23	131½	109 24 42	0 25 41
109½	97 59 49	0 36 9	132	109 37 32	0 25 26
110	98 17 54	0 35 56	132½	109 50 15	0 25 11
110½	98 35 52	0 35 42	133	110 2 50	0 24 56
111	98 53 43	0 35 29	133½	110 15 18	0 24 41
111½	99 11 27	0 35 15	134	110 27 39	0 24 26
112	99 29 5	0 35 1	134½	110 39 52	0 24 10
112½	99 46 35	0 34 48	135	110 51 57	0 23 55

Crd 72°, seconds, γ (with all mss. except D) for δ (4) at H54,10 (cf. H35,1 and p. 81 n. 19; corrected by Manitius)
Crd 88½°, minutes, μδ (with Ar) for μα (41) at H55,43.
Crd 97°, seconds, κθ (with D, Ar) for κζ (27) at H56,15
Crd 108°, seconds, νε (with D, Ar) for νς (56) at H57,37
Crd 118½°, seconds, μδ (with Ar) for μα (41) at H58,13
Crd 143°, seconds, νς (with D, Ar) for κς (26) at H60,17.

Arcs	Chords	Sixtieths	Arcs	Chords	Sixtieths
135½	111 3 54	0 23 40	158	117 47 43	0 11 51
136	111 15 44	0 23 25	158½	117 53 39	0 11 35
136½	111 27 26	0 23 9	159	117 59 27	0 11 19
137	111 39 1	0 22 54	159½	118 5 7	0 11 3
137½	111 50 28	0 22 39	160	118 10 37	0 10 47
138	112 1 47	0 22 24	160½	118 16 1	0 10 31
138½	112 12 59	0 22 8	161	118 21 16	0 10 14
139	112 24 3	0 21 53	161½	118 26 23	0 9 58
139½	112 35 0	0 21 37	162	118 31 22	0 9 42
140	112 45 48	0 21 22	162½	118 36 13	0 9 25
140½	112 56 29	0 21 7	163	118 40 55	0 9 9
141	113 7 2	0 20 51	163½	118 45 30	0 8 53
141½	113 17 25	0 20 36	164	118 49 56	0 8 37
142	113 27 44	0 20 20	164½	118 54 15	0 8 20
142½	113 37 54	0 20 4	165	118 58 25	0 8 4
143	113 47 56	0 19 49	165½	119 2 26	0 7 48
143½	113 57 50	0 19 33	166	119 6 20	0 7 31
144	114 7 37	0 19 17	166½	119 10 6	0 7 15
144½	114 17 15	0 19 2	167	119 13 44	0 6 59
145	114 26 46	0 18 46	167½	119 17 13	0 6 42
145½	114 36 9	0 18 30	168	119 20 34	0 6 26
146	114 45 24	0 18 14	168½	119 23 47	0 6 10
146½	114 54 31	0 17 59	169	119 26 52	0 5 53
147	115 3 30	0 17 43	169½	119 29 49	0 5 37
147½	115 12 22	0 17 27	170	119 32 37	0 5 20
148	115 21 6	0 17 11	170½	119 35 17	0 5 4
148½	115 29 41	0 16 55	171	119 37 49	0 4 48
149	115 38 9	0 16 40	171½	119 40 13	0 4 31
149½	115 46 29	0 16 24	172	119 42 28	0 4 14
150	115 54 40	0 16 8	172½	119 44 35	0 3 58
150½	116 2 44	0 15 52	173	119 46 35	0 3 42
151	116 10 40	0 15 36	173½	119 48 26	0 3 26
151½	116 18 28	0 15 20	174	119 50 8	0 3 9
152	116 26 8	0 15 4	174½	119 51 43	0 2 53
152½	116 33 40	0 14 48	175	119 53 10	0 2 36
153	116 41 4	0 14 32	175½	119 54 27	0 2 20
153½	116 48 20	0 14 16	176	119 55 38	0 2 3
154	116 55 28	0 14 0	176½	119 56 39	0 1 47
154½	117 2 28	0 13 44	177	119 57 32	0 1 30
155	117 9 20	0 13 28	177½	119 58 18	0 1 14
155½	117 16 4	0 13 12	178	119 58 55	0 0 57
156	117 22 40	0 12 56	178½	119 59 24	0 0 41
156½	117 29 8	0 12 40	179	119 59 44	0 0 25
157	117 35 28	0 12 24	179½	119 59 56	0 0 9
157½	117 41 40	0 12 7	180	120 0 0	0 0 0

12. {*On the arc between the solstices*}[69] H64

Now that we have tabulated the chords, our first task, as we said, is to determine the inclination of the ecliptic to the equator, that is, the ratio of the great circle through the poles of both to the arc intercepted between the poles. It is obvious that this is equal to the distance from the equator to either of the solsticial points. This quantity can be determined directly by an instrumental method, using the following simple apparatus.[70] [See Fig. C.]

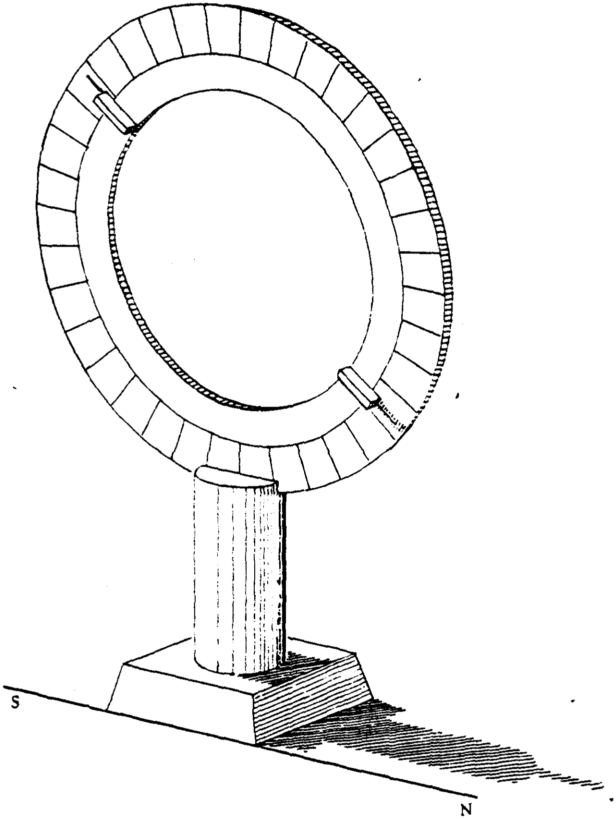

Fig. C

We make a bronze ring of a suitable size, turned on the lathe so that its surface is accurately squared off [i.e. has a rectangular cross-section]. We use this as a meridian circle, by dividing it into the normal 360° of a great circle, and subdividing each degree into as many parts as [the size of the instrument] allows. Then we take another smaller ring, and fit it inside the first in such a

[69] On Ptolemy's determination of the obliquity of the ecliptic see Britton[2].

[70] On the instruments described by Ptolemy here see Price, *Precision Instruments*, 587–9. There is a very detailed ancient description of the construction and use of the ring instrument by Proclus, *Hypotyposis* III 5–27 (ed. Manitius pp. 42–52).

manner that the lateral faces of both are in the same plane, while the smaller ring can rotate freely inside the larger, with a north-south motion, [always] in the same plane. At two diametrically opposite points on one lateral face of the
H65 smaller ring we fix [two] little plates, of equal size, pointing towards each other and the centre of the rings, and exactly in the middle of the width of each plate we fix small pointers, which graze the surface of the larger, graduated ring. To serve all the necessary purposes we fix this ring firmly on a pillar of appropriate size, and set it up in the open air, so that the base of the pillar is on a foundation which is not inclined to the plane of the horizon. We take care that the [lateral] plane of the rings is perpendicular to the plane of the horizon and parallel to the plane of the meridian. The first of these [desiderata] is achieved by suspending a plumb-line from a point [on the outer ring] chosen as zenith, and adjusting supporting elements[71] until the plumb-line points towards the point diametrically opposite [the zenith-point]. The second is achieved by marking a meridian line[72] clearly in the plane below the pillar and moving the rings laterally until one can sight their [lateral] plane as parallel to that line. Having set the instrument up in that way, we observed the sun's movement towards the north and south by turning the inner ring at noon until
H66 the lower plate was completely enshadowed by the upper one. When this was the case, the tips of the pointers indicated to us the distance of the sun from the zenith in degrees,[73] measured along the meridian.

We found an even handier way of making this kind of observation by constructing, instead of the rings, a plaque [see Fig. D] of stone or wood, square

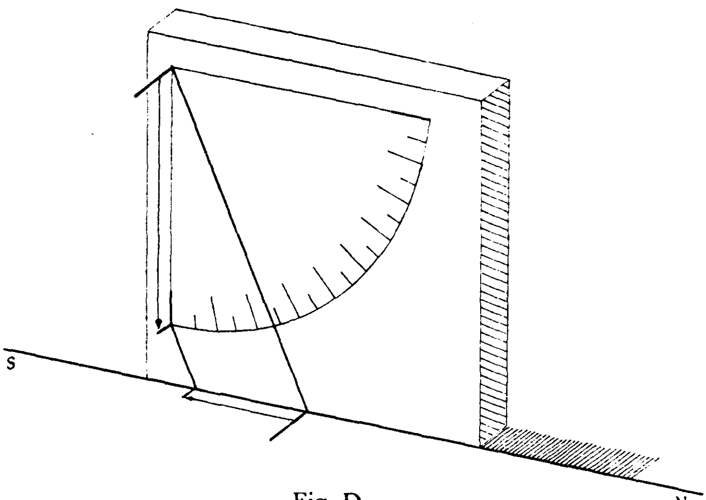

Fig. D

[71] Reading ὑποθεματίων (with D) for ὑποθεμάτων at H65,13. Cf. H67,7. Both readings are found in mss. of Proclus, *Hypotyposis* p. 50,10.

[72] Ptolemy assumes that one can draw a meridian line, without explaining how. Diodorus of Alexandria (first century B.C.) in his (lost) treatise *Analemma*, gave an ingenious method for determining the meridian line from any three gnomon shadows (see *HAMA* II 841–2).

[73] τμήματα, literally 'divisions', and it could be so interpreted here ('divisions of the graduated arc'), cf. p. 61. But there are many places in the Almagest where it means simply 'degrees'.

and rigid, with one of its faces smooth and accurately squared off. On this we drew a quadrant, using as centre a point near one of the corners, and drew from the centre to the inscribed arc the lines enclosing the right angle forming the quadrant. We divided the arc, as we had [the other instrument], into 90 degrees and subdivisions of those degrees. Next, on that line which was chosen to be perpendicular to the plane of the horizon and towards the south, we fixed two small cylindrical pegs, with their sides at right angles to their bases and exactly circular, machined to be of equal size: one of them we fixed on the centre-point itself, positioning the mid-point of the peg precisely on it, the other at the lower end of the line. Then we set this inscribed face of the plaque up along the H67
meridian line which we had drawn on the foundation-plane, so as to be parallel to the plane of the meridian, and, using a plumb-line suspended between the pegs, set up the line between them precisely at right angles to the plane of the horizon, again correcting any deficiency by adjusting thin supporting elements underneath. In the same way as before, we observed the shadow cast at midday by the peg at the centre. In order to determine its position more accurately, we placed some object on the inscribed arc [where the shadow crossed it]. Marking the mid-point of the shadow, we took that division of the quadrant as indicating the position of the sun on the meridian in the north-south direction.[74]

From observations of this kind, and especially from comparing observations near the actual solstices, which revealed that, over a number of returns [of the sun], the distance from the zenith was in general the same number of degrees of the meridian circle at the [same] solstice, whether summer or winter, we found that the arc between the northernmost and southernmost points, which is the arc between the solstitial points, is always greater than $47\frac{2}{3}°$ and less than $47\frac{3}{4}°$. H68
From this we derive very much the same ratio as Eratosthenes, which Hipparchus also used. For [according to this] the arc between the solstices is approximately 11 parts where the meridian is 83.[75]

From the preceding kind of observation it is easy to derive immediately the latitude of the region in which the observation is made, wherever it is: one takes the point halfway between the two extrema; this point lies on the equator; then one takes the distance between this point and the zenith, which is the same, obviously, as the distance of the poles from the horizon.

[74] κατὰ πλάτος, literally 'in latitude'. Ptolemy, following common Greek usage, uses πλάτος for any 'vertical' direction, including that normal to the equator, as here. See Introduction p. 21.

[75] $\frac{11}{83}$ of $360° \approx 47;42,39,2° = 2\varepsilon$, hence $\varepsilon \approx 23;51,20°$, which is what Ptolemy actually adopts (his 2ε lies between 47;40° and 47;45°, but is not the mean).

The text could equally well mean, not that Eratosthenes and Hipparchus used the ratio 11:83, but that the ratio 11:83 is Ptolemy's value, which is close to the actual ratio used by them [namely 2:15, i.e. $\varepsilon = 24°$]. That interpretation has the advantage of agreeing with the only value otherwise attested for Eratosthenes (in his *Geography*, see Berger Frg. II B 23, Strabo 2.5.7) and Hipparchus (in his *Geography* and in his *Commentary on Aratus*, ed. Manitius p. 96,20; cf. *HAMA* 303, 335). It was proposed by Berger, *Eratosthenes* 131, followed by Heath, *Aristarchus* 131 n. 4. I prefer the traditional interpretation, since I find it inconceivable that Ptolemy would not mention what the ratio was to which his own was close, and also because of his expression at I 14 (p. 70). Eratosthenes' peculiar ratio is due not to a perverse division of the circle into 83rds, as Theon supposes (Rome II 529), but to a pre-trigonometrical derivation from gnomon measurements, as I shall show elsewhere.

13. {*Preliminaries for spherical proofs*}[76]

Our next task is to demonstrate the sizes of the individual arcs cut off between the equator and the ecliptic along a great circle through the poles of the equator. As a preliminary we shall set out some short and useful theorems which will enable us to carry out most demonstrations involving spherical theorems in the simplest and most methodical way possible.

H69 [See Fig. 1.8.] Let two straight lines, BE and GD, which are drawn to meet two straight lines. AB and AG, cut each other at point Z.

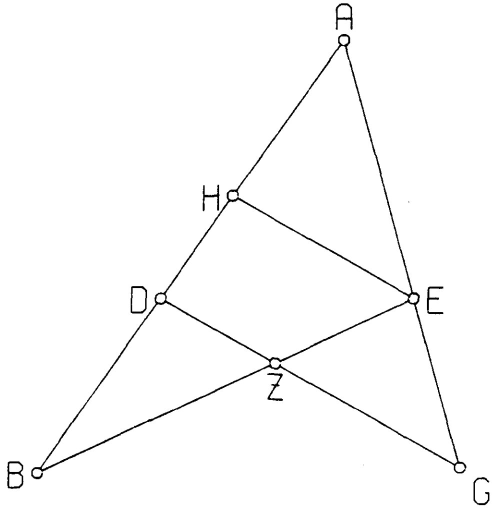

Fig. 1.8

I say that

$$GA:AE = (GD:DZ).(ZB:BE).$$[77]

[Proof:] Let EH be drawn through E parallel to GD.
Then, since GD and EH are parallel,

$$GA:AE = GD:EH.$$

If we bring ZD in [as auxiliary],

$$GD:EH = (GD:DZ).(DZ:HE).$$

$$\therefore\ GA:AE = (GD:DZ).(DZ:HE).$$

But DZ:HE = ZB:BE (EH parallel to ZD).

$$\therefore\ GA:AE = (GD:DZ).(ZB:BE). \qquad [13.1]$$

Q.E.D.

In the same way, *dividendo*, we shall prove that

$$GE:EA = (GZ:DZ).(DB:BA).$$

[76] On the spherical trigonometry in this chapter see *HAMA* 26–30, Pedersen 72–8.
[77] Literally (here and in general) this kind of ratio is expressed as 'the ratio of GA to AE is combined from (συνῆπται ἐκ, συγκεῖται ἐκ) the ratio of GD to DZ and the ratio of ZB to BE'.

[See Fig. 1.9.] Draw a line through A parallel to EB and produce GD to cut it at H. Again, since AH is parallel to EZ, H70

GE:EA = GZ:ZH.

But, if we bring in ZD [as auxiliary],

GZ:ZH = (GZ:ZD).(DZ:ZH).

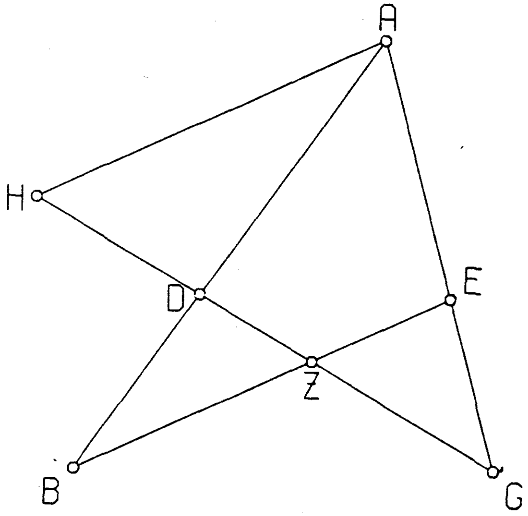

Fig. 1.9

But DZ:ZH = DB:BA (BA and ZH drawn to meet the parallel lines AH and ZB).

∴ GZ:ZH = (GZ:DZ).(DB:BA).

But GZ:ZH = GE:EA.

∴ GE:EA = (GZ:DZ).(DB:BA). [13.2]

Q.E.D.

Again [Fig. 1.10] on circle ABG, with centre D, take any three points A,B,G, on the circumference, provided that each of the arcs AB and BG is less than a semi-circle (let the same condition be understood to apply to all subsequent arcs we take). Draw AG and DEB.

I say that H71

Crd arc 2AB:Crd arc 2BG = AE:EG.

[Proof:] Drop perpendiculars AZ and GH from points A and G on to DB. Then, since AZ is parallel to GH, and they meet line AEG,

AZ:GH = AE:EG.

But AZ:GH = Crd arc 2AB : Crd arc 2BG

(for AZ = $\frac{1}{2}$ Crd arc 2AB and GH = $\frac{1}{2}$ Crd arc 2BG).

∴ AE:EG = Crd arc 2AB:Crd arc 2BG. [13.3]

Q.E.D.

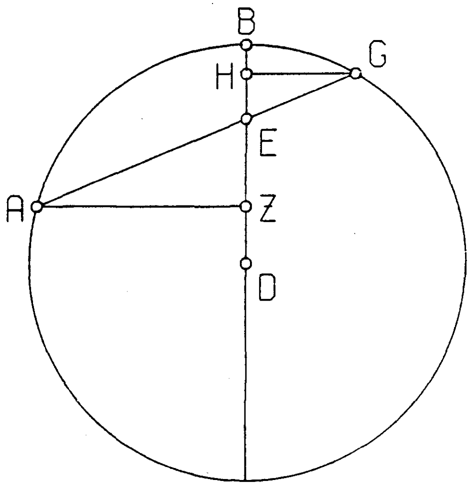

Fig. 1.10

It immediately follows that if we are given the whole of arc AG and the ratio (Crd arc 2AB:Crd arc 2BG), both arc AB and arc BG will be given.

For, repeating the same figure [see Fig. 1.11], join AD, and drop perpendicular DZ from D on to AEG.

H72 It is obvious that, if arc AG be given, ∠ ADZ, which subtends half arc AG, will be given, and hence the whole triangle ADZ.[78] Now, since the whole chord AG is

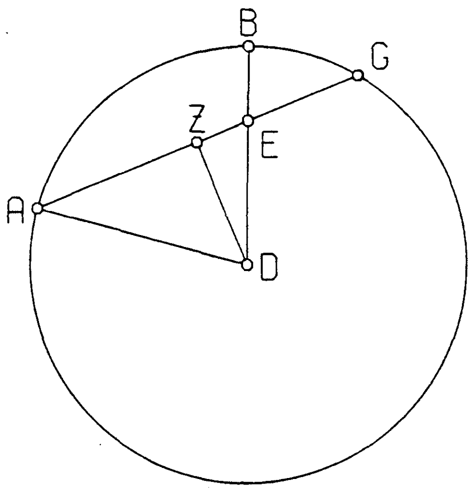

Fig. 1.11

[78] For one already knows ∠ AZD, a right angle, and AD, a radius.

given, and (AE:EG) is given (for it equals (Crd arc 2AB:Crd arc 2BG)), AE will be given,[79] and so will ZE, by subtraction [of AZ from AE]. Hence, since DZ too is given, in the right-angled triangle EDZ, ∠ EDZ will be given, and hence the whole angle ADB. Hence arc AB will be given and (by subtraction) arc BG.

Q.E.D.

Again [see Fig. 1.12] on circle ABG with centre D take three points on the circumference, A,B,G.[80] Join DA and GB and produce them to meet at E.

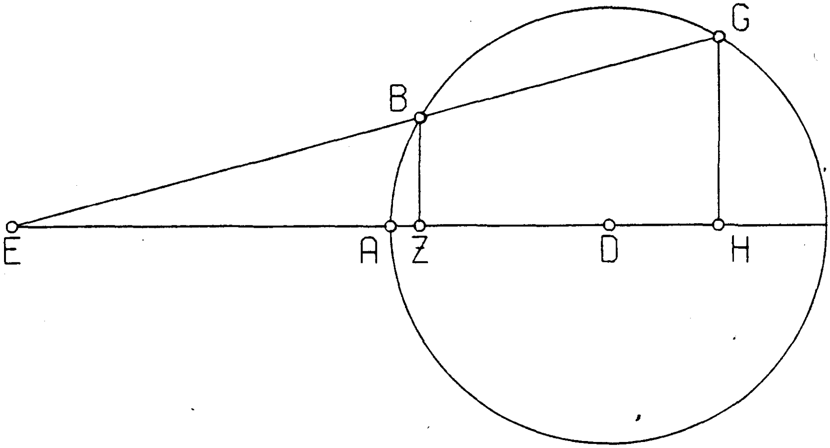

Fig. 1.12

I say that H73

Crd arc 2GA:Crd arc 2AB = GE:BE.

By a similar argument to the previous theorem, if we drop perpendiculars BZ and GH from B and G on to DA, since they are parallel,

GH:BZ = GE:EB.

∴ Crd arc 2GA:Crd arc 2AB = GE:EB. [13.4]

Q.E.D.

In this case too it follows immediately that if we are given just the arc GB and the ratio (Crd arc 2GA:Crd arc 2AB), arc AB will also be given.

For, if we repeat the same figure [see Fig. 1.13], and join DB and drop DZ perpendicular to BG, then ∠BDZ, which subtends half arc BG, will be given. H74

Hence the whole of the right-angled triangle[81] BDZ will be given. Now, since the ratio (GE:EB) and line GB are given, EB will be given, and hence, by addition, line EBZ. So, since DZ is given, in the right-angled triangle EDZ,

[79] Euclid *Data* 7 (if a given magnitude is divided in a given ratio, each part is given).

[80] Omitting (with D, Is), at H72, 13–15, ὥστε ἑκατέραν τῶν AB, AΓ περιφερειῶν ἐλάσσονα εἶναι ἡμικυκλίου. καὶ ἐπὶ τῶν ἑξῆς δὲ λαμβανομένων περιφερειῶν το ὅμοιον ὑπακουέσθω, which is an otiose repetition of H70, 21–5.

[81] Here (H74,3) and elsewhere (e.g. H74,7) D has the fuller form ὀρθογώνιον τρίγωνον for Heiberg's ὀρθογώνιον. This may be right, but I have not recorded it as a correction, following the principle enunciated *Introduction* p. 4.

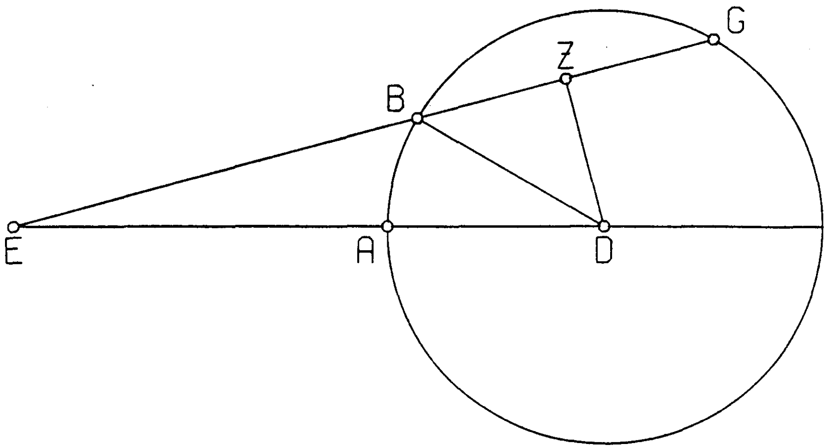

Fig. 1.13

∠ EDZ is given, and, by subtraction [of the given ∠ BDZ] ∠ EDB is given. Hence arc AB will be given.

Having established these preliminary theorems, let us draw [Fig. 1.14][82] the following arcs of great circles on a sphere: BE and GD are drawn to meet AB and AG, and cut each other at Z. Let each of them be less than a semi-circle (and let the same condition be understood to apply to all the figures).
I say that

Crd arc 2GE:Crd arc 2EA =

(Crd arc 2GZ:Crd arc 2ZD). (Crd arc 2DB:Crd arc 2BA).

[Proof:] Let us take the centre of the sphere, H, and draw from it to the intersections of the circles, B, Z, E, lines HB, HZ, HE. Join AD and produce it to
H75 meet HB, also produced, at Θ. Similarly, join DG and AG, and let them cut HZ and HE at points[83] K and L.

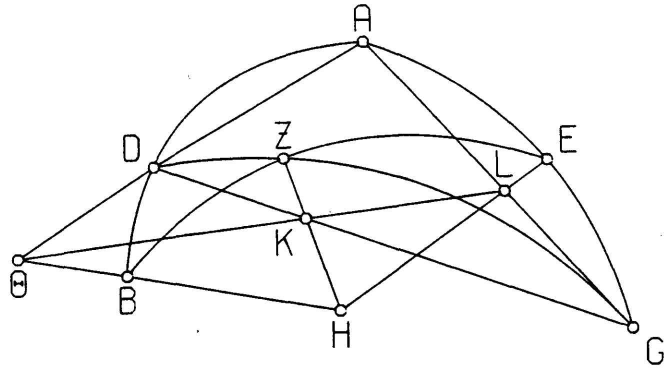

Fig. 1.14

[82] For an adaptation of this figure useful in visualizing the various planes involved see *HAMA* Fig. 17 p. 1213.
[83] Reading τὰ . . . σημεῖα (with D) at H75,2 for τὸ . . . σημεῖον. Corrected by Manitius.

Then Θ, K and L lie on a straight line, since they all lie simultaneously in two planes, the plane of triangle AGD, and the plane of circle BZE.

Draw this line [ΘKL]. The result will be that there are two straight lines, ΘL and GD, drawn to meet two straight lines, ΘA and GA, and intersecting each other at K.

∴ GL:LA = (GK:KD).(DΘ:ΘA). [from 13.2]
But GL:LA = Crd arc 2GE:Crd arc 2EA [from 13.3]
and GK:KD = Crd arc 2GZ:Crd arc 2ZD [from 13.3]
and DΘ:ΘA = Crd arc 2DB:Crd arc 2BA. [from 13.4]

∴ Crd arc 2GE:Crd arc 2EA =
(Crd arc 2GZ:Crd arc 2ZD).(Crd arc 2DB:Crd arc 2BA). [13.5] H76

In the same way, corresponding to the straight lines in the plane figure [Fig. 1.8], it can be shown that

Crd arc 2GA:Crd arc 2EA =
(Crd arc 2GD:Crd arc 2DZ).(Crd arc 2ZB:Crd arc 2BE).[84] [13.6]

Q.E.D.

14. {*On the arcs between the equator and the ecliptic*}[85]

Having set out this preliminary theorem, we shall first of all demonstrate the amounts of the arcs we set ourselves to determine,[86] as follows.

[See Fig. 1.15.] Let the circle through both poles, that of the equator and that of the ecliptic, be ABGD; let the semi-circle representing the equator be AEG, and that representing the ecliptic BED, and let point E be the intersection of the two at the spring equinox, so that B is the winter solstice and D the summer solstice. On arc ABG take the pole of the equator AEG: let it be point Z. Cut off H77 arc EH on the ecliptic: let us suppose it to be 30°, and draw through Z and H an arc of a great circle ZHΘ. Our problem, obviously, is to determine HΘ. Let us take for granted both here and in general for all such demonstrations (to avoid repeating ourselves on each occasion), that when we speak of the sizes of arcs or chords in terms of 'degrees' or 'parts' we mean (for arcs) those degrees of which the circumference of a great circle contains 360, and (for chords) those parts of which the diameter of the circle contains 120.

Now since, in the figure, the two great circle arcs ZΘ and EB are drawn to meet the two great circle arcs AZ and AE, and intersect each other at H,

Crd arc 2ZA:Crd arc 2AB =
(Crd arc 2ΘZ:Crd arc 2ΘH). (Crd arc 2HE:Crd arc 2EB). [M.T.I]

[84] The theorem connecting six great circle arcs on the surface of the sphere in a Menelaus Configuration (see Introduction p. 18), of which the enunciations 13.5 and 13.6 are examples, is due to Menelaus, whom Ptolemy mentions in the Almagest only as an observer (see index s.v.). It appears (in both forms) as Prop. III 1 of his *Sphaerica* (ed. Krause pp. 194–7). These two forms have been labelled by Neugebauer (*HAMA* 28) as Theorem I (= 13.6), where four inner parts of the Menelaus Configuration are related to two outer parts, and Theorem II (= 13.5), where four outer parts are related to two inner parts. We shall use this terminology in what follows (M.T. I and M.T. II for brevity).

[85] See *HAMA* 30–1, Pedersen 95–6.

[86] Reference back to I 13 p. 64.

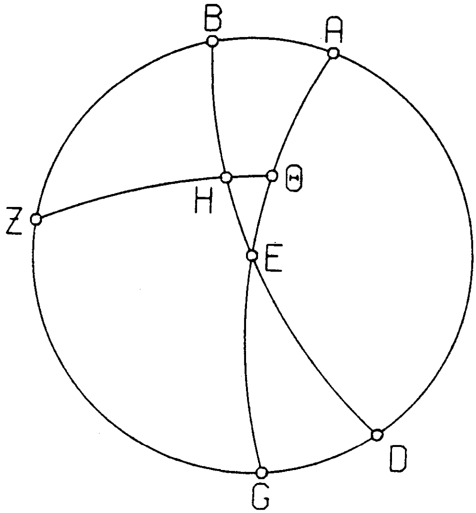

Fig. 1.15

But arc 2ZA = 180°, so Crd arc 2ZA = 120^p,
and arc 2AB = 47;42,40° (according to the ratio 11:83, with which we agreed [p. 63]).
so Crd arc 2AB = $48;31,55^p$.

H78 Again, arc 2HE = 60°, so Crd arc 2HE = 60^p,
and arc 2EB = 180°, so Crd arc 2EB = 120^p.
∴ Crd arc 2ZΘ:Crd arc 2ΘH = (120 : 48;31,55)/(60 : 120)
= 120 : 24;15,57.
And arc 2ZΘ = 180°, so Crd arc 2ZΘ = 120^p.
∴ Crd arc 2ΘH = $24;15,57^p$.
∴ arc 2ΘH = 23;19,59°.
and arc ΘH ≈ 11;40°.

Again, let arc EH be taken as 60°. Then the other magnitudes will remain unchanged, but

arc 2EH = 120°, so Crd arc 2EH = $103;55,23^p$.
∴ Crd arc 2ZΘ:Crd arc 2ΘH = (120 : 48;31,55)/(103;55,23 : 120)
= 120 : 42;1,48.
But Crd arc 2ZΘ = 120^p.
∴ Crd arc 2ΘH = $42;1,48^p$.
∴ arc 2ΘH = 41;0,18°,
and arc ΘH = 20;30,9°.

Q.E.D.

H79 In the same way we shall compute the sizes of [the other] individual arcs, and set out a table giving for each degree of the quadrant the arc corresponding to those computed above. The table is as follows.

15. {*Table of Inclination*}[87] H80—6

[See p. 72.]

16. {*On rising-times at* sphaera recta}[88] H82

Our next task is to show how to compute the size of an arc of the equator determined by a circle drawn through the poles of the equator and a given point on the ecliptic. In this way we can find how long, in equinoctial time-degrees, it takes a given section of the ecliptic to cross the meridian at any point on earth and the horizon at *sphaera recta* (for only in that situation does the horizon pass through the poles of the equator).

Repeat the previous figure [see Fig. 1.16]. Let the ecliptic arc EH again be given, first as 30°. We have to find arc EΘ of the equator.

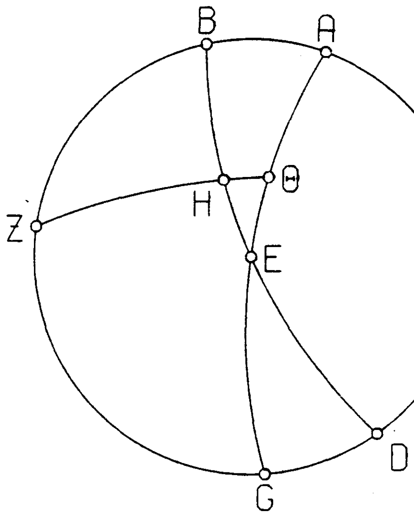

Fig. 1.16

By the same argument as the preceding,

Crd arc 2ZB:Crd arc 2BA =

(Crd arc 2ZH:Crd arc 2HΘ). (Crd arc 2ΘE:Crd arc 2EA). [M.T.II]

But arc 2ZB = 132;17,20°,

so Crd arc 2ZB = 109;44,53^p.

[87] Corrections to Heiberg in Table I 15:
45°, seconds, α (with D, Ar) for κ (20) at H81,50 (computed: 2)
69°, seconds, α (with D, Ar) for ια (11) at H81,29 (computed: 10,59 for 11,1).
Possible emendations are:
27°, seconds μζ (47) for νζ (57) (computed: 48). No ms. authority.
51°, seconds ε (5) for ιε (15) (computed: 7). No ms. authority.
59°, seconds α (1) for δ (4) (computed: 0). Only variant is '0' in L.

[88] See *HAMA* 31-2, Pedersen 97-9.

TABLE OF INCLINATION

ARCS of the Ecliptic	ARCS of the Meridian	ARCS of the Ecliptic	ARCS of the Meridian
1	0 24 16	46	16 54 47
2	0 48 31	47	17 12 16
3	1 12 46	48	17 29 27
4	1 37 0	49	17 46 20
5	2 1 12	50	18 2 53
6	2 25 22	51	18 19 15
7	2 49 30	52	18 35 5
8	3 13 35	53	18 50 41
9	3 37 37	54	19 5 57
10	4 1 38	55	19 20 56
11	4 25 32	56	19 35 28
12	4 49 24	57	19 49 42
13	5 13 11	58	20 3 31
14	5 36 53	59	20 17 4
15	6 0 31	60	20 30 9
16	6 24 1	61	20 42 58
17	6 47 26	62	20 55 24
18	7 10 45	63	21 7 21
19	7 33 57	64	21 18 58
20	7 57 3	65	21 30 11
21	8 20 0	66	21 41 0
22	8 42 50	67	21 51 25
23	9 5 32	68	22 1 25
24	9 28 5	69	22 11 1
25	9 50 29	70	22 20 11
26	10 12 46	71	22 28 57
27	10 34 57	72	22 37 17
28	10 56 44	73	22 45 11
29	11 18 25	74	22 52 39
30	11 39 59	75	22 59 41
31	12 1 20	76	23 6 17
32	12 22 30	77	23 12 27
33	12 43 28	78	23 18 11
34	13 4 14	79	23 23 28
35	13 24 47	80	23 28 16
36	13 45 6	81	23 32 30
37	14 5 11	82	23 36 35
38	14 25 2	83	23 40 2
39	14 44 39	84	23 43 2
40	15 4 4	85	23 45 34
41	15 23 10	86	23 47 39
42	15 42 2	87	23 49 16
43	16 0 38	88	23 50 25
44	16 18 58	89	23 51 6
45	16 37 1	90	23 51 20

And arc 2BA = 47;42,40°,
so Crd arc 2BA = 48;31,55^p. H83
Again, arc 2ZH = 156;40,1° [180° − arc 2ΘH, p. 70]
so Crd arc 2ZH = 117;31,15^p,
and arc 2HΘ = 23;19,59°,
so Crd arc 2HΘ = 24;15,57^p.
∴ Crd arc ΘE:Crd arc 2EA = (109;44,53 : 48;31,55)/(117;31,15 : 24;15,57)
= 54;52,26 : 117;31,15 = 56;1,53 : 120.
But arc 2EA = 180°, so Crd arc 2EA = 120^p.
∴ Crd arc 2ΘE = 56;1,53^p.[89]
So arc 2ΘE ≈ 55;40° and arc ΘE ≈ 27;50°.

Again, let arc EH be taken as 60°. Then the other magnitudes will remain unchanged, but

arc 2ZH = 138;59,42°, [180° − arc 2ΘH, p. 70]
so Crd arc 2ZH = 112;23,56^p.
And arc 2ΘH = 41;0,18°.
so Crd arc 2ΘH = 42;1,48^p.
∴ Crd arc 2ΘE:Crd arc 2EA = (109;44,53 : 48;31,55)/(112;23,56 : 42;1,48)
= 95;2,40 : 112;23,56 H84
= 101;28,20 : 120.
But Crd arc 2EA = 120^p.
∴ Crd arc 2ΘE = 101;28,20^p.
∴ arc 2ΘE ≈ 115;28°.
∴ arc ΘE ≈ 57;44°.

Thus it has been shown that the first sign of the ecliptic, counted from the equinox,[90] rises in the aforementioned manner [i.e. at *sphaera recta*] in the same time as 27;50° of the equator; and that the second sign rises with 29;54° (for the sum of both arcs was shown to be 57;44°). It is obvious that the third sign will rise at *sphaera recta* in the same time as 32;16° (which is the complement [of 57;44°]), since each whole quadrant of the ecliptic[91] rises in the same time as the corresponding quadrant of the equator as defined by circles drawn through the poles of the equator.

Following the same method as demonstrated above, we calculated the arc of the equator which rises in the same time as each 10-degree section of the ecliptic. (The [true] rising times of arcs smaller than 10° are not noticeably different from those derived by linear interpolation [from those of 10° arcs]). We shall set these too out, then, in order to be able to reckon conveniently the time which each arc takes, as we said, to cross the meridian at any point on earth and H85
the horizon at *sphaera recta*. We begin with the 10° arc starting at [either] equinoctial point.

[89] Here and just above (H83,13 and 10) Heiberg's text gives 56;1,25 (κε̄ for νγ̄). The correct reading is given by D and Is.

[90] From considerations of symmetry, it makes no difference which equinox one starts from.

[91] A 'quadrant' here is understood to start at equinox or solstice.

		Time-degrees
1st	ten-degree section rises in	9;10°
2nd	ten-degree section rises in	9;15°
3rd	ten-degree section rises in	9;25°
For 1st sign sum is		27;50°.
4th	ten-degree section rises in	9;40°
5th	ten-degree section rises in	9;58°
6th	ten-degree section rises in	10;16°
For 2nd sign sum is		29;54°
7th	ten-degree section rises in	10;34°
8th	ten-degree section rises in	10;47°
9th	ten-degree section rises in	10;55°.
For 3rd sign, ending at either solstice, sum is		32;16°.

The sum for the whole quadrant is 90°, as it should be.[92]

It is immediately obvious that the arrangement [of the rising-times] is the same for the other [three] quadrants, since the same relationships hold in each at *sphaera recta*, that is when the equator has no inclination to the horizon [i.e. is vertical to it].

[92] These data are repeated in tabular form in the table of rising-times, II 8.

Book II

1. {*On the general location of our part of the inhabited world*}

In Book I of our treatise we discussed such preliminary notions about the situation of the universe as had to be summarily disposed of, and such theorems concerning *sphaera recta* as might be thought useful for the investigations which we propose. In what follows we shall try to develop the more important theorems concerning *sphaera obliqua* too, in the most convenient way possible.

On that topic, then, we must first make the following general introductory
remark. If one considers the earth to be divided into four quarters by the H88
equator and a circle drawn through the poles of the equator, our part of the
inhabited world[1] is approximately bounded by one of the two northern
quarters. The main proof of this in the case of latitude (that is in the north-south
direction) is that the noon shadows of gnomons at equinox always point towards
the north and never towards the south. In the case of longitude (that is in the
east-west direction) the main proof is that observations of the same eclipse
(especially a lunar eclipse) by those at the extreme western and extreme eastern
regions of our part of the inhabited world (which occur at the same [absolute]
time), never differ[2] by more than twelve equinoctial hours [in local time];[3] and
the quarter [of the earth] contains a twelve-hour interval in longitude, since it is
bounded by one of the two halves of the equator.

The individual points [concerning *sphaera obliqua*] which might be considered
most appropriate to study for the subject we have undertaken are the more
important phenomena which are particular to each of the northern parallels to
the equator and to the region of the earth directly beneath each. These are
[1] the distance of the poles of the first motion [i.e. the equator] from the H89
horizon, or [in other words] the distance of the zenith from the equator,
measured along the meridian;[4]

[1] So one must translate ἡ καθ' ἡμᾶς οἰκουμένη : καθ' ἡμᾶς can mean 'in our neighbourhood' or 'in our time'. Manitius takes the expression to be temporal (e.g. here, 58,17 'des zurzeit bewohnten Gebietes der Erde'). This implausible interpretation is contradicted by VI 6 (p. 294) where Ptolemy talks about 'different parts of the inhabited world' (ἐπὶ διαφόρου οἰκουμένης, H498,2), and mentions the 'so-called antipodes' (τῶν ἀντιχθόνων καλουμένων). In using the expression he is implicitly allowing the possibility of an inhabited zone in the southern hemisphere. On the meaning and history of the concept οἰκουμένη see Campanus 396–7.

[2] 'differ': literally 'are earlier or later'.

[3] One should not infer that Ptolemy possessed records of lunar eclipses observed simultaneously at eastern and western ends of the known world. In fact it seems probable that the *only* eclipse observed at places widely separated in longitude for which he had records of both observations was that of −330 Sept. 20 (cf. *HAMA* 668 n.30), observed at Arbela and Carthage.

[4] In modern terms, the terrestrial latitude, in antiquity usually known as ἔξαρμα τοῦ πόλου, 'elevation of the pole'.

[2] for those regions where the sun reaches the zenith, when and how often this occurs;
[3] the ratios of the equinoctial and solsticial noon shadows to the gnomon;
[4] the size of the difference of the longest and shortest day from the equinoctial day;[5] and all other additional phenomena which are [commonly] studied concerning
[5] the individual increases and decreases in the length of the days and nights,[6]
[6] and the arcs of the equator which rise or set with [given] arcs of the ecliptic,[7]
[7] and the particulars and quantities of angles between the more important great circles.[8]

2. {*Given the length of the longest day, how to find the arcs of the horizon cut off between the equator and the ecliptic*}[9]

Let us take as a general basis for our examples the parallel circle to the equator
H90 through Rhodes, where the elevation of the pole is 36°, and the longest day 14½ equinoctial hours. Let [Fig. 2.1] ABGD represent the meridian, BED the eastern half of the horizon, AEG, likewise, the [eastern] half of the equator, with its south pole at Z. Let us suppose that the winter solstice on the ecliptic is rising at H. Draw through Z and H the great circle quadrant ZHΘ.

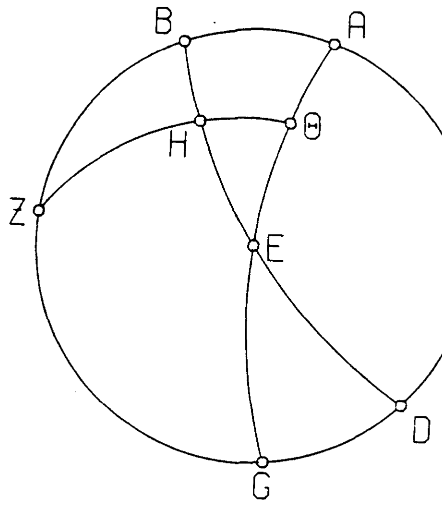

Fig. 2.1

[5] Details of [1] to [4] are given for numerous parallels in II 6.
[6] See II 9.
[7] See II 7–8.
[8] See II 10–13.
[9] On chapters 2 and 3 see *HAMA* 37–8, Pedersen 101–4.

First of all let the length of the longest day be given, and let the problem be to find arc EH of the horizon.[10]

Now, since the revolution of the [heavenly] sphere takes place about the poles of the equator, it is obvious that points H and Θ will be on the meridian ABGD at the same time. Thus the time from the rising of H to its upper culmination is given by the equatorial arc ΘA, and the time from its lower culmination to its rising is given by [the equatorial arc] GΘ. It follows that the length of daylight is H91
twice the time corresponding to arc ΘA, and the length of night twice the time corresponding to arc GΘ. For every parallel circle to the equator has both sections alike, that above the earth and that below it, bisected by the meridian.

Therefore arc EΘ, which is half the difference between longest or shortest day and equinoctial day, is $1\frac{1}{4}^h$ at the parallel in question, or 18;45 time-degrees. Hence its complement, arc ΘA, is 71;15 time-degrees.

Then since, in accordance with the previous theorems, the two great circle arcs EB and ZΘ have been drawn to meet the two great circle arcs AE and AZ, and intersect each other at H,

Crd arc 2ΘA:Crd arc 2AE =
(Crd arc 2ΘZ:Crd arc 2ZH). (Crd arc 2HB:Crd arc 2BE). [M.T.I]
But arc 2ΘA = 142;30°,
so Crd arc 2ΘA = 113;37,54^p
and arc 2AE = 180°,
so Crd arc 2AE = 120^p. H92
Again, arc 2ΘZ = 180°, so Crd arc 2ΘZ = 120^p,
and arc 2ZH = 132;17,20°, so Crd arc 2ZH = 109;44,53^p.
∴ Crd arc 2HB:Crd arc 2BE = (113;37,54 : 120)/(120 : 109;44,53)
= 103;55,26 : 120.
But arc 2BE = 120^p, since arc BE is a quadrant.
∴ Crd arc 2HB = 103;55,26^p.[11]
∴ arc 2HB ≈ 120°,
and arc HB ≈ 60°.
∴ arc HE, its complement, is 30° where the horizon is 360°.

Q.E.D.

3. {*If the same quantities be given, how to find the elevation of the pole, and* vice versa}

Next let the problem be, given the same quantity [i.e. the length of the longest day] again, to find the elevation of the pole, that is arc BZ of the meridian [in Fig. 2.1]. Now, in the same figure,

Crd arc 2EΘ:Crd arc 2ΘA =
(Crd arc 2EH:Crd arc 2HB). (Crd arc 2BZ:Crd arc 2ZA). [M.T.II] H93

[10] In modern terms, arc EH is the ortive amplitude of the sun.

[11] Here and just above (H92,11 and 8) Heiberg's text gives 103;55,23 (κγ̄ for κϛ̄). The correct reading is given by ACDAr at H92,8 and by all mss. at H92,11. Heiberg prefers the reading '23' because it is given by all mss. at H93,10. But the comparison is illegitimate, since there the amount is taken from the chord table, whereas here it is derived by calculation.

But arc 2EΘ = 37;30°,
so Crd arc 2EΘ = 38;34,22^p,
and arc 2ΘA = 142;30°,
so Crd arc 2ΘA = 113;37,54^p.
Furthermore arc 2EH = 60°,
so Crd arc 2EH = 60^p,
and arc 2HB = 120°,
so Crd arc 2HB = 103;55,23^p.
∴ Crd arc 2BZ:Crd arc 2ZA = (38;34,22 : 113;37,54)/(60 : 103;55,23)
≈ 70;33 : 120.
And again, Crd arc 2ZA = 120^p,
so Crd arc 2BZ = 70;33^p.
∴ arc 2BZ = 72;1°
and arc BZ ≈ 36°.

To do the reverse, in the same figure again [Fig. 2.1] let BZ, the arc of the
H94 pole's elevation, be given, having been observed to be 36°. Let the problem be to find the difference between the shortest or longest day and the equinoctial day, i.e. arc 2EΘ.

Now, from the same considerations,
Crd arc 2ZB:Crd arc 2BA =
(Crd arc 2ZH:Crd arc 2HΘ). (Crd arc 2ΘE:Crd arc 2EA). [M.T.II]
But arc 2ZB = 72°
so Crd arc 2ZB = 70;32,3^p,
and arc 2BA = 108°,
so Crd arc 2BA = 97;4,56^p.
Furthermore arc 2ZH = 132;17,20°,
so Crd arc 2ZH = 109;44,53^p,
and arc 2HΘ = 47;42,40°,
so Crd arc 2HΘ = 48;31,55^p.
∴ Crd arc 2ΘE:Crd arc 2EA = (70;32,3 : 97;4,56)/(109;44,53 : 48;31,55)
= 31;11,23 : 97;4,56
≈ 38;34 : 120.
H95 But Crd arc 2EA = 120^p,
∴ Crd arc 2EΘ = 38;34^p.
∴ arc 2EΘ ≈ 37;30°, or 2½ equinoctial hours.[12]

Q.E.D.

In the same way arc EH of the horizon can be determined. For
Crd arc 2ZA:Crd arc 2AB =
(Crd arc 2ZΘ:Crd arc 2ΘH). (Crd arc 2HE:Crd arc 2EB), [M.T.I]
and (Crd arc 2ZA:Crd arc 2AB) is a given ratio,
and so is (Crd arc 2ZΘ:Crd 2ΘH),
so, since arc EB is given, so is the amount of arc EH.

It is obvious that if we suppose H to be, instead of the place of the winter solstice, any other degree of the ecliptic, by similar reasoning both of the arcs

[12] There has been selective rounding at different stages of this calculation to achieve this nice result. Accurate calculation of arc 2EΘ would give (to the nearest minute) 37;29°.

EΘ and EH will be given, since we have already set out, in the 'Table of Inclination', the arc of the meridian intercepted between ecliptic and equator for every degree of the ecliptic: this arc[13] corresponds to HΘ [in Fig. 2.1].

It immediately follows that points on the ecliptic cut by the same parallel H96
circle, i.e. points equidistant from the same solstice, cut off [between ecliptic and equator] arcs of the horizon which are equal and on the same side of the equator. They also make the length of the day equal to that of the day [at the corresponding point], and the length of the night equal to that of the [corresponding] night.

It likewise follows that points [on the ecliptic] cut by equal parallel circles, that is points equidistant from the same equinox, cut off arcs of the horizon which are equal, but on opposite sides of the equator. They also make the length of the day equal to the length of the night at the opposite [corresponding] point, and the length of the night equal to that of the [corresponding] day.

For, in the figure already drawn [see Fig. 2.2], we put K as the point in which the parallel circle equal to the parallel through H cuts the semi-circle BED of the horizon; we draw in arcs HL and KM of the parallel circles: these will, clearly, be equal and opposite. We draw through K and the north pole the [great circle] quadrant NKX. Then

arc ΘA = arc XG (arc ΘA || arc LH, and arc XG || arc MK).

∴ arc EΘ = arc EX (complements [of arc ΘA and arc XG]).

Then, in the two similar spherical triangles[14] EHΘ and EKX we have two H97
pairs of corresponding sides equal, EΘ to EX, and HΘ to KX,[15] and both of the angles at Θ and X are right, so the base EH equals the base KE.

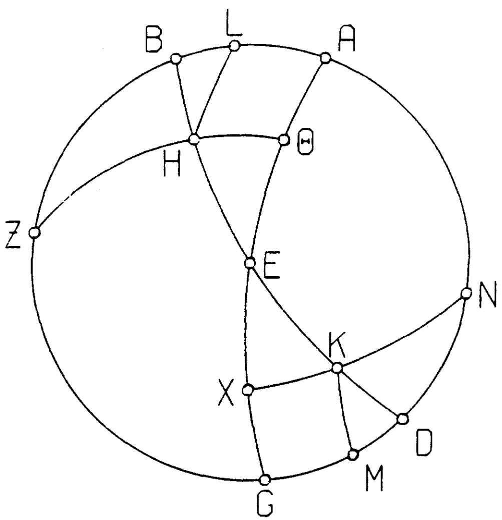

Fig. 2.2

[13] Reading προεκτεθειμένων (with D) for προεκτιθεμένων at H95,18, and περιφερειῶν (with DL, adopted by Manitius), for περιφερείᾳ at H95,22.

4. {*How to compute for what regions, when, and how often the sun reaches the zenith*}[16]

Once the above quantities are given, it is a straightforward computation to determine for what regions, when, and how often the sun reaches the zenith. For it is immediately obvious that for those beneath a parallel which is farther away from the equator than the 23;51,20° (approximately), which represents the distance of the summer solstice [from the equator], the sun never reaches the zenith at all, while for those beneath the parallel which is exactly that distance [from the equator], it reaches the zenith once [a year], precisely at the summer solstice. It is furthermore clear that for those beneath a parallel less far [from the equator] than the above-mentioned amount the sun reaches the zenith twice [a year]. The time when this happens is readily supplied from the Table of Inclination which we have set out [I 15]. For we take the distance from the equator, in degrees, of the parallel in question (which must, obviously, lie within the [parallel of the] summer solstice), and enter with it the second set of columns; we take the corresponding argument, in degrees from 1° to 90°, in the
H98 first set of columns; this gives us the distance of the sun from each of the equinoxes towards the summer solstice when it is in the zenith for those beneath the parallel in question.

5. {*How one can derive the ratios of the gnomon to the equinoctial and solsticial noon shadows from the above-mentioned quantities*}[17]

The required ratios of shadow to gnomon[18] can be found quite simply once one is given the arc between the solstices and the arc between the horizon and the pole: this can be shown as follows.

[See Fig. 2.3.] Let the meridian circle be ABGD, on centre E. Let A be taken as the zenith, and draw the diameter AEG. At right angles to this, in the plane of the meridian, draw GKZN: clearly, this will be parallel to the intersection of horizon and meridian. Now, since the whole earth has, to the senses, the ratio of a point and centre to the sphere of the sun, so that the centre E can be considered as the tip of the gnomon, let us imagine GE to be the gnomon, and
H99 line GKZN to be the line on which the tip of the shadow falls at noon. Draw through E the equinoctial noon ray and the [two] solsticial noon rays: let BEDZ represent the equinoctial ray, HEΘK the summer solsticial ray, and LEMN the winter solsticial ray. Thus GK will be the shadow at the summer solstice, GZ the equinoctial shadow, and GN the shadow at the winter solstice.

Then, since arc GD, which is equal to the elevation of the north pole from the horizon, is 36° (where meridian ABG is 360°) at the latitude in question, and

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[14] The word Ptolemy uses for 'spherical triangle', τρίπλευρον, was, according to Pappus *Synagoge* VI 2, Hultsch p. 476, 16–7, the term used by Menelaus.

[15] Arc HΘ = arc KX because they are the declinations of points equidistant from an equinox.

[16] See Pedersen 104–5 and Appendix A, Example 1*a*.

[17] See Pedersen 105–6.

[18] Reference back to II 1 [3] p. 76. They are the equinoctial and solsticial noon shadows.

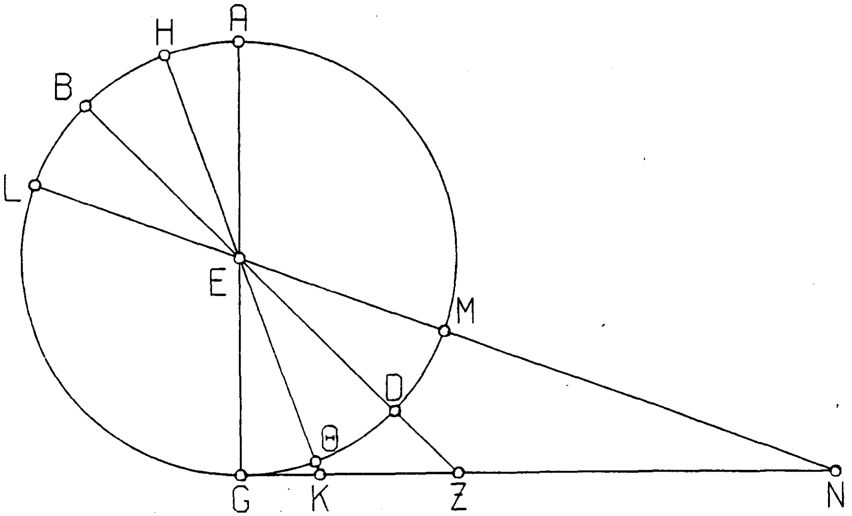

Fig. 2.3

both arc ΘD and arc DM are 23;51,20°, by subtraction arc GΘ = 12;8,40°, and by addition arc GM = 59;51,20°.

Therefore the corresponding angles

∠ KEG = 12;8,40°
∠ ZEG = 36°
∠ NEG = 59;51,20°
} where 4 right angles = 360°

and

∠ KEG = 24;17,20°°
∠ ZEG = 72°°
∠ NEG = 119;42,40°°
} where 2 right angles = 360°°.

Therefore in the circles about right-angled triangles KEG, ZEG, NEG, H100

arc GK = 24;17,20°
and arc GE = 155;42,40° (supplement),
arc GZ = 72°
and arc GE = 108°, similarly [as supplement],
arc GN = 119;42,40°
and arc GE = 60;17,20° (again as supplement).

Therefore where Crd arc GK = 25;14,43^p, Crd arc GE = 117;18,51^p,
and where Crd arc GZ = 70;32,4^p[19], Crd arc GE = 97;4,56^p,
and where Crd arc GN = 103;46,16^p, Crd arc GE = 60;15,42^p.

Therefore, where the gnomon GE has 60^p, in the same units
the summer [solsticial] shadow, GK ≈ 12;55^p,
the equinoctial shadow, GZ ≈ 43;36^p
and the winter [solsticial] shadow, GN ≈ 103;20^p.

[19] The chord table gives, for 72°, 70;32,3^p (wrongly changed to 70;32,4^p by Heiberg on the basis of this passage). All mss. (including the Arabic tradition, except for Gerard, who has 3) have 4 here. The inconsistency probably goes back to Ptolemy. It has no effect on the final result. Cf. p. 93.

It is immediately clear that the reverse process is possible. That is, provided only that any two of the three above ratios of the gnomon GE to the shadow be given, the elevation of the pole and the arc between the solstices are determined. For if any two of the angles at E are given, so is the third, since arcs ΘD and DM
H101 are equal. However, in so far as accuracy of the observation is concerned, the former quantities [elevation of the pole and 2ε] can be exactly determined in the way we explained; but the ratios of the shadows in question to the gnomon cannot be determined with equal accuracy, since the moment of the equinoxes is, in itself, somewhat indeterminate, and the tip of the shadow at winter solstice is hard to discern.

6. {*Exposition of the special characteristics, parallel by parallel*}[20]

By the same method we also found the above-mentioned general characteristics for the other parallels [to the equator]. We calculated for latitudes at intervals of ¼-hour [of longest daylight], considering that sufficient. Before we deal with particulars,[21] we shall set out these general characteristics.

1. We begin with the parallel beneath the equator itself, which forms, approximately, the southern boundary of the [earth's] quarter which comprises our part of the inhabited world. This is the only parallel which has every day equal to every night, since only in that case [i.e. at the equator] are all parallel circles bisected by the horizon, so that every section above the earth is an arc of the same size, and is equal to the corresponding section below the earth. This does not occur at any other latitude:[22] [elsewhere] only the equator is bisected at
H102 every place on earth by the horizon, so that it makes the night sensibly equal to the day [when the sun is] in it. For the equator too is a great circle. All the other [parallels] are divided [by the horizon] into unequal parts.[23] As the sphere is inclined in our part of the inhabited world, parallels south of the equator make the sections above the earth smaller than those below the earth, and the days shorter than the nights, while the northern [parallels], on the contrary, make the sections above the earth larger, and the days longer.

This parallel [of the equator] also has the shadow going both ways:[24] the sun

[20] The information given in this chapter is a gesture towards the traditional topics of Hellenistic geography. Most of it is irrelevant to the rest of the Almagest and is never mentioned or used again. In particular, the definition of latitude by the gnomon-shadow ratio at equinox or solstices is known to have been much used in earlier works (see *HAMA* II 746–8), and, to judge from Sanskrit astronomical works, had important applications in earlier Hellenistic astronomy, but is a mere fossil in the *Almagest* (although Ptolemy probably introduced the norm of 60^p for the gnomon).

The shadow lengths in this chapter are all rounded to the nearest neat fraction or whole number. For higher latitudes there are considerable inaccuracies.

[21] By 'particulars' he refers to rising-times at *sphaera obliqua* and other matters treated in the latter part of Book II.

[22] 'at any other latitude': literally 'at any of the inclinations'. See Introduction p. 19.

[23] Proved Theodosius, *Sphaerica* II 19.

[24] ἀμφίσκιος, meaning that the *noon* shadow is to the south for part of the year. This term, and the corresponding ἑτερόσκιος and περίσκιος (see p. 85 n.36 and p. 89 n.67) were used by Posidonius (early first century B.C.) in his geographical work (Edelstein-Kidd frs. 49,44–8 and 208) as reported

comes into the zenith twice [a year] for those living beneath it, when it reaches the intersections of ecliptic and equator; only at those [two times] do the gnomons cast no shadow at noon; while the sun is traversing the northern semi-circle [of the ecliptic] the shadows of the gnomons point towards the south, and while it is traversing the southern semi-circle they point towards the north. In that region a gnomon of 60^p has a shadow of $26\frac{1}{2}^p$ at both summer and winter solstices. (When we say 'shadow' we mean, in general, the noon shadow; it makes no significant difference that the equinoxes and solstices do not, in general, take place exactly at noon.)

For those who live beneath the equator those stars come into the zenith which revolve on the equator itself, but all stars are seen to rise and set, since the poles H103 of the sphere are exactly on the horizon, and thus it is impossible for any of the parallel circles to appear always visible or always invisible, or for any meridian to be a colure[25] [i.e. always partly invisible]. It is said that the regions beneath the equator could be inhabited, since the climate must be quite temperate. For the sun does not stay long in the neighbourhood of the zenith, since its motion in declination is swift round about the equinoctial points, and hence the summer would be temperate; furthermore, it is not very far from the zenith at the solstices, so the winter would not be harsh. But what these inhabited regions are we have no reliable grounds for saying. For up to now they are unexplored by men from our part of the inhabited world, and what people say about them must be considered guesswork rather than report. In any case, such, in sum, are the characteristics of the parallel beneath the equator.

As for the other parallels, which, according to some authorities, comprise the inhabited regions, we shall make the following general observations, to avoid repeating ourselves in every case. For each of them in order those stars come into the zenith whose distance from the equator, measured along the circle H104 through the poles of the equator, is equal to the distance of the parallel in question [from the equator]. Furthermore the circle which has the north pole of the equator as its pole, and the elevation of the pole [at that parallel] as its radius, is always visible, and all stars within that circle are always visible. [Likewise], the circle which has the south pole as its pole, and the same radius [as the former], is always invisible, and the stars within it are always invisible.

2. The second is the parallel with a longest day of $12\frac{1}{4}$ equinoctial hours. This is $4\frac{1}{4}°$ from the equator, and passes through the island Taprobane.[26] This too is one of the parallels with the shadow going both ways: the sun comes into the zenith for those beneath it twice [a year], and makes the gnomons shadowless at noon, when it is $79\frac{1}{2}°$ distant from the summer solstice on either side. Thus while it is traversing these 159°, the gnomon shadows point towards the south; and while

by Strabo 2.2.3 and 2.5.43. Whether Posidonius actually coined the terms, as Strabo implies (ἐκάλεσεν, wrongly denied by me, Toomer[3] 146) seems improbable, but we have no earlier attestation.

25 On this term see Introduction p. 19.

26 Ceylon. For this and the rest of the geographical data in this chapter help is provided by Kiepert's reconstruction of Ptolemy's world map, 'Orbis Terrarum secundum Cl. Ptolemaeum', *Formae Orbis Antiquae* no. XXXVI, 1911.

it is traversing the other 201°, they point towards the north. In this region, for a gnomon of 60^p, the equinoctial shadow is $4\frac{5}{12}{}^p$, the summer [solsticial] shadow $21\frac{1}{3}{}^p$, and the winter [solsticial] shadow 32^p.

H105 3. The third is the parallel with a longest day of $12\frac{1}{2}$ equinoctial hours. This is 8;25° from the equator and goes through the Avalite gulf.[27] This too is one of the parallels with the shadow going both ways: the sun comes into the zenith for those beneath it twice [a year], and makes the gnomons shadowless at noon, when it is 69° distant from the summer solstice on either side. Thus while it is traversing these 138°, the gnomon shadows point towards the south; and while it is traversing the other 222°, they point towards the north. In this region, for a gnomon of 60^p, the equinoctial shadow is $8\frac{5}{6}{}^p$, the summer [solsticial] shadow $16\frac{7}{12}{}^p$,[28] and the winter [solsticial] shadow $37\frac{9}{10}{}^p$.

4. The fourth is the parallel with a longest day of $12\frac{3}{4}$ equinoctial hours. This is $12\frac{1}{2}°$ from the equator, and goes through the Adulitic gulf.[29] This too is one of the parallels with the shadow going both ways: the sun comes into the zenith twice [a year] for those beneath it, and makes the gnomons shadowless at noon,
H106 when it is $57\frac{2}{3}°$ from the summer solstice on either side. Thus while it is traversing these $115\frac{1}{3}°$ the gnomon shadows point towards the south, and while it is traversing the remaining $244\frac{2}{3}°$ they point towards the north. In this region, for a gnomon of 60^p, the equinoctial shadow is $13\frac{1}{3}°$, the summer [solsticial] shadow 12^p, and the winter [solsticial] shadow $44\frac{1}{6}{}^p$.

5. The fifth is the parallel with a longest day of 13 equinoctial hours. This is 16;27° from the equator, and goes through the island of Meroe.[30] This too is one of the parallels with the shadow going both ways: the sun comes into the zenith for those beneath it twice [a year], and makes the gnomons shadowless at noon, when it is 45° from the summer solstice on either side. Thus while it is traversing these 90° the gnomon shadows point towards the south, and while it is traversing the remaining 270° they point towards the north. In this region, for a gnomon of 60^p, the equinoctial shadow is $17\frac{3}{4}{}^p$, the summer [solsticial] shadow $7\frac{3}{4}{}^p$, and the winter [solsticial]) shadow 51^p.[31]

6. The sixth is the parallel with a longest day of $13\frac{1}{4}$ equinoctial hours. This is

[27] Avalites was a trading-post on the African coast just outside the mouth of the Red Sea. It is identified with the mediaeval and modern Zeila, just south of Djibouti. The 'Avalite gulf' is surely the nearby Gulf of Tajura, rather than the Gulf of Aden, as asserted by Tomaschek (R-E s.v. Aualites).

[28] Reading $\overline{\iota\varsigma}\ \angle'\ \iota\beta'$ (with Is) for $\overline{\iota\varsigma}\ \angle'\ \gamma'$ ($16\frac{5}{6}$) at H105,13. Computed: 16;34,28.

[29] Adule or Adulis was a town on the Aethiopic coast of the Red Sea. The gulf is the modern Gulf of Zula (formerly Annesley Bay).

[30] Meroe is not an island in the modern sense, but was so called by the Greek geographers because it was roughly bounded by the rivers Nile, Atbara (ancient Astaboras), Blue Nile (ancient Astopus) and possibly some of their tributaries. Cf. Ptolemy, *Geography* IV 7 20 (νησοποιεῖται Meroe, bounded by Nile to the west and Astaboras to the east), and the confused account of Strabo, 17.2.2.

[31] Computed: 50;53,4. 51 is probably correct as a rounding to the nearest whole number, but one might consider D's 50;51 or T's $50\frac{5}{6}$ (H106,18).

20;14° from the equator, and goes through Napata.[32] This too is one of the parallels with the shadow going both ways: the sun comes into the zenith for H107
those beneath it twice [a year], and makes the gnomons shadowless at noon, when it is 31° from the summer solstice on either side. Thus while it is traversing these 62° the gnomon shadows point towards the south, and while it is traversing the remaining 298° they point towards the north. In this region, for a gnomon of 60^p, the equinoctial shadow is $22\frac{1}{6}^p$, the summer [solsticial] shadow $3\frac{3}{4}^p$, and the winter [solsticial] shadow $58\frac{1}{6}^p$.[33]

7. The seventh is the parallel with a longest day of $13\frac{1}{2}$ equinoctial hours. This is 23;51° from the equator[34] and goes through Soene.[35] This is the first of the so-called 'one-way-shadow'[36] parallels. For in this region the noon shadows of the gnomon never point towards the south. Only at the actual summer solstice does the sun come into the zenith for those beneath this parallel, so that the gnomons appear shadowless. For they are exactly the same distance from the equator as the summer solstice is. At every other time the shadows of the gnomons point towards the north. In this region, for a gnomon of 60^p, the equinoctial shadow is $26\frac{1}{2}^p$, the winter [solsticial] shadow is $65\frac{5}{6}^p$, and the summer [solsticial] shadow is H108
zero.[37] Furthermore, all parallels north of this up to the northern boundary of our part of the inhabited world have the shadows going one way. For in those regions the gnomons at noon neither become shadowless nor point their shadows towards the south: they always point them towards the north, since the sun never comes into the zenith for them, either.

8. The eighth is the parallel with a longest day of $13\frac{3}{4}$ equinoctial hours. This is 27;12° from the equator, and goes through Ptolemais in the Thebaid, which is called Ptolemais Hermeiou. In this region, for a gnomon of 60^p, the summer [solsticial] shadow is $3\frac{1}{2}^p$, the equinoctial shadow $30\frac{5}{6}^p$,[38] and the winter [solsticial] shadow $74\frac{1}{6}^p$.

9. The ninth is the parallel with a longest day of 14 equinoctial hours. This is 30;22° from the equator, and goes through lower Egypt. In this region, for a gnomon of 60^p, the summer [solsticial] shadow is $6\frac{5}{6}^p$, the equinoctial shadow $35\frac{1}{12}^p$, and the winter [solsticial] shadow $83;12^p$.[39]

[32] Napata is the modern Gebel Barkal, near Merowe in the Sudan.

[33] Computed: 22;6,7 for the equinoctial shadow, and 58;5,55 for the winter solsticial shadow. One would expect $\frac{1}{10}$ instead of $\frac{1}{6}$ in both places. Perhaps one should interpret ς′ as $\overline{\varsigma}$, i.e. 6 minutes; but this would normally be written as an aliquot fraction (ι′).

[34] Computed: 23;48,20. The discrepancy is interesting, because it is due, not to rounding, but to the desire to make the parallel with M = $13\frac{1}{2}^h$ exactly coincide with the parallel with a latitude equal to the obliquity of the ecliptic, i.e. where the sun is in the zenith at summer solstice. The difference is negligible, but instead of saying so Ptolemy fudges the result.

[35] Also known as Syene: the modern Assuan in upper Egypt.

[36] ἑτερόσκιος, the opposite of ἀμφίσκιος; see p. 82 n.24.

[37] Literally 'shadowless'.

[38] Reading $\overline{\lambda}$ ∠′ γ′ (with D, Is) for $\overline{\lambda\varsigma}$ ∠′ γ′ ($36\frac{5}{6}$) at H108,13. Computed: 30;48,36.

[39] Reading $\overline{\pi\gamma}$ $\overline{\iota\beta}$ (with L) for $\overline{\pi\gamma}$ ιβ′ (i.e. 12 minutes instead of $\frac{1}{12}$) at H108,20. Computed: 83;10,39. Ptolemy does not often use the aliquot fraction ε′ ($\frac{1}{5}$).

10. The tenth is the parallel with a longest of $14\frac{1}{4}$ equinoctial hours. This is
H109 33;18° from the equator, and goes through the middle of Phoenicia. In this region, for a gnomon of 60^p, the summer [solsticial] shadow is 10^p, the equinoctial shadow $39\frac{1}{2}^p$, and the winter [solsticial] shadow $93\frac{1}{12}^p$.[40]

11. The eleventh is the parallel with a longest day of $14\frac{1}{2}$ equinoctial hours. This is 36° from the equator, and goes through Rhodes. In this region, for a gnomon of 60^p, the summer [solsticial] shadow is $12\frac{11}{12}^p$, the equinoctial shadow $43\frac{3}{5}^p$,[41] and the winter [solsticial] shadow $103\frac{1}{3}^p$.

12. The twelfth is the parallel with a longest day of $14\frac{3}{4}$ equinoctial hours. This is 38;35° from the equator, and goes through Smyrna. In this region, for a gnomon of 60^p, the summer [solsticial] shadow is $15\frac{2}{3}^p$, the equinoctial shadow is $47\frac{5}{6}^p$, and the winter [solsticial] shadow is $114\frac{11}{12}^p$.

13. The thirteenth is the parallel with a longest day of 15 equinoctial hours. This is 40;56° from the equator, and goes through the Hellespont. In this region, for a gnomon of 60^p, the summer [solsticial] shadow is $18\frac{1}{2}^p$, the equinoctial shadow $52\frac{1}{6}^p$, and the winter [solsticial] shadow $127\frac{5}{6}^p$.[42]

H110 14. The fourteenth is the parallel with a longest day of $15\frac{1}{4}$ equinoctial hours. This is 43;1°[43] from the equator, and goes through Massalia.[44] In this region, for a gnomon of 60^p, the summer [solsticial] shadow is $20\frac{5}{6}^p$, the equinoctial shadow $55\frac{11}{12}^p$, and the winter [solsticial] shadow $140\frac{1}{4}^p$.[45]

15. The fifteenth is the parallel with a longest day of $15\frac{1}{2}$ equinoctial hours. This is 45;1° from the equator, and goes through the middle of Pontus.[46] In this region, for a gnomon of 60^p, the summer [solsticial] shadow is $23\frac{1}{4}^p$, the equinoctial shadow 60^p, and the winter [solsticial] shadow $155\frac{1}{12}^p$.[47]

[40] All the values for the shadow at this parallel are rather inaccurate. For $M = 14\frac{1}{4}^h$ one finds 9;57,43, 39;23,11 and 92;52,51. Ptolemy's figures fit a latitude of $33\frac{1}{4}$° much better.

[41] Reading $\overline{\mu\gamma}$ ∠′ ι′ (with Ar) for $\overline{\mu\gamma}$ ∠′ γ′ ($43\frac{5}{6}$) at H109,9. Corrected by Manitius. Cf. 43;36 at II 5 p. 81.

[42] There is a strange discrepancy here. For $M = 15^h$, one finds $\varphi = 40;52,21°$. However, the shadow lengths fit neither $M = 15^h$ nor $\varphi = 40;56°$, but $\varphi = 41°$. Computations:

	$M = 15^h$	$\varphi = 40;56°$	$\varphi = 41°$	text
summer shadow	18;21,47	18;25,58	18;30,34	18;30
equinoctial shadow	51;55,23	52;2,5	52;9,26	52;10
winter shadow	127;5,30	127;26,32	127;49,41	127;50

The parallel through the Hellespont is Clima V in the traditional '7 climata' (see Introduction p. 19). Possibly, an older round number for the latitude underlies Ptolemy's values here.

[43] Reading μγ $\bar{\alpha}$ for $\overline{\mu\gamma}$ $\bar{\delta}$ (43;4) at H110,3. Although not supported by any ms. reading (Ar has $43\frac{1}{4}$), 43;1 is confirmed by the values for the shadow lengths. Furthermore, 4′ would normally be written as an aliquot fraction, ιε′ (but cf. H111,6 where 50;4 is certainly correct, and is written $\bar{\nu}$ $\bar{\delta}$, i.e. 50;4 and not $50\frac{1}{15}$).

[44] Modern Marseilles.

[45] Reading $\overline{\rho\mu}$ δ′ (with BCIs) for $\overline{\rho\mu\delta}$ (144) at H110,6. Computed: 140;31,31. One might also consider $\overline{\rho\mu\alpha}$ (141), as a rounding to the nearest whole number, but this has no ms. support.

[46] The Black Sea.

[47] Computed: 155;10,32. Possibly one should read 155;12 (with L, ιβ for ιβ′). Cf. p. 85 n.39.

16. The sixteenth is the parallel with a longest day of $15\frac{3}{4}$ equinoctial hours. This is 46;51° from the equator and goes through the sources of the river Istros.[48] In this region, for a gnomon of 60^p, the summer [solsticial] shadow is $25\frac{1}{2}^p$, the equinoctial shadow $63\frac{11}{12}^p$, and the winter [solsticial] shadow $171\frac{1}{6}^p$.

17. The seventeenth is the parallel with a longest day of 16 equinoctial hours. This is 48;32° from the equator, and goes through the mouths of the H111 Borysthenes.[49] In this region, for a gnomon of 60^p, the summer [solsticial] shadow is $27\frac{1}{2}^p$, the equinoctial shadow $67\frac{5}{6}^p$, and the winter [solsticial] shadow $188\frac{7}{12}^p$.[50]

18. The eighteenth is the parallel with a longest day of $16\frac{1}{4}$ equinoctial hours. This is 50;4° from the equator, and goes through the middle of the Maiotic lake.[51] In this region, for a gnomon of 60^p, the summer [solsticial] shadow is $29\frac{7}{12}^p$,[52] the equinoctial shadow $71\frac{2}{3}^p$, and the winter [solsticial] shadow $208\frac{1}{3}^p$.[53]

19. The nineteenth is the parallel with a longest day of $16\frac{1}{2}$ equinoctial hours. This is $51\frac{1}{2}$°[54] from the equator and goes through the southernmost parts of Brittania. In this region, for a gnomon of 60^p, the summer [solsticial] shadow is $31\frac{5}{12}^p$, the equinoctial shadow $75\frac{5}{12}^p$, and the winter [solsticial] shadow $229\frac{1}{3}^p$.

20. The twentieth is the parallel with a longest day of $16\frac{3}{4}$ equinoctial hours. This is 52;50° from the equator and goes through the mouths of the Rhine. In this region, for a gnomon of 60^p, the summer [solsticial] shadow is $33\frac{1}{3}^p$, the equinoctial shadow $79\frac{1}{12}^p$, and the winter [solsticial] shadow $253\frac{1}{6}^p$.[55]

21. The twenty-first is the parallel with a longest day of 17 equinoctal hours. H112
This is 54;1° from the equator,[56] and goes through the mouths of the Tanais.[57] In this region, for a gnomon of 60^p, the summer [solsticial] shadow is $34\frac{11}{12}^p$, the equinoctial shadow $82\frac{7}{12}^p$, and the winter [solsticial] shadow $278\frac{3}{4}^p$.

[48] The Danube.

[49] The modern river Dnieper.

[50] These shadow lengths accord better with a latitude of $48\frac{1}{2}$°. However, $\varphi = 48;32°$ is abundantly attested for this parallel, which is Clima VII of the 7 climata. There are variants $188\frac{5}{8}$ (T) and $188\frac{1}{2}\frac{1}{8}$ (≈ 188;38, L) for the winter shadow. Computed: 188;44,49.

[51] Modern Sea of Azov.

[52] Reading $\overline{\kappa\theta}$ ∠′ ιβ′ (with Ar) for $\overline{\kappa\theta}$ ∠′ γ′ ιβ′ ($29\frac{11}{12}$) at H111,9. Computed: 29;31,31.

[53] Computed: 208;2,32. Perhaps one should read 208;3 (interpreting γ′ as $\overline{\gamma}$, i.e. 3 minutes, at H111,10).

[54] Reading $\overline{\nu\alpha}$ ∠′ (with D, Ar) for $\overline{\nu\alpha}$ ∠′ ϛ′ ($51\frac{1}{2} + \frac{1}{6}$) at H111,13. Computed: 51;28,54. Corrected by Manitius.

[55] For $\varphi = 52;50°$ one finds the winter shadow as 253;35,53. L has 253;36. Hence one might consider emending ϛ′ to ∠′ ι′ at H111, 23. However, there are increasing inaccuracies in the winter shadows from here on.

[56] Reading $\overline{\nu\delta}$ $\overline{\alpha}$ (with BCDAr) for $\overline{\nu\delta}$ $\overline{\lambda}$ (54;30) at H112,3. Computed: 54;0,18. Corrected by Manitius.

[57] The modern river Don. For the great error in the latitude assigned to this region here and in the *Geography* see Toomer[3] 148.

22. The twenty-second is the parallel with a longest day of $17\frac{1}{4}$ equinoctial hours. This is 55° from the equator[58] and goes through Brigantium in Great Brittania.[59] In this region, for a gnomon of 60^p, the summer [solsticial] shadow is $36\frac{1}{4}^p$, the equinoctial shadow is $85\frac{2}{3}^p$, and the winter [solsticial] shadow is $304\frac{1}{2}^p$.

23. The twenty-third is the parallel with a longest day of $17\frac{1}{2}$ equinoctial hours. This is 56° from the equator, and goes through the middle of Great Brittania. In this region, for a gnomon of 60^p, the summer [solsticial] shadow is $37\frac{2}{3}^p$, the equinoctial shadow $88\frac{5}{6}^p$, and the winter [solsticial] shadow $335\frac{1}{4}^p$.

H113 24. The twenty-fourth is the parallel with a longest day of $17\frac{3}{4}$ equinoctial hours. This is 57° from the equator, and goes through Caturactonium in Brittania.[60] In this region, for a gnomon of 60^p, the summer [solsticial] shadow is $39\frac{1}{6}^p$,[61] the equinoctial shadow is $92\frac{5}{12}^p$, and the winter [solsticial] shadow is $372\frac{2}{3}^p$.[62]

25. The twenty-fifth is the parallel with a longest day of 18 equinoctial hours. This is 58° from the equator and goes through the southern part of Little Brittania.[63] In this region, for a gnomon of 60^p, the summer [solsticial] shadow is $40\frac{2}{3}^p$, the equinoctial shadow 96^p, and the winter [solsticial] shadow $419\frac{1}{12}^p$.[64]

26. The twenty-sixth is the parallel with a longest day of $18\frac{1}{2}$ equinoctial hours. This is $59\frac{1}{2}$° from the equator, and goes through the middle of Little Brittania.

From here on we no longer used $\frac{1}{4}$-hour increments, since [at intervals of $\frac{1}{4}$-hour for the longest daylight] the parallels are now close together, and the difference in the elevation of the pole is no longer as much as a whole degree. Furthermore, for the points even further north there is not the same need for detail. Hence we considered it superfluous to list the ratios of the shadows to the gnomon, as if it were for some well-defined place.

H114 27. The parallel where the longest day is 19 equinoctial hours is 61° from the equator and goes through the northern parts of Little Brittania.

[58] Computed: 55;7,16. From here on the roundings become much more drastic.

[59] By 'Great Brittania' and 'Little Brittania' Ptolemy refers to the two principal islands of the British isles, namely modern 'Great Britain' (England, Wales and Scotland) and Ireland. None of the places called Brigantium were in Britain. However, there was in Britain a tribe of Brigantes, whose kingdom was sometimes known as Brigantia (which was further to the north than this latitude would imply). Ptolemy presumably made an error here. He seems to have corrected it by the time he came to write the *Geography*, which does mention the Brigantes, but no Brigantium in Britain.

[60] Modern Catterick in Yorkshire. The usual Latin form is 'Cataractonium'.

[61] Reading $\overline{\lambda\theta}$ ς′ (with D, Is) for $\overline{\lambda\theta}$ γ′ ($39\frac{1}{3}$) at H113,4. Computed for $\varphi = 57°$: 39;10,48.

[62] Reading $\overline{\tau o \beta}$ ⌐ο (with B^3D^2, Ar) for $\overline{\tau o \beta}$ ιβ′ ($372\frac{1}{12}$) at H113,5. Computed: for $\varphi = 59°$: 372;44,27.

[63] Ireland: see above n.59.

[64] Computed for $\varphi = 58°$: 419;15,1. Perhaps one should emend to $419\frac{1}{4}$ (δ′ for ιβ′ at H113,11). Cf. '$119\frac{1}{4}$', Ger.

28. The parallel where the longest day is 19½ equinoctial hours is 62° from the equator and goes through the islands called 'Eboudae'.[65]

29. The parallel where the longest day is 20 equinoctial hours is 63° from the equator and goes through the island Thule.[66]

30. The parallel where the longest day is 21 equinoctial hours is 64½° from the equator and goes through unknown Scythian peoples.

31. The parallel where the longest day is 22 equinoctial hours is 65½° from the equator.

32. The parallel where the longest day is 23 equinoctial hours is 66° from the equator.

33. The parallel where the longest day is 24 equinoctial hours is 66;8,40° from the equator. This is the first of the [parallels] where the shadow goes full circle.[67] For on that parallel, at the summer solstice (and then only), the sun does not set, so the shadow of the gnomon points towards every part of the horizon [in turn]. H115
There the parallel of the summer solstice is ever-visible, and the parallel of the winter solstice is ever-invisible, since both are tangent to the horizon, on opposite sides. And the ecliptic coincides with the horizon when the spring equinoctial point on it is rising.

If, purely theoretically, one were to investigate some of the general characteristics of the latitudes even farther north, one would find the following.

34. Where the elevation of the north pole is about 67°, the 15° of the ecliptic on either side of the summer solstice do not set at all. So the longest day and the period when the shadow turns to point in all directions on the horizon is about a month long. This too can easily be seen from the Table of Inclination set out [above]. For we take a parallel, e.g. the parallel which cuts off [a segment of the ecliptic] 15° either side of the solstice (at which point it is either ever-visible or ever-invisible). The distance from the equator corresponding to that segment of the ecliptic will, obviously, give the amount by which the elevation of the north pole differs from the 90° of the quadrant.[68]

35. Thus, where the elevation of the pole is 69½°, one would find that the 30° on H116
either side of the summer solstice do not set at all. So the longest day and the

[65] By this name (which possibly ought to be aspirated, as 'Hebudae' in Pliny *NH* 4.30) Ptolemy refers to the Hebrides, which he supposed to lie north of Ireland.

[66] By 'Thule' Ptolemy refers to the modern Shetlands, as is clear from his *Geography* (II 3 32). It has been a matter of great dispute to what place (if any) the man who first introduced the name 'Thule' to the Greek world, Pytheas of Massalia, was referring. For ancient information on Pytheas' voyage to Thule, a discussion of its identification and references to modern literature see Hennig, *Terrae Incognitae* I 119–24, 129–35.

[67] περίσκιος. Cf. p. 82 n.24.

[68] See Appendix A, Example 1*b*.

period when the gnomons throw shadows in all directions last about two months.

36. Where the elevation of the pole is $73\frac{1}{3}°$, one would find that the 45° on either side of the summer solstice do not set at all. So the longest day and the period when the gnomons throw shadows in all directions last about three months.

37. Where the elevation of the pole is $78\frac{1}{3}°$, one would find that the 60° on either side of the same solstice do not set at all. So the longest day and the period when the shadow turns through a full circle would last about four months.

38. Where the elevation of the pole is 84°, one would find that the 75° on either side of the summer solstice do not set at all. So in this case the longest day would be about five months long, and the gnomon would throw shadows in all directions for the same period.

39. Where the north pole is elevated from the horizon through the 90° of the complete quadrant, the whole semi-circle of the ecliptic which is north of the equator never goes below the earth, and the whole semi-circle south of it never
H117 comes above the earth. Therefore every year contains only one day and one night, each about six months long, and the gnomons always throw shadows in all directions. Further special characteristics of this latitude are that the north pole is in the zenith, and that the equator coincides with the position of the ever-visible circle, and also with that of the ever-invisible circle and with the horizon; thus the whole hemisphere north of the equator is always above the earth, and the whole hemisphere south of the equator is always below the earth.

7. {*On simultaneous risings of arcs of the ecliptic and equator at* sphaera obliqua}[69]

After we have thus set out the general characteristics which can be theoretically deduced for the [various] latitudes, our next task is to show how to calculate, for each latitude, the arcs of the equator, measured as time-degrees, which rise together with [given] arcs of the ecliptic. From this we shall systematically derive all the other special characteristics [of the climata]. We shall use the names of the signs of the zodiac for the twelve [30°-] divisions of the ecliptic, according to the system in which the divisions begin at the solsticial and equinoctial points.[70]
H118 We call the first division, beginning at the spring equinox and going towards the rear with respect to the motion of the universe, 'Aries', the second 'Taurus', and so on for the rest, in the traditional order of the 12 signs.

We shall first prove that arcs of the ecliptic which are equidistant from the same equinox always rise with equal arcs of the equator.

[69] See *HAMA* 34–7, Pedersen 110–13.

[70] I.e. the spring equinox defines 'Aries 0°', etc. This specification was necessary because other norms existed in antiquity, notably those where the spring equinox was at ♈ 8° and ♈ 10° (derived from Babylonian practice). See *HAMA* II 594–8.

[See Fig. 2.4.] Let ABGD be a meridian, BED the semi-circle of the horizon, AEG the semi-circle of the equator, and ZH and ΘK two arcs of the ecliptic such that points Z and Θ are each supposed to be the spring equinox, and equal arcs have been cut off on opposite sides of [that equinox]: these are arcs ZH and ΘK, which are rising at points K and H [respectively]. I say, that the arcs of the equator which rise with them, namely ZE and ΘE respectively, are equal.
[Proof.] Let points L and M represent the poles of the equator, and draw H119
through them the great-circle arcs LEM, LΘ, LK, ZM and MH. Then since

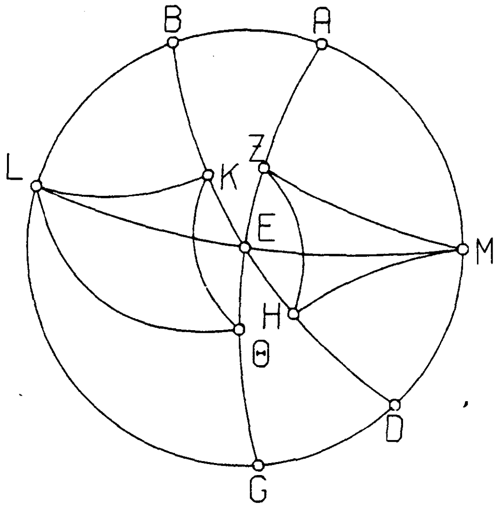

Fig. 2.4

arc ZH = arc ΘK,
and arc LK = arc MH
and arc EK = arc EH } because the parallels through K and H are equidistant from the equator on opposite sides,[71]
[spherical triangle] LKΘ ≡ [spherical triangle] MHZ
and [spherical triangle] LEK ≡ [spherical triangle] MEH.
∴ ∠ KLE = ∠ HME,
and ∠ KLΘ = ∠ HMZ.
Therefore, by subtraction, ∠ ELΘ = ∠ EMZ.
∴ EΘ = EZ, bases [of congruent triangles ELΘ, EMZ].
Q.E.D.

Again, we shall prove that if two arcs of the ecliptic are equal and are equidistant from the same solstice, the sum of the two arcs of the equator which

[71] Cf. II 3 (p. 79).

rise with them is equal to the sum of the rising-times [of the same two arcs of the ecliptic] at *sphaera recta*.

[See Fig. 2.5.] Let ABGD be a meridian, and let semi-circle BED represent the horizon, and semi-circle AEG the equator. Draw two arcs of the ecliptic,
H120 equal and equidistant from the winter solstice, ZH (where Z is taken as the autumnal equinox) and ΘH (where Θ is taken as the spring equinox).

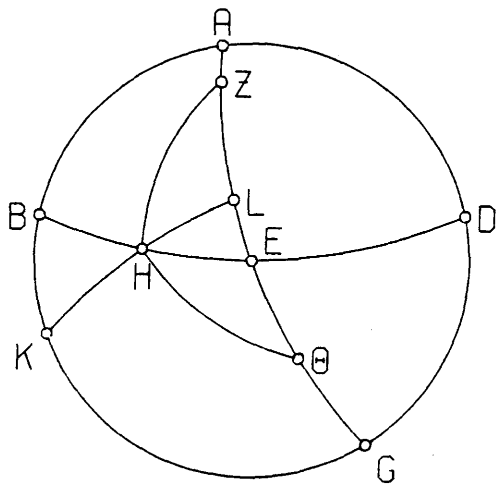

Fig. 2.5

Thus H is the point on the horizon which is common to the rising of both, since arcs ZH and ΘH are both bounded by the same parallel circle to the equator. Therefore, obviously, arc ΘE rises with arc ΘH, and arc EZ with arc ZH. Then it is immediately obvious that the whole arc ΘEZ is equal to the sum of the rising-times of arc ZH and arc ΘH at *sphaera recta*.
[Proof.] For if we take K as the south pole of the equator, and draw through it and H the great-circle quadrant KHL, which represents the horizon at *sphaera recta*, then ΘL is the arc which rises with arc ΘH at *sphaera recta*, and similarly LZ is the arc which rises with arc ZH. Thus the sum of the arcs (ΘL + LZ) equals the sum of the arcs (ΘE + EZ), and both are comprised in the arc ΘZ.

Q.E.D.

From the above we have shown that, if we can calculate the individual rising-
H121 times at any latitude for just a single quadrant, we will simultaneously have solved the problem for the remaining three quadrants as well.

This being the case, let us again take as a paradigm the parallel through Rhodes, where the longest day is $14\frac{1}{2}$ equinoctial hours, and the elevation of the north pole from the horizon is 36°.

[See Fig. 2.6.] Let ABGD be a meridian, BED the semi-circle of the horizon, AEG the semi-circle of the equator, and ZHΘ the semi-circle of the ecliptic, positioned so that H represents the spring equinox. Take K as the north pole of

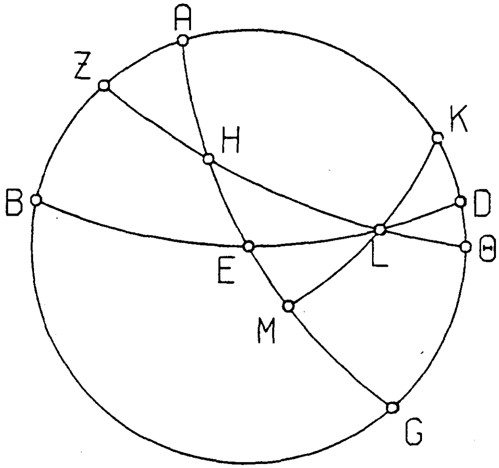

Fig. 2.6

the equator, and draw through K and L, which is the intersection of the ecliptic and the horizon, the great-circle quadrant KLM.

Let the problem be, given arc HL, to find the arc of the equator which rises with it, that is arc EH.

First let arc HL comprise the sign of Aries.

Then since, in the diagram, the two great-circle arcs ED and KM are drawn to meet the two great-circle arcs EG and GK, and intersect each other at L,

Crd arc 2KD:Crd arc 2DG =

(Crd arc 2KL:Crd arc 2LM). (Crd arc 2ME:Crd arc 2EG). [M.T. II] H122

But arc 2KD = 72°, so Crd arc 2KD = 70;32,4^{p};[72]

arc 2GD = 108°, so Crd arc 2GD = 97;4,56^{p}.

And arc 2KL = 156;40,1°,[73] so Crd arc 2KL = 117;31,15^{p};

arc 2LM = 23;19,59°, so Crd arc 2LM = 24;15,57^{p}.

∴ Crd arc 2ME:Crd arc 2EG = (70;32,4 : 97;4,56)/(117;31,15 : 24;15,57)

= 18;0,5 : 120.

And Crd arc 2EG = 120^{p}.

∴ Crd arc 2ME = 18;0,5^{p}

∴ arc 2ME ≈ 17;16°

and arc ME = 8;38°

And since the whole arc HM rises with the whole arc HL at *sphaera recta*, it is 27;50°, as was shown above. [p. 73.]

Therefore, by subtraction, EH is 19;12°.

We have simultaneously proved that the sign Pisces rises in the same time (in H123

[72]Here (H122,4) and at H122,10 and H123,13 the Greek and Arabic ms. traditions give 70;32,4^{p} as the chord of 72°, whereas in the chord table it is 70;32,3^{p} (found here only in Ger.). Is this an indication that there was an earlier version of the chord table? Cf. p. 81 n.19.

[73]Reading $\overline{\rho\nu\varsigma}\ \overline{\mu}\ \overline{\alpha}$ (with B.Is) for $\overline{\rho\nu\varsigma}\ \overline{\mu\alpha}$ (156;41) at H122,7. Corrected by Manitius.

degrees) of 19;12°, and that each of the signs Virgo and Libra rises in 36;28°, which is the remainder [of 19;12° taken] from twice the rising-time at *sphaera recta.*

Q.E.D.

Secondly, let arc HL comprise the 60° of the two signs Aries and Taurus. Then, from our assumptions, the other quantities will remain the same, but

$$\text{arc } 2KL = 138;59,42^\circ, \text{ so Crd arc } 2KL = 112;23,56^p,$$

$$\text{and arc } 2LM = 41;0,18^\circ,^{74} \text{ so Crd arc } 2LM = 42;1,48^p.$$

$$\therefore \text{Crd arc } 2ME:\text{Crd arc } 2EG = (70;32,4 : 97;4,56)/(112;23,56 : 42;1,48)$$

$$= 32;36,4 : 120.$$

$$\text{And Crd arc } 2EG = 120^p.$$

$$\therefore \text{Crd arc } 2ME = 32;36,4^p.$$

$$\therefore \text{arc } 2ME \approx 31;32^\circ,$$

$$\text{and arc } ME \approx 15;46^\circ.$$

But the whole arc MH[75] was previously shown to be 57;44° [p. 73.]

Therefore, by subtraction, arc HE = 41;58°.

Therefore the combined signs of Aries and Taurus rise in 41;58 time degrees,
H124 of which 19;12° was shown to belong to the rising-time of Aries. Therefore the sign of Taurus by itself rises in 22;46 time-degrees.

By the same reasoning as before, the sign of Aquarius will rise in the same time of 22;46°, and each of the signs of Leo and Scorpio in 37;2°, which is the remainder [of 22;46° taken] from twice the rising-time at *sphaera recta.*

Now since the longest day is 14½ equinoctial hours, and the shortest 9½ equinoctial hours, it is obvious that the semi-circle [of the ecliptic] from Cancer to Sagittarius will rise with 217;30° of the equator, and the semi-circle from Capricorn to Gemini with 142;30°. Therefore each of the quadrants on either side of the spring equinox will rise in 71;15 time-degrees, and each of the quadrants on either side of the autumnal equinox will rise in 108;45 time-degrees. Therefore the remaining signs [in each quadrant], Gemini and Capricorn, will each rise in 29;17 time-degrees, which is the difference [of 19;12° + 22;46°] from the 71;15° in which the quadrant rises, and the remaining signs Cancer and Sagittarius will each rise in 35;15 time-degrees, which is the difference [of 36;28° + 37;2°] from the 108;45° in which that quadrant rises.

H125 It is obvious that we could also calculate the rising-times of smaller arcs of the ecliptic [than whole signs] by exactly the same method. But we can also compute them by another easier and more practical procedure, as follows.

[See Fig. 2.7.] First let ABGD represent a meridian, BED the semi-circle of the horizon, AEG the semi-circle of the equator, and ZEH the semi-circle of the ecliptic, with the intersection E taken as the spring equinox. Cut off an arbitrary arc EΘ on [the ecliptic], and draw the segment ΘK of the parallel to the equator through Θ. Taking L as the [south] pole of the equator, draw through it the great-circle quadrants LΘM, LKN and LE.

[74] Reading $\overline{\mu\alpha}$ $\overline{o}$ $\overline{\iota\eta}$ (with Ar and variants in Greek mss.) for $\overline{\mu\alpha}$ $\overline{\theta}$ $\overline{\iota\eta}$ (41;9,18) at H123,11. Corrected by Manitius.

[75] Correcting the misprint 'ME' at H123,21, with Manitius.

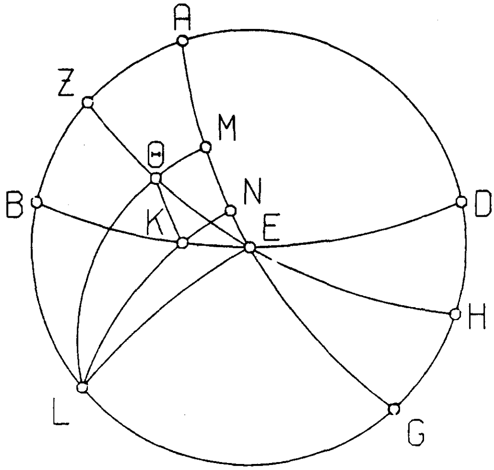

Fig. 2.7

Then it is immediately obvious that the segment EΘ of the ecliptic rises with arc EM of the equator at *sphaera recta*, and with NM at *sphaera obliqua*, since arc KΘ of the parallel circle, with which segment EΘ rises [at *sphaera obliqua*], is similar to arc NM of the equator and similar arcs of parallel circles rise in equal times everywhere. Therefore arc EN is the difference between the rising-times H126 of segment EΘ at *sphaera obliqua* and at *sphaera recta*. Thus we have shown that, for arcs of the ecliptic bounded by point E and the parallel circle through K, in every case, if the great-circle arc corresponding to LKN is drawn, segment EN will comprise the difference between that arc's rising-times at *sphaera recta* and at *sphaera obliqua*.[76]

Q.E.D.

Having established this as a preliminary, let us draw [see Fig. 2.8] a diagram containing only the meridian and the semi-circles of the horizon [BED] and of the equator [AEG]; through Z, the south pole of the equator, let us draw the two great-circle quadrants ZHΘ and ZKL. Let us take H as the intersection of the horizon with the parallel circle through the winter solstice, and K as the intersection [of the horizon] with the parallel circle through, e.g., the beginning of Pisces, or any other given point on the quadrant [from the beginning of H127 Capricorn to the end of Pisces].

Then, again, the great-circle arcs ZKL and EKH are drawn to meet the great-circle arcs ZΘ and EΘ, and intersect each other at K. Therefore

Crd arc 2ΘH:Crd arc 2ZH =

(Crd arc 2ΘE:Crd arc 2EL). (Crd arc 2KL:Crd arc 2KZ) [M.T. II]

But at every latitude arc 2ΘH is given and is the same, since it is the arc between the solstices. Hence arc 2HZ, its supplement, is also given. Similarly,

[76] This arc EN is known in mediaeval astronomy as the 'ascensional difference'. See *HAMA* 36 and 980–2, and Neugebauer-Schmidt.

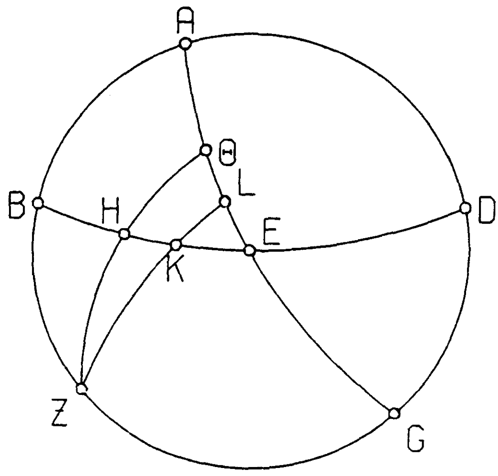

Fig. 2.8

for the same arc of the ecliptic, arc 2LK is the same at all latitudes, and is given from the Table of Inclination [I 15]; and thence again its supplement, arc 2KZ, is given. Therefore, by division [of the above members], (Crd arc 2ΘE:Crd arc 2EL) is found to be the same at all latitudes (for the same arc of that quadrant [of the ecliptic]).

Since this is so, we take the different values of arc KL at every 10° [of the ecliptic] through the quadrant from the spring equinox to the winter solstice (for subdivision down to arcs of this size [10°] will be sufficient for practical purposes). Then in every case

H128 arc 2ΘH = 47;42,40°, and Crd arc 2ΘH = 48;31,55^p,
arc 2HZ = 132;17,20°, and Crd arc 2HZ = 109;44,53^p.

Then, for the 10° [of the ecliptic] from the spring equinox towards the winter solstice,
arc 2KL = 8;3,16°, and Crd arc 2KL = 8;25,39^p,
arc 2KZ = 171;56,44°, and Crd arc 2KZ = 119;42,14^p.

For the arc 20° from the equinox
arc 2KL = 15;54,6°, Crd arc 2KL = 16;35,56^p,
arc 2KZ = 164;5,54°, Crd arc 2KZ = 118;50,47^p.

For the arc 30° from the equinox
arc 2LK = 23;19,58°, Crd arc 2LK = 24;15,56^p,
arc 2KZ = 156;40,2°, Crd arc 2KZ = 117;31,15^p.

For the arc 40° from the equinox
H129 arc 2LK = 30;8,8°, Crd arc 2LK = 31;11,43^p.
arc 2KZ = 149;51,52°, Crd arc 2KZ = 115;52,19^p.

For the arc 50° from the equinox
arc 2LK = 36;5,46°, Crd arc 2LK = 37;10,39^p,
arc 2KZ = 143;54,14°, Crd arc 2KZ = 114;5,44^p.

For the arc 60° from the equinox

arc 2LK = 41;0,18°, Crd arc 2LK = 42;1,48^p,
arc 2KZ = 138;59,42°, Crd arc 2KZ = 112;23,57^p.

For the arc 70° from the equinox
arc 2LK = 44;40,22°, Crd arc 2LK = 45;36,18^p
arc 2KZ = 135;19,38°, Crd arc 2KZ = 110;59,47^p.

For the arc 80° from the equinox
arc 2LK = 46;56,32°, Crd arc 2LK = 47;47,40^p,
arc 2KZ = 133;3,28°, Crd arc 2KZ = 110;4,16^p.

From the above we find that if we divide the ratio (Crd arc 2ΘH:Crd arc 2HZ), namely (48:31,55 : 109;44,53), by the ratio (Crd arc 2LK:Crd arc 2KZ), H130 as given above, at each of the 10° intervals, we will get the ratio (Crd arc 2ΘE:Crd arc 2EL), which is the same at all latitudes.

For the 10° arc it is	60 : 9;33
for the 20° arc	60 : 18;57
for the 30° arc	60 : 28;1
for the 40° arc	60 : 36;33[77]
for the 50° arc	60 : 44;12
for the 60° arc	60 : 50;44
for the 70° arc	60 : 55;45
and for the 80° arc	60 : 58;55.

It is immediately obvious that for each latitude we will have arc 2ΘE as a given arc, since it is, in degrees, the difference in time-degrees of the equinoctial day from the shortest day. Hence, from Crd arc 2ΘE and the ratio (Crd arc 2ΘE:Crd arc 2EL), Crd arc 2EL will be given, and [hence] arc 2EL. We will subtract half of this, namely arc EL, which comprises the above-mentioned difference [between rising-times at *sphaera recta* and *sphaera obliqua*], from the rising-time of the ecliptic arc in question at *sphaera recta*, and thus obtain the rising-time of the same arc at the given latitude.

As an example, let us again take the latitude of the parallel through Rhodes. H131 Here

arc 2EΘ = 37;30°, so Crd arc 2EΘ ≈ 38;34^p.

Then since 60 : 38;34 = 9;33 : 6;8
= 18;57 : 12;11
= 28;1 : 18;0
= 36;33 : 23;29[78]
= 44;12 : 28;25
= 50;44 : 32;37
= 55;45 : 35;52[79]
= 58;55 : 37;52,

[77] Computed from Ptolemy's figures: 36;31,42. For the arc 40° above, a more accurate value for Crd arc 2KZ would be 115;52,26^p. However, substituting that leads to 36;31,40 here. In either case, 36;32 would be the correct result to the nearest minute. This is the reading of Ger, but the rest of the tradition is unanimous for 36;33.

[78] Accurate computation with 36;33 here gives 23;29,36, while 36;32 (see n.77) gives 23;28,58. This speaks in favour of the reading 36;32, but not decisively.

[79] Computed: 35;50,6. However 35;52 is guaranteed by 17;24 for the seventh 10° arc below (35;50 leads to 17;23°).

and since Crd arc 2EL equals the above amount [6;8^p, etc.] at each of the above-mentioned 10° intervals, half of the arc it subtends, namely arc EL, will assume the following values:

for the first 10°	2;56°
up to the end of the second	5;50°
up to the end of the third	8;38°
up to the end of the fourth	11;17°
up to the end of the fifth	13;42°
up to the end of the sixth	15;46°
up to the end of the seventh	17;24°
up to the end of the eighth	18;24°
up to the end of the ninth, obviously,	18;45°.

Since the corresponding rising-times at *sphaera recta* are as follows:

for the first 10°	9;10°
up to the end of the second	18;25°
up to the end of the third	27;50°
H132 up to the end of the fourth	37;30°
up to the end of the fifth	47;28°
up to the end of the sixth	57;44°
up to the end of the seventh	68;18°
up to the end of the eighth	79;5°
and up to the end of the ninth	90° (the time-degrees of the whole quadrant),

it is clear that by subtracting the difference, given by the arc EL, from the corresponding rising-time at *sphaera recta* in each case, we get the rising-times of the same arcs at the latitude in question. These are

for the first 10°	6;14°
up to the end of the second	12;35°
up to the end of the third	19;12°
up to the end of the fourth	26;13°
up to the end of the fifth	33;46°
up to the end of sixth	41;58°
up to the end of the seventh	50;54°
up to the end of the eighth	60;41°
up to the end of the ninth	71;15°
(i.e. for the whole quadrant)	(which corresponds to the length of half of the [shortest] day).

The ten-degree segments will rise in the following time-degrees:

1st	6;14°
2nd	6;21°
3rd	6;37°
4th	7;1°
5th	7;33°
6th	8;12°
7th	8;56°

8th	9;47°
9th	10;34°.

Once we have established the above, the corresponding rising-times of the remaining quadrants will immediately be established on the same basis, by means of the theorems set out above. H133

In the same way we calculated the rising-times at every 10° interval for all other parallels which one might come upon in actual practice. For future use we shall set these out in tabular form, beginning with the parallel directly beneath the equator, and going as far as the parallel with a longest day of 17 hours. The parallels are taken at intervals of ½-hour [of longest day], since the difference [of exact computations] from results derived from linear interpolation [between half-hour intervals] is negligible. In the first column we put the 36 ten-degree intervals of the circle, in the next the corresponding time-degrees of the rising-time of that 10-degree arc at the latitude in question, and in the third the accumulated sum, as follows.

8. {*Table of rising-times at ten-degree intervals*}[80] H134—41

[See pp. 100-3.]

9. {*On the particular features which follow from the rising-times*}[81] H142

Now that we have set out the rising-times in the above manner, all the other problems associated with this subject will be easily soluble, and we shall not need to go through geometrical proofs or construct special tables to solve each problem. This will become clear from the actual methods described below.

First, one can find the length of a given day or night as follows. Take the rising-times of the appropriate latitude; for the day, count from the degree in which the sun is to the degree diametrically opposite, going towards the rear through the signs; for the night, count from the degree opposite the sun to the sun's degree. Form the sum of the rising-times [of the relevant 180°], and divide by 15: this will give the relevant interval in equinoctial hours. If we take $\frac{1}{12}$th [of the sum of the rising-times] we will have the length of the seasonal hour of that interval [i.e. day or night] in time-degrees.

One can also find the length of the [seasonal] hour more conveniently by taking, from the above Table of Rising-times [II 8], the total rising-time corresponding to the sun's degree for the day (or the degree opposite the sun for the night) both at the parallel beneath the equator [i.e. *sphaera recta*] and at the relevant latitude, and forming the difference. Take $\frac{1}{6}$th of the latter, and add it to the 15 time-degrees of one equinoctial hour for points on the northern semicircle [of the ecliptic], or subtract it from 15° for points on the southern semicircle: the result will be the length of the relevant seasonal hour in time-degrees.[82] H143

[80] Correction to text: at H138,2 (latitude for M = 16^h) read $\overline{\mu\eta}\ \overline{\lambda\beta}$ (with Ar) for $\overline{\mu\eta}$ (48°). Cf. II 6 p. 87.

[81] See *HAMA* 40-3 (with worked examples) and Pedersen 113-15.

[82] See Appendix A, Example 2.

TABLE OF RISING-TIMES AT 10° INTERVALS

SIGNS	10° Inter-vals	SPHAERA RECTA 12^h ° ′	$0°$ Accumulated Time-Degrees	AVALITE GULF $12\frac{1}{2}^h$ ° ′	$8;25°$ Accumulated Time-Degrees	MEROE 13^h ° ′	$16;27°$ Accumulated Time-Degrees
ARIES	10	9 10	9 10	8 35	8 35	7 58	7 58
	20	9 15	18 25	8 39	17 14	8 5	16 3
	30	9 25	27 50	8 52	26 6	8 17	24 20
TAURUS	10	9 40	37 30	9 8	35 14	8 36	32 56
	20	9 58	47 28	9 29	44 43	9 1	41 57
	30	10 16	57 44	9 51	54 34	9 27	51 24
GEMINI	10	10 34	68 18	10 15	64 49	9 56	61 20
	20	10 47	79 5	10 35	75 24	10 23	71 43
	30	10 55	90 0	10 51	86 15	10 47	82 30
CANCER	10	10 55	100 55	10 59	97 14	11 3	93 33
	20	10 47	111 42	10 59	108 13	11 11	104 44
	30	10 34	122 16	10 53	119 6	11 12	115 56
LEO	10	10 16	132 32	10 41	129 47	11 5	127 1
	20	9 58	142 30	10 27	140 14	10 55	137 56
	30	9 40	152 10	10 12	150 26	10 44	148 40
VIRGO	10	9 25	161 35	9 58	160 24	10 33	159 13
	20	9 15	170 50	9 51	170 15	10 25	169 38
	30	9 10	180 0	9 45	180 0	10 22	180 0
LIBRA	10	9 10	189 10	9 45	189 45	10 22	190 22
	20	9 15	198 25	9 51	199 36	10 25	200 47
	30	9 25	207 50	9 58	209 34	10 33	211 20
SCORPIUS	10	9 40	217 30	10 12	219 46	10 44	222 4
	20	9 58	227 28	10 27	230 13	10 55	232 59
	30	10 16	237 44	10 41	240 54	11 5	244 4
SAGITTARIUS	10	10 34	248 18	10 53	251 47	11 12	255 16
	20	10 47	259 5	10 59	262 46	11 11	266 27
	30	10 55	270 0	10 59	273 45	11 3	277 30
CAPRICORNUS	10	10 55	280 55	10 51	284 36	10 47	288 17
	20	10 47	291 42	10 35	295 11	10 23	298 40
	30	10 34	302 16	10 15	305 26	9 56	308 36
AQUARIUS	10	10 16	312 32	9 51	315 17	9 27	318 3
	20	9 58	322 30	9 29	324 46	9 1	327 4
	30	9 40	332 10	9 8	333 54	8 36	335 40
PISCES	10	9 25	341 35	8 52	342 46	8 17	343 57
	20	9 15	350 50	8 39	351 25	8 5	352 2
	30	9 10	360 0	8 35	360 0	7 58	360 0

SIGNS	10° Inter-vals	SOENE 13½ʰ	23;51°	LOWER EGYPT 14ʰ	30;22°	RHODES 14½ʰ	36;0°
		° ′	Accumulated Time-Degrees	° ′	Accumulated Time-Degrees	° ′	Accumulated Time-Degrees
ARIES	10	7 23	7 23	6 48	6 48	6 14	6 14
	20	7 29	14 52	6 55	13 43	6 21	12 35
	30	7 45	22 37	7 10	20 53	6 37	19 12
TAURUS	10	8 4	30 41	7 33	28 26	7 1	26 13
	20	8 31	39 12	8 2	36 28	7 33	33 46
	30	9 3	48 15	8 37	45 5	8 12	41 58
GEMINI	10	9 36	57 51	9 17	54 22	8 56	50 54
	20	10 11	68 2	10 0	64 22	9 47	60 41
	30	10 43	78 45	10 38	75 0	10 34	71 15
CANCER	10	11 7	89 52	11 12	86 12	11 16	82 31
	20	11 23	101 15	11 34	97 46	11 47	94 18
	30	11 32	112 47	11 51	109 37	12 12	106 30
LEO	10	11 29	124 16	11 55	121 32	12 20	118 50
	20	11 25	135 41	11 54	133 26	12 23	131 13
	30	11 16	146 57	11 47	145 13	12 19	143 32
VIRGO	10	11 5	158 2	11 40	156 53	12 13	155 45
	20	11 1	169 3	11 35	168 28	12 9	167 54
	30	10 57	180 0	11 32	180 0	12 6	180 0
LIBRA	10	10 57	190 57	11 32	191 32	12 6	192 6
	20	11 1	201 58	11 35	203 7	12 9	204 15
	30	11 5	213 3	11 40	214 47	12 13	216 28
SCORPIUS	10	11 16	224 19	11 47	226 34	12 19	228 47
	20	11 25	235 44	11 54	238 28	12 23	241 10
	30	11 29	247 13	11 55	250 23	12 20	253 30
SAGITTARIUS	10	11 32	258 45	11 51	262 14	12 12	265 42
	20	11 23	270 8	11 34	273 48	11 47	277 29
	30	11 7	281 15	11 12	285 0	11 16	288 45
CAPRICORNUS	10	10 43	291 58	10 38	295 38	10 34	299 19
	20	10 11	302 9	10 0	305 38	9 47	309 6
	30	9 36	311 45	9 17	314 55	8 56	318 2
AQUARIUS	10	9 3	320 48	8 37	323 32	8 12	326 14
	20	8 31	329 19	8 2	331 34	7 33	333 47
	30	8 4	337 23	7 33	339 7	7 1	340 48
PISCES	10	7 45	345 8	7 10	346 17	6 37	347 25
	20	7 29	352 37	6 55	353 12	6 21	353 46
	30	7 23	360 0	6 48	360 0	6 14	360 0

SIGNS	10° Inter-vals	HELLESPONT 15^h 40;56° ° ′	Accumulated Time-Degrees	MIDDLE OF PONTUS $15\frac{1}{2}^h$ 45;1° ° ′	Accumulated Time-Degrees	MOUTHS OF BORYSTHENES 16^h 48;32° ° ′	Accumulated Time-Degrees
	10	5 40	5 40	5 8	5 8	4 36	4 36
ARIES	20	5 47	11 27	5 14	10 22	4 43	9 19
	30	6 5	17 32	5 33	15 55	5 1	14 20
	10	6 29	24 1	5 58	21 53	5 26	19 46
TAURUS	20	7 4	31 5	6 34	28 27	6 5	25 51
	30	7 46	38 51	7 20	35 47	6 52	32 43
	10	8 38	47 29	8 15	44 2	7 53	40 36
GEMINI	20	9 32	57 1	9 19	53 21	9 5	49 41
	30	10 29	67 30	10 24	63 45	10 19	60 0
	10	11 21	78 51	11 26	75 11	11 31	71 31
CANCER	20	12 2	90 53	12 15	87 26	12 29	84 0
	30	12 30	103 23	12 53	100 19	13 15	97 15
	10	12 46	116 9	13 12	113 31	13 40	110 55
LEO	20	12 52	129 1	13 22	126 53	13 51	124 46
	30	12 51	141 52	13 22	140 15	13 54	138 40
	10	12 45	154 37	13 17	153 32	13 49	152 29
VIRGO	20	12 43	167 20	13 16	166 48	13 47	166 16
	30	12 40	180 0	13 12	180 0	13 44	180 0
	10	12 40	192 40	13 12	193 12	13 44	193 44
LIBRA	20	12 43	205 23	13 16	206 28	13 47	207 31
	30	12 45	218 8	13 17	219 45	13 49	221 20
	10	12 51	230 59	13 22	233 7	13 54	235 14
SCORPIUS	20	12 52	243 51	13 22	246 29	13 51	249 5
	30	12 46	256 37	13 12	259 41	13 40	262 45
	10	12 30	269 7	12 53	272 34	13 15	276 0
SAGITTARIUS	20	12 2	281 9	12 15	284 49	12 29	288 29
	30	11 21	292 30	11 26	296 15	11 31	300 0
	10	10 29	302 59	10 24	306 39	10 19	310 19
CAPRICORNUS	20	9 32	312 31	9 19	315 58	9 5	319 24
	30	8 38	321 9	8 15	324 13	7 53	327 17
	10	7 46	328 55	7 20	331 33	6 52	334 9
AQUARIUS	20	7 4	335 59	6 34	338 7	6 5	340 14
	30	6 29	342 28	5 58	344 5	5 26	345 40
	10	6 5	348 33	5 33	349 38	5 1	350 41
PISCES	20	5 47	354 20	5 14	354 52	4 43	355 24
	30	5 40	360 0	5 8	360 0	4 36	360 0

SIGNS	10° Intervals	SOUTHERNMOST BRITTANIA $16\frac{1}{2}^h$ ° ′	51;30° Accumulated Time-Degrees	MOUTHS OF TANAIS 17^h ° ′	54;1° Accumulated Time-Degrees
	10	4 5	4 5	3 36	3 36
ARIES	20	4 12	8 17	3 43	7 19
	30	4 31	12 48	4 0	11 19
	10	4 56	17 44	4 26	15 45
TAURUS	20	5 34	23 18	5 4	20 49
	30	6 25	29 43	5 56	26 45
	10	7 29	37 12	7 5	33 50
GEMINI	20	8 49	46 1	8 33	42 23
	30	10 14	56 15	10 7	52 30
	10	11 36	67 51	11 43	64 13
CANCER	20	12 45	80 36	13 1	77 14
	30	13 39	94 15	14 3	91 17
	10	14 7	108 22	14 36	105 53
LEO	20	14 22	122 44	14 52	120 45
	30	14 24	137 8	14 54	135 39
	10	14 19	151 27	14 50	150 29
VIRGO	20	14 18	165 45	14 47	165 16
	30	14 15	180 0	14 44	180 0
	10	14 15	194 15	14 44	194 44
LIBRA	20	14 18	208 33	14 47	209 31
	30	14 19	222 52	14 50	224 21
	10	14 24	237 16	14 54	239 15
SCORPIUS	20	14 22	251 38	14 52	254 7
	30	14 7	265 45	14 36	268 43
	10	13 39	279 24	14 3	282 46
SAGITTARIUS	20	12 45	292 9	13 1	295 47
	30	11 36	303 45	11 43	307 30
	10	10 14	313 59	10 7	317 37
CAPRICORNUS	20	8 49	322 48	8 33	326 10
	30	7 29	330 17	7 5	333 15
	10	6 25	336 42	5 56	339 11
AQUARIUS	20	5 34	342 16	5 4	344 15
	30	4 56	347 12	4 26	348 41
	10	4 31	351 43	4 0	352 41
PISCES	20	4 12	355 55	3 43	356 24
	30	4 5	360 0	3 36	360 0

Next, one can convert seasonal hours for a given date into equinoctial hours by multiplying them by the length in time-degrees of the hour of the day in question at the relevant latitude (if they are hours of the day), or by the length in time-degrees of the hour of the night in question (if they are hours of the night). Then division of that product by 15 will give the total of equinoctial hours. *Vice versa*, one can convert equinoctial hours to seasonal by multiplying by 15 and dividing by the length of the hour of the relevant interval in time-degrees.[83]

Furthermore, given a date and any time whatever, expressed in seasonal hours, on that date, we can find, first, the degree of the ecliptic rising at that moment. We do this by multiplying the number of hours, counted from sunrise by day, and from sunset by night, by the relevant length of the [seasonal] hour
H144 in time-degrees. We add this product to the rising-time at the latitude in question of the sun's degree by day (or the degree opposite the sun by night): the degree [of the ecliptic] with rising-time corresponding to the total will be rising at that moment.[84]

[Secondly], if we want to find the point at upper culmination [at the given moment], we take in every case [i.e. for both day and night] the total of seasonal hours from the last midday to the given time, multiply it by the appropriate length(s) of the hour(s) in time-degrees, and add the product to the rising-time at *sphaera recta* of the sun's degree: the degree [of the ecliptic] with rising-time at *sphaera recta* equal to the total will be at upper culmination at that moment.[85]

Similarly, we can find the culminating point from the rising point as follows: find from the table of rising-times for the relevant latitude the cumulative rising-times corresponding to the degree which is rising. Subtract from it, in every case, the 90° of the quadrant [of the equator between horizon and meridian]. The degree corresponding to the result in the column for rising-times at *sphaera recta* will be at upper culmination at that moment.[86] *Vice versa*, one can find the rising point from the culminating point by taking the degree corresponding to the culminating point in the column for rising-times at *sphaera*
H145 *recta*, adding to it, in every case, the above 90°, and finding the degree corresponding to the result in the column for rising-times for the latitude in question: this degree will be rising at that moment.

It is also obvious that for those living beneath the same meridian the sun is the same distance from noon or midnight, counted in equinoctial hours, while for those living beneath different meridians the sun's distance from noon or midnight differs by an amount, counted in time-degrees, equal to the distance of one meridian from the other in degrees.

[83] See Appendix A, Example 3.

[84] This sentence, like the corresponding one in the next problem, is a paraphrase giving the sense of Ptolemy's ambiguous expression. Literally 'we count off this product towards the rear through the signs, beginning from the sun's degree . . . by night, according to the rising-times of the latitude in question: we say that whatever degree this amount reaches is the degree rising at that moment'. See Appendix A, Example 4.

[85] See Appendix A, Example 5.

[86] See Appendix A, Example 6.

10. {*On the angles between the ecliptic and the meridian*}[87]

The remaining topic in the present theory is the discussion of angles formed at the ecliptic. We must first make clear that we define an angle between [two] great circles as follows: we say that [two] great circles form a right angle when a circle having as pole the intersection of the great circles and as radius any distance whatever has [exactly] a quadrant intercepted between the segments of the great circles forming the angle; in general, whatever ratio the intercepted H146
arc of a circle described in the above manner bears to the whole circle is the same as the ratio of the angle between the planes [of the two great circles] to 4 right angles. Thus, since we set the circumference of the circle as 360°, the angle subtending the intercepted arc will contain the same number of degrees as the arc, in the system where one right angle contains 90°.

For the purposes of our present investigation, the most useful of the angles at the ecliptic are those formed by

[1] the intersection of the ecliptic and the meridian,

[2] the intersection of the ecliptic and the horizon for all positions [of the ecliptic], and

[3] the intersection of the ecliptic and a great circle drawn through the poles of the horizon [i.e. an altitude circle];

the process of finding the latter will also produce the arc of this [altitude] circle cut off between its intersection with the ecliptic and the pole of the horizon, i.e. the zenith. Computation of each of the above angles, besides being a most suitable topic for the theory proper, also plays a very important part in the requirements for lunar parallax: it is impossible to make any progress in that subject without having first understood how to compute these angles.

Now there are four angles at the intersection of the two circles (I mean the H147
ecliptic and any of the [above] circles meeting it). Since we shall [always] discuss only one of these, which always occupies the same relative position, we must make the following preliminary definition. In general, when we demonstrate in what follows the characteristics and size of an angle, we refer to that angle [of the four possible] which lies to the rear of the intersection of the circles and to the north of the ecliptic.[88]

The computation of the angles between the meridian and the ecliptic is simpler, so we shall start with that, and first we shall show that points on the ecliptic equidistant from the same equinox produce angles of the above kind equal to each other.

[See Fig. 2.9.] Let ABG be an arc of the equator, DBE an arc of the ecliptic, and Z the pole of the equator. Cut off equal arcs, BH and BΘ, on opposite sides of the equinox B, and draw through pole Z and points H, Θ the meridian arcs ZKH and ZΘL. I say that H148

$$\angle \text{KHB} = \angle \text{Z}\Theta\text{E}. \qquad [10.1]$$

[Proof:] This is immediately obvious. For the spherical triangle BHK has all its

[87] On chapters 10 and 11 see *HAMA* 45–8, Pedersen 115–18.

[88] Literally 'that one of the two angles on the arc to the rear of the intersection of the circles which is to the north of the ecliptic'. See *HAMA* 45 with Fig. 38.

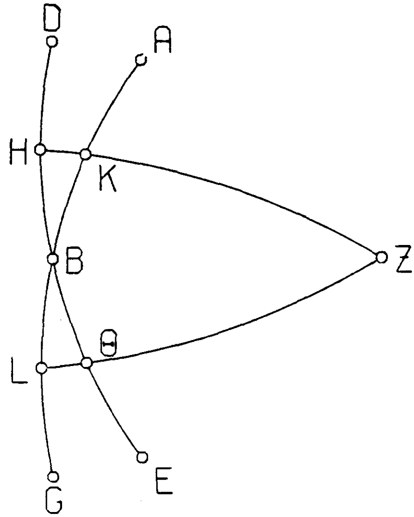

Fig. 2.9

angles equal to the angles of spherical triangle BΘL, since the three corresponding sides in each triangle are equal, HB to BΘ, HK to ΘL, and BK to BL. All this has been proven previously.[89]

Therefore ∠ KHB = ∠ BΘL = ∠ ZΘE.

Q.E.D.

Secondly, we must prove that the sum of the angles between ecliptic and meridian at points on the ecliptic equidistant from the same solstice is equal to two right angles.

[See Fig. 2.10.] Let ABG be an arc of the ecliptic, with B taken as solstice. Let equal arcs, BD and BE, be taken on opposite sides of it, and draw through Z, the
H149 pole of the equator, and points D, E the meridian arcs ZD and ZE. I say that

∠ ZDB + ∠ ZEG = 2 right angles [10.2]

[Proof:] This too is immediately obvious. For since points D and E are equidistant from the same solstice,

arc DZ = arc ZE.

∴ ∠ ZDB = ∠ ZEB.

But ∠ ZEB + ∠ ZEG = 2 right angles.

∴ ∠ ZDB + ∠ ZEG = 2 right angles.

Q.E.D.

Having established these preliminary theorems, let us draw [Fig. 2.11] the meridian circle ABGD and the semi-circle of the ecliptic AEG (taking A as the winter solstice); then with pole A and radius the side of the [inscribed] square draw semi-circle BED. Then, since meridian ABGD goes through the poles of AEG and the poles of BED, arc ED is a quadrant.[90]

[89]HB = BΘ by construction; HK = ΘL, declinations of points equidistant from an equinox (cf. p. 80 n.15); BK = BL, cf. II 7 (arc EΘ = arc EZ p 91).

[90] Derivable from Theodosius *Sphaerica* II 9.

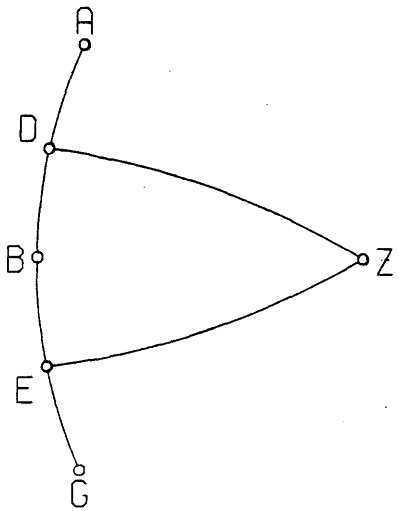

Fig. 2.10

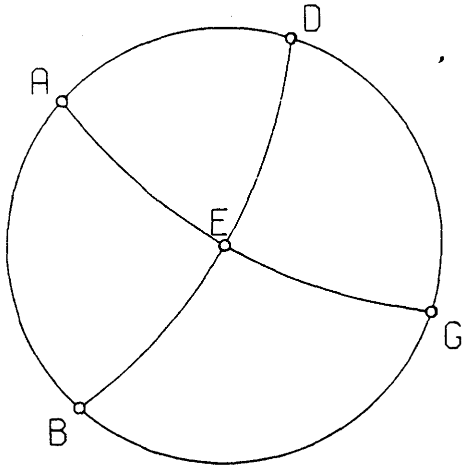

Fig. 2.11

Therefore ∠ DAE is right.

And the angle at the summer solstice is also right, from the previous theorem [10.2].

Q.E.D.

Again, [see Fig.2.12] let ABGD be a meridian circle, AEG a semi-circle of the equator, and AZG a semi-circle of the ecliptic in such a position that A is the autumnal equinox. Then with pole A and radius the side of the [inscribed] square draw semi-circle BZED. H150

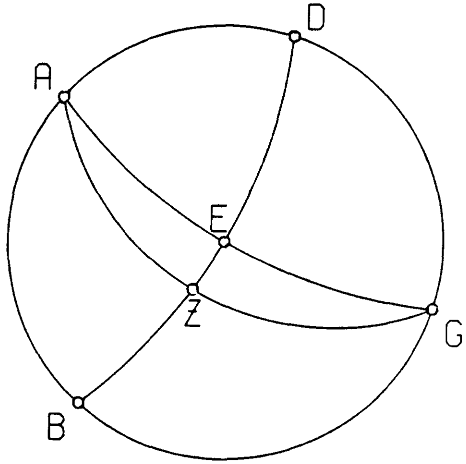

Fig. 2.12

By the same reasoning [as above], since ABGD goes through the poles of [circles] AEG and BED, AZ and ED are quadrants. Hence point Z is the winter solstice, and

arc ZE ≈ 23;51°, as was shown previously [I 12 p. 63].

Therefore, by addition, arc ZED = 113;51°

and ∠ DAZ = 113;51° where one right angle = 90°.

And again, from the previous theorem [10.2], the angle at the spring equinoctial point is the supplement, 66;9°.

Again [see Fig. 2.13] let ABGD be a meridian circle, AEG a semi-circle of the equator, and BZD a semi-circle of the ecliptic in such a position that point Z is

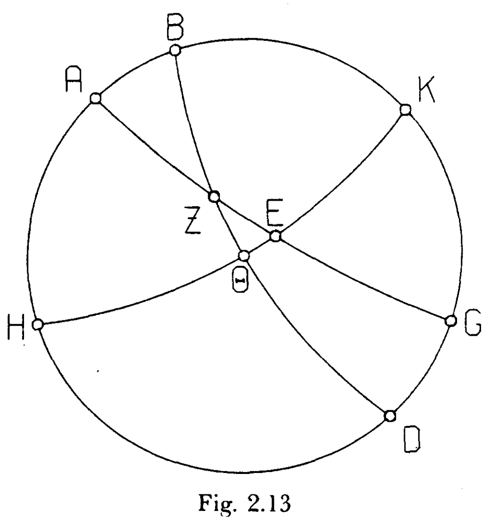

Fig. 2.13

the autumnal equinox, and arc BZ is (first of all) the length of one sign, that of Virgo; thus point B, obviously, is the beginning of Virgo. Again, with pole B and radius the side of the [inscribed] square, draw semi-circle HΘEK. H151

Let the problem be to find ∠ KBΘ.

Now since meridian ABGD goes through the poles of [circles] AEG and HEK, arc BH, arc BΘ and arc EH are all quadrants.

And, from the figure,

Crd arc 2BA:Crd arc 2AH =

(Crd arc 2BZ:Crd arc 2ΘZ). (Crd arc 2ΘE:Crd arc 2EH). [M.T. II]

But, as was shown previously,[91] arc 2BA = 23;20°, so Crd arc 2BA = 24;16^p,

arc 2AH = 156;40°, so Crd arc 2AH = 117;31^p,

and arc 2ZB = 60°, so Crd arc 2ZB ≈ 60^p, H152

arc 2ZΘ = 120°, so Crd arc 2ZΘ = 103;55,23^p.

∴ Crd arc 2ΘE:Crd arc 2EH = (24;16 : 117;31)/(60 : 103;55,23)

≈ 42;58 : 120.

But Crd arc 2EH = 120^p.

∴ Crd arc 2ΘE ≈ 42;58^p

∴ arc 2ΘE ≈ 42°

and arc ΘE ≈ 21°.[92]

Therefore, by addition [of a quadrant] arc ΘEK = ∠ KBΘ = 111°, and the angle at the beginning of Scorpius is also 111°, and the angles at the beginning of Taurus and Pisces are each 69°, the supplement, by the theorems proved above [10.1 and 10.2].

Q.E.D.

Next, in the same figure [2.13], let arc ZB represent two signs, so that point B is the beginning of Leo. Then, with the [other] quantities remaining the same,

arc 2BA = [2δ(60°)=] 41°, so Crd arc 2BA = 42;2^p

and arc 2AH = 139°, so Crd arc 2AH = 112;24^p;

furthermore arc 2ZB = 120°, so Crd arc 2ZB = 103;55,23^p H153

and arc 2ZΘ = 60°, so Crd arc 2ZΘ = 60^p.

∴ Crd arc 2ΘE:Crd arc 2EH = (42;2 : 112;24)/(103;55,23 : 60)

= 25;53 : 120.

∴ Crd arc 2ΘE = 25;53^p

∴ arc 2ΘE ≈ 25°

and arc ΘE ≈ 12½°.[93]

Therefore, by addition, arc ΘEK = ∠ KBΘ = 102½°.

Therefore the angle at the beginning of Sagittarius is also 102½°, and the angle at both the beginning of Gemini and the beginning of Aquarius is the supplement, 77½°.

We have [thus] calculated what we set out to do. It is sufficient for practical use to display [the results] for each sign, although the same procedure would apply to even smaller sections of the ecliptic.

[91] Reference to II 7 p. 93. The quantities are rounded here.

[92] Accurate computation would give 20;58° to the nearest minute.

[93] Accurate computation would give 12;28° to the nearest minute.

H154 ## 11. {*On the angles between the ecliptic*[94] *and the horizon*}

Next we shall show how to calculate, for any given latitude, the angles formed by the ecliptic at the horizon. These too can be derived by a procedure which is simpler than that for the remaining angles [between ecliptic and altitude circles].

Now it is obvious that the angles [between ecliptic and] meridian are the same as those [between ecliptic and] horizon at *sphaera recta*. But, in order to calculate these angles also at *sphaera obliqua*, we must first prove that points on the ecliptic equidistant from the same equinox produce equal angles at the same horizon.

[See Fig. 2.14.] Let ABGD be a meridian circle, AEG the semi-circle of the equator and BED the semi-circle of the horizon. Draw two segments of the ecliptic, ZHΘ and KLM, such that points Z and K both represent the autumnal equinox, and arc ZH equals arc KL.

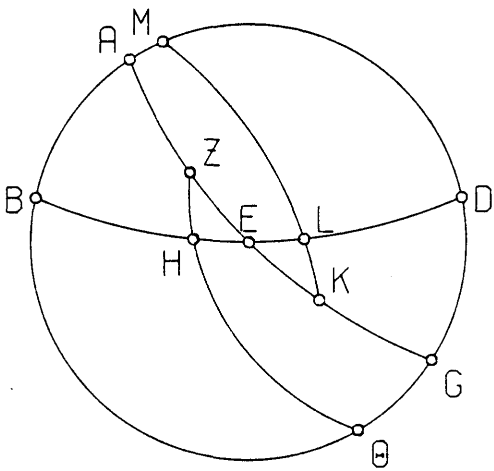

Fig. 2.14

H155 I say that ∠ EHΘ = ∠ DLK.

[Proof:] This is immediately obvious.

For spherical triangle EZH ≡ spherical triangle EKL, since, from what was proven above, the corresponding sides are equal:

ZH = KL

HE = EL ([arcs cut off by] the intersection of the horizon [with the ecliptic])

EZ = EK (rising-time arcs).[95]

∴ ∠ EHZ = ∠ ELK

∴ ∠ EHΘ = ∠ DLK (supplements).

Q.E.D.

[94] 'ecliptic': literally 'the same inclined circle'.

[95] ZH = KL by hypothesis; HE = EL from II 3 (p. 79); EZ = EK from II 7 (p. 91).

I also say that, if two points [of the ecliptic] are diametrically opposite, the sum of the angles [between ecliptic and horizon] at the rising-point of one and the setting-point of the other is equal to two right angles.
[Proof: see Fig. 2.15.] If we draw ABGD as the circle of the horizon, and AEGZ as the circle of the ecliptic, so that they intersect at A and G, then

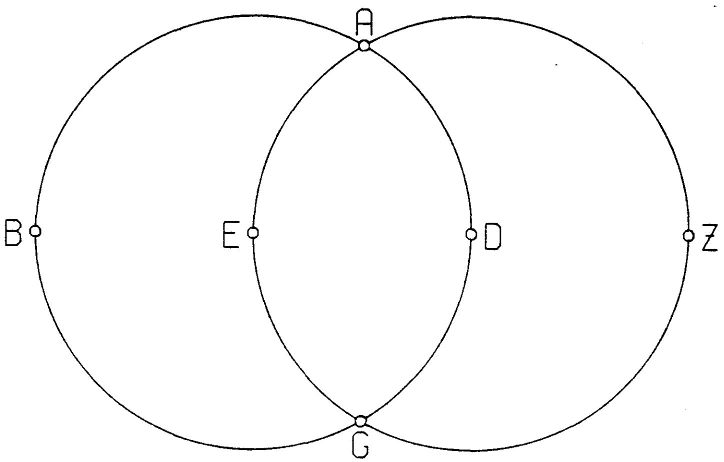

Fig. 2.15

$$\angle\, ZAD + \angle\, DAE = 2 \text{ right angles.}$$
$$\text{But } \angle\, ZAD = \angle\, ZGD$$ H156
$$\therefore \angle\, ZGD + \angle\, DAE = 2 \text{ right angles.}$$

Q.E.D.

Since this is so, and since we have also proven that angles at the same horizon formed by points [on the ecliptic] equidistant from the same equinox are equal, a further consequence will be that, for points equidistant from the same solstice, the sum of the rising-angle at one and the setting-angle at the other will be equal to two right angles.[96]

Hence, if we find the rising-angles from Aries to Libra [inclusive], we will simultaneously have found the rising-angles on the other semi-circle and the setting-angles on both semi-circles. We shall explain briefly how to do the calculation, again taking as example the same parallel, at which the elevation of the north pole from the horizon is 36°.

As for the angles between ecliptic and horizon at the equinoctial points, they can be calculated simply. For if [see Fig. 2.16] we draw ABGD as the meridian circle, AED as the eastern semi-circle of the horizon in question, EZ as a H157

[96] Proof: see Fig. E, in which the ecliptic EXT intersects the horizon SR in the setting-point S and the rising-point R. T is the solstice, E the equinox (hence ET = 90°) and the two points X and R are the same distance, d, from T. Then EX = TE − TX = 90° − d. ES = RS − RE = 180° − (90° + d) = 90° − d. ∴ EX = ES. Therefore setting-angle at X equals setting-angle at S (p. 110). But the sum of the angles at the rising-point R and the setting-point S is 2 right angles (p. 111). Therefore the sum of the rising-angle at R and the setting-angle at X equals 2 right angles.

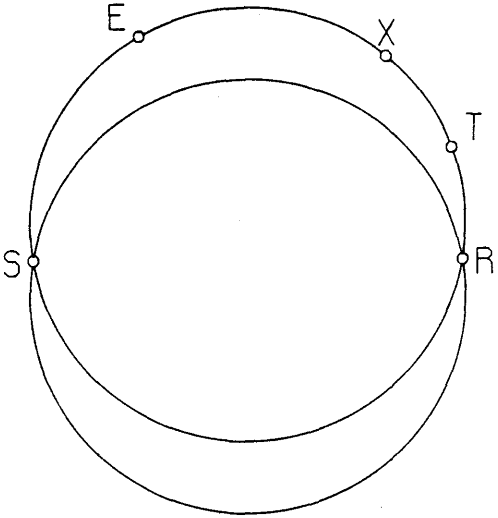

Fig. E

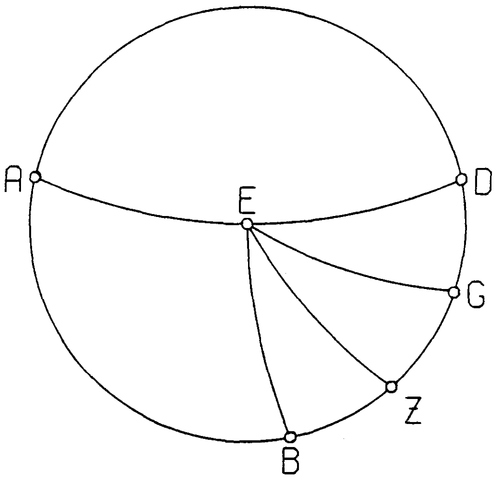

Fig. 2.16

quadrant of the equator, and EB and EG as two quadrants of the ecliptic such that point E is the autumnal equinox with respect to EB, and the spring equinox with respect to EG (thus B is the winter solstice and G the summer solstice), we can conclude as follows.

Ex hypothesi, arc DZ = 54° [colatitude of 36°]
and arc BZ = arc ZG ≈ 23;51°.
∴ arc GD = 30;9°
and arc BD = 77;51°.

Thus, since E is the pole of meridian ABG,

$\angle$ DEG, the angle at the beginning of Aries, is 30;9° } where 1 right
and $\angle$ DEB, the angle at the beginning of Libra, is 77;51° } angle = 90°.

In order to explain the procedure for finding the angles at other points, let us take, for example, the problem of finding the rising-angle formed at the beginning of Taurus and the horizon.

[See Fig. 2.17.] Let ABGD be the circle of the meridian, and BED the eastern semi-circle of the horizon in question. Draw semi-circle AEG of the ecliptic, so H158
that point E represents the beginning of Taurus. Now at this latitude, when the beginning of Taurus is rising, ♋ 17;41° is at lower culmination (we have shown

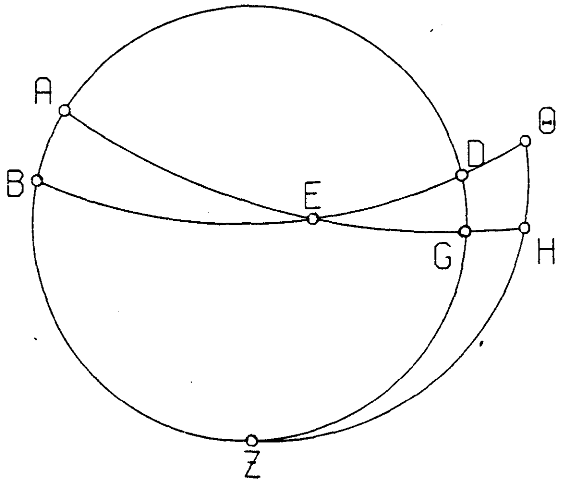

Fig. 2.17

how such a problem can readily be solved by means of the tabulated rising-times).[97] Therefore arc EG is less than a quadrant. So with pole E and radius the side of the [inscribed] square draw the great circle segment ΘHZ, and complete the quadrants EGH and EDΘ. Both DGZ and ZHΘ are also quadrants, because the horizon BEΘ goes through the poles of meridian ZGD and of the great circle ZHΘ. Furthermore, ♋ 17;41° is 22;40° north of the equator, measured along the great circle through the poles of the equator (we have set out a table [I 15] for that too); and the equator is 36° from pole Z of the horizon, measured along the same arc, ZGD. Therefore arc ZG = 58;40°. These quantities being given, it then follows from the figure that

Crd arc 2GD:Crd arc 2DZ =

(Crd arc 2GE:Crd arc 2EH). (Crd arc 2HΘ:Crd arc 2ZΘ). [M.T. I] H159

But, from the above,

arc 2GD = 62;40°, so Crd arc 2GD = 62;24^p,

arc 2DZ = 180°, so Crd arc 2DZ = 120^p,

[97] II 9 p. 104 (simply add 180° to the point of upper culmination, which is calculated for this example in *HAMA*, 42).

arc 2GE = 155;22°, so Crd arc 2GE = 117;14ᵖ,
arc 2EH = 180°, so Crd arc 2EH = 120ᵖ.
∴ Crd arc 2ΘH:Crd arc 2ZΘ = (62;24 : 120)/(117;14 : 120)
= 63;52 : 120.
And Crd arc 2ΘZ = 120ᵖ.
∴ Crd arc 2HΘ = 63;52ᵖ
∴ arc 2HΘ = 64;20°
and arc HΘ = ∠ HEΘ = 32;10°.

Q.E.D.

To avoid lengthening the explanatory part of this treatise by continual repetition of the procedure, we will take the same method for granted for the remaining signs and latitudes.[98]

H160 12. {*On the angles and arcs formed with the same circle* [*i.e. the ecliptic*] *by a circle drawn through the poles of the horizon*}[99]

It remains [to describe] the method by which we can compute the angles formed between the ecliptic and a circle through the poles of the horizon [i.e. an altitude circle] for any latitude and any position [of the ecliptic relative to the altitude circle]. As we said, this method also produces the size of the arc of the circle through the poles of the horizon cut off between the zenith and the intersection of that circle with the ecliptic. We shall again set out the preliminary theorems for this topic too: we shall prove, first, that if two points of the ecliptic are equidistant from the same solstice, and cut off an equal number of time-degrees on either side of the meridian, one to the east and the other to the west, then the great circle arcs from the zenith to those two points are equal, and the sum of the [two] angles at those points, chosen according to our [previous] definition,[100] is equal to two right angles.

[See Fig. 2.18.] Let ABG be a segment of the meridian, with point B on it taken as the zenith, and point G as the pole of the equator. Draw two segments
H161 of the ecliptic, ADE and AZH, such that points D and Z are equidistant from the same solstice, and cut off, on either side of meridian ABG, equal arcs of the parallel circle which passes through them. Furthermore, draw through points D and Z the following great circle arcs: arc GD and arc GZ from the pole of the equator G, and arc BD and arc BZ from the zenith B.

I say that

arc BD = arc BZ
and ∠ BDE + ∠ BZA = 2 right angles.

[Proof:] Since points D and Z cut off equal arcs of the parallel circle through them on either side of meridian ABG,

∠ BGD = ∠ BGZ.

[98] The angles between ecliptic and horizon are not explicitly tabulated by Ptolemy, but can be derived from the angles between ecliptic and altitude circle at the rising-point tabulated in Table II 13. See *HAMA* 47, which also tabulates them explicitly.
[99] See *HAMA* 48–52, Pedersen 118–21 (with my correction, Toomer[3] 139).
[100] II 10 p. 105, with n.88.

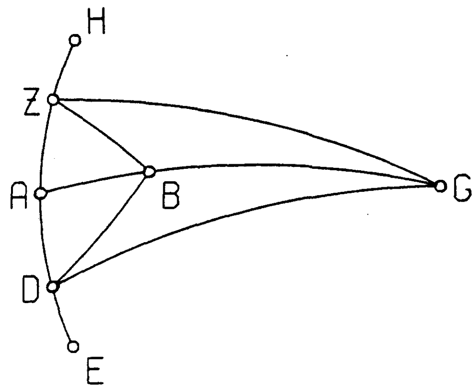

Fig. 2.18

Therefore, in the two spherical triangles BGD, BGZ

GD = GZ [D, Z equidistant from solstice]

BG = BG (common)

and ∠ BGD = ∠ BGZ,

so they have two sides and the included angle equal.

∴ BD = BZ (bases)

and ∠ BZG = ∠ BDG. H162

But since we showed just above that the sum of the two angles formed by a circle through the poles of the equator at points [of the ecliptic] equidistant from the same solstice is equal to two right angles [10.2],

∠ GDE + ∠ GZA = 2 right angles.

But we proved that ∠ BDG = ∠ BZG.

∴ ∠ BDE + ∠ BZA = 2 right angles.[101]

Q.E.D.

Next we must prove that if we take the same point of the ecliptic at two positions equidistant from the meridian (as measured in time-degrees) on opposite sides of it, the great-circle arcs from the zenith to these two positions are equal, and the sum of the two angles [between altitude circle and ecliptic] east and west [of the meridian] is equal to twice the angle formed by the same point [of the ecliptic] at the meridian, provided that for both positions [i.e. when the point is east and west of the meridian] the points [of the ecliptic] which are [then] culminating are either both north or both south of the zenith.

Let us suppose, first, that both are south. [See Fig. 2.19.] Let ABGD be a segment of the meridian, with point G on it as the zenith, and D as the pole of the equator. Draw two segments of the ecliptic, AEZ and BHΘ, such that points E and H represent the same point, and cut off equal arcs of the parallel circle through that point on opposite sides of meridian ABGD. Again, draw through them [points E and H] the great-circle arcs GE and GH from G, and DE and H163

[101] For ∠ BDE = ∠ GDE + ∠ BDG; ∠ BZA = ∠ GZA − ∠ BZG. So, by addition (since ∠ BDG = ∠ BZG), ∠ BDE + ∠ BZA = ∠ GDE + ∠ GZA = 2 right angles.

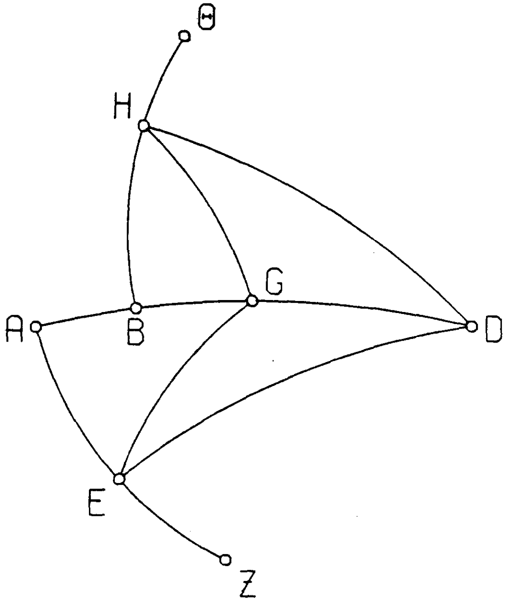

Fig. 2.19

DH from D. By the same reasoning as before, since points E and H generate the same parallel circle and cut off equal arcs of it on either side of the meridian,

spherical triangle GDE ≡ spherical triangle GDH.

∴ arc GE = arc GH.

Then I say that

$$\angle GEZ + \angle GHB = 2\angle DEZ = 2\angle DHB.$$

[Proof:] Since ∠ DEZ is the same as ∠ DHB [E and H the same point]

and ∠ GED = ∠ DHG [from congruent spherical triangles],

H164 ∠ GED + ∠ GHB[= ∠ DHG + ∠ GHB = ∠ DHB] = ∠ DEZ.

Therefore, by addition ∠ GEZ + ∠ GHB = 2 ∠ DEZ = 2 ∠ DHB

Q.E.D.

Next, draw the same segments of the above circles again [Fig. 2.20], except that points A and B should be north of point G. I say that here too the same will apply, namely

$$\angle KEZ + \angle LHB = 2\angle DEZ.$$

[Proof:] Since ∠ DEZ is the same as ∠ DHB,

and ∠ DEK = ∠ DHL [supplements of equal angles DEG, DHG],

by addition [of ∠ DHB to ∠ DHL], ∠ LHB = ∠ DEZ + ∠ DEK.

∴ ∠ LHB + ∠ KEZ = 2 ∠ DEZ.

Now again draw a similar figure [Fig. 2.21], except that the culminating point on the segment [of the ecliptic] east [of the meridian], namely A, should be
H165 south of the zenith G, while the culminating point on the segment west [of the meridian], namely B, should be north of the zenith.

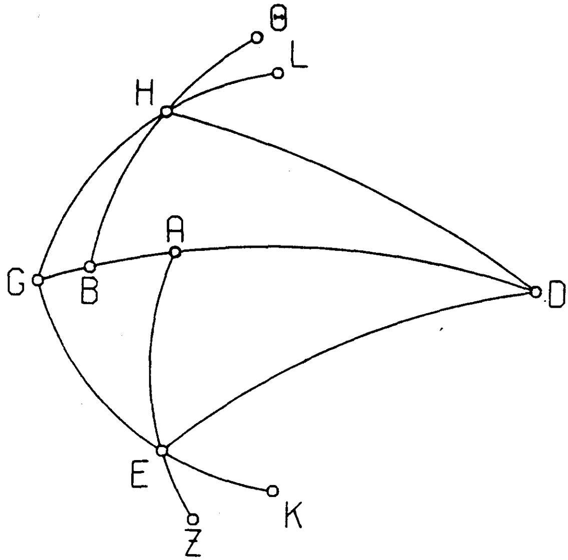

Fig. 2.20

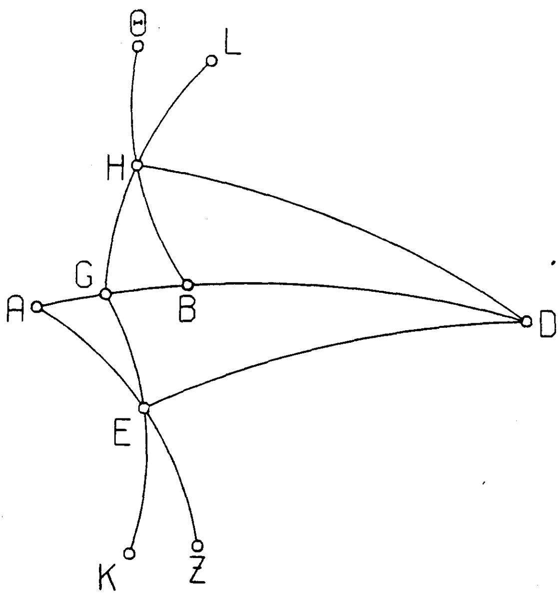

Fig. 2.21

I say that

$$\angle GEZ + \angle LHB = 2\angle DEZ \text{ plus 2 right angles.}$$

[Proof:] Since

$$\angle DHG = \angle DEG$$

and $\angle DHG + \angle DHL = 2$ right angles,

$\therefore \angle DEG + \angle DHL = 2$ right angles.

But $\angle DEZ$ is the same as $\angle DHB$.

$$\begin{aligned}\therefore \angle GEZ + \angle LHB &[= (\angle DEZ + \angle DEG) + (\angle DHB + \angle DHL)]\\ &= (\angle DEZ + \angle DHB) + (\angle DEG + \angle DHL)\\ &= (\angle DEZ + \angle DHB) \text{ plus 2 right angles}\\ &= 2\angle DEZ \text{ plus 2 right angles.}\end{aligned}$$

Q.E.D.

For the remaining case, draw a similar figure [Fig. 2.22], in which point A,
H166 which is culminating on the section east [of the meridian], is north of G, while B, which is culminating on the section west [of the meridian], is south of [the zenith].

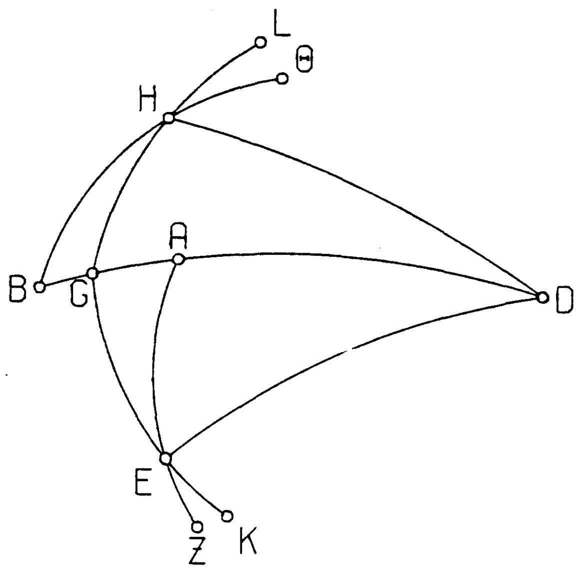

Fig. 2.22

I say that

$$\angle KEZ + \angle GHB = 2\angle DEZ \text{ minus 2 right angles.}$$

[Proof:] By the same reasoning as before

$$\begin{aligned}\angle KEZ + \angle GHB &= (\angle DEZ + \angle DHB) - (\angle DEK + \angle DHG)\\ &= 2\angle DEZ - (\angle DEK + \angle DHG).\end{aligned}$$

But $\angle DEK + \angle DHG = 2$ right angles, since

$\angle DEK + \angle DEG = 2$ right angles, and $\angle DEG = \angle DHG$.

Q.E.D.

Of the angles and arcs formed in the way defined between the ecliptic and an altitude circle, those at the meridian and the horizon can be computed readily, as can be shown immediately in the following way.

Draw [Fig. 2.23] the meridian circle ABGD, the semi-circle of the horizon BED, and the semi-circle of the ecliptic in any position, ZEH. Then if we H167
imagine the altitude circle through the zenith A and the culminating point of the ecliptic Z, it coincides with the meridian ABGD, and ∠ DZE will immediately be given, since the point Z and the angle that [the ecliptic makes] with the meridian at point Z are given.[102] Arc AZ will also be given, since we know the distance in degrees of point Z from the equator (measured along the meridian), and the distance of the equator from the zenith A.[103]

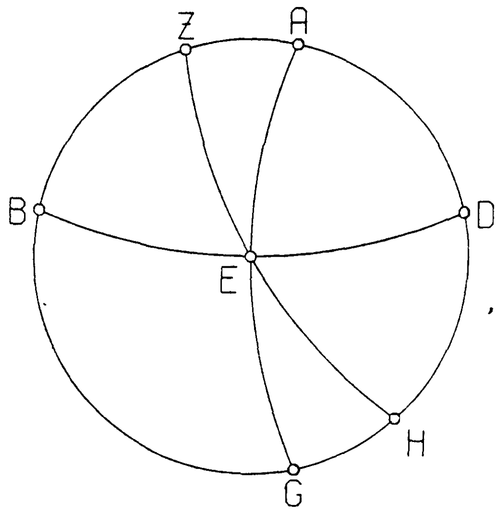

Fig. 2.23

Next, if we imagine the altitude circle AEG, drawn through the rising-point of the ecliptic, E, and [the zenith] A, here too it is immediately obvious that arc AE is always a quadrant, since point A is the pole of the horizon BED. For the same reason, ∠ AED is always right; and since the angle which the ecliptic makes with the horizon, namely ∠ DEH, is given,[104] the sum, angle AEH, will also be given.

Q.E.D.

Thus it is clear that, since the above relationships hold, if we compute, for H168
each latitude, just the angles and arcs before [i.e. to the east of] the meridian, and just for the signs from the beginning of Cancer to the beginning of Capricorn, we will simultaneously have found the angles and arcs for the same

[102] By II 10 (p. 109).
[103] δ and φ respectively, so arc $AZ = \varphi - \delta$.
[104] By II 11 (pp. 113–14).

signs [Cancer to Capricorn] after the meridian too, and also the angles and arcs both before and after the meridian for the remaining signs. But in order to make clear the procedure in this case too for any position [of the ecliptic], as an example we shall display the general method by means of a single solution of the problem.[105] At the same latitude, namely where the elevation of the north pole from the horizon is 36°, we suppose that the beginning of Cancer is, e.g., one equinoctial hour to the east of the meridian. In this situation, at the above latitude, ♊ 16;12° is culminating, and ♍ 17;37° is rising.

Then let [Fig. 2.24] ABGD be the meridian circle, BED the semi-circle of the horizon, and ZHΘ the semi-circle of the ecliptic in such a position that point H
H169 is the beginning of Cancer, while Z represents ♊ 16;12° and Θ ♍ 17;37°. Draw through A, the zenith, and H, the beginning of Cancer, segment AHEG of the [altitude] great circle. Let the first problem be to find arc AH.

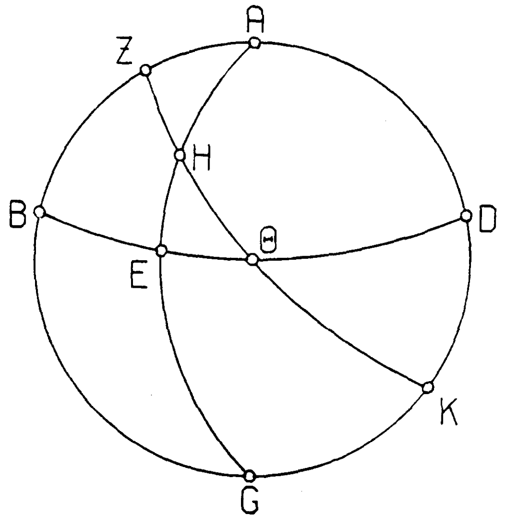

Fig. 2.24

Now it is clear that arc ZΘ = 91;25° [♍ 17;37° – ♊ 16;12°]

and arc HΘ = 77;37° [♍ 17;37° – ♋ 0°].

Similarly, since ♊ 16;12° cut off 23;7° of the meridian to the north of the equator, and the equator cuts off 36° [of the meridian] from the zenith A,

arc AZ = 12;53°

and arc ZB = 77;7° (complement).

When these quantities are given, from the figure

Crd arc 2ZB:Crd arc 2BA =

(Crd arc 2ZΘ:Crd arc 2ΘH). (Crd arc 2HE:Crd arc 2EA). [M.T. I]

But arc 2ZB = 154;14°, so Crd arc 2ZB = 116;59^p

H170 and arc 2BA = 180°, so Crd arc 2BA = 120^p.

[105] This example is worked through *HAMA* 49–50.

Furthermore arc 2ZΘ = 182;50°, so Crd arc 2ZΘ = 119;58^p
and arc 2ΘH = 155;14°, so Crd arc 2ΘH = 117;12^p.
∴ Crd arc 2EH: Crd arc 2EA = (116;59 : 120)/(119;58 : 117;12)
≈ 114;16 : 120.
But Crd arc 2EA = 120^p
∴ Crd arc 2EH = 114;16^p
∴ arc 2EH ≈ 144;26°
and arc EH = 72;13°.
∴ arc AH = 17;47° (complement).

Q.E.D.

Next we shall find ∠ AHΘ, as follows.

Draw the same figure [Fig. 2.25], and with pole H and radius the side of the [inscribed] square draw the great circle segment KLM.

Then, since circle AHE is drawn through the poles of EΘM and KLM, both EM and KM are quadrants. Again, from the figure H171

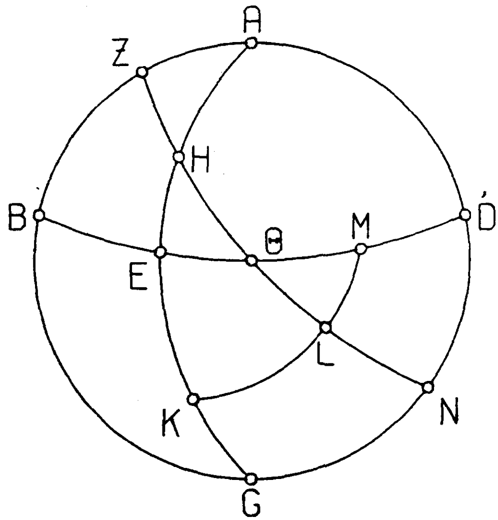

Fig. 2.25

Crd arc 2HE:Crd arc 2EK =
(Crd arc 2HΘ:Crd arc 2ΘL). (Crd arc 2LM:Crd arc 2KM). [M.T. II]
But arc 2HE = 144;26° [above], so Crd arc 2HE = 114;16^p
and arc 2EK = 35;34°, so Crd arc 2EK = 36;38^p.
Furthermore arc 2ΘH = 155;14°, so Crd arc 2ΘH = 117;12^p
and arc 2ΘL = 24;46°, so Crd arc 2ΘL = 25;44^p.
∴ Crd arc 2LM:Crd arc 2MK = (114;16 : 36;38)/(117;12 : 25;44)
≈ 82;11 : 120.
But Crd arc 2MK = 120^p
∴ Crd arc 2LM = 82;11^p

$$\therefore \text{ arc } 2LM = 86;28^\circ$$
$$\text{and arc } LM = 43;14^\circ.$$
$$\therefore \text{ arc } LK = \angle\, LHK = 46;46^\circ \text{ (complement).}$$
$$\therefore \angle\, AH\Theta = 133;14^\circ \text{ (supplement).}$$

Q.E.D.

H172 The same method as was used for finding the above also applies to the remaining [arcs and angles]. But in order to have conveniently displayed all the other arcs and angles which it is reasonable to suppose we may need in our particular investigations, we computed these too geometrically, beginning from the parallel through Meroe, at which the longest day is 13 equinoctial hours, and going up to the parallel above Pontus [the Black Sea], through the mouths of the Borysthenes, where the longest day is 16 equinoctial hours.[106] The intervals which we used were half an hour [of length of longest day] between parallels of latitude (as for the rising-times), one sign for the sections of the ecliptic, and one equinoctial hour for the position [of the altitude circles] to east and west of the meridian. We shall display the results in tabular form, one set of tables for each parallel of latitude, and one table for each sign. In the first column we put, first, the meridian situation, then the distance before or after the meridian, measured in equinoctial hours. In the second column we put the amount of the corresponding arc (as explained above) from the zenith to the beginning of the sign in question. In the third and fourth columns we put the
H173 amount of the angles formed by the above-mentioned intersection [between ecliptic and altitude circle], defined in the way we explained: the angles at positions to the east of the meridian in the third column, and those at positions to the west of the meridian in the fourth column. One must bear in mind that, according to our original definition,[107] we always took the angle which lies to the rear of the intersection of the circles and to the north of the ecliptic, and expressed its magnitude in the system in which one right angle is 90 [degrees].

The layout of the tables is as follows.

H174—87

13. {*Layout of angles and arcs, parallel by parallel*}[108]

[See pp. 123-9.]

H188 Now that the treatment of the angles [between ecliptic and principal circles] has been methodically discussed, the only remaining topic in the foundations [of the rest of the treatise] is to determine the coordinates in latitude and longitude of the cities in each province which deserve note, in order to calculate

[106] The seven parallels selected here are in fact the canonical '7 climata', for which see Introduction p. 19.

[107] II 10 p. 105 with n.88.

[108] The table for Clima I (Meroe) has a peculiarity. Since, alone of the parallels tabulated, its latitude is less than ε, it is possible for the point of the ecliptic which is culminating to fall north of the zenith. When this occurs at a tabulated position, the corresponding eastern or western angle is marked 'N' (for 'north'). This is a modification of the system in the mss., where BO (for βόρειος) is written *above* the *first* value in each column where the ecliptic is north of the zenith, and NO (for νότιος) above the value where it changes back to south. Since Ptolemy makes no mention of this

PARALLEL THROUGH MEROE 13^h 16;27°

CANCER				CAPRICORNUS			
Hour	Arc	East Angle	West Angle	Hour	Arc	East Angle	West Angle
noon	7 24	90 0 N		noon	40 18	90 0	
1	15 55	25 16 N	154 44 N	1	42 54	111 24	68 36
2	29 3	9 15 N	170 45 N	2	49 48	128 51	51 9
3	42 42	1 38 N	178 22 N	3	59 35	141 49	38 11
4	56 25	175 7	4 53	4	71 4	151 25	28 35
5	70 2	170 18	9 42	5	83 31	158 48	21 12
6	83 27	164 41	15 19	5 30	90 0	161 57	18 3
6 30	90 0	161 57	18 3				

LEO				AQUARIUS			
Hour	Arc	East Angle	West Angle	Hour	Arc	East Angle	West Angle
noon	4 3	102 30 N		noon	36 57	77 30	
1	14 20	26 3 N	178 57 N	1	39 46	100 12	54 48
2	28 42	15 28 N	9 32	2	47 15	118 5	36 55
3	42 43	10 5 N	14 55	3	57 33	131 3	23 57
4	56 49	6 19 N	18 41	4	69 30	139 48	15 12
5	70 38	2 33 N	22 27	5	82 18	146 43	8 17
6	84 17	177 0	28 0	5 35	90 0	149 51	5 9
6 25	90 0	174 51	30 9				

VIRGO				PISCES			
Hour	Arc	East Angle	West Angle	Hour	Arc	East Angle	West Angle
noon	4 47	111 0		noon	28 7	69 0	
1	15 20	0 0 N	42 0	1	31 46	97 0	41 0
2	29 28	8 0 N	34 0	2	40 52	115 59	22 1
3	43 40	9 15 N	32 45	3	52 30	127 23	10 37
4	58 13	8 39 N	33 21	4	65 40	134 41	3 19
5	72 36	6 53 N	35 7	5	79 18	139 41	178 19 N
6	86 41	5 37 N	36 23	5 46	90 0	142 9	175 51 N
6 14	90 0	4 9 N	37 51				

LIBRA				ARIES			
Hour	Arc	East Angle	West Angle	Hour	Arc	East Angle	West Angle
noon	16 27	113 51		noon	16 27	66 9	
1	22 8	154 53	72 49	1	22 8	107 11	25 7
2	33 50	173 17	54 25	2	33 50	125 35	6 43
3	47 20	1 23 N	46 19	3	47 20	133 41	178 37 N
4	61 22	5 8 N	42 34	4	61 22	137 26	174 52 N
5	75 39	7 9 N	40 33	5	75 39	139 27	172 51 N
6	90 0	7 24 N	40 18	6	90 0	139 42	172 36 N

SCORPIUS				TAURUS			
Hour	Arc	East Angle	West Angle	Hour	Arc	East Angle	West Angle
noon	28 7	111 0		noon	4 47	69 0	
1	31 46	139 0	83 0	1	15 20	138 0	180 0 N
2	40 52	157 59	64 1	2	29 28	146 0	172 0 N
3	52 30	169 23	52 37	3	43 40	147 15	170 45 N
4	65 40	176 41	45 19	4	58 13	146 39	171 21 N
5	79 18	1 41 N	40 19	5	72 36	144 53	173 7 N
5 46	90 0	4 9 N	37 51	6	86 41	143 37	174 23 N
				5 14	90 0	142 9	175 51 N

SAGITTARIUS				GEMINI			
Hour	Arc	East Angle	West Angle	Hour	Arc	East Angle	West Angle
noon	36 57	102 30		noon	4 3	77 30 N	
1	39 46	125 12	79 48	1	14 20	1 3 N	153 57 N
2	47 15	143 5	61 55	2	28 42	170 28	164 32 N
3	57 33	156 3	48 57	3	42 43	165 5	169 55 N
4	69 30	164 48	40 12	4	56 49	161 19	173 41 N
5	82 18	171 43	33 17	5	70 38	157 33	177 27 N
5 35	90 0	174 51	30 9	6	84 17	152 0	3 0
				6 25	90 0	149 51	5 9

PARALLEL THROUGH SOENE $13\frac{1}{2}^h$ 23;51°

CANCER				CAPRICORNUS			
Hour	Arc	East Angle	West Angle	Hour	Arc	East Angle	West Angle
noon	0 0	90 0		noon	47 42	90 0	
1	13 43	176 15	3 45	1	49 52	108 3	71 57
2	27 23	173 51	6 9	2	55 52	123 31	56 29
3	41 20	168 15	11 45	3	64 37	135 37	44 23
4	54 27	166 51	13 9	4	75 12	144 57	35 3
5	67 42	162 42	17 18	5	86 54	152 0	28 0
6	80 36	157 59	22 1	5 15	90 0	153 46	26 14
6 45	90 0	153 46	26 14				

LEO				AQUARIUS			
Hour	Arc	East Angle	West Angle	Hour	Arc	East Angle	West Angle
noon	3 21	102 30		noon	44 21	77 30	
1	14 18	176 4	28 56	1	46 40	96 30	58 30
2	27 56	180 0	25 0	2	53 4	112 16	42 44
3	41 44	179 3	25 57	3	62 18	124 25	30 35
4	55 14	177 18	27 42	4	73 20	132 58	22 2
5	68 43	173 40	31 20	5	85 23	139 46	15 14
6	81 52	168 56	36 4	5 22	90 0	141 53	13 7
6 38	90 0	166 53	38 7				

VIRGO				PISCES			
Hour	Arc	East Angle	West Angle	Hour	Arc	East Angle	West Angle
noon	12 11	111 0		noon	35 31	69 0	
1	18 42	158 40	63 20	1	38 25	91 15	46 45
2	30 57	173 44	48 16	2	46 2	108 18	29 42
3	44 22	178 3	43 57	3	56 30	119 41	18 19
4	58 1	180 0	42 0	4	68 31	127 5	10 55
5	71 43	179 15	42 45	5	81 22	132 30	5 30
6	85 20	177 39	44 21	5 39	90 0	134 41	3 19
6 21	90 0	176 41	45 19				

LIBRA				ARIES			
Hour	Arc	East Angle	West Angle	Hour	Arc	East Angle	West Angle
noon	23 51	113 51		noon	23 51	66 9	
1	27 56	144 10	83 32	1	27 56	96 28	35 50
2	37 36	162 13	65 29	2	37 36	114 31	17 47
3	49 42	171 45	55 57	3	49 42	124 3	8 15
4	62 47	176 59	50 43	4	62 47	129 17	3 1
5	76 20	179 3	48 39	5	76 20	131 21	0 57
6	90 0	180 0	47 42	6	90 0	132 18	0 0

SCORPIUS				TAURUS			
Hour	Arc	East Angle	West Angle	Hour	Arc	East Angle	West Angle
noon	35 31	111 0		noon	12 11	69 0	
1	38 25	133 15	88 45	1	18 42	116 40	21 20
2	46 2	150 18	71 42	2	30 57	131 44	6 16
3	56 30	161 41	60 19	3	44 22	136 3	1 57
4	68 31	169 5	52 55	4	58 1	138 0	0 0
5	81 22	174 30	47 30	5	71 43	137 15	0 45
5 39	90 0	176 41	45 19	6	85 20	135 39	2 21
				6 21	90 0	134 41	3 19

SAGITTARIUS				GEMINI			
Hour	Arc	East Angle	West Angle	Hour	Arc	East Angle	West Angle
noon	44 21	102 30		noon	3 21	77 30	
1	46 40	121 30	83 30	1	14 18	151 4	3 56
2	53 4	137 16	67 44	2	27 56	155 0	0 0
3	62 18	149 25	55 35	3	41 44	154 3	0 57
4	73 20	157 58	47 2	4	55 14	152 18	2 42
5	85 23	164 46	40 14	5	68 43	148 40	6 20
5 22	90 0	166 53	38 7	6	81 52	143 56	11 4
				6 38	90 0	141 53	13 7

PARALLEL THROUGH LOWER EGYPT 14^h 30;22°

CANCER				CAPRICORNUS			
Hour	Arc	East Angle	West Angle	Hour	Arc	East Angle	West Angle
noon	6 31	90 0		noon	54 13	90 0	
1	14 56	150 0	30 0	1	56 6	105 34	74 26
2	27 23	159 38	20 22	2	61 22	119 23	60 37
3	40 19	160 30	19 30	3	69 17	130 46	49 14
4	53 14	158 51	21 9	4	78 59	139 30	40 30
5	65 55	156 0	24 0	5	90 0	146 28	33 32
6	78 15	151 49	28 11				
7	90 0	146 28	33 32				

LEO				AQUARIUS			
Hour	Arc	East Angle	West Angle	Hour	Arc	East Angle	West Angle
noon	9 52	102 30		noon	50 52	77 30	
1	16 45	153 13	51 47	1	52 53	93 39	61 21
2	28 44	166 22	38 38	2	58 27	107 51	47 9
3	41 31	169 26	35 34	3	66 44	119 1	35 59
4	54 27	169 8	35 52	4	76 51	127 37	27 23
5	67 17	167 1	37 59	5	88 9	133 43	21 17
6	79 48	163 46	41 14	5 9	90 0	134 49	20 11
6 51	90 0	159 49	45 11				

VIRGO				PISCES			
Hour	Arc	East Angle	West Angle	Hour	Arc	East Angle	West Angle
noon	18 42	111 0		noon	42 2	69 0	
1	23 18	145 18	76 42	1	44 26	87 32	50 28
2	33 30	162 25	59 35	2	50 58	102 38	35 22
3	45 36	169 34	52 26	3	60 19	113 33	24 27
4	58 21	172 10	49 50	4	71 20	120 56	17 4
5	71 15	172 28	49 32	5	83 19	125 54	12 6
6	84 7	171 5	50 55	5 32	90 0	127 55	10 5
6 28	90 0	169 55	52 5				

LIBRA				ARIES			
Hour	Arc	East Angle	West Angle	Hour	Arc	East Angle	West Angle
noon	30 22	113 51		noon	30 22	66 9	
1	33 35	137 52	90 10	1	33 35	89 50	42 28
2	41 39	154 19	73 23	2	41 39	106 37	25 41
3	52 25	164 10	63 32	3	52 25	116 28	15 50
4	64 28	169 47	57 55	4	64 28	122 5	10 13
5	77 6	172 21	55 21	5	77 6	124 39	7 39
6	90 0	173 29	54 13	6	90 0	125 47	6 31

SCORPIUS				TAURUS			
Hour	Arc	East Angle	West Angle	Hour	Arc	East Angle	West Angle
noon	42 2	111 0		noon	18 42	69 0	
1	44 26	129 32	92 28	1	23 18	103 18	34 42
2	50 58	144 38	77 22	2	33 30	120 25	17 35
3	60 19	155 33	66 27	3	45 36	127 34	10 26
4	71 20	162 56	59 4	4	58 21	130 10	7 50
5	83 19	167 54	54 6	5	71 15	130 28	7 32
5 32	90 0	169 55	52 5	6	84 7	129 5	8 55
				6 28	90 0	127 55	10 5

SAGITTARIUS				GEMINI			
Hour	Arc	East Angle	West Angle	Hour	Arc	East Angle	West Angle
noon	50 52	102 30		noon	9 52	77 30	
1	52 53	118 39	86 21	1	16 45	128 13	26 47
2	58 27	132 51	72 9	2	28 44	141 22	13 38
3	66 44	144 1	60 59	3	41 31	144 26	10 34
4	76 51	152 37	52 23	4	54 27	144 8	10 52
5	88 9	158 43	46 17	5	67 17	142 1	12 59
5 9	90 0	159 49	45 11	6	79 48	138 46	16 14
				6 51	90 0	134 49	20 11

PARALLEL THROUGH RHODES $14\frac{1}{2}^{h}$ 36°

CANCER				CAPRICORNUS			
Hour	Arc	East Angle	West Angle	Hour	Arc	East Angle	West Angle
noon	12 9	90 0		noon	59 51	90 0	
1	17 47	133 14	46 46	1	61 30	103 45	76 15
2	28 22	147 45	32 15	2	66 12	116 10	63 50
3	40 27	151 46	28 14	3	73 22	126 36	53 24
4	52 36	151 52	28 8	4	82 24	134 56	45 4
5	64 36	149 54	30 6	4 45	90 0	140 1	39 59
6	76 16	146 25	33 35				
7	87 23	141 30	38 30				
7 15	90 0	140 1	39 59				

LEO				AQUARIUS			
Hour	Arc	East Angle	West Angle	Hour	Arc	East Angle	West Angle
noon	15 30	102 30		noon	56 30	77 30	
1	20 20	139 32	65 28	1	58 14	91 39	63 21
2	30 28	155 19	49 41	2	63 13	104 23	50 37
3	42 6	160 37	44 23	3	70 41	114 47	40 13
4	54 12	162 11	42 49	4	80 2	122 47	32 13
5	66 17	161 5	43 55	4 56	90 0	128 36	26 24
6	78 7	158 10	46 50				
7	89 27	153 39	51 21				
7 4	90 0	153 36	51 24				

VIRGO				PISCES			
Hour	Arc	East Angle	West Angle	Hour	Arc	East Angle	West Angle
noon	24 20	111 0		noon	47 40	69 0	
1	27 51	137 38	84 22	1	49 42	84 50	53 10
2	36 24	153 59	68 1	2	55 26	98 20	39 40
3	47 14	162 10	59 50	3	63 48	108 34	29 26
4	59 0	165 40	56 20	4	73 55	115 51	22 9
5	71 5	166 34	55 26	5	85 5	120 28	17 32
6	83 9	165 30	56 30	5 25	90 0	122 7	15 53
6 35	90 0	164 7	57 53				

LIBRA				ARIES			
Hour	Arc	East Angle	West Angle	Hour	Arc	East Angle	West Angle
noon	36 0	113 51		noon	36 0	66 9	
1	38 37	133 23	94 19	1	38 37	85 41	46 37
2	45 31	148 23	79 19	2	45 31	100 41	31 37
3	55 6	158 9	69 33	3	55 6	110 27	21 51
4	66 9	163 58	63 44	4	66 9	116 16	16 2
5	77 56	116 36	61 6	5	77 56	118 54	13 24
6	90 0	167 51	59 51	6	90 0	120 9	12 9

SCORPIUS				TAURUS			
Hour	Arc	East Angle	West Angle	Hour	Arc	East Angle	West Angle
noon	47 40	111 0		noon	24 20	69 0	
1	49 42	126 50	95 10	1	27 51	95 38	42 22
2	55 26	140 20	81 40	2	36 24	111 59	26 1
3	63 48	150 34	71 26	3	47 14	120 10	17 50
4	73 55	157 51	64 9	4	59 0	123 40	14 20
5	85 5	162 28	59 32	5	71 5	124 34	13 26
5 25	90 0	164 7	57 53	6	83 9	123 30	14 30
				6 35	90 0	122 7	15 53

SAGITTARIUS				GEMINI			
Hour	Arc	East Angle	West Angle	Hour	Arc	East Angle	West Angle
noon	56 30	102 30		noon	15 30	77 30	
1	58 14	116 39	88 21	1	20 20	114 32	40 28
2	63 13	129 23	75 37	2	30 28	130 19	24 41
3	70 41	139 47	65 13	3	42 6	135 37	19 23
4	80 2	147 47	57 13	4	54 12	137 11	17 49
4 56	90 0	153 36	51 24	5	66 17	136 5	18 55
				6	78 7	133 10	21 50
				7	89 27	128 39	25 21
				7 4	90 0	128 36	26 24

PARALLEL THROUGH THE HELLESPONT 15^{h} 40;56°

CANCER				CAPRICORNUS			
Hour	Arc	East Angle	West Angle	Hour	Arc	East Angle	West Angle
noon	17 5	90 0		noon	64 47	90 0	
1	21 18	122 32	57 28	1	66 15	102 27	77 33
2	30 17	138 29	41 31	2	70 30	113 35	66 25
3	41 37	144 18	35 42	3	77 4	122 55	57 5
4	52 25	145 38	34 22	4	85 18	130 58	49 2
5	63 47	144 28	35 32	4 30	90 0	134 16	45 44
6	74 48	141 30	38 30				
7	85 9	137 5	42 55				
7 30	90 0	134 16	45 44				

LEO				AQUARIUS			
Hour	Arc	East Angle	West Angle	Hour	Arc	East Angle	West Angle
noon	20 26	102 30		noon	61 26	77 30	
1	24 5	131 6	73 54	1	63 0	90 5	64 55
2	32 37	147 0	58 0	2	67 24	101 29	53 31
3	43 8	153 50	51 10	3	74 13	111 10	43 50
4	54 19	156 5	48 55	4	82 48	118 45	36 15
5	65 36	155 8	49 52	4 44	90 0	123 6	31 54
6	76 46	153 24	51 36				
7	87 24	149 6	55 54				
7 16	90 0	148 6	56 54				

VIRGO				PISCES			
Hour	Arc	East Angle	West Angle	Hour	Arc	East Angle	West Angle
noon	29 16	111 0		noon	52 36	69 0	
1	32 5	132 30	89 30	1	54 23	82 46	55 14
2	39 22	147 30	74 30	2	59 25	94 55	43 5
3	49 3	156 0	66 0	3	66 58	104 24	33 36
4	59 50	160 7	61 53	4	76 15	111 10	26 50
5	71 5	161 24	60 36	5	86 38	115 45	22 15
6	82 22	160 40	61 20	5 18	90 0	116 59	21 1
6 42	90 0	158 59	63 1				

LIBRA				ARIES			
Hour	Arc	East Angle	West Angle	Hour	Arc	East Angle	West Angle
noon	40 56	113 51		noon	40 56	66 9	
1	43 8	129 57	97 45	1	43 8	82 15	50 3
2	49 7	143 38	84 4	2	49 7	95 56	36 22
3	57 42	153 8	74 34	3	57 42	105 26	26 52
4	67 50	158 47	68 55	4	67 50	111 5	21 13
5	78 45	161 59	65 43	5	78 45	114 17	18 1
6	90 0	162 55	64 47	6	90 0	115 13	17 5

SCORPIUS				TAURUS			
Hour	Arc	East Angle	West Angle	Hour	Arc	East Angle	West Angle
noon	52 36	111 0		noon	29 16	69 0	
1	54 23	124 46	97 14	1	32 5	90 30	47 30
2	59 25	136 55	85 5	2	39 22	105 30	32 30
3	66 58	146 24	75 36	3	49 3	114 0	24 0
4	76 15	153 10	68 50	4	59 50	118 7	19 53
5	86 38	157 45	64 15	5	71 5	119 24	18 36
5 18	90 0	158 59	63 1	6	82 22	118 40	19 20
				6 42	90 0	116 59	21 1

SAGITTARIUS				GEMINI			
Hour	Arc	East Angle	West Angle	Hour	Arc	East Angle	West Angle
noon	61 26	102 30		noon	20 26	77 30	
1	63 0	115 5	89 55	1	24 5	106 6	48 54
2	67 24	126 29	78 31	2	32 37	122 0	33 0
3	74 13	136 10	68 50	3	43 8	128 50	26 10
4	82 48	143 45	61 15	4	54 19	131 5	23 55
4 44	90 0	148 6	56 54	5	65 36	130 8	24 52
				6	76 46	128 24	26 36
				7	87 24	124 6	30 54
				7 16	90 0	123 6	31 54

PARALLEL THROUGH THE MIDDLE OF PONTUS 15½ʰ 45;1°

CANCER				CAPRICORNUS			
Hour	Arc	East Angle	West Angle	Hour	Arc	East Angle	West Angle
noon	21 10	90 0		noon	68 52	90 0	
1	24 32	116 5	63 55	1	70 14	101 11	78 49
2	32 12	131 30	48 30	2	74 5	111 30	68 30
3	42 1	138 17	41 43	3	80 6	120 29	59 31
4	52 29	140 31	39 29	4	87 42	128 13	51 47
5	63 4	140 2	39 58	4 15	90 0	129 21	50 39
6	73 24	137 32	42 28				
7	83 17	133 26	46 34				
7 45	90 0	129 21	50 39				

LEO				AQUARIUS			
Hour	Arc	East Angle	West Angle	Hour	Arc	East Angle	West Angle
noon	24 31	102 30		noon	65 31	77 30	
1	27 29	124 49	80 11	1	66 55	88 50	66 10
2	34 48	140 47	64 13	2	70 58	99 21	55 39
3	44 20	148 5	56 55	3	77 14	108 19	46 41
4	54 37	151 5	53 55	4	85 10	115 20	39 40
5	65 15	151 7	53 53	4 32	90 0	118 25	36 35
6	75 39	149 20	55 40				
7	85 39	145 39	59 21				
7 28	90 0	143 25	61 35				

VIRGO				PISCES			
Hour	Arc	East Angle	West Angle	Hour	Arc	East Angle	West Angle
noon	33 21	111 0		noon	56 41	69 0	
1	35 43	129 15	92 45	1	58 19	81 31	56 29
2	42 4	142 50	79 10	2	62 49	92 16	45 44
3	50 46	151 9	70 51	3	69 42	101 12	36 48
4	60 44	155 31	66 29	4	78 16	107 31	30 29
5	71 12	157 3	64 57	5	87 56	112 6	25 54
6	81 46	156 31	65 29	5 12	90 0	112 43	25 17
6 48	90 0	154 43	67 17				

LIBRA				ARIES			
Hour	Arc	East Angle	West Angle	Hour	Arc	East Angle	West Angle
noon	45 1	113 51		noon	45 1	66 9	
1	46 55	128 19	99 23	1	46 55	80 37	51 41
2	52 17	140 26	87 16	2	52 17	92 44	39 34
3	60 1	149 4	78 38	3	60 1	101 22	30 56
4	69 19	154 48	72 54	4	69 19	107 6	25 12
5	79 28	157 55	69 47	5	79 28	110 13	22 5
6	90 0	158 50	68 52	6	90 0	111 8	21 10

SCORPIUS				TAURUS			
Hour	Arc	East Angle	West Angle	Hour	Arc	East Angle	West Angle
noon	56 41	111 0		noon	33 21	69 0	
1	58 19	123 31	98 29	1	35 43	87 15	50 45
2	62 49	134 16	87 44	2	42 4	100 50	37 10
3	69 42	143 12	78 48	3	50 46	109 9	28 51
4	78 16	149 31	72 29	4	60 44	113 31	24 29
5	87 56	154 6	67 54	5	71 12	115 3	22 57
5 12	90 0	154 43	67 17	6	81 46	114 31	23 29
				6 48	90 0	112 43	25 17

SAGITTARIUS				GEMINI			
Hour	Arc	East Angle	West Angle	Hour	Arc	East Angle	West Angle
noon	65 31	102 30		noon	24 31	77 30	
1	66 55	113 50	91 10	1	27 29	99 49	55 11
2	70 58	124 21	80 39	2	34 48	115 47	39 13
3	77 14	133 19	71 41	3	44 20	123 5	31 55
4	85 10	140 20	64 40	4	54 37	126 5	28 55
4 32	90 0	143 25	61 35	5	65 15	126 7	28 53
				6	75 39	124 20	30 40
				7	85 39	120 39	34 21
				7 28	90 0	118 25	36 35

PARALLEL THROUGH BORYSTHENES 16^h 48;32°

CANCER				CAPRICORNUS			
Hour	Arc	East Angle	West Angle	Hour	Arc	East Angle	West Angle
noon	24 41	90 0		noon	72 23	90 0	
1	27 30	111 44	68 16	1	73 38	100 15	79 45
2	34 9	126 7	53 53	2	77 10	109 47	70 13
3	43 2	133 18	46 42	3	82 44	118 3	61 57
4	52 44	136 6	43 54	4	90 0	124 58	55 2
5	62 40	136 4	43 56				
6	72 24	134 0	46 0				
7	81 38	130 16	49 44				
8	90 0	124 58	55 2				

LEO				AQUARIUS			
Hour	Arc	East Angle	West Angle	Hour	Arc	East Angle	West Angle
noon	28 2	102 30		noon	69 2	77 30	
1	30 32	122 9	82 51	1	70 20	87 49	67 11
2	36 55	135 54	69 6	2	74 2	97 31	57 29
3	45 30	143 28	61 32	3	79 48	105 49	49 11
4	55 3	146 50	58 10	4	87 14	112 25	42 35
5	64 59	147 19	57 41	4 20	90 0	114 20	40 40
6	74 47	145 46	59 14				
7	84 10	142 27	62 33				
7 40	90 0	139 20	65 40				

VIRGO				PISCES			
Hour	Arc	East Angle	West Angle	Hour	Arc	East Angle	West Angle
noon	36 52	111 0		noon	60 12	69 0	
1	38 56	126 45	95 15	1	61 38	80 5	57 55
2	44 31	139 7	82 53	2	65 36	90 16	47 44
3	52 25	147 9	74 51	3	72 5	98 26	39 54
4	61 35	151 36	70 24	4	80 3	104 28	33 32
5	71 22	153 23	68 37	5	89 3	109 2	28 58
6	81 17	152 58	69 2	5 6	90 0	109 22	28 38
6 54	90 0	151 22	70 38				

LIBRA				ARIES			
Hour	Arc	East Angle	West Angle	Hour	Arc	East Angle	West Angle
noon	48 32	113 51		noon	48 32	66 9	
1	50 21	126 30	101 12	1	50 21	78 48	53 30
2	54 59	137 40	90 2	2	54 59	89 58	42 20
3	62 5	145 46	81 56	3	62 5	98 4	34 14
4	70 41	151 18	76 24	4	70 41	103 36	28 42
5	80 8	154 23	73 19	5	80 8	106 41	25 37
6	90 0	155 19	72 23	6	90 0	107 37	24 41

SCORPIUS				TAURUS			
Hour	Arc	East Angle	West Angle	Hour	Arc	East Angle	West Angle
noon	60 12	111 0		noon	36 52	69 0	
1	61 38	122 5	99 55	1	38 56	84 45	53 15
2	65 36	132 16	89 44	2	44 31	97 7	40 53
3	72 5	140 26	81 34	3	52 25	105 9	32 51
4	80 3	146 28	75 32	4	61 35	109 36	28 24
5	89 3	151 2	70 58	5	71 22	111 23	26 37
5 6	90 0	151 22	70 38	6	81 17	110 58	27 2
				6 54	90 0	109 22	28 38

SAGITTARIUS				GEMINI			
Hour	Arc	East Angle	West Angle	Hour	Arc	East Angle	West Angle
noon	69 2	102 30		noon	28 2	77 30	
1	70 20	112 49	92 11	1	30 32	97 9	57 51
2	74 2	122 31	82 29	2	36 55	110 54	44 6
3	79 48	130 49	74 11	3	45 30	118 28	36 32
4	87 14	137 25	67 35	4	55 3	121 50	33 10
4 20	90 0	139 20	65 40	5	64 59	122 19	32 41
				6	74 47	120 46	34 14
				7	84 10	117 27	37 33
				7 40	90 0	114 20	40 40

the [astronomical] phenomena for those cities. However, the discussion of this subject belongs to a separate, geographical treatise, so we shall expose it to view by itself [in such a treatise], in which we shall use the accounts of those who have elaborated this field to the extent which is possible. We shall [there] list for each of the cities its distance in degrees from the equator, measured along its meridian, and the distance in degrees of that meridian from the meridian through Alexandria, to the east or west, measured along the equator (for that [Alexandria] is the meridian for which we establish the times of the positions [of the heavenly bodies]).[109]

For the time being we take the locations [of the cities] for granted, and [therefore] think it appropriate to add no more than the following. Whenever we are given the time at some standard place, and we undertake to determine what the corresponding time is at another place, then, if they lie on different meridians, we have to take the distance between the two places in degrees,
H189 measured along the equator, and determine which of them is to the east or west, and then increase or decrease the time at the standard place by the same number of time-degrees, to get the corresponding time at the required place. We increase if the required place is the further east, and decrease if the standard [place is the further east].[110]

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notation, it may be a later addition, but it is a useful one, since it affects the sign of the parallax (see V 19 p. 266). *It is easy to verify that Ptolemy's rules on pp. 115–18 hold good according as N is* appended to the eastern angle, the western angle, or both.

Because of the symmetries demonstrated in II 12 (see also *HAMA* 51) we have a means of checking most of the entries in these tables. The only entries which cannot be thus checked are the zenith distances for the signs of Cancer and Capricorn. This shows that there are very few scribal errors in Heiberg's text here. However, recomputation of the data using modern formulas reveals considerable inaccuracies in Ptolemy's values. The zenith distances are generally correct to within 2′, although occasional errors of up to 10′ occur; but the angles regularly show errors of 10′, and occasionally as much as 1° (e.g. Parallel through Middle Pontus, Gemini, 1 hour from noon, eastern angle: text 99;49°; computed 100;54°).

Corrections to Heiberg's text:

Clima I, ♑, 2^h (H175,7) μθ νη (49;58): μθ μη, with BCDL (computed: 49;49).

Clima IV, ♈, 2^h (H181,7) ρ μζ (100;47), λα λα (31;31): ρ μα, λα λζ with Ar. Cf. supplementary angles at Libra: 148;23, 79;19. Corrected by Manitius.

Clima V, ♉, 2^h (H183,17) λβ (32): λβ λ. Cf. supplementary angle for Virgo: 167;30. This is simply a misprint, corrected by Manitius.

Clima VII, ♏, 2^h (H186,17) ρλβ ι (132;10), πθ ν (89;50): ρλβ ις, πθ μδ, as Ger. Cf. supplementary angles at Pisces: 90;16, 47;44. Manitius noticed the discrepancy, but changed the Pisces entries. My correction is closer to the accurately computed values (132;15°, 89;39°). Most of the Arabic tradition agrees with Heiberg here; L has 47;50 at Pisces, 2^h, west angle.

[109] This promise is fulfilled in Ptolemy's *Geography*. However, by the time he came to write that, he decided to give distances in longitude, not from the meridian through Alexandria, but from one at the extreme west of the known world (through the Fortunate Isles), so that all longitudes could be counted in the same direction. A remnant of the original plan survives in *Geography* VIII, which includes a summary of time differences from Alexandria to east or west.

[110] Excising δυσμικώτερος at H189,6. Heiberg's text would mean 'and decrease if the standard place is the further west', which is the opposite of what is required. Manitius' excision of ὁ ὑποκείμενος produces a good sense ('if the required place is the further west'), and the same sense is found in part of the Arabic tradition (L, Ger, P, but not T, Q). But the word order favours my correction.

Book III

{*Preface*}[1]

In the preceding part of our treatise we have dealt with those aspects of heaven and earth which required, in outline, a preliminary mathematical discussion; H191
also the inclination of the sun's path through the ecliptic, and the resultant particular phenomena, both at *sphaera recta* and at *sphaera obliqua* for every inhabited region. We think that we should [now] discuss, as the subject which appropriately follows the above, the theory of the sun and moon, and go through the phenomena which are a consequence of their motions. For none of the phenomena associated with the [other] heavenly bodies can be completely investigated without the previous treatment of these [two]. Furthermore, we find that the subject of the sun's motion must take first place amongst these [sun and moon], since without that it would, again, be impossible to give a complete discussion of the moon's theory from start to finish.

1. {*On the length of the year*}[2]

The very first of the theorems concerning the sun is the determination of the length of the year. The ancients were in disagreement and confusion in their pronouncements on this topic, as can be seen from their treatises, especially those of Hipparchus, who was both industrious and a lover of truth. The main cause of the confusion on this topic which even he displayed is the fact that, when one examines the apparent returns [of the sun] to [the same] equinox or solstice, one finds that the length of the year exceeds 365 days by less than $\frac{1}{4}$-day, but when H192
one examines its return to [one of] the fixed stars it is greater [than $365\frac{1}{4}$ days]. Hence Hipparchus comes to the idea that the sphere of the fixed stars too has a very slow motion, which, just like that of the planets, is towards the rear with respect to the revolution producing the first [daily] motion, which is that of a [great] circle drawn through the poles of both equator and ecliptic.[3]

As for us, we shall show this is indeed the case, and how it takes place, in our discussion of the fixed stars[4] (the theory of the fixed stars, too, cannot be

[1] D and part of the Arabic tradition (L, P, but not Q, T) begin chapter 1 at this point. On such variations, and the conclusion to be drawn, see Introduction p. 5.

[2] See *HAMA* 54–5, Pedersen 128–34.

[3] This characterisation of the daily motion by means of the rotation of a great circle through the poles of equator and ecliptic refers back to I 8 p. 47.

[4] VII 2–3.

thoroughly investigated without previously establishing the theory of the sun and moon). However, for the purposes of the present investigation, it is our judgment that the only reference point we must consider when examining the length of the solar year is the return of the sun to itself, that is [the period in which it traverses] the circle of the ecliptic defined by its own motion. We must define the length of the year as the time the sun takes to travel from some fixed point on this circle back again to the same point. The only points which we can consider proper starting-points for the sun's revolution are those defined by the equinoxes and solstices on that circle. For if we consider the subject from a mathematical viewpoint, we will find no more appropriate way to define a
H193 'revolution' than that which returns the sun to the same relative position, both in place and in time, whether one relates it to the [local] horizon, to the meridian, or to the length of the day and night; and the only starting-points on the ecliptic which we can find are those which happen to be defined by the equinoxes and solstices. And if, instead, we consider what is appropriate from a physical point of view, we will not find anything which could more reasonably be considered a 'revolution' than that which returns the sun to a similar atmospheric condition and the same season; and the only starting-points one could find [for this revolution] are those which are the principal means of marking off the seasons from one another [i.e. solsticial and equinoctial points]. One might add that it seems unnatural to define the sun's revolution by its return to [one of] the fixed stars, especially since the sphere of the fixed stars is observed to have a regular motion of its own towards the rear with respect to the [daily] motion of the heavens. For, this being the case, it would be equally appropriate to say that the length of the solar year is the time it takes the sun to go from one conjunction with Saturn, let us say, (or any other of the planets) to the next. In this way many different 'years' could be generated. For the above reasons we think it appropriate to define the solar year as the time from one
H194 equinox or solstice to the next of the same kind, as determined by observations taken at the greatest possible interval.

Now since Hipparchus is somewhat disturbed by the suspicion, derived from a series of observations which he made in close succession, that this same revolution [of the sun] is not of constant length, we shall try to show succinctly that there is nothing to be disturbed about here. We became convinced that these intervals [from solstice to solstice etc.] do not vary, from the successive solstices and equinoxes which we ourselves have observed by means of our instruments. For we find that [the times of the observed solstices etc.] do not differ by a significant amount from those derivable from the [365¼]-day [year][5] (sometimes they differ by an amount roughly corresponding to the error which is explicable by the construction and positioning of the intruments). But we also guess from Hipparchus' own calculations that his suspicion concerning the irregularity [in the length of the tropical year] is an error due mainly to the observations he used.

For, in his treatise 'On the displacement of the solsticial and equinoctial points', he first sets out those summer and winter solstices which he considers to

[5] Literally 'from the surplus due to the ¼-day'.

have been observed accurately, in succession, and himself admits that these do not display enough discrepancies to allow one to use them to assert the existence of any irregularity[6] in the length of the year. He comments on them as follows: 'Now from the above observations it is clear that the differences in the year-length are very small indeed. However, in the case of the solstices, I have to H195
admit that both I and Archimedes may have committed errors of up to a quarter of a day in our observations and calculations [of the time]. But the irregularity in the length of the year can be accurately perceived from the [equinoxes] observed on the bronze ring situated in the place at Alexandria called the "Square Stoa". This is supposed to indicate the equinox on the day when the direction from which its concave surface is illuminated changes from one side to the other'.[7]

Then he sets out, first, the times of autumnal equinoxes which he considers to have been very accurately observed:

[1] In the seventeenth year of the Third Kallippic Cycle, Mesore 30 [-161 Sept. 27], about sunset.
[2] 3 years later, in the twentieth year, on the first epagomenal day [-158 Sept. 27], at dawn. This should have been at noon, so there is a $\frac{1}{4}$-day discrepancy.
[3] 1 year later, in the twenty-first year, [on the first epagomenal day, -157 Sept. 27], at the sixth hour. This was in agreement with the preceding observation.[8]
[4] 11 years later, in the thirty-second year, at the midnight between the third and fourth epagomenal days [-146 Sept. 26/27]. This should have been at dawn, so again there is a $\frac{1}{4}$-day discrepancy.
[5] 1 year later, in the thirty-third year, on the fourth epagomenal day [-145 Sept. 27], at dawn. This was in agreement with the previous observation. H196
[6] 3 years later, in the thirty-sixth year, on the fourth epagomenal day [-142 Sept. 26], in the evening. This should have been at midnight, so again there is only a $\frac{1}{4}$-day discrepancy.

Next he sets out the spring equinoxes which have been observed with a similar accuracy:

[6] Manitius claims that the reading ἀνισότητά τινα for ἀνισότητα at H194,21 is 'absolutely necessary'. It is Halma's text, adopted from the *editio princeps*. However, it is not found in any of the principal mss., and Heiberg's text as it stands can mean the same thing.

[7] For a diagram of this 'equatorial armillary' see Price, 'Precision Instruments' Fig. 343C on p. 589. It is simply a ring permanently fixed in the plane of the equator. From Ptolemy (p. 134) we learn that there were two such rings at Alexandria in his time, in the Palaestra. Whether either was identical with the one mentioned by Hipparchus cannot be discussed here. For what little is known about the 'Square Stoa' and the Palaestra (presumably in the great gymnasium mentioned in Strabo 17.1.10) see Fraser[1] II 98 n.222 and 223, I 28–9, and Fraser[2] 144–5.

[8] While there is general agreement that all the other equinox observations reported from Hipparchus were made by him in person, there is considerable dispute whether these three were observed by him or merely used by him. They are separated by an interval of 11 years from the next attested observation, which also falls into the period for which other types of observation by Hipparchus are recorded (the lunar eclipse of -145 Apr. 21, p. 135). My own view is that this group of three early observations was not made by Hipparchus himself, but was simply adduced by him for comparison.

[1] In the thirty-second year of the Third Kallippic Cycle, Mechir 27 [-145 Mar. 24], at dawn. Furthermore, he says, the ring at Alexandria was illuminated equally from both sides at about the fifth hour.[9] Thus we can already see two different observations of the same equinox with a discrepancy of approximately 5 hours.

[2 to 6] He says that the subsequent observations up to the thirty-seventh year [-144 to -140] were all in agreement with the times derivable from the [365]¼-day [year].

[7] 11 years later [than 1], in the forty-third year, he says, the spring equinox occurred after midnight Mechir 29/30 [-134 Mar. 23/24]. This was in agreement[10] with the observation [1] in the thirty-second year, and, he says, again agrees with the observations [8 to 13, -133 to -128] in the subsequent years up to the fiftieth year [14]. This took place on Phamenoth 1 [-127 Mar. 23], about sunset. This is approximately 1¾ days later [in the Egyptian year] than the [equinox] in the forty-third year. This also fits the 7-year interval.

H197 Thus in these observations too there is no discrepancy worth noticing, even though it is possible for an error of up to a quarter of a day to occur not only in observations of solstices, but even in equinox observations. For suppose that the instrument, due to its positioning or graduation, is out of true by as little as $\frac{1}{3600}$th of the circle through the poles of the equator: then, to correct an error of that size in declination, the sun, [when it is] near the intersection [of the ecliptic] with the equator, has to move ¼° in longitude on the ecliptic. Thus the discrepancy comes to about ¼ of a day.[11] The error could be even greater in the case of an instrument which, instead of being set up for the specific occasion and positioned accurately at the time of the actual observation, has been fixed once for all on a base intended to preserve it in the same position for a long period: [the error occurs when] the instrument is affected by a [gradual] displacement which is unnoticed because of the length of time over which it takes place. One can see this in the case of the bronze rings in our Palaestra, which are supposed to be fixed in the plane of the equator. When we observe with them, the distortion in their positioning is apparent, especially that of the larger and older of the two, to such an extent that sometimes the direction of illumination of the concave surface in them shifts from one side to the other twice on the same equinoctial day.[12]

[9] This statement has occasionally been used (most recently by Fraser[1] I 423) as evidence that Hipparchus observed in Alexandria. On the contrary, Ptolemy's expression makes it clear that this Alexandrian observation was different (and discrepant) from Hipparchus' own. Whenever the place of an observation by Hipparchus is known, it is Rhodes (except for his weather prognostications reported in Ptolemy's *Phaseis*, for which the place was Bithynia, presumably Hipparchus' native Nicaea).

[10] Reading ἀκόλουθον at H196,15 for the misprint ἀκόλουσθον.

[11] Ptolemy says that an observational error of 6′ in declination corresponds, near equinox, to an ecliptic motion of ¼° or (since the sun moves about 1° per day in the ecliptic) to an error of ¼ day in the time of observation. This is easily verified by linear interpolation in the declination table I 15, where the declination for 1° is 0;24,16°.

[12] For the ring see p. 133 n.7. If the instrument was correctly set up, at the moment of equinox the direction of illumination would shift from below the shadowing part to above it in spring (and vice

However, Hipparchus himself does not think that there is anything in the above observations which provides convincing support for his suspicion that there is an irregularity in the length of the year. Instead he makes computations on the basis of certain lunar eclipses, and declares that he finds that the variation in the length of the year, with respect to the mean value, is no more than $\frac{3}{4}$ of a day. This would be sufficiently great to take some account of, if it were indeed so; but it can be seen to be false from the very considerations which he adduces [to support it]. For he uses certain lunar eclipses which were observed to take place near [specific] fixed stars to compare the distance of the star called Spica in advance of the autumnal equinox at each [eclipse]. By this means he thinks he finds, on one occasion, a distance of $6\frac{1}{2}°$, the maximum in his time, and on another a distance of $5\frac{1}{4}°$, the minimum [in his time]. Thence he concludes that, since it is impossible for Spica [itself] to move so much in such a short time, it is plausible to suppose that the sun, which Hipparchus uses to determine the positions of the fixed stars, does not have a constant period of revolution. But this kind of computation cannot be made without using the sun's position at the eclipse as a basis. Thus, though he does not realise it, at each eclipse he is applying for this purpose [determination of the sun's position] the accurate observations of solstices and equinoxes which he himself has made[13] in these same years. By the very act of doing this he shows that, when one compares the length of those years, there is no discrepancy from the [$365\frac{1}{4}$-day interval. H198 H199

To take a single example: from the eclipse observation in the thirty-second year of the Third Kallippic Cycle which he adduces, he claims to find that Spica is $6\frac{1}{2}°$ in advance of the autumnal equinox, whereas from the eclipse observation in the forty-third year of that cycle he claims to find that it is $5\frac{1}{4}°$ in advance.[14] Likewise,[15] in order to carry out the computations for the above, he adduces the spring equinoxes which he had accurately observed in those years. This was in order that from the latter he could find the position of the sun at the middle of each eclipse, from these the positions of the moon, and from the positions of the moon those of the stars. He says that the spring equinox in the thirty-second year took place on Mechir 27 [−145 March 24] at dawn, and the one in the forty-third year on [Mechir] 29/30 [−134 March 23/24] after midnight, later [in the Egyptian year] than that in the thirty-second by approximately $2\frac{3}{4}$ days, which is the same amount as is produced by the addition of precisely $\frac{1}{4}$-day in each of

versa in autumn). Manitius (I 427 n.21) explains the phenomenon reported here by Ptolemy as due to the effect of refraction on a correctly placed ring. His argument is dismissed by Rome[5] I 230–5 and [1] II p. 818 n., on the grounds that the true one of the two 'equinoxes' could easily be determined by the direction of shift. This does not of course invalidate Manitius' explanation. The only good detailed discussion is Britton[1] 29–42, correcting both Manitius and Rome, and concluding (p. 34) that multiple "equinoxes" on a well-aligned ring would be normal.

[13] Reading ὑφ' ἑαυτοῦ (with D, Ar) at H198,24 for ἐφ' ἑαυτοῦ ('which were made in his time').

[14] The eclipses in question are those of −145 Apr. 21 and −134 Mar. 21 (misprinted March 31 in Pedersen Appendix A, 414). We have no further data on Hipparchus' observations of these eclipses. For a detailed discussion of the procedures involved see Rome[5] II. From VII 2 (p. 327) it seems that Hipparchus eventually settled on a compromise figure of 6° from the autumnal equinox in his own time.

[15] Meaning 'as *in the other* similar calculations'. D's reading is ὅμως, 'however', which makes good sense, but is not supported by the Arabic tradition.

the intervening 11 years. Since, then, the sun has been shown to complete its revolution (as measured with respect to those equinoxes) in a time neither greater nor less than the [365¼]-day interval, and since it is impossible for Spica to move
H200 $1\frac{1}{4}°$ in such a small number of years, surely it is perverse to use calculations based on the above foundations to impugn the very foundations on which they were based. It is perverse to ascribe the reason for such an impossibly large motion of Spica solely to the equinoxes on which the calculations are based (which entails the simultaneous assumptions, both that they are accurately observed, and that they have been inaccurately observed), when there are several possible causes for so great an error. It is more plausible to suppose, either that the distances of the moon from the nearest stars at the eclipses have been too crudely estimated, or that there has been an error or inaccuracy in the determinations of the moon's parallax with respect to its apparent position, or of the motion of the sun from the equinox to the time of mid-eclipse.

However, it is my opinion that Hipparchus himself realised that this kind of argumentation provides no persuasive evidence for the attribution of a second anomaly to the sun, but his love of truth led him not to suppress anything which might in any way lead some people to suspect [such an anomaly]. At any rate, he himself, in his theories of the sun and moon, assumes that the sun has a single and invariable anomaly, the period of which is the length of the year as
H201 defined by [return to] solstices and equinoxes. Furthermore, when we assume that the period of these revolutions of the sun is constant, we see that there is never any significant difference between the phenomena observed at eclipses and those calculated on the above assumption. Yet there would be a very perceptible difference if there were some correction due to a variation in the length of the year which we failed to take into account, even if that correction were as little as a single degree, which corresponds to approximately two equinoctial hours.[16]

From all the above considerations, and from our own determination of the period of the [solar] revolution, by means of a series of observations of the sun's position, we conclude that the length of the year is constant, provided that it is always defined with respect to the same criterion, and not with respect to the solsticial and equinoctial points at one time and to the fixed stars at another. We also conclude that the most natural definition of the revolution is that in which the sun, starting from one solstice or equinox or any point on the ecliptic, returns to the same point again. And in general, we consider it a good principle to explain the phenomena by the simplest hypotheses possible, in so far as there is nothing in the observations to provide a significant objection to such a procedure.[17]

Now it was already clear to us from Hipparchus' demonstrations that the length of the year, defined with respect to the solstices and equinoxes, is less than
H202 ¼-day in excess of 365 days. The amount by which it falls short [of ¼-day] cannot

[16] The time of an eclipse depends on the speeds of sun and moon. Assuming, with Ptolemy, round figures of $1^{ο/d}$ for the sun's motion and $13^{ο/d}$ for the moon's, we get a relative motion of $12^{ο/d}$, or $\frac{1}{2}°$ per hour. Thus a shift of 1° in the position of the sun at an eclipse leads to a shift of 2 hours in the time.

[17] This general principle of the desirability of simplicity in the hypotheses is repeated, but modified, at XIII 2 p. 600. Cf. also III 4 p. 153.

be determined with absolute certainty, since the difference is so small that for many years in succession the increment [over 365 days] remains sensibly the same as a constant $\frac{1}{4}$-day increment. Hence it is possible, when comparing observations taken over quite a long period, that the surplus days [over 365], which have to be obtained by distributing [the total surplus] over the years of the interval [between the observations], may appear to be the same whether one takes [observations over] a greater or lesser number of years. However, the longer the time between the observations compared, the greater the accuracy of the determination of the period of revolution. This rule holds good not only in this case, but for all periodic revolutions. For the error due to the inaccuracy inherent in even carefully performed observations is, to the senses of the observer, small and approximately the same at any [two] observations, whether these are taken at a large or a small interval. However, this same error, when distributed over a smaller number of years, makes the inaccuracy in the yearly motion [comparatively] greater (and [hence increases] the error accumulated over a longer period of time), but when distributed over a larger number of years makes the inaccuracy [comparatively] less. Hence we must consider it sufficient if we endeavour to take into account only that increase in the accuracy H203
of our hypotheses concerning periodic motions which can be derived from the length of time between us and those observations we have which are both ancient and accurate. We must not, if we can avoid it, neglect the proper examination [of such records]; but as for assertions of validity 'for eternity', or even for a length of time which is many times that over which the observations have been taken, we must consider such as alien to a love of science and truth.[18]

Now, as far as concerns antiquity [of the observations], the summer solstices observed by the school of Meton and Euktemon, and, later, the school of Aristarchus, deserve to be compared with those of our own time.[19] However, since observations of solstices are, in general, hard to determine accurately, and since, furthermore, the observations handed down by the above-mentioned people were conducted rather crudely (as Hipparchus too seems to think), we abandoned those, and have used instead, for the comparison we propose, equinox observations, choosing amongst them, for the sake of accuracy, those which Hipparchus especially noted as very securely determined by him, and those which we ourselves have made with the greatest accuracy using the instruments for such purposes described at the beginning of our treatise [I 12]. For these we find that the solstices and equinoxes occur earlier than [one would expect from a year of 365]$\frac{1}{4}$ days by one day in approximately 300 years. H204

For Hipparchus noted that in the thirty-second year of the Third Kallippic

[18] This remarkably sensible attitude towards the validity of mean motions derived from observations was not imitated by most of Ptolemy's successors throughout antiquity and the middle ages. The contemptuous remark about 'eternity' may be a glance at the αἰώνιοι κανόνες mentioned at IX 2 p. 422 (see n.12 there).

[19] The only solstices known to have been observed by these men are that of −431 June 27, ascribed below (p. 138) to 'the school of Meton and Euktemon', and that of −279 (no further details known) ascribed below (p. 138) to 'the school of Aristarchus'. The latter is Aristarchus of Samos, now famous mainly for his 'heliocentric hypothesis'. See Heath, *Aristarchus*. On Meton see Toomer[7]. By 'the school of . . .' I translate οἱ περὶ . . . The precise way to interpret the phrase here and elsewhere in the Almagest remains obscure.

Cycle he had made a very accurate observation of the autumnal equinox, and says that he calculated that it occurred at midnight, third-fourth epagomenal day [-146 Sept. 26/27]. The year is the 178th from the death of Alexander.[20] 285 years later, in the third year of Antoninus, which is the 463rd from the death of Alexander, we observed, again very securely, that the autumnal equinox occurred on Athyr 9 [139 Sept. 26], approximately one hour after sunrise.[21] Therefore the period of return comprised, over 285 complete Egyptian years (that is years of 365 days), $70\frac{1}{4}$ days plus approximately $\frac{1}{20}$th of a day, instead of the $71\frac{1}{4}$ days corresponding to the $\frac{1}{4}$-day surplus for the above [285] years. Thus the return took place earlier than it would have with the [365]$\frac{1}{4}$-day year by one day less about $\frac{1}{20}$th day.

Similarly, Hipparchus says that the spring equinox in the same thirty-second year of the Third Kallippic Cycle, which he observed most accurately, took place on Mechir 27 [-145 Mar. 24] at dawn. The year is the 178th from the
H205 death of Alexander. We find that the corresponding spring equinox 285 years later, in the 463rd year from the death of Alexander, took place on Pachon 7 [140 Mar. 22], approximately 1 hour after noon. Thus this period too comprised an increment [over 285 Egyptian years] of the same amount, $70\frac{1}{4}$ + about $\frac{1}{20}$ days, instead of the $71\frac{1}{4}$ days corresponding to the $\frac{1}{4}$-day surplus for the 285 years. Here too, then, the return of the spring equinox took place earlier than it would have with the [365]$\frac{1}{4}$-day year by $\frac{19}{20}$ths of a day. Hence, since

$$1 \text{ day} : \tfrac{19}{20} \text{ day} = 300 : 285,$$

we conclude that the return of the sun to the equinoctial points takes place earlier than it would for a [365]$\frac{1}{4}$-day year by approximately one day in 300 years.

Furthermore if, because of its antiquity, we compare the summer solstice observed by the school of Meton and Euktemon (though somewhat crudely recorded) with the solstice which we determined as accurately as possible, we will get the same result. For that [solstice] is recorded as occurring in the year when Apseudes was archon at Athens, on Phamenoth 21 in the Egyptian
H206 calendar [-431 June 27], at dawn.[22] We determined securely that the [summer solstice] in the above-mentioned 463rd year from the death of Alexander occurred on Mesore 11/12 [140 June 24/25] about 2 hours after midnight. Now there are 152 years (as Hipparchus too reckons) from the summer solstice recorded in the archonship of Apseudes to the solstice observed by the school of Aristarchus in the fiftieth year of the First Kallippic Cycle [-279], and from that fiftieth year, which corresponds to the 44th year from the death of Alexander, to the 463rd year, in which our observation was made, is 419 years. Therefore in

[20] On this (−323, not −322, the actual year of Alexander's death) see Introduction p. 10 n.16. '178th' is inclusive reckoning.

[21] Notoriously, like Ptolemy's spring equinox and summer solstice observations below, this is about 1 day later than the actual event. This is the strongest argument of those modern critics who have maintained that Ptolemy 'faked' observations. See Toomer[5] 189. The best discussion of this difficult problem is Britton[1] Chapter II.

[22] The Egyptian date of this observation was not given by Meton himself, who dated it to Skirophorion 13 in his calendar, but is a later conversion (found in the Milesian parapegma of the late second century B.C., see Samuel, *Greek and Roman Chronology* 44 or Toomer[7] 338, but no doubt already made by Hipparchus).

the whole interval of 571 years, if the summer solstice observed by the school of Euktemon took place around the dawning of Phamenoth 21, there is an increment of approximately $140\frac{5}{6}$ days over complete Egyptian years,[23] instead of the $142\frac{3}{4}$ days corresponding to the $\frac{1}{4}$-day surplus for 571 years. Thus the return in question took place earlier than it would have with the $[365]\frac{1}{4}$-day year by $1\frac{11}{12}$ days. Here too, then, it is clear that in a round 600 years the [true] year-length accumulates a decrement of approximately 2 complete days against the $[365]\frac{1}{4}$-day year.

We find the same result from a number of other observations of our own, and we see that Hipparchus agrees with it on more than one occasion. For in his work 'On the length of the year' he compares the summer solstice observed by Aristarchus at the end of the fiftieth year of the First Kallippic Cycle [-279] with H207
the one which he himself had determined, again with accuracy, at the end of the forty-third year of the Third Kallippic Cycle [-134], and then says: 'It is clear, then, that over 145 years the solstice occurs sooner than it would have with a $[365]\frac{1}{4}$-day year by half the sum of the length of day and night'. Again, in 'On intercalary months and days' also, after remarking that according to the school of Meton and Euktemon the length of the year comprises $365\frac{1}{4}+\frac{1}{76}$ days, but according to Kallippos only $365\frac{1}{4}$ days,[24] he comments, in his own words, as follows: 'As for us, we find the number of whole months comprised in 19 years to be the same as they found, but we find the year to be even less than $\frac{1}{4}$-[day beyond 365], by approximately $\frac{1}{300}$th of a day. Thus in 300 years its [accumulated] deficit is 5 days compared with Meton['s figure], and 1 day compared with Kallippos'.' And when he more or less sums up his opinions in his list of his own writings,[25] he says: 'I have also composed a work on the length of the year in one book, in which I show that the solar year (by which I mean the time in which the sun goes from a solstice back to the same solstice, or from an equinox back to the same equinox) contains 365 days, plus a fraction which is less than $\frac{1}{4}$ by about $\frac{1}{300}$th of the sum of one day and night, and not, as the H208
mathematicians[26] suppose, exactly $\frac{1}{4}$-day beyond the above-mentioned number [365] of days.'

Thus I think it appears plainly from the agreement of present-day [observations] with earlier ones, that all phenomena observed up to the present

[23] Ptolemy apparently reckons 'dawn' (πρωίας) in the earlier observation as 6 a.m. in equinoctial hours (despite the fact that at Athens sunrise at summer solstice occurs at about 4;45 a.m.), and means '2 hours after midnight' in his own observation to be 2 a.m., i.e. equinoctial hours. Then the increment over whole days between the observations is 20 equinoctial hours $=\frac{5}{6}$ day. If we were to take the times as 'precisely sunrise' and '2 seasonal hours', the interval would be closer to 21 hours, or $\frac{7}{8}$ day.

[24] These accord with the Metonic and Kallippic cycles respectively. See Introduction pp. 12-13.

[25] This phrase, which appears to have been misunderstood by all earlier translators, but is correctly interpreted by Rehm, 'Hipparchos' col. 1666, shows that Hipparchus published a catalogue of his own works with a summary of the contents of each. An example of this kind of publication which has come down to us is Galen's 'On his own Books' (περὶ τῶν ἰδίων βιβλίων), *Scripta Minora* II 91 ff. From Galen's work it is apparent that for a prolific writer of monographs, like Hipparchus, such a catalogue was necessary as a check on the ascription of his works (perhaps circulating in unauthorised versions) to others.

[26] οἱ μαθηματικοί, which includes astronomers. One might almost think from Hipparchus' tone that he means 'astrologers' (this is a standard meaning in later Greek). Ptolemy, however, does not use the word in this sense (cf. pp. 175 and 421, where I have translated it 'astronomers').

time having to do with the length of the solar year accord with the above-mentioned figure for the return to solstices or equinoxes. This being so, if we distribute the one day over the 300 years, every year gets 12 seconds of a day. Subtracting these from the $365;15^{d}$ of the $\frac{1}{4}$-day increment, we get the required length of the year as $365;14,48^{d}$. Such, then, is the closest possible approximation which we can derive from the available data.

Now, with regard to the determination of the positions of the sun and the other [heavenly bodies] for any given time, which the construction of individual tables is designed to provide in a handy and as it were readymade form: we think that the mathematician's task and goal ought to be to show all the heavenly phenomena being reproduced by uniform circular motions, and that the tabular form most appropriate and suited to this task is one which separates the individual uniform motions from the non-uniform [anomalistic] motion which [only] seems to take place, and is [in fact] due to the circular models; the apparent places of the bodies are then displayed by the combination of these two motions into one.[27] In order to have this type of table in a form which shall be usable and ready to hand for the actual proofs [which are to come], we shall now set out the individual uniform motions of the sun in the following manner.

H209 Since we have shown that one revolution contains $365;14,48^{d}$, dividing the latter into the 360° of the circle, we find the mean daily motion of the sun as approximately 0;59,8,17,13,12,31° (it will be sufficient to carry out divisions to this number [i.e. 6] of sexagesimal places).

Next, taking $\frac{1}{24}$th of the daily motion, we find the hourly motion as approximately 0;2,27,50,43,3,1°.

Similarly, we multiply the daily motion by 30, the number of days in one month, and get as the mean monthly motion 29;34,8,36,36,15,30°;

and, multiplying it by 365, the number of days in one Egyptian year, we get the mean annual motion as 359;45,24,45,21,8,35°.

Then we multiply the yearly motion by 18 years, since this number will produce symmetry in the layout of the tables,[28] and, after reduction of complete circles, we find the increment over 18 years to be 355;37,25,36,20,34,30°.

So we have set out three tables for the uniform motion of the sun, each again containing 45 lines, and each having two [vertical] sections. The first table will contain the mean motions of the 18-year periods, the second will contain the

[27] This is an implicit polemic against the ephemeris kind of astronomical table which gives the true positions of the planets (their 'apparent places'). To judge from the surviving papyri, the most common kind of planetary table was that giving the entries of the heavenly bodies into the zodiacal signs for a period of years (see *HAMA* II 785 ff.). Ptolemy was perhaps thinking of a kind of 'perpetual almanac' which gives the true positions of the planets at regular intervals over a whole planetary period. His argument is that his approach (mean motion tables modified by equation tables) gives a truer picture of the actual motions, which are uniform and circular.

[28] Despite Ptolemy's clear statement here of his motivation for choosing the 18-year period, it has been the subject of much fruitless debate. Starting from a standard height of 45 lines (see I 10 p. 56 n.67), and allowing some space for headings, he is led by the combination of single years on the same sheet with hours to 18 lines for that table (18 + 24 = 42 = 12 + 30 [months and days]). That is also the reason why the table for 18-year periods goes up to only 810 years (45 × 18), even though this does not reach Ptolemy's own time from his epoch. By the time he came to compose the Handy Tables, he had realised the inconvenience of this arrangement, and switched to 25-year periods and an epoch closer to his own time (Era Philip, -323 Nov. 12).

yearly motions above and the hourly motions below, and the third will contain the monthly motions above and the daily motions below. The numbers representing time will be in the first [i.e. left-hand] section, and the corresponding degrees, obtained by successive addition of the appropriate amount for each [time-unit], in the second [i.e. right-hand] section. The tables are as follows.

2. {*Table of the mean motion of the sun*}[29] H210—15

[See pp. 142–3.]

3. {*On the hypotheses for uniform circular motion*}[30] H216

Our next task is to demonstrate the apparent anomaly of the sun. But first we must make the general point that the rearward displacements of the planets with respect to the heavens are, in every case, just like the motion of the universe in advance, by nature uniform and circular. That is to say, if we imagine the bodies or their circles being carried around by straight lines, in absolutely every case the straight line in question describes equal angles at the centre of its revolution in equal times. The apparent irregularity [anomaly] in their motions is the result of the position and order of those circles in the sphere of each by means of which they carry out their movements, and in reality there is in essence nothing alien to their eternal nature in the 'disorder' which the phenomena are supposed to exhibit. The reason for the appearance of irregularity can be explained by two hypotheses, which are the most basic and simple. When their motion is viewed with respect to a circle imagined to be in the plane of the ecliptic, the centre of which coincides with the centre of the universe (thus its centre can be considered to coincide with our point of view), then we can suppose, either that the uniform motion of each [body] takes place on a circle which is not concentric with the universe, or that they have such a concentric circle, but their uniform motion takes place, not actually on that circle, but on H217
another circle, which is carried by the first circle, and [hence] is known as the 'epicycle'. It will be shown that either of these hypotheses will enable [the planets] to appear, to our eyes, to traverse unequal arcs of the ecliptic (which is concentric to the universe) in equal times.

In the eccentric hypothesis: [see Fig. 3.1] we imagine the eccentric circle, on which the body travels with uniform motion, to be ABGD on centre E, with diameter AED, on which point Z represents the observer.[31] Thus A is the apogee, and D the perigee. We cut off equal arcs AB and DG, and join BE, BZ, GE and GZ. Then it is immediately obvious that the body will traverse the arcs

[29] Corrections to Heiberg's text: H210, 23–5, column of fourths (for arguments 342, 360 and 378). A misprint has disrupted the order, which should be λ, να, ιβ, but has become να, ιβ, λ (51, 12, 30). H215,38, thirds : λε (35): λς, as Is.

[30] See *HAMA* 55–7, Pedersen 134–44.

[31] 'the observer'; literally 'our point of view'.

TABLE OF THE SUN'S MEAN MOTION

Distance [in Anomaly] from the Sun's Apogee in ♊ 5;30° to its Mean Longitude in the 1st Year of Nabonassar, ♓ 0;45° : 265;15°

18-Year Periods	°	′	″	‴	′′′′	′′′′′	′′′′′′
18	355	37	25	36	20	34	30
36	351	14	51	12	41	9	0
54	346	52	16	49	1	43	30
72	342	29	42	25	22	18	0
90	338	7	8	1	42	52	30
108	333	44	33	38	3	27	0
126	329	21	59	14	24	1	30
144	324	59	24	50	44	36	0
162	320	36	50	27	5	10	30
180	316	14	16	3	25	45	0
198	311	51	41	39	46	19	30
216	307	29	7	16	6	54	0
234	303	6	32	52	27	28	30
252	298	43	58	28	48	3	0
270	294	21	24	5	8	37	30
288	289	58	49	41	29	12	0
306	285	36	15	17	49	46	30
324	281	13	40	54	10	21	0
342	276	51	6	30	30	55	30
360	272	28	32	6	51	30	0
378	268	5	57	43	12	4	30
396	263	43	23	19	32	39	0
414	259	20	48	55	53	13	30
432	254	58	14	32	13	48	0
450	250	35	40	8	34	22	30
468	246	13	5	44	54	57	0
486	241	50	31	21	15	31	30
504	237	27	56	57	36	6	0
522	233	5	22	33	56	40	30
540	228	42	48	10	17	15	0
558	224	20	13	46	37	49	30
576	219	57	39	22	58	24	0
594	215	35	4	59	18	58	30
612	211	12	30	35	39	33	0
630	206	49	56	12	0	7	30
648	202	27	21	48	20	42	0
666	198	4	47	24	41	16	30
684	193	42	13	1	1	51	0
702	189	19	38	37	22	25	30
720	184	57	4	13	43	0	0
738	180	34	29	50	3	34	30
756	176	11	55	26	24	9	0
774	171	49	21	2	44	43	30
792	167	26	46	39	5	18	0
810	163	4	12	15	25	52	30

Single Years	°	′	″	‴	⁗	′′′′′	′′′′′′
1	359	45	24	45	21	8	35
2	359	30	49	30	42	17	10
3	359	16	14	16	3	25	45
4	359	1	39	1	24	34	20
5	358	47	3	46	45	42	55
6	358	32	28	32	6	51	30
7	358	17	53	17	28	0	5
8	358	3	18	2	49	8	40
9	357	48	42	48	10	17	15
10	357	34	7	33	31	25	50
11	357	19	32	18	52	34	25
12	357	4	57	4	13	43	0
13	356	50	21	49	34	51	35
14	356	35	46	34	56	0	10
15	356	21	11	20	17	8	45
16	356	6	36	5	38	17	20
17	355	52	0	50	59	25	55
18	355	37	25	36	20	34	30

Hours	°	′	″	‴	⁗	′′′′′	′′′′′′
1	0	2	27	50	43	3	1
2	0	4	55	41	26	6	2
3	0	7	23	32	9	9	3
4	0	9	51	22	52	12	5
5	0	12	19	13	35	15	6
6	0	14	47	4	18	18	7
7	0	17	14	55	1	21	9
8	0	19	42	45	44	24	10
9	0	22	10	36	27	27	11
10	0	24	38	27	10	30	12
11	0	27	6	17	53	33	14
12	0	29	34	8	36	36	15
13	0	32	1	59	19	39	16
14	0	34	29	50	2	42	18
15	0	36	57	40	45	45	19
16	0	39	25	31	28	48	20
17	0	41	53	22	11	51	21
18	0	44	21	12	54	54	23
19	0	46	49	3	37	57	24
20	0	49	16	54	21	0	25
21	0	51	44	45	4	3	27
22	0	54	12	35	47	6	28
23	0	56	40	26	30	9	29
24	0	59	8	17	13	12	31

Months	°	′	″	‴	⁗	′′′′′	′′′′′′
30	29	34	8	36	36	15	30
60	59	8	17	13	12	31	0
90	88	42	25	49	48	46	30
120	118	16	34	26	25	2	0
150	147	50	43	3	1	17	30
180	177	24	51	39	37	33	0
210	206	59	0	16	13	48	30
240	236	33	8	52	50	4	0
270	266	7	17	29	26	19	30
300	295	41	26	6	2	35	0
330	325	15	34	42	38	50	30
360	354	49	43	19	15	6	0

Days	°	′	″	‴	⁗	′′′′′	′′′′′′
1	0	59	8	17	13	12	31
2	1	58	16	34	26	25	2
3	2	57	24	51	39	37	33
4	3	56	33	8	52	50	4
5	4	55	41	26	6	2	35
6	5	54	49	43	19	15	6
7	6	53	58	0	32	27	37
8	7	53	6	17	45	40	8
9	8	52	14	34	58	52	39
10	9	51	22	52	12	5	10
11	10	50	31	9	25	17	41
12	11	49	39	26	38	30	12
13	12	48	47	43	51	42	43
14	13	47	56	1	4	55	14
15	14	47	4	18	18	7	45
16	15	46	12	35	31	20	16
17	16	45	20	52	44	32	47
18	17	44	29	9	57	45	18
19	18	43	37	27	10	57	49
20	19	42	45	44	24	10	20
21	20	41	54	1	37	22	51
22	21	41	2	18	50	35	22
23	22	40	10	36	3	47	53
24	23	39	18	53	17	0	24
25	24	38	27	10	30	12	55
26	25	37	35	27	43	25	26
27	26	36	43	44	56	37	57
28	27	35	52	2	9	50	28
29	28	35	0	19	23	2	59
30	29	34	8	36	36	15	30

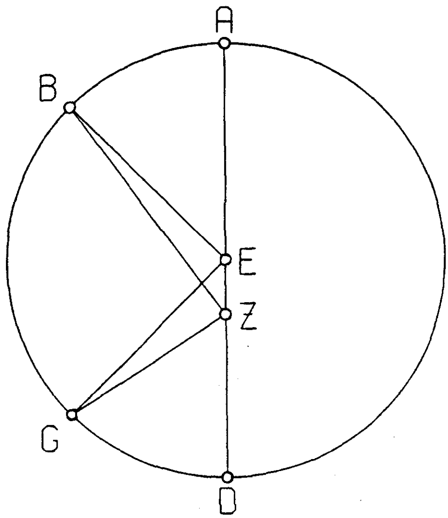

Fig. 3.1

AB and GD in equal times, but will [in so doing] appear to have traversed unequal arcs of a circle drawn on centre Z. For

$$\angle \text{BEA} = \angle \text{GED}.$$
$$\text{But } \angle \text{BZA} < \angle \text{BEA (or } \angle \text{GED)},$$
$$\text{and } \angle \text{GZD} > \angle \text{GED (or } \angle \text{BEA)}.$$

H218 In the epicyclic hypothesis: we imagine [see Fig. 3.2] the circle concentric with the ecliptic as ABGD on centre E, with diameter AEG, and the epicycle carried by it, on which the body moves, as ZHΘK on centre A.

Then here too it is immediately obvious that, as the epicycle traverses circle ABGD with uniform motion, say from A towards B, and as the body traverses the epicycle with uniform motion, then when the body is at points Z and Θ, it will appear to coincide with A, the centre of the epicycle, but when it is at other points it will not. Thus when it is, e.g., at H, its motion will appear greater than the uniform motion [of the epicycle] by arc AH, and similarly when it is at K its motion will appear less than the uniform by arc AK.

Now in this kind of eccentric hypothesis[32] the least speed always occurs at the apogee and the greatest at the perigee, since ∠ AZB [in Fig. 3.1] is always less than ∠ DZG. But in the epicyclic hypothesis both this and the reverse are possible. For the motion of the epicycle is towards the rear with respect to the
H219 heavens, say from A towards B [in Fig. 3.2]. Now if the motion of the body on the epicycle is such that it too moves rearwards from the apogee, that is from Z towards H, the greatest speed will occur at the apogee, since at that point both

[32] Ptolemy is hinting at the existence of another kind of eccentric hypothesis, one which is geometrically equivalent to that epicyclic hypothesis in which the sense of rotation is the same for both planet and epicycle. But he does not discuss this until XII 1 (p. 555), where we learn that the equivalence was already known to Apollonius of Perge (*c.* 200 B.C.). See *HAMA* 149–50.

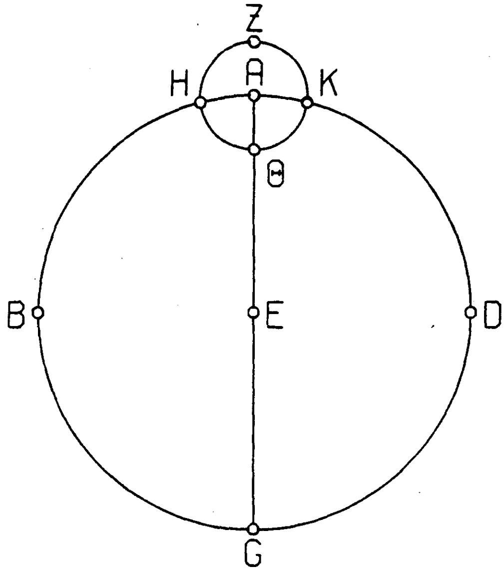

Fig. 3.2

epicycle and body are moving in the same direction. But if the motion of the body from the apogee is in advance on the epicycle, that is from Z towards K, then the reverse will occur: the least speed will occur at the apogee, since at that point the body is moving in the opposite direction to the epicycle.

Having established that, we must next make the additional preliminary point that for bodies which exhibit a double anomaly both the above hypotheses may be combined, as we shall prove in our discussions of such bodies, but for a body which displays a single invariant anomaly, a single one of the above hypotheses will suffice; and [in this case] all the phenomena will be represented, with no difference, by either hypothesis, provided that the same ratios are preserved in both. By this I mean that the ratio, in the eccentric hypothesis, of the distance between the centre of vision and the centre of the eccentre to the radius of the eccentre, must be the same as the ratio, in the epicyclic hypothesis, of the radius of the epicycle to the radius of the deferent;[33] and furthermore that the time taken by the body, travelling towards the rear, to H220 traverse the immovable eccentre, must be the same as the time taken by the epicycle, also travelling towards the rear, to traverse the circle with the observer as centre [the deferent], while the body moves with equal [angular] speed about the epicycle, but so that its motion at the apogee [of the epicycle] is in advance.

If these conditions are fulfilled, the identical phenomena will result from either hypothesis. We shall briefly show this [now] by comparing the ratios in abstract, and later by means of the actual numbers we shall assign to them for

[33] 'deferent': see Introduction p. 21.

the sun's anomaly.[34] I say then, first, that in both hypotheses, the greatest difference between the uniform motion and the apparent, non-uniform motion (which is also the notional position of the mean speed for the bodies)[35] occurs when the apparent distance from the apogee comprises a quadrant, and that the time between apogee [position] and the above-mentioned mean speed [position] is greater than the time between mean speed and perigee. Hence, for the eccentric hypothesis always, and for the epicyclic hypothesis when the motion at apogee is in advance, the time from least speed to mean is greater
H221 than the time from mean speed to greatest; for in both hypotheses the slowest motion takes place at the apogee. But [for the epicyclic hypothesis] when the sense of revolution of the body is rearwards from the apogee on the epicycle, the reverse is true: the time from greatest speed to mean is greater than the time from mean to least, since in this case the greatest speed occurs at the apogee.

First, then, [see Fig. 3.3] let the body's eccenter be ABGD on centre E, with diameter AEG. On this diameter take the centre of the ecliptic, that is, the position of the observer, at Z, and draw BZD through Z at right angles to AEG. Let the positions of the body be B and D, so that, obviously, its apparent distance from apogee A is a quadrant on either side. We have to prove that the greatest difference between mean and anomalistic motion takes place at points B and D.

Join EB and ED.

It is immediately obvious that the ratio of ∠ EBZ to 4 right angles equals the

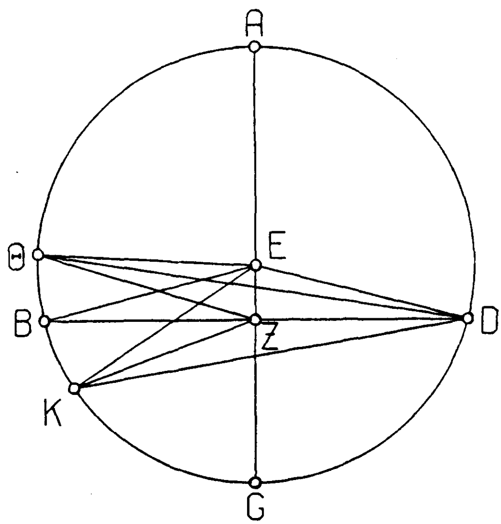

Fig. 3.3

[34] Reference to III 4 p. 157.

[35] Ptolemy never attempts to prove this statement about the position where the apparent motion equals the mean motion, but it is intuitively seen to be true from the epicyclic model. See *HAMA* 57, Pedersen 143.

ratio of the arc of the difference due to the anomaly[36] to the whole circle; for ∠ AEB subtends the arc of the uniform motion, and ∠ AZB subtends the arc of the apparent, non-uniform motion, and the difference between them is ∠ EBZ. H222

I say, then, that no angle greater than these two [∠ EBZ and ∠ EDZ] can be constructed on line EZ at the circumference of circle ABGD.

[Proof:] Construct at points Θ and K angles EΘZ and EKZ, and join ΘD, KD. Then since, in any triangle, the greater side subtends the greater angle,[37]

and ΘZ > ZD,

∴ ∠ ΘDZ > ∠ DΘZ.

But ∠ EDΘ = ∠ EΘD, since EΘ = ED [radii].

Therefore, by addition, ∠ EDZ (=∠ EBD) > ∠ EΘZ.

Again, since DZ > KZ,

∠ ZKD > ∠ ZDK.

But ∠ EKD = ∠ EDK, since EK = ED.

Therefore, by subtraction, ∠ EDZ (= ∠ EBZ) > ∠ EKZ.

Therefore it is impossible for any other angle to be constructed in the way defined greater than those at points B and D. H223

Simultaneously it is proven that arc AB, which represents the time from least speed to mean, exceeds BG, which represents the time from mean speed to greatest, by twice the arc comprising the equation of anomaly. For ∠ AEB exceeds a right angle (∠ EZB) by ∠ EBZ, and ∠ BEG falls short of a right angle by the same amount.

To prove the same theorem again for the other hypothesis, let [Fig. 3.4] the circle concentric with the universe be ABG on centre D and diameter ADB, and let the epicycle which is carried around it in the same plane be EZH on centre A. Let us suppose the body to be at H when its apparent distance from the apogee is a quadrant. Join AH and DHG.

I say that DHG is tangent to the epicycle; for that is the position in which the difference between uniform and anomalistic motion is greatest. H224

[Proof:] The mean motion, counted from the apogee, is represented by ∠ EAH; for the body traverses the epicycle with the same [angular] speed as the epicycle traverses circle ABG. Furthermore the difference between mean and apparent motion is represented by ∠ ADH. Therefore it is clear that the amount by which ∠ EAH exceeds ∠ ADH (namely ∠ AHD) represents the apparent distance of the body from the apogee. But this distance is, by hypothesis, a quadrant. Therefore ∠ AHD is a right angle, and hence line DHG is tangent to epicycle EZH. Therefore arc AG, since it comprises the distance between the centre A and the tangent, is the greatest possible difference due to the anomaly.

By the same reasoning, arc EH, which according to the sense of rotation on

[36] This expression is later used as a technical term for the angle corresponding to ∠ EBZ here, and is usually translated 'equation of anomaly'. See Introduction pp. 21–2.

[37] Precisely this statement, that the greater angle is subtended by the greater side, is the enunciation of Euclid I 19 (which Heiberg refers to ad loc.). But in fact what underlies Ptolemy's statement is that, if side a is greater than side b, angle A is greater than angle B, which is Euclid I 18. Perhaps we should adopt the reading of D, ὑπὸ τὴν μείζονα πλευρὰν ἡ μείζων γωνία ὑποτείνει ('the greater angle subtends the greater side'), and assume that the text has been assimilated to the (wrong) Euclidean wording.

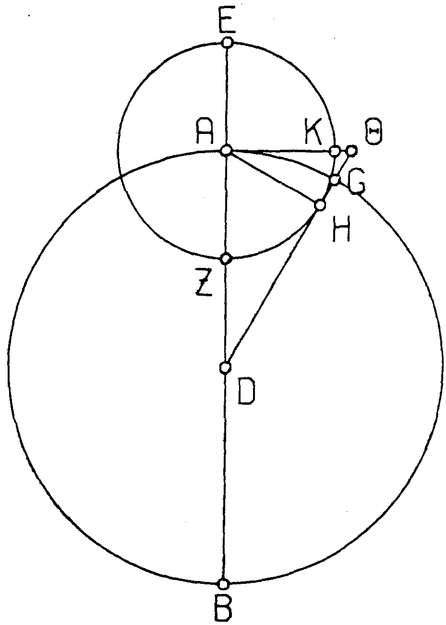

Fig. 3.4

the epicycle assumed here, represents the time from least speed to mean, exceeds arc HZ, which represents the time from mean speed to greatest, by
H225 twice arc AG. For if we produce DH to Θ and draw AKΘ at right angles to EZ,

$\angle$ KAH = $\angle$ ADG,[38]

and arc KH = arc AG.[39]

And arc EKH is greater than a quadrant by arc KH,
while arc ZH is less than a quadrant by arc KH.

Q.E.D.

It is also true that the same effects will be produced by both hypotheses if one takes a partial motion over the same stretch of time for both, whether one considers the mean motion or the apparent, or the difference between them, that is the equation of anomaly. The best way to see that is as follows.

[See Fig. 3.5.][40] Let the circle concentric with the ecliptic be ABG on centre D, and let the circle which is eccentric but equal to the concentre ABG be EZH on centre Θ. Let the common diameter through their centres D, Θ and the

[38] Euclid VI 8.

[39] To get a grammatical text I excise ὁμοία at H225,4. It was introduced (at an early period, since it is reflected in the Arabic translations) as a correction of Ptolemy's inaccurate (to the scholastic mind) statement that arc KH *equals* arc AG. Since the arcs are on circles of different sizes, they are technically only 'similar'. An alternative correction would be ἴσαι μὲν γίγνονται αἵ τε ὑπὸ ΚΑΗ καὶ ΑΔΗ γωνίαι (which is actually found in Theon's commentary ad loc., Rome III 868,8, but is probably a paraphrase; it also seems to be behind L).

[40] The figure in Heiberg (p. 225) wrongly omits the letter corresponding to L (though this is found in all mss.). Manitius, misled by this, 'emended' ΑΛ at H226,23 to the nonsensical 'AB'.

apogee E be EAΘD. Cut off at random an arc AB on the concentre, and with centre B and radius DΘ draw the epicycle KZ. Join KBD.

I say that the body will be carried by both kinds of motion [i.e. according to both hypotheses] to point Z, the intersection of the eccentre and the epicycle, in the same time in all cases (that is, the three arcs, EZ on the eccentre, AB on the H226

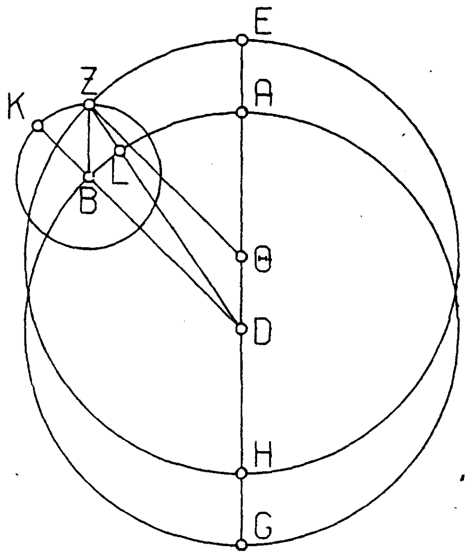

Fig. 3.5

concentre, and KZ on the epicycle, are all similar), and that the difference between uniform and anomalistic motion, and the apparent positions of the body, will turn out to be one and the same according to both hypotheses. [Proof:] Join ZΘ, BZ and DZ.

Since, in the quadrilateral BDΘZ, the opposite sides are equal, ZΘ to BD and BZ to DΘ, BDΘZ is a parallelogram.

Therefore $\angle$ EΘZ = $\angle$ ADB = $\angle$ ZBK.

Therefore, since they are angles at the centre [of circles], the arcs subtended by them are also similar, i.e.

Arc EZ of the eccentre || arc AB of the concentre || arc KZ of the epicycle.

Therefore the body will be carried by both kinds of motions in the same time to the same point, Z, and will appear to have traversed the same arc AL of the ecliptic from the apogee, and accordingly the equation of anomaly will be the same in both hypotheses; for we showed that that equation is represented by $\angle$ DZΘ in the eccentric hypothesis and by $\angle$ BDZ in the epicyclic hypothesis, and these two angles are alternate and equal, since, as we have shown, ZΘ is parallel to BD. H227

It is obvious that the same results will hold good for all distances [of the body from the apogee]. For quadrilateral ΘDZB will always be a parallelogram, and [hence] the motion of the body on the epicycle will actually describe the eccentric circle, provided the ratios[41] are similar and their members equal in both hypotheses.

Moreover, even if the members are unequal in size, provided their ratios are similar, the same phenomena will result. This can be shown as follows.

As before [see Fig. 3.6] let the circle concentric with the universe be ABG on centre D and the diameter, on which the body reaches apogee and perigee positions, ADG. Let the epicycle be drawn on point B, at an arbitrary distance, arc AB, from apogee A. Let the arc traversed by the body [on the epicycle] be
H228 EZ, which is, obviously, similar to AB, since the revolutions on [both] circles have the same period. Join DBE, BZ, DZ.

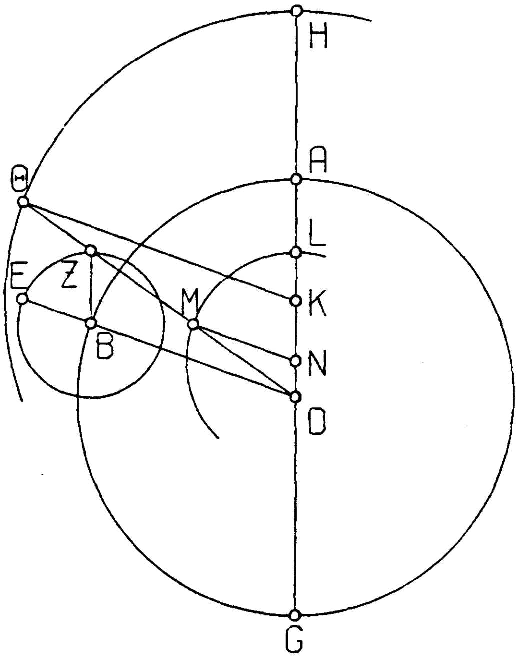

Fig. 3.6

Now it is immediately obvious that, according to this [epicyclic] hypothesis, ∠ ADE will always equal ∠ ZBE, and the body will appear to lie on line DZ.

But I say that the body will also appear to lie on the same line DZ according to the eccentric hypothesis, whether the eccentre is greater or smaller than the concentre ABG, provided only that one assumes that the ratios are similar and that the periods of revolution are the same.

[Proof:] Let the eccentre be drawn under the conditions we have described, greater [than the concentre] as HΘ on centre K ([which must lie] on AG), and

[41] The ratios are e:R and r:R.

smaller [than the concentre] as LM on centre N (this too [must lie on AG]). Produce DZ as DMZΘ, and DA as DLAH, and join ΘK, MN.

Then since

$$DB:BZ = \Theta K:KD = MN:ND \text{ [by hypothesis]},$$
$$\text{and } \angle BZD = \angle MDN \text{ (since DA is parallel to BZ);}$$ H229

the three triangles [ZDB,DΘK,DMN] are equiangular,
and $\angle BDZ = \angle D\Theta K = \angle DMN$ (angles subtended by corresponding sides).

Therefore DB, ΘK and MN are parallel.

$$\therefore \angle ADB = \angle AK\Theta = \angle ANM.$$

Since these angles are at the centres of their circles, the arcs on them, AB, HΘ and LM, will also be similar.

So it is true, not only that the epicycle has traversed arc AB in the same time as the body has traversed arc EZ, but also that the body will have traversed arcs HΘ and LM on the eccentres in that same time; hence in every case it will be seen along the same line DMZΘ, according to the epicyclic [hypothesis] at point Z, according to the greater eccentre at point Θ, and according to the smaller eccentre at point M. The same will hold true in all positions.

A further consequence is that where the apparent distance of the body from apogee [at one moment] equals its apparent distance from perigee [at another], the equation of anomaly will be the same at both positions.
[Proof:] In the eccentric hypothesis [see Fig. 3.7], we draw the eccentric circle

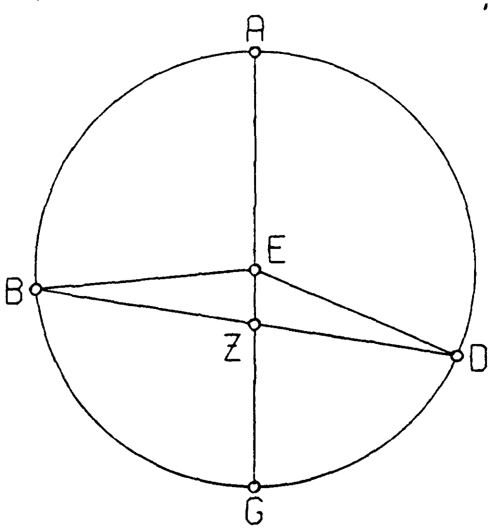

Fig. 3.7

ABGD on centre E and diameter AEG through apogee A. We suppose the observer to be located at Z, and draw an arbitrary [chord] BZD through Z, and join EB and ED. Then the apparent positions [of the body at B and D] will be equal and opposite, that is the angle AZB from the apogee will be equal and H230 opposite to angle GZD from the perigee; and the equation of anomaly will be the same [in both cases], since

$$BE = ED, \text{ and } \angle\, EBZ = \angle\, EDZ.$$

So the arc [AB] of mean motion counted from the apogee A will exceed the arc of apparent motion (i.e. the arc subtended by angle AZB) by the same equation [equal to $\angle$ EBZ] as the arc of mean motion counted from the perigee G is exceeded by the arc of apparent motion (i.e. the [equal] arc subtended by $\angle$ GZD). For

$$\angle\, AEB > \angle\, AZB, \text{ and } \angle\, GED < \angle\, GZD.$$

In the epicyclic hypothesis [see Fig. 3.8] if, as before, we draw the concentre
ABG on centre D and diameter ADG, and the epicycle EZH on centre A, draw
H231 an arbitrary line DHBZ, and join AZ and AH, then the arc AB representing the
equation of anomaly will be the same at both positions, i.e. whether the body is

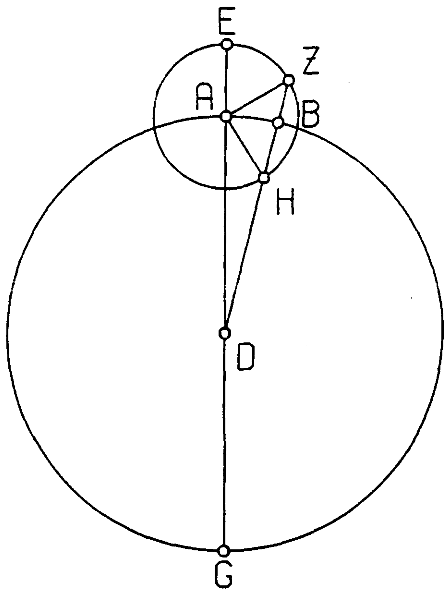

Fig. 3.8

at Z or at H. And the distance of the body from the point on the ecliptic corresponding to the apogee when it is at Z will be equal to its distance from the point corresponding to the perigee when it is at H. For the arc of its apparent distance from the apogee is represented by $\angle$ DZA, since, as we showed, this is the difference between the mean motion and the equation of anomaly.[42] And the arc of its apparent distance from the perigee is represented by $\angle$ ZHA (for this, too, is equal to the mean motion from the perigee plus the equation of anomaly).

$$\text{But } \angle\, DZA = \angle\, ZHA, \text{ since } AZ = AH.$$

[42] $\angle$ DZA = $\angle$ EAZ−$\angle$ ADZ. Shown p. 147.

Thus here too we conclude that the mean motion exceeds the apparent near the apogee (i.e. ∠ EAZ exceeds ∠ AZD) by the same equation (namely ∠ ADH) as the mean motion is exceeded by the (same) apparent motion (i.e. ∠ HAD by H232
∠ AHZ) near the perigee.

Q.E.D.

4. {*On the apparent anomaly of the sun*}[43]

Having set out the above preliminary theorems, we must add a further preliminary thesis concerning the apparent anomaly of the sun. This has to be a single anomaly, of such a kind that the time taken from least speed to mean shall always be greater than the time from mean speed to greatest, for we find that this accords with the phenomena. Now this could be represented by either of the hypotheses described above, though in case of the epicyclic hypothesis the motion of the sun on the apogee arc of the epicycle would have to be in advance. However, it would seem more reasonable to associate it with the eccentric hypothesis, since that is simpler and is performed by means of one motion instead of two.[44]

Our first task is to find the ratio of the eccentricity of the sun's circle, that is, the ratio which the distance between the centre of the eccentre and the centre of the ecliptic (located at the observer) bears to the radius of the eccentre. We must also find the degree of the ecliptic in which the apogee of the eccentre is located. These problems have been solved by Hipparchus with great care.[45] He assumes H233
that the interval from spring equinox to summer solstice is 94½ days, and that the interval from summer solstice to autumnal equinox is 92½ days, and then, with these observations as his sole data, shows that the line segment between the above-mentioned centres [of eccentre and ecliptic] is approximately $\frac{1}{24}$th of the radius of the eccentre, and that the apogee is approximately 24½° (where the ecliptic is divided into 360°) in advance of the summer solstice. We too, for our own time, find approximately the same values for the times [taken by the sun to traverse] the above-mentioned quadrants, and for those ratios. Hence it is clear to us that the sun's eccentre always maintains the same position relative to the solsticial and equinoctial points.[46]

In order not to neglect this topic, but rather to display the theorem worked out according to our own numerical solution, we too shall solve the problem, for the eccentre, using the same observed data, namely, as already stated, that the interval from spring equinox to summer solstice comprises 94½ days, and that

[43] See *HAMA* 57–8, Pedersen 144–9.

[44] On the desirability of simplicity in hypotheses see III 1 p. 136 with n.17.

[45] Reading μετὰ πάσης σπουδῆς (with D, Ar) at H233,1–2 for μετὰ σπουδῆς ('with care').

[46] According to Ptolemy the sun's apogee (unlike those of the five planets, as it later turns out, IX 7) does not share in the motion of precession. The reproaches that have been cast on Ptolemy (e.g. by Manitius I 428–9) for failing to discover that the sun's apogee too has a motion through the ecliptic are unjustified. To do that he would have needed observations of the time of equinox and solstice far more accurate than those available (to the nearest ¼-day), and not only for his own time but also for an earlier time. See the papers by Rome[3] and Petersen and Schmidt for a mathematical demonstration of this.

from summer solstice to autumnal equinox $92\frac{1}{2}$ days. For our own very precise
H234 observations of the equinoxes and the summer solstice in the 463rd year from the death of Alexander confirm the day-totals in these intervals: as we said, [III 1, p. 138], the autumnal equinox occurred on Athyr [III] 9, [139 Sept. 26], after sunrise, the spring equinox on Pachon [IX] 7 [140 March 22], after noon (thus the interval [between them] is $178\frac{1}{4}$ days), and the summer solstice on Mesore [XII] 11/12, [140 June 24/25], after midnight. Thus this interval, from spring equinox to summer solstice, comprises $94\frac{1}{2}$ days, which leaves approximately $92\frac{1}{2}$ days to complete the year; this number represents the interval from the summer solstice to the following autumnal equinox.[47]

[See Fig. 3.9.] Let the ecliptic be ABGD on centre E. In it draw two diameters, AG and BD, at right angles to each other, through the solsticial and equinoctial points. Let A represent the spring [equinox], B the summer [solstice], and so on in order.

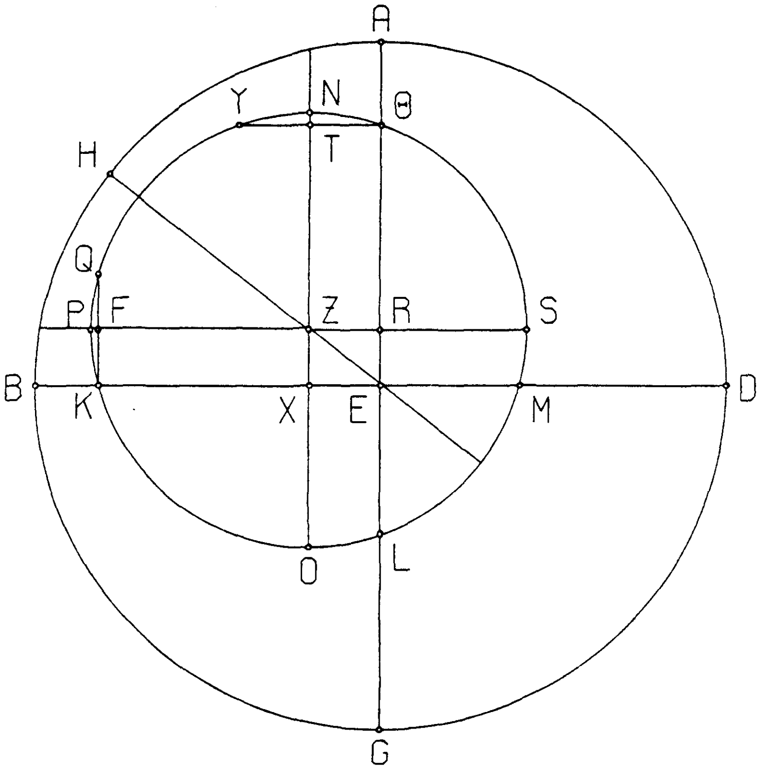

Fig. 3.9

[47] In III 1 the precise times of day given are '1 hour after sunrise', '1 hour after noon' and '2 hours after midnight'. Thus the precise intervals are $178\frac{1}{4}$ days and $94^d\ 13^h$, leading to corrected figures of $94^d\ 13^h$ and $92^d\ 11^h$ for the intervals used in the computation. But see p. 139 n.23 for the possibility that the time of solstice is '2 seasonal hours' ($\approx 1\frac{2}{3}$ equinoctial hours). Even as small a change as 1 hour in an interval has an effect of about 1° in the location of the apogee (cf. Petersen and Schmidt 80-3 and Rome[3] 13-15).

Now it is clear that the centre of the eccentre will be located between lines EA and EB. For semi-circle ABG comprises more than half of the length of the year, and hence cuts off more than a semi-circle of the eccentre; and quadrant AB too comprises a longer time and cuts off a greater arc of the eccentre than quadrant H235
BG. This being so, let point Z represent the centre of the eccentre, and draw the diameter through both centres and the apogee, EZH. With centre Z and arbitrary radius draw the sun's eccentre ΘKLM, and draw through Z lines NXO parallel to AG and PRS parallel to BD. Draw perpendicular ΘTY from Θ to NXO and perpendicular KFQ from K to PRS.

Now since the sun traverses circle ΘKLM with uniform motion, it will traverse arc ΘK in 94½ days, and arc KL in 92½ days. In 94½ days its mean motion is approximately 93;9°, and in 92½ days 91;11°. Therefore

arc ΘKL = 184;20°

and, by subtraction of the semi-circle NPO [from arc ΘKL], H236
arc NΘ + arc LO [= 184;20° − 180°] = 4;20°

So arc ΘNY = 2 arc ΘN = 4;20° also,

$\therefore$ ΘY = Crd arc ΘNY $\approx$ 4;32^p
and EX = ΘT = ½ΘY = 2;16^p } where the diameter of the eccentre = 120^p.

Now since arc ΘNPK = 93;9°,
and arc ΘN = 2;10° and quadrant NP = 90°,
by subtraction, arc PK = 0;59°,
and arc KPQ = 2 arc PK = 1;58°.

$\therefore$ KFQ = Crd arc KPQ = 2;4^p,
and ZX = KF = ½KFQ = 1;2^p } where the diameter of the eccentre = 120^p.

And we have shown that EX = 2;16^p in the same units.

Now since $EZ^2 = ZX^2 + EX^2$,

EZ $\approx$ 2;29½^p where the radius of the eccentre = 60^p.

Therefore the radius of the eccentre is approximately 24 times the distance between the centres of the eccentre and the ecliptic.

Now, since EZ:ZX = 2;29½ : 1;2, H237
ZX will be about 49;46^p where hypotenuse EZ = 120^p.

Therefore, in the circle about right-angled triangle EZX,

arc ZX $\approx$ 49°,

$$\therefore \angle \text{ZEX} = \begin{cases} 49^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \\ 24;30^\circ \text{ where 4 right angles} = 360^{\circ\circ}. \end{cases}$$

So, since ∠ ZEX is an angle at the centre of the ecliptic, arc BH, which is the amount by which the apogee at H is in advance of the summer solstice at B, is also 24;30°.

Furthermore, since quadrants OS and SN are each 90°,
and arc OL = arc ΘN = 2;10°,
and arc MS = 0;59°,
$\therefore$ arc LM = 86;51°,
and arc MΘ = 88;49°.

But the sun in its uniform motion travels
86;51° in about 88⅛ days,
and 88;49° in about 90⅛ days.

Hence it is clear that the sun will traverse arc GD, which extends from the

H238 autumnal equinox to the winter solstice, in about 88⅛ days, and arc DA, which extends from the winter solstice to the spring equinox, in about 90⅛ days. The above conclusions are in agreement with what Hipparchus says.

Using these quantities, then, let us first see what the greatest difference between mean and anomalistic motions is, and at what points it will occur.

[See Fig. 3.10.] Let the eccentric circle be ABG on centre D and diameter ADG through the apogee A, on which E represents the centre of the ecliptic. Draw EB at right angles to AG, and join DB.

Now since, where BD, the radius, equals 60^p, DE, the eccentricity, equals $2;30^p$ (according to the ration 24:1),

in the circle about right-angled triangle BDE,

$$DE = 5^p \text{ where hypotenuse } BD = 120^p,$$

$$\text{and arc } DE \approx 4;46^\circ.$$

Therefore $\angle$ DBE, which represents the greatest equation of anomaly,

H239
$$= \begin{cases} 4;46^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \\ 2;23^\circ \text{ where 4 right angles} = 360^\circ. \end{cases}$$

In the same units, right angle $\text{BED} = 90^\circ$,

$$\text{and } \angle BDA = \angle DBE + \angle BED = 92;23^\circ.$$

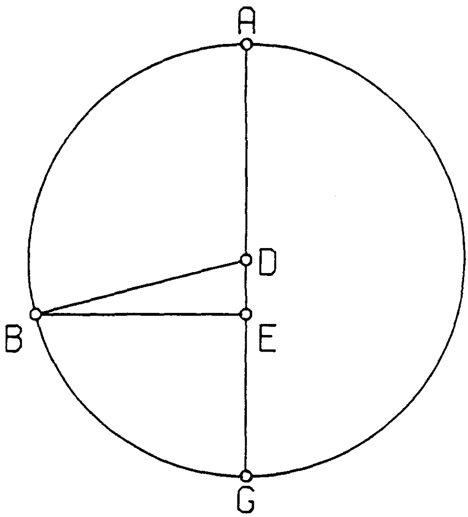

Fig. 3.10

Thus, since $\angle$ BDA is at the centre of the eccentre and $\angle$ BED is at the centre of the ecliptic, we conclude that the greatest equation of anomaly is 2;23°, and the position where it occurs is 92;23° from the apogee, measured along the eccentre in uniform motion, and (as we proved earlier) a quadrant, or 90° [from the apogee], measured along the ecliptic in anomalistic motion. It is obvious from our previous results that in the opposite semi-circle[48] the mean speed and the greatest equation of anomaly will occur at 270° of apparent motion, and at 267;37° of mean motion on the eccentre.

[48] Reading ἡμικύκλιον (with D,Ar) for τμῆμα ('segment') at H239,12.

We now want to use numerical computation, as we promised [pp. 145–6], to show that one derives the same quantities from the epicyclic hypothesis too, provided the same ratios are preserved in the way we explained.

[See Fig. 3.11.] Let the circle concentric to the ecliptic be ABG on centre D and diameter ADG, and the epicycle circle EZH on centre A. From D draw a tangent to the epicycle, DZB, and join AZ. Then, as before, in the right-angled triangle ADZ, AD is 24 times AZ, so that, in the circle about right-angled triangle ADZ, AZ is, again, 5^p where hypotenuse AD is 120^p, and the arc on AZ is 4;46°. H240

$$\therefore \angle \text{ADZ} = \begin{cases} 4;46^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \\ 2;23^{\circ} \text{ where 4 right angles} = 360^{\circ}. \end{cases}$$

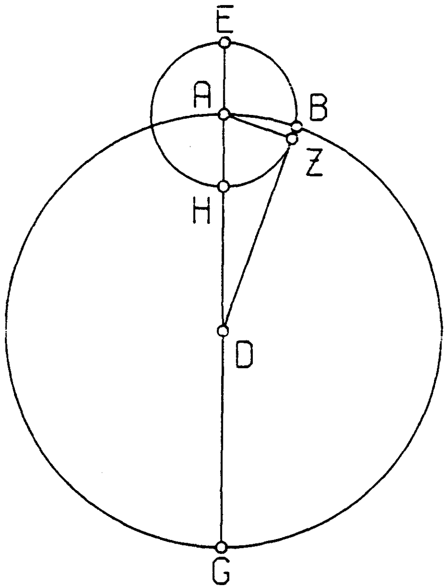

Fig. 3.11

Therefore the greatest equation of anomaly, namely arc AB, has been found to be 2;23° here too, in agreement with [the previous result], and the arc of anomalistic motion is 90°, since it is represented by the right angle AZD, while the arc of mean motion, which is represented by $\angle$ EAZ, is again 92;23°.

5. {*On the construction of a table for individual subdivisions of the anomaly*}[49]

In order to enable one to determine the anomalistic motion over any

[49] Reading τῶν ἀνωμαλιῶν κανονοποιίας at H240.16–17, with D (cf. all Greek mss. in the table of contents, H190.9–10) for τῆς ἀνωμαλίας ἐπισκέψεως ('investigation of the anomaly for partial stretches', which is the reading of Ar in both places).

On chs. 5 and 6 see *HAMA* 58–60, Pedersen 149–51.

subdivision [of the circle], we shall show, again for both hypotheses, how, given one of the arcs in question, we can compute the others.

H241 [See Fig. 3.12.] First, let the circle concentric to the ecliptic be ABG on centre D, the eccentre EZH on centre Θ, and let the diameter through both centres and the apogee E be EAΘDH. Cut off arc EZ, and join ZD, ZΘ. First, let arc EZ be given, e.g. as 30°.

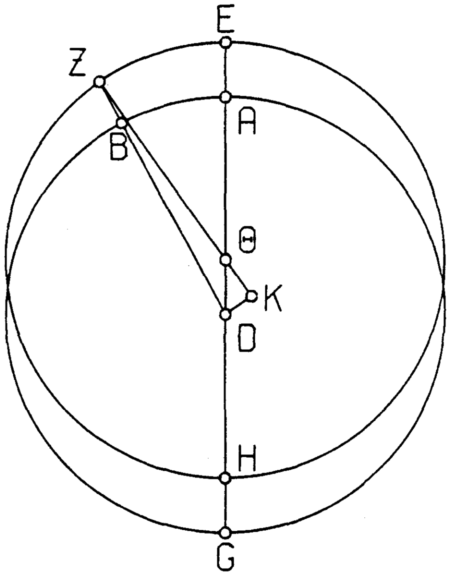

Fig. 3.12

Produce ZΘ and drop the perpendicular to it from D, DK.

Then, since arc EZ is, by hypothesis, 30°,

$$\angle\ \mathrm{E\Theta Z} = \angle\ \mathrm{D\Theta K} = \begin{cases} 30^\circ & \text{where 4 right angles} = 360^\circ \\ 60^{\circ\circ} & \text{where 2 right angles} = 360^{\circ\circ}. \end{cases}$$

Therefore, in the circle about right-angled triangle DΘK,

arc DK = 60°

and arc KΘ = 120° (supplement).

Therefore the corresponding chords

$$\left.\begin{array}{r} \mathrm{DK} = 60^p \\ \text{and } \mathrm{K\Theta} = 103;55^p \end{array}\right\} \text{ where hypotenuse } \mathrm{D\Theta} = 120^p.$$

Therefore, where DΘ = 2;30^p and radius ZΘ = 60^p,

DK = 1;15^p and ΘK = 2;10^p.

H242 Therefore, by addition [of ΘK to radius ZΘ], KΘZ = 62;10^p.

Now since $\mathrm{DK}^2 + \mathrm{K\Theta Z}^2 = \mathrm{ZD}^2$,

the hypotenuse ZD ≈ 62;11^p.

Therefore, where ZD = 120^p, DK = 2;25^p,

and, in the circle about right-angled triangle ZDK,

arc DK = 2;18°.

$$\therefore \angle \text{DZK} = \begin{cases} 2;18^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \\ 1;9^{\circ} \text{ where 4 right angles} = 360^{\circ}. \end{cases}$$

That [1;9°] will be the amount of the equation of anomaly at this position.

And ∠ EΘZ was taken as 30°.

Therefore, by subtraction, ∠ ADB (which equals arc AB of the ecliptic) equals 28;51°.

Furthermore, if any other of the [relevant] angles be given [instead of ∠ EΘZ], the remaining angles will be given, as is immediately obvious if, in the same figure [see Fig. 3.13] we drop perpendicular ΘL from Θ on to ZD.

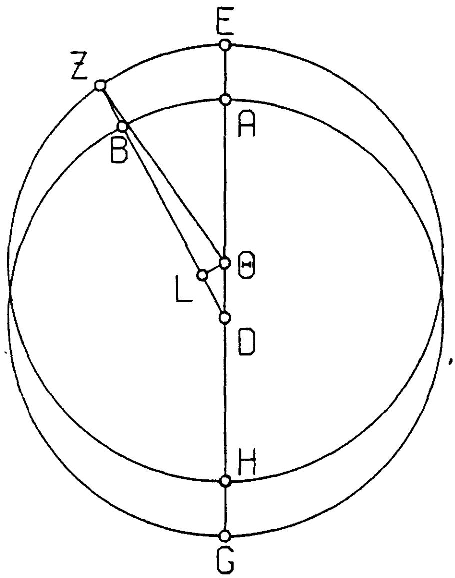

Fig. 3.13

For suppose first that arc AB of the ecliptic, i.e. ∠ ΘDL, is given. Then the ratio DΘ:ΘL will be given.[50] And since DΘ:ΘZ is also given, ΘZ:ΘL will be given.[51] Hence ∠ ΘZL, the equation of anomaly, will be given.[52] and so will ∠ EΘZ, i.e. arc EZ of the eccentre. H243

Or suppose, secondly, that the equation of anomaly, i.e. ∠ ΘZD, is given: we will get the same results in reverse order. For from ∠ ΘZD the ratio ΘZ:ΘL will be given, and ΘZ:ΘD is given from the beginning. Hence DΘ:ΘL will be given, and hence ∠ ΘDL, i.e. arc AB of the ecliptic, and [hence] ∠ EΘZ, i.e. arc EZ of the eccentre.

[50] Euclid *Data* 40: if the angles of a triangle are given, its sides are given in form (i.e. the ratio of the sides is given, cf. *Data* 3).

[51] Euclid *Data* 8: magnitudes having a given ratio to the same magnitude have a given ratio to each other. DΘ:ΘZ is given as the ratio of eccentricity.

[52] Euclid *Data* 43: if, in a right-angled triangle, the sides about one of the acute angles have a given ratio, the triangle is given in form (cf. n.50).

Next [see Fig. 3.14] let the circle concentric with the ecliptic be ABG on centre D and diameter ADG, and let the epicycle (in the same ratio [to circle ABG as the eccentricity to the eccentre]) be EZHΘ on centre A. Cut off arc EZ and join ZBD and ZA. Let arc EZ again be taken in the same amount, 30°. Drop perpendicular ZK from Z on to AE.

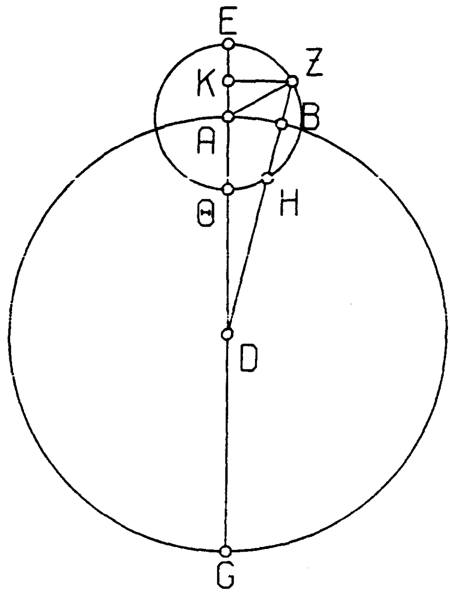

Fig. 3.14

Since arc EZ = 30°,

$$\angle \text{EAZ} = \begin{cases} 30^\circ \text{ where 4 right angles} = 360^\circ \\ 60^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ}. \end{cases}$$

Therefore in the circle about right-angled triangle AZK,

H244 arc ZK = 60°

and arc AK = 120° (supplement).

Therefore the corresponding chords

$$\left.\begin{array}{r} \text{ZK} = 60^p \\ \text{and KA} = 103;55^p \end{array}\right\} \text{where the diameter AZ} = 120^p.$$

Therefore where hypotenuse AZ = $2;30^p$ and radius AD = 60^p

ZK = $1;15^p$, KA = $2;10^p$,

and, by addition, KAD = $62;10^p$.

And since $\text{ZK}^2 + \text{KD}^2 = \text{ZBD}^2$,

ZD = $62;11^p$, where ZK = $1;15^p$.

So where hypotenuse DZ = 120^p, ZK = $2;25^p$,

and, in the circle about right-angled triangle DZK,

arc ZK = 2;18°.

$$\therefore \angle \text{ZDK} = \begin{cases} 2;18^{\circ\circ} \text{ where 4 right angles} = 360^{\circ\circ} \\ 1;9^\circ \text{ where 2 right angles} = 360^\circ. \end{cases}$$

This is, again, the amount of the equation of anomaly, which is represented by arc AB.

And ∠ EAZ was taken as 30°. H245

Therefore, by subtraction, ∠ AZD, which represents the arc of apparent motion on the ecliptic, is 28;51°.

These amounts are in agreement with what we found for the eccentric hypothesis.

Here too, if any other angle be given [instead of ∠ EAZ], the remaining angles will be given, [as can be seen] on the same figure [see Fig. 3.15] if the perpendicular AL is dropped from A on to DZ.

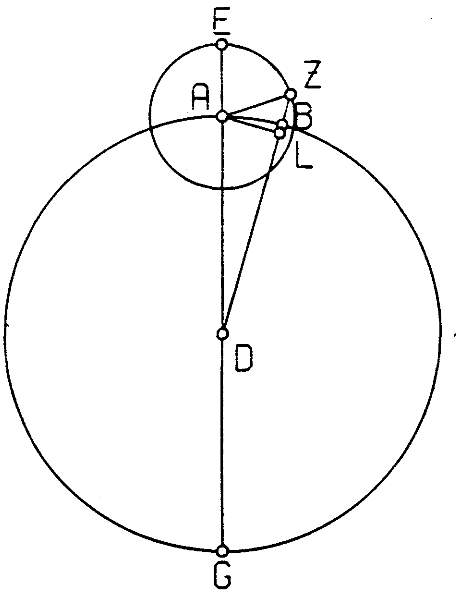

Fig. 3.15

For if, as before, we first take the arc of apparent motion on the ecliptic, i.e. ∠ AZD, as given, from this the ratio ZA:AL will be given. And since ZA:AD was given from the beginning, DA:AL will be given. Hence ∠ ADB will be given, i.e. arc AB, the arc of the equation of anomaly, and so will ∠ EAZ, i.e. arc EZ of the epicycle.

Of if, secondly, we take the equation of anomaly, i.e. ∠ADB, as given, then, in the same way but in reverse order, from this AD:AL will be given; and since DA:AZ was given from the beginning, ZA:AL will also be given; and hence H246 ∠ AZD will be given, which corresponds to the arc of apparent motion on the ecliptic, and so will ∠ EAZ, i.e. arc EZ of the epicycle.

Let us again take the previous figure for the eccentre [see Fig. 3.16], and cut off from H, the perigee of the eccentre, arc HZ, which we again take as 30°. Join DZB and ZΘ, and drop perpendicular DK from D on to ΘZ.

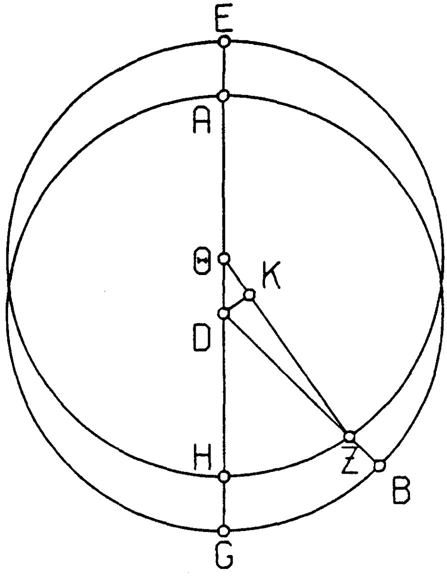

Fig. 3.16

Then since arc ZH = 30°,

$$\angle Z\Theta H = \begin{cases} 30^\circ \text{ where 4 right angles} = 360^\circ \\ 60^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ}. \end{cases}$$

Therefore in the circle about right-angled triangle DΘK,

arc DK = 60°

and arc KΘ = 120° (supplement).

Therefore the corresponding chords

$$\left.\begin{aligned} DK &= 60^p \\ \text{and } K\Theta &= 103;55^p \end{aligned}\right\} \text{ where diameter } D\Theta = 120^p.$$

Therefore where hypotenuse DΘ = 2;30^p and radius ΘZ = 60^p,

H247 DK = 1;15^p and ΘK = 2;10^p,

and KZ = 57;50^p by subtraction [of ΘK from ΘZ]

And since $DZ^2 = DK^2 + KZ^2$,

DZ ≈ 57;51^p where DK = 1;15^p.

Therefore where hypotenuse DZ = 120^p, DK = 2;34^p.[53]

And, in the circle about right-angled triangle DZK,

arc DK = 2;27°.

$$\therefore \angle DZK = \begin{cases} 2;27^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \\ 1;14^\circ \text{ (approximately) where 4 right angles} = 360^\circ. \end{cases}$$

[53] Reading $\bar{\beta}\ \overline{\lambda\delta}$ for $\bar{\beta}\ \overline{\lambda\delta}\ \overline{\lambda\varsigma}$ (2;34,36) at H247,6, with Ar. Accurate computation gives 2;35,34 (cf. reading of D[2]), but Ptolemy gives his results here only to minutes, and 2;34 is correct, since Crd 2;27° = 2;33,55^p ≈ 2;34^p. The 36 was presumably a marginal correction to the 34 (cf. reading of D at H249,20), which was later mistakenly incorporated as an extra place. The same correction has to be made at H249,20 (both made by Manitius).

This [1;14°], then, is the equation of anomaly.

And since ∠ ZΘH was taken as 30°,

by addition, ∠ BDG, i.e. arc GB of the ecliptic, equals 31;14°.

Here too, in the same way [as before], [see Fig. 3.17], we produce BD and drop perpendicular ΘL on to it.

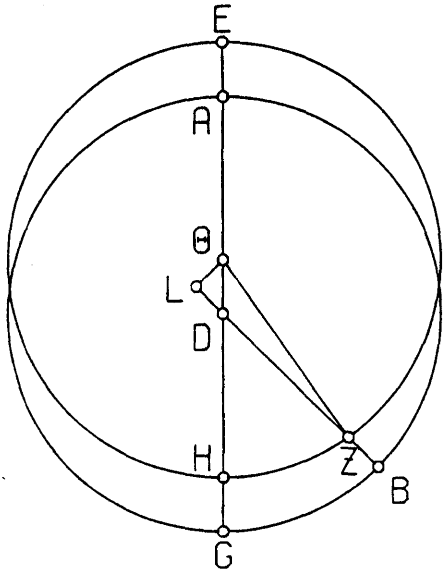

Fig. 3.17

Then if, first, we take arc GB of the ecliptic, i.e. ∠ ΘDL, as given, from this the ratio DΘ:ΘL will be given. And since ΘD:ΘZ was also given from the beginning, ZΘ:ΘL will be given. Hence we will have as given angles H248

∠ ΘZD, i.e. the equation of anomaly

and ∠ ZΘD, i.e. arc HZ of the eccentre.

Or if, secondly, we take the equation of anomaly, i.e. ∠ ΘZD, as given, then conversely, from this ZΘ:ΘL will be given. And since ZΘ:ΘD was also given from the beginning, DΘ:ΘL will be given. Hence we will have, as given angles,

∠ ΘDL, which corresponds to arc GB of the ecliptic

and ∠ ZΘH, i.e. arc HZ of the eccentre.

Similarly, on the previous figure of concentre and epicycle [see Fig. 3.18], we cut off arc ΘH from the perigee, in the same amount of 30°, join AH and DHB, and drop perpendicular HK from H on to AD.

Then since arc ΘH is again 30°,

$$\angle\,\Theta AH = \begin{cases} 30^\circ \text{ where 4 right angles} = 360^\circ \\ 60^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ}. \end{cases}$$ H249

Therefore in the circle about right-angled triangle HKA,

arc HK = 60°

and arc AK = 120° (supplement).

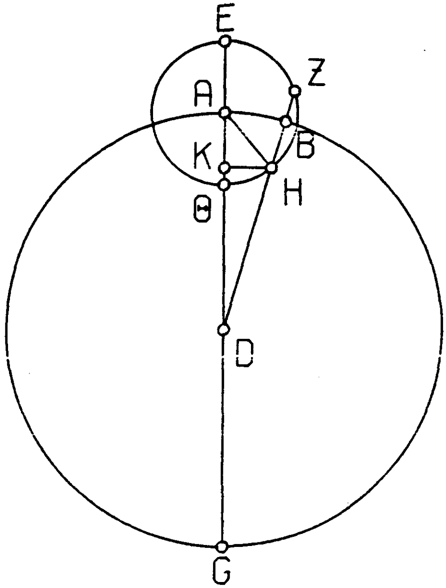

Fig. 3.18

Therefore the corresponding chords

$$\left.\begin{aligned} HK &= 60^p \\ \text{and } AK &= 103;55^p \end{aligned}\right\} \text{ where hypotenuse } AH = 120^p.$$

Therefore where AH = 2;30ᵖ and radius AD = 60ᵖ,

HK = 1;15ᵖ, AK = 2;10ᵖ and KD = 57;50ᵖ, by subtraction.

and since $HK^2 + KD^2 = DH^2$,

DH ≈ 57;51ᵖ where KH = 1;15ᵖ.

Therefore where hypotenuse DH = 120ᵖ

HK = 2;34ᵖ,

and, in the circle about DHK, arc HK = 2;27°.

H250 $$\therefore \angle HDK = \begin{cases} 2;27^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \\ 1;14^{\circ} \text{ (approximately) where 4 right angles} = 360^{\circ}. \end{cases}$$

Here too, then, that is the size of the equation of anomaly, i.e. arc AB.

And since ∠ KAH was taken as 30°, by addition, ∠ BHA, which represents the apparent motion on the ecliptic [counted from perigee], is 31;14°. These amounts agree with those found for the eccentric [hypothesis].

Here too, in the same way [as before], we drop perpendicular AL on to DB [see Fig. 3.19].

Then if, first, we take the arc of the ecliptic, i.e. ∠ AHL, as given, from this the ratio HA:AL will be given. And since HA:AD was given from the beginning, DA:AL will be given. Thence we will have as given angles

∠ ADB, i.e. arc AB, representing the equation of anomaly

and ∠ ΘAH, i.e. arc ΘH of the epicycle.

Or if, secondly, we take as given arc AB, representing the equation of

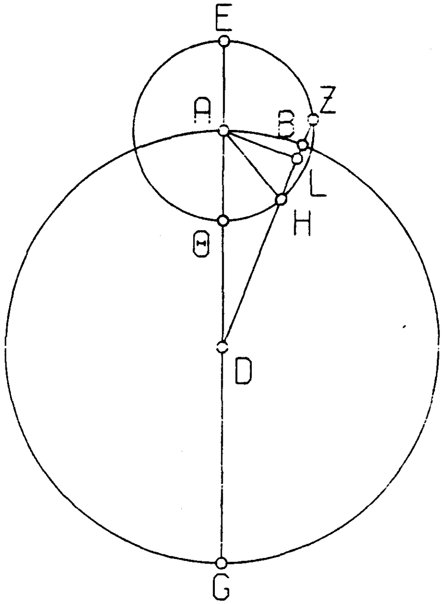

Fig. 3.19

anomaly, i.e. ∠ ADB, then, in the same way but in reverse order, from this the ratio DA:AL will be given. And since DA:AH is given from the beginning, HA:AL will also be given. Hence we will have as given angles H251

∠ AHL, i.e. the arc of the ecliptic

and ∠ ΘAH, i.e. arc ΘH of the epicycle.

Thus we have proved what we set out to do.

In order to have conveniently available the amount of the correction for any given position, [we want] to establish a table, subdivided into [appropriate] sections, for the computation of the apparent positions from the anomaly. The above theorems would allow a wide variety in the form of such a table,[54] but we prefer that form in which the argument is the mean motion and the function is the equation of anomaly.[55] For this form accords well with the actual theories, and it also provides a simple but highly practical way of computing any desired result. So using the first set of theorems [i.e. with the eccentric hypothesis] which we used in the numerical examples above, we computed geometrically, in the way described, for the individual subdivisions [of the circle], the equation of anomaly corresponding to the arc of mean motion. In general, both for the sun and for the other bodies, we divided the quadrants near the apogee[56] into 15 H252
subdivisions (thus in these quadrants the interval of tabulation is 6°), and the

[54] Ptolemy means that theoretically one could take as argument either the mean motion ($\bar{\kappa}$), the true position (κ), or the equation (θ).

[55] Literally 'which contains the equations of anomaly corresponding to the arcs of mean motion'.

[56] Reading πρὸς τοῖς ἀπογείοις (with all mss.) for πρὸς ἀπογείοις (misprint in Heiberg) at H251.24. Corrected by Manitius.

quadrants near the perigee into 30 subdivisions (thus in these the interval of tabulation is 3°). The reason is that the differences between [successive] equations of anomaly, for equal subdivisions [of the argument], are greater near the perigee than near the apogee.

We shall set out the table of the sun's anomaly, then, in 45 lines, as before, and 3 columns. The first two columns will contain the numbers of the mean motion through 360°: the first 15 lines will comprise the two quadrants near the apogee, the next 30 the two quadrants near the perigee. The third column will contain the degrees of equation of anomaly to be added or subtracted, corresponding to the appropriate mean motion. The table is as follows.

H253

6. {*Table of the sun's anomaly*}

[See p. 167.]

H254

7. {*On the epoch of the sun's mean motion*}[57]

It remains to establish the epoch of the sun's mean motion, in order to be able to compute the particular position for any given time. In making our exposition of that matter, we shall again use[58] those positions of the body which we ourselves have observed most accurately (this is our general rule both for the sun and for the other planets), but we use the mean motions we have derived to compute back to the beginning of the reign of Nabonassar for the epochs we establish. For that is the era beginning from which the ancient observations are, on the whole, preserved down to our own time.[59]

[See Fig. 3.20.] Let the circle concentric with the ecliptic be ABG on centre D, and the sun's eccentre EZH on centre Θ, and let the diameter through both centres and the apogee E be EAHG. Let B represent the autumnal equinox on the ecliptic. Join BZD and ZΘ, and drop perpendicular ΘK from Θ on to ZD produced.

H255 Then since B, the autumnal equinox, is located at the beginning of Libra, and G, the perigee, at ♐ 5½°,

$$\text{arc BG} = 65;30°.$$

$$\therefore \angle \text{BDG} = \angle \Theta\text{DK} = \begin{cases} 65;30° \text{ where 4 right angles} = 360° \\ 131^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ}. \end{cases}$$

Therefore in the circle about right-angled triangle DΘK,

$$\text{arc } \Theta\text{K} = 131°,$$

$$\text{and its chord } \Theta\text{K} = 109;12^{p} \text{ where the diameter D}\Theta = 120^{p}.$$

[57] See *HAMA* 58-60, Pedersen 151-3.

[58] Reading ποιησόμεθα (with D) for ἐποιησάμεθα ('we used') at H254,5. It is unclear what reading(s) lie behind the Arabic translations.

[59] This statement is borne out not only by the Babylonian observations preserved in the Almagest (the earliest of which is the lunar eclipse of −720 Mar. 19, in the 1st year of Mardokempad, or the 27th year of the era Nabonassar, IV 6 p. 191, but also by the extant cuneiform records: the earliest surviving astronomical observations (apart from the special case of the Venus tablets of Ammisaduqa) are from -651 (Sachs[1] 44).

TABLE OF THE SUN'S ANOMALY

1	2	3
Common Numbers		Equation
6	354	0 14
12	348	0 28
18	342	0 42
24	336	0 56
30	330	1 9
36	324	1 21
42	318	1 32
48	312	1 43
54	306	1 53
60	300	2 1
66	294	2 8
72	288	2 14
78	282	2 18
84	276	2 21
90	270	2 23
93	267	2 23
96	264	2 23
99	261	2 22
102	258	2 21
105	255	2 20
108	252	2 18
111	249	2 16
114	246	2 13
117	243	2 10
120	240	2 6
123	237	2 2
126	234	1 58
129	231	1 54
132	228	1 49
135	225	1 44
138	222	1 39
141	219	1 33
144	216	1 27
147	213	1 21
150	210	1 14
153	207	1 7
156	204	1 0
159	201	0 53
162	198	0 46
165	195	0 39
168	192	0 32
171	189	0 24
174	186	0 16
177	183	0 8
180	180	0 0

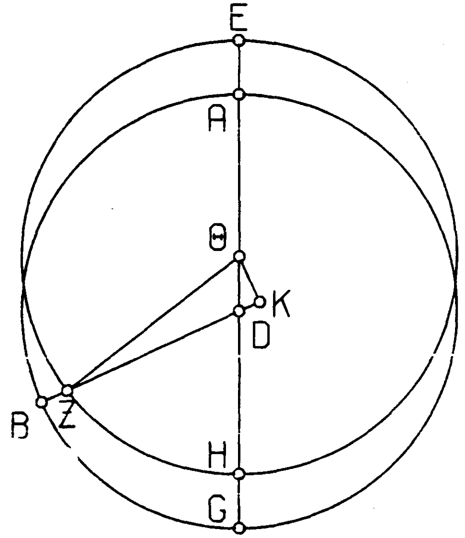

Fig. 3.20

Therefore where DΘ = 5^p and the hypotenuse ZΘ = 120^p,
ΘK = $4;33^p$.

And, in the circle about right-angled triangle ΘZK,
arc ΘK = 4;20°.

$$\therefore \angle \Theta ZK = \begin{cases} 4;20^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \\ 2;10^{\circ} \text{ where 4 right angles} = 360^{\circ}. \end{cases}$$

And we found ∠ BDG = 65;30°.

Therefore, by subtraction, ∠ ZΘH (i.e. arc ZH of the eccentre) = 63;20°.
Therefore, when the sun is at the autumnal equinox, it is 63;20° in mean motion
H256 in advance of the perigee (i.e. ♐ 5½°), and 116;40° in mean motion to the rear of the apogee (i.e. ♊ 5;30°).

Now that we have established that, among the first of the equinoxes observed by us, one of the most accurately determined was the autumnal equinox which occurred in the seventeenth year of Hadrian, on Athyr [III] 7 in the Egyptian calendar [132 Sept. 25], about 2 equinoctial hours after noon. [From the above computation] it is clear that at that time the sun, in its mean motion, was 116;40° to the rear of the apogee on the eccentre. Now from [the beginning of] the reign of Nabonassar [−746 Feb. 26] to the death of Alexander [−323 Nov. 12] is a total of 424 Egyptian years, and from the death of Alexander to [the beginning of] the reign of Augustus [−29 Aug. 31] 294 years, and from the first year of Augustus, Thoth 1 in the Egyptian calendar, noon (for we establish all epochs at noon), to the seventeenth year of Hadrian, Athyr 7, 2 equinoctial hours after noon, is 161 years 66 days 2 equinoctial hours. Therefore the sum total from the first year of Nabonassar, Thoth 1 in the Egyptian calendar, noon, up to the time of the above autumnal equinox, is 879 Egyptian years 66 days and 2 equinoctial
H257 hours. In that interval the mean motion of sun is approximately 211;25° beyond

complete revolutions. Therefore, if to the 116;40°, which is the [sun's] distance from the apogee of the eccentre at the above autumnal equinox, we add the 360° of one revolution, and subtract from the result the 211;25° of the increment in mean motion over the interval [in question], we find for the epoch in mean motion in the first year of Nabonassar, Thoth 1 in the Egyptian calendar, noon, that the sun's distance in mean motion is 265;15° to the rear of the apogee. Thus its mean position is ♓ 0;45°.[60]

8. {*On the calculation of the solar position*}[61]

So whenever we want to know the sun's position for any required time, we take the time from epoch to the given moment (reckoned with respect to the local time at Alexandria), and enter with it into the table of mean motion. We add up the degrees [and their subdivisions] corresponding to the various arguments [18-year periods, years, months, etc.], add to this the elongation [from apogee at epoch],[62] 265;15°, subtract complete revolutions from the total, and count the result from ♊ 5;30° rearwards through [i.e. in the order of] the signs. The point we come to will be the mean position of the sun. Next we enter with the same H258 number, that is the distance from apogee to the sun's mean position, into the table of anomaly, and take the corresponding amount in the third column. If the argument falls in the first column, that is if it is less than 180°, we subtract the [equation] from the mean position; but if the argument falls in the second column, i.e. is greater than 180°, we add it to the mean position. Thus we obtain the true or apparent [position of the] sun.

9. {*On the inequality in the [solar] days*}[63]

Such, then, we may say, are the theories concerning the sun alone. Following this it seems appropriate to add a brief discussion of the subject of the inequality of the solar day.[64] A grasp of this topic is a necessary prerequisite, since the mean motions which we tabulate for each body are all arranged on the simple system of equal increments, as if all solar days were of equal length. However, it can be seen that this is not so. The revolution of the universe takes place uniformly about the poles of the equator. The more prominent ways of marking that revolution are by its return to the horizon, or to the meridian. Thus one revolution of the universe is, clearly, the return of a given point on the equator from some place on either the horizon or the meridian to the same place; and a H259 solar day, simply defined, is the return of the sun from some point either on the

[60] Literally '45 minutes of the *first* degree of Pisces'.

[61] See *HAMA* 58–61, Pedersen 153–4, and Appendix A, Example 7.

[62] The reading of D,Ar at H257,18, ἐποχῆς (for ἀποχῆς) is possible. The meaning would be the same, but one would have to understand '[the elongation from apogee] at epoch', which is rather obscure.

[63] See *HAMA* 61–8, Pedersen 154–8.

[64] νυχθήμερον, literally 'a night plus a day'. See Introduction p. 23.

horizon or on the meridian to the same point. On this definition, a mean solar day is the period comprising the passage of the 360 time-degrees of one revolution of the equator plus approximately 0;59 time-degrees, which is the amount of the mean motion of the sun during that period; and an anomalistic solar day is the period comprising the passage of the 360 time-degrees of one revolution of the equator plus that stretch of the equator which rises with, or crosses the meridian with, the anomalistic motion of the sun [in that period].

This additional stretch of the equator, beyond the 360 time-degrees, which crosses [the horizon or meridian] cannot be a constant, for two reasons: firstly, because of the sun's apparent anomaly; and secondly, because equal sections of the ecliptic do not cross either the horizon or the meridian in equal times. Neither of these effects causes a perceptible difference between the mean and the anomalistic return for a single solar day, but the accumulated difference over a number of solar days is quite noticeable.

As far as the effect of the solar anomaly is concerned, the greatest
H260 [accumulated] difference occurs between the two positions of the sun where its [true] speed equals its mean speed. The sum of the [anomalistic] solar days [over either of the two such intervals] will differ from the sum of the mean solar days [over the same interval] by about $4\frac{3}{4}$ time-degrees, and from the sum of [anomalistic] solar days over the other [such] interval by twice that amount, about $9\frac{1}{2}$ time-degrees. For the apparent motion of the sun over the semi-circle containing the apogee is $4\frac{3}{4}°$ less than the mean, and its apparent motion over the semi-circle containing the perigee is the same amount [$4\frac{3}{4}°$] greater than the mean.[65]

As far as the effect of the variation in the time taken to cross the horizon at rising or setting is concerned, the greatest [accumulated] difference occurs between the ends of the semi-circles bounded by the solsticial points. For here too the rising-times of either of those semi-circles will differ from the 180° of the mean interval by the amount by which the longest or shortest day differs from the equinoctial day (measured in time-degrees); and they will differ from each other by the amount by which the longest day (or night) differs from the shortest. As far as the effect of the variation in the time taken to cross the meridian is concerned, the greatest [accumulated] difference will occur between two points enclosing two signs which are on either side of either a solsticial or an equinoctial point. For the sum of [the rising-times at *sphaera recta* of] the two such signs on either side of a solstice will differ from the mean interval by about $4\frac{1}{2}$ time-degrees, and from [the sum of the rising-times of] the two signs on either side of an equinox by 9 time-degrees, since the latter fall short of, and
H261 the former exceed the amount for the mean by about the same quantity.[66] Hence we establish the beginning of the solar day at [astronomical] epochs from the meridian-crossing of the sun, and not from its rising or setting, since the [time-] difference with respect to the horizon can reach several hours, and is not the same everywhere but varies according to the difference in longest or shortest

[65] The sun's maximum equation of anomaly is 2;23° (II 6). Thus from mean speed (90° or 270° from apogee) to mean speed the mean motion is ($2 \times 2;23 \approx 4\frac{3}{4}$) greater or less than the true.

[66] From the table of rising-times at *sphaera recta*, II 8, the sum of the rising-times of e.g. ♊ and ♋ is 64;32 ($\approx 60° + 4\frac{1}{2}°$), while that of e.g. ♍ and ♎ is 55;40 ($\approx 60° - 4\frac{1}{2}°$).

day at the different latitudes, whereas the [time-]difference with respect to the meridian is the same at every place on earth, and is no greater than the time-variation due to the sun's anomaly.

The greatest[67] [accumulated] difference [between mean and anomalistic solar days] resulting from the combination of both these effects, namely that due to the sun's anomaly and that due to the [variation in the time of] meridian-crossing, occurs over intervals where the above effects are either both additive or both subtractive. Now the [maximum] subtractive result from both effects occurs over the interval from the middle of Aquarius to [the end of] Libra, and the [maximum] additive one over the interval from [the beginning of] Scorpio to the middle of Aquarius. Both of these intervals produce a maximum additive or subtractive result which is composed of about $3\frac{2}{3}°$ due to the effect of the solar anomaly, and about $4\frac{2}{3}°$ due to the [variation in the time of] meridian-crossing.[68] Thus the maximum difference arising from the combination of both the above effects is $8\frac{1}{3}$ time-degrees, or $\frac{5}{9}$ths of an hour, between the [true] solar H262
days over either of these intervals and the [corresponding] mean solar days, and twice as much, $16\frac{2}{3}$ time-degrees, or $1\frac{1}{9}$ hours, between the [true] solar days of one such interval and those of the other. Neglect of a difference of this order would, perhaps, produce no perceptible error in the computation of the phenomena associated with the sun or the other [planets]; but in the case of the moon, since its speed is so great, the resulting error could no longer be overlooked, since it could amount to $\frac{3}{5}$ of a degree.[69]

Therefore, to state once for all the rule for converting any interval whatever, given in [true] solar days (by which I mean days counted from noon to noon or midnight to midnight), into mean solar days: we determine the ecliptic position of the sun in both mean and anomalistic motion at the beginning and end of the given interval of solar days; then we take the increment, in degrees, from [the first] anomalistic (i.e. apparent) position to [the second] apparent position, enter with it into the table of rising-times at *sphaera recta*, and [thus] determine the time taken by this apparent distance [of the sun between the first and second H263
positions] to cross the meridian, measured in degrees of the equator. We then take the difference between this number of time-degrees and the mean distance [of the sun from first to second positions], measured in degrees, and convert this difference, which is in time-degrees, to a fraction of an equinoctial hour. We add the result to the number of [true] solar days given if the amount of the time-degrees [corresponding to the rising-time of the apparent motion] was greater than the mean motion, or subtract it if less. The interval we arrive at will be corrected for expression in mean solar days. We shall use this type of interval particularly in computing the mean motions of the moon from its tables. One can immediately comprehend that, given mean solar days, one can find the [corresponding] civil solar days, i.e. days defined by simple observation, by

[67] Reading το πλεῖστον διάφορον (with DB³Ar) at H261,14 for το διάφορον ('the difference').

[68] For a graphical verification of the amounts and positions given here by Ptolemy see *HAMA* III Fig. 57 on p. 1222.

[69] The hourly mean motion of the moon (IV 3 p. 179) is about 0;32,56. So in $1\frac{1}{9}$ hours it moves $0;36,36 \approx \frac{3}{5}°$.

performing the above computation of addition or subtraction of time-degrees in reverse.[70]

At our epoch, that is, Year 1 of Nabonassar, Thoth 1 in the Egyptian calendar, noon, the position of the sun was in mean motion, as we showed just above, ♓ 0;45°, and in anomalistic motion about ♓ 3;8°.[71]

[70] If we call the interval in true solar days between times t_1 and t_2 Δt, and the interval in mean solar days $\Delta\bar{t}$, then Ptolemy's rule, expressed algebraically, is $\Delta\bar{t} = \Delta t + E$ (E corresponds, in a certain sense, to the modern 'equation of time'), and $E = (\alpha(t_2) - \alpha(t_1)) - (\bar{\lambda}(t_2) - \bar{\lambda}(t_1))$. For proofs of the validity of this rule see *HAMA* 65–6, Pedersen 156–7. Pedersen shows that the rule is in fact an approximation, since one should take the motion in mean longitude, not over the interval $(t_2 - t_1) = \Delta t$, but over the interval in mean solar days $\Delta\bar{t}$ (which is in practice impossible). Since, however, the difference between Δt and $\Delta\bar{t}$ never exceeds about 33 minutes, during which the sun moves less than 2′, the error is utterly negligible. For examples of computation see *HAMA* 63–5 and Appendix A, Example 8.

[71] Ptolemy gives the data for era Nabonassar because they will be required every time one needs to compute the lunar position accurately (i.e. in mean solar days) from his tables (e.g. for the series of observations of fixed stars with respect to the moon in VII 3). Neugebauer notes (*HAMA* 63) that the epoch value for the mean longitude, ♓ 0;45°, seems itself to be corrected for the equation of time, since reckoning backwards 'simply' from Ptolemy's observation would give ♓ 0;44° to the nearest minute.

Book IV

1. {*The kind of observations which one must use to examine lunar phenomena*}[1]

In the preceding book we treated all the phenomena associated with the sun's motion. We now begin our discussion of the moon, as is appropriate to the logical order. In doing so we think it our first duty not to take a naive or arbitrary approach in our use of the relevant observations. Rather, to establish our general notions [on this topic], we should rely especially on those demonstrations which depend on observations which not only cover a long period, but are actually made at lunar eclipses. For these are the only observations which allow one to determine the lunar position precisely: all others, whether they are taken from passages [of the moon] near fixed stars, or from [sightings with] instruments, or from solar eclipses, can contain a considerable error due to lunar parallax. It is only for particular further developments [of the theory] that we should use these other kinds of observations for our investigations. For the distance between the sphere of the H266
moon and the centre of the earth, unlike the distance to the ecliptic, is not so great that the earth's bulk has the ratio of a point to it. Hence it necessarily follows that the straight line drawn from the centre of the earth (which is the centre of the ecliptic) through the centre of the moon[2] to a point on the ecliptic, which determines the true position ([as it does] for all bodies), does not in this case always coincide, even sensibly, with the line drawn from some point on the earth's surface, that is, the observer's point of view, to the moon's centre, which determines its apparent position. Only when the moon is in the observer's zenith do the lines from the earth's centre and the observer's eye through the moon's centre to the ecliptic coincide. But when the moon is displaced from the zenith position in any way whatever, the directions of the above lines become different, and hence the apparent position cannot be the same as the true, but [differs from it], as the [line through] the observer's eye assumes various positions with respect to the line drawn through the centre of the earth, [by an amount] proportional to the varying angle of inclination [between the two lines].

This is the reason why in the case of solar eclipses, which are caused by the H267

[1] On Chs 1-3 see *HAMA* 68-73, 308-15, Pedersen 160-4.

[2] Reading ἀπὸ τοῦ κέντρου τῆς γῆς τουτέστι τοῦ ζῳδιακοῦ διὰ τοῦ κέντρου τῆς σελήνης (with D, Ar) for ἀπὸ τοῦ κέντρου τῆς σελήνης ('the straight line drawn from the moon's centre', which is nonsense) at H266,5. The error in most Greek mss. is due to haplography, and is an important indication that all except D and its descendants come from a single (?Byzantine) ms. Corrected by Manitius.

moon passing below and blocking [the sun] (for when the moon falls into the cone from the observer's eye to the sun it produces the obscuration which lasts until it has passed out [of the cone] again), the same[3] eclipse does not appear identical, either in size or in duration,[4] in all places. For the moon does not produce obscuration for all observers, for the reasons stated above, and [even for those for whom it does produce obscuration] does not appear to obscure the same parts of the sun [for all alike]. Whereas in the case of lunar eclipses there is no such variation due to parallax, since the observer's position is not a contributory cause to what happens at a lunar eclipse. For the moon's light is at all times caused by the illumination from the sun. Thus when it is diametrically opposite to the sun, it normally appears to us as lighted over its whole surface, since the whole of its illuminated hemisphere is turned towards us as well [as towards the sun] at that time. However, when its position at opposition is such that it is immersed in the earth's shadow-cone (which revolves with the same speed as the sun, but opposite it), then the moon loses the light over a part of its surface corresponding to the amount of its immersion, as the earth obstructs the illumination by the sun. Hence it appears to be eclipsed for all parts of the earth
H268 alike, both in the size [of the eclipse] and the length of the intervals [of the various phases].

Now to establish our general theory we need to use true, and not apparent, positions of the moon; for the ordered and regular must necessarily precede and serve as a foundation for the disordered and irregular. So, for the above reasons, we declare that we must not use, for this purpose, observations of the moon into which the observer's position enters, but only lunar eclipse observations, since [only] in these does the observer's position have no effect on the determination of the moon's position. For it is obvious that, if we find the point on the ecliptic which the sun occupies at the time of mid-eclipse (which is, as accurately as we can determine, the moment at which the moon's centre is diametrically opposite the sun's in longitude), then at the same time of mid-eclipse the precise position of the moon's centre will be the point diametrically opposite.

2. *On the periods of the moon*

The above may serve as an outline of the kind of observations which must be
H269 examined to determine the general theory of the moon. We shall now endeavour to describe the method which was used by the ancients in their attempts at establishing a [lunar] theory, and which we will find a most convenient tool in deciding which hypotheses accord with the phenomena.

The moon's motion appears anomalistic both in longitude and in latitude: the time it takes to traverse the ecliptic is not constant, and neither is the time it

[3] Reading τὰς αὐτὰς (with D, Ar) for ταύτας ('these eclipses') at H267,4. Corrected by Manitius.

[4] 'duration': the Greek has the vague 'times' (τοῖς χρόνοις). This is elucidated by H268,1 τοῖς τῶν διαστάσεων χρόνοις, 'the duration of the intervals [of partial and total phases]'. Ptolemy may also be alluding, in both places, to the fact that the actual moments of e.g. the beginning or middle of a solar eclipse are different at different places, and by an amount which does not correspond directly to the difference in longitude.

takes to return to the same latitude.[5] Now unless one finds the period of its return in anomaly it is, necessarily, impossible to determine the period of the other motions [in longitude and latitude]. However, from individual observations it is apparent that the moon's mean speed can occur in any part of the ecliptic, as can its greatest speed and its least speed, and that it can reach its greatest northern or southern latitude, or appear exactly in the ecliptic, anywhere, too. Hence the ancient astronomers, with good reason, tried to find some period in which the moon's motion in longitude would always be the same, on the grounds that only such a period could produce a return in anomaly. So they compared observations of lunar eclipses (for the reasons mentioned above), and tried to see whether there was an interval, consisting of an integer number of months, such that, between whatever points one took that interval of months,[6] the length in time was always the same, and so was the motion [of the moon] in longitude, [i.e.] either the same number of integer revolutions, or the same number of revolutions plus the same arc. H270

The even more ancient [astronomers] used the somewhat crude estimate that such a period could be found in $6585\frac{1}{3}$ days. For they saw that in that interval occurred approximately 223 lunations, 239 returns in anomaly, 242 returns in latitude, and 241 revolutions in longitude plus $10\frac{2}{3}$°, which is the amount the sun travels beyond the 18 revolutions which it performs in the above time (that is when the motion of sun and moon is measured with respect to the fixed stars). They called this interval the 'Periodic', since it is the smallest single period which contains (approximately) an integer number of returns of the various motions.[7] In order to obtain a period with an integer number of days, they tripled the $6585\frac{1}{3}$ days, obtaining 19756 days, which they called 'Exeligmos'. Similarly, by tripling the other numbers, they obtained 669 lunations, 717 returns in anomaly, 726 returns in latitude, and 723 revolutions in longitude plus 32°, which is the amount the sun travels beyond its 54 revolutions.[8]

However, Hipparchus already proved, by calculations from observations made by the Chaldaeans and in his time, that the above relationships were not accurate. For from the observations he set out he shows that the smallest constant interval defining an ecliptic period in which the number of months and the amount of [lunar] motion is always the same, is 126007 days plus 1 equinoctial hour. In this interval he finds comprised 4267 months, 4573 complete returns in anomaly, and 4612 revolutions on the ecliptic less about $7\frac{1}{2}$°, which is the amount by which the sun's motion falls short of 345 revolutions (here too the revolution of sun and moon is taken with respect to the fixed stars). (Hence, dividing the above number of days by the 4267 months, he finds the H271

[5] Reading κατὰ πλάτος (with D) for κατὰ τὸ πλάτος at H269,9.

[6] 'months' here means 'true synodic months'. This is generally true throughout the Almagest (except where the context makes it obvious that the reference is strictly calendaric). In the translation I usually make the meaning explicit.

[7] This period, generally, but wrongly, called 'Saros' in modern times (see Neugebauer[1]), was well-known in Babylonian astronomy. See *HAMA* 497 ff. We do not know to whom Ptolemy refers by 'the even more ancient people', except that they are earlier than Hipparchus.

[8] The ἐξελιγμός (meaning 'turn of the wheel') is also mentioned by Geminus (Cap. XVIII, ed. Manitius pp. 200–2), who gives exactly the same numbers as Ptolemy, including the excess in sidereal longitude of 32°.

mean length of the [synodic] month as approximately 29;31,50,8,20 days). He shows, then, that the corresponding interval between two lunar eclipses is always precisely the same when they are taken over the above period [$126007^d 1^h$]. So it is obvious that it is a period of return in anomaly, since [from whatever eclipse it begins], it always contains the same number [4267] of months, and 4611 revolutions in longitude plus $352\frac{1}{2}°$, as determined by its syzygies with the sun.

But if one were to look for the number of months [which always cover the same time-interval], not between two lunar eclipses, but merely between one
H272 conjunction or opposition and another syzygy of the same type, he would find an even smaller integer number of months containing a return in anomaly, by dividing the above numbers by 17 (which is their only common factor). This produces 251 months and 269 returns in anomaly.

However, it was found that the above period [of $126007^d 1^h$] did not contain an integer number of returns in latitude too. For it was apparent that the [pairs of] corresponding eclipses exhibited equality only with respect to the interval [between the pair] in time and revolution in longitude, but not with respect to the size and type of the obscuration,[9] which is the criterion for [a return in] latitude. Nevertheless, having already determined the period of return in anomaly, Hipparchus again adduces intervals containing [an integer number of] months which have at each end eclipses which were identical in every respect, both in size and in duration [of the various phases], and in which there was no difference due to the anomaly. Thus it is apparent that there is a return in latitude too. He shows that such a period is contained in 5458 months and 5923 returns in latitude.[10]

That, then, is the method which our predecessors used for the determination of such [periods]. It is not simple or easy to carry out, but demands a great deal of extraordinary care, as we can see from the following considerations.[11] Let us grant that [two] intervals [between pairs of eclipses] are found to be precisely
H273 equal in time. In the first place, this is no use to us unless the sun too exhibits no effect due to anomaly, or exhibits the same over both intervals: for if this is not the case, but instead, as I said, the equation of anomaly has some effect, the sun will not have travelled equal distances over [the two] equal time-intervals, nor, obviously, will the moon. For example, let us suppose that each of the two intervals being compared comprises half a year beyond the same number of complete years, and that in this time the motion of the sun in the first interval

[9] By 'type' Ptolemy means whether the obscuration begins from the north or south of the lunar disk.

[10] Ptolemy's account here is not historically accurate. In fact Hipparchus took from Babylonian sources the parameters [1] 1 synodic month = $29;31,50,8,20^d$, [2] 251 synodic months = 269 anomalistic months, and [3] 5458 synodic months = 5923 returns in latitude (Kugler, *Babylonische Mondrechnung* 4–46). Multiplying [2] by 17, he constructed an eclipse-period (Aaboe[1955], whence *HAMA* 310–2). An input of some value for the length of the year produced the solar motion over this period, rounded by Hipparchus to the nearest $\frac{1}{4}$-sign (on which see Neugebauer[2], 251). Then Hipparchus *confirmed* (not *derived*, as Ptolemy says) the above by comparison of eclipses from his own time with Babylonian ones 345 years earlier (see Toomer[11] for the method and identification of the eclipses he used).

[11] The following (to p. 178) is well explained and illustrated by Neugebauer, *HAMA* 71–2.

starts from the position of mean speed in Pisces, and in the second interval from the position of mean speed in Virgo.[12] Then over the first interval the sun will have traversed about $4\frac{3}{4}$° less than a semi-circle [beyond complete revolutions], but over the second about $4\frac{3}{4}$° more than a semi-circle. Thus the moon too will have traversed over the first interval $175\frac{1}{4}$° beyond complete revolutions and over the second $184\frac{3}{4}$°, although both intervals cover an equal time. Therefore we define as the first necessary condition [for a return in lunar anomaly] that the intervals must exhibit one of the following characteristics with respect to the sun:

[1] It must complete an integer number of revolutions [in both intervals]; or
[2] traverse the semi-circle beginning at the apogee over one interval and the semi-circle beginning at the perigee over the other; or
[3] begin from the same point [of the ecliptic] in each interval; or
[4] be the same distance from apogee (or perigee) at the first eclipse of one interval H274
as it is at the second eclipse of the other interval, [but] on the other side.[13]

For only under one of these conditions will there be no effect due to the anomaly, or the same effect over both intervals, so that the arc traversed beyond complete revolutions over one interval is equal to that traversed over the other, or even equal to the mean motion of the sun [over the intervals] as well.

Secondly, it is our opinion that we must pay no less attention to the moon's [varying] speed.[14] For if this is not taken into account, it will be possible for the moon, in many situations, to cover equal arcs in longitude in equal times which do not at all represent a return in lunar anomaly as well. This will come to pass

[1] if in both intervals the moon starts from the same speed (either both increasing or both decreasing), but does not return to that speed; or
[2] if in one interval it starts from its greatest speed and ends at its least speed, while in the other interval it starts from its least speed and ends at its greatest speed; or
[3] if the distance of [the position of] its speed at the beginning of one interval is the same distance from the [position of] greatest or least speed as [the position of] its speed at the end of the other interval, [but] on the other side.[15]

In each of these situations there will again be either no effect or the same effect [in both intervals] of the lunar anomaly, and hence equal increments in H275
longitude will be produced [over both intervals], but there will be no return in anomaly at all. So the intervals adduced must avoid all the above situations if

[12] That is, from the positions where the equation of anomaly reaches its positive maximum (Pisces) and negative maximum (Virgo). Illustrated by *HAMA* Fig. 59 p. 1223.

[13] That is, if the sun has an anomaly of α° at the beginning of the first interval, it must have an anomaly of (360−α)° at the end of the second interval. This situation (and the others listed here) is illustrated by *HAMA* Fig. 60 p. 1223.

[14] δρόμος is often used in early Greek astronomy for the (varying) amount which the moon travels in one day. The earliest example seems to be the 'Eudoxus' papyrus (ed. Blass p. 14). Where Ptolemy uses δρόμος for the moon (e.g. V 2, H355,14; V 3, H361,16) 'speed' seems the best translation. For a special use of the term by Hipparchus see V 3 p. 224 with n.14.

[15] Illustrated (in the order [1], [3], [2]) by *HAMA* Fig. 61 p. 1224, which utilizes the lunar epicycle model. One must presume that Ptolemy avoids talking in geometrical terms (which is the most convenient way to visualize the situation) because he has not yet established a lunar model. However, it is hard to give any sense to ἑκατέρωθεν (literally 'on opposite sides', translated here as 'on the other side') which does not involve an epicycle model.

they are to provide us directly with a period of return in anomaly. On the contrary, we should select intervals [the ends of which are situated] so as to best indicate [whether the interval is or is not a period of anomaly], by displaying the discrepancy [between two intervals] when they do not contain an integer number of returns in anomaly. Such is the case when the intervals begin from speeds which are not merely different, but greatly different either in size or in effect. By 'in size' I mean when in one interval [the moon] starts from its least speed and does not end at the greatest speed, while in the other it starts from its greatest speed and does not end at its least speed. For in this case, unless the intervals contain an integer number of revolutions in anomaly, the difference in the increments in longitude over the two intervals will be very great; when the increment in anomaly is about one or three quadrants of a revolution, the intervals will differ by twice the [maximum] equation of anomaly. By 'in effect' I mean when [the moon] starts from mean speed in both positions, not, however, from the same mean speed, but from the mean speed during the period of increasing speed at one interval, and from that during the period of
H276 decreasing speed at the other. Here too, if there is not a return in anomaly, there will be a great difference in the increment in longitude [over the two intervals]; again, when the increment in anomaly is one or three quadrants of a revolution, the difference will again amount to twice the [maximum] equation of anomaly, and when the increment in anomaly is a semi-circle, the difference will be four times that amount.[16]

That is why, as we can see, Hipparchus too used his customary extreme care in the selection of the intervals adduced for his investigation of this question: he used [two intervals], in one of which the moon started from its greatest speed and did not end at its least speed, and in the other of which it started from its least speed and did not end at its greatest speed. Furthermore he also made a correction, albeit a small one, for the sun's equation of anomaly, since the sun fell short of an integer number of revolutions by about $\frac{1}{4}$ of a sign, and this sign was different, and produced a different equation of anomaly, in each of the two intervals.[17]

We have made the above remarks, not to disparage the preceding method of determining the periodic returns, but to show that, while it can achieve its goal if applied with due care and the appropriate kind of calculations, if any of the conditions we set out above are omitted from consideration, even the least of
H277 them, it can fail utterly in its intended effect; and that, if one does use the proper criteria in making one's selection of observational material, it is difficult to find corresponding [pairs of eclipse] observations which precisely fulfil all the required conditions.

In any case, when we take the above periodic returns, as determined by Hipparchus' calculations, we find that the period [containing an integer number] of months has, as we said, been calculated as correctly as possible, and has no perceptible difference from the true value. But there is an error in the

[16] These two situations (of maximum effect due to the anomaly when there is not a return in anomaly) are illustrated by *HAMA* Fig. 62 p. 1225.

[17] On the eclipses used by Hipparchus see Toomer[11].

periods of anomaly and latitude, so considerable as to become quite apparent to us from the procedures we devised to check these values in simpler and more practical ways; we shall soon explain these, in connection with our demonstration of the size of the lunar anomaly. But first, for convenience [of calculation] in what follows, we set out the individual mean motions [of the moon] in longitude, anomaly and latitude, in accordance with the above periods of their returns, and [also the mean motions] calculated on the basis of the corrections which we shall derive later.[18]

3. {*On the individual mean motions of the moon*} H278

If, then, we multiply the mean daily motion of the sun which we derived, ca. 0;59,8,17,13,12,31$^{\circ/d}$, by the number of days in one [mean synodic] month, 29;31,50,8,20^{d}, and add to the result the 360° of one revolution, we will get the mean motion of the moon in longitude during one synodic month as ca. 389;6,23,1,24,2,30,57°. Dividing this by the above number of days in a month, we get the mean daily motion of the moon in longitude as ca. 13;10,34,58,33,30,30°.

Next, multiplying the 269 revolutions in anomaly by the 360° of one revolution, we get 96840°. Dividing this by the number of days in 251 months, 7412;10,44,51,40^{d}, we get the mean daily motion in anomaly as 13;3,53,56,29,38,38°.

Similarly, multiplying the 5923 returns in latitude by the 360° of one revolution, we get 2132280°. Dividing this by the number of days in 5458 H279
months, 161177;58,58,3,20^{d}, we get the mean daily motion in latitude as 13;13,45,39,40,17,19°.

Next, subtracting the mean daily motion of the sun from the mean daily motion of the moon in longitude, we get the mean daily motion in elongation as 12;11,26,41,20,17,59°.

However, from the methods which, as we said, we shall employ in what follows for investigation of this topic, we find that the mean daily motion in longitude (and hence, obviously, that in elongation), is practically identical to the above, but the mean daily motion in anomaly is 0;0,0,0,11,46,39° less: thus it is 13;3,53,56,17,51,59°; and the mean daily motion in latitude is 0;0,0,0,8,39,18° more; thus it is 13;13,45,39,48,56,37°.[19]

Using the latter daily motions, and taking $\frac{1}{24}$th of each, we get the following mean hourly motions:

in longitude:	0;32,56,27,26,23,46,15°
in anomaly:	0;32,39,44,50,44,39,57,30°
in latitude:	0;33,4,24,9,32,21,32,30°
in elongation:	0;30,28,36,43,20,44,57,30°. H280

[18] Ptolemy's corrections to the mean motions in anomaly and latitude, given below, are justified at IV 7 (p. 204) and IV 9 (p. 207).

[19] All the above computations have been carried out very precisely, and are correct to the nearest sixth (60^{-6} degree). In the following computations of the mean motions for the greater units, however, Ptolemy operates as if the last place in the mean daily motions were precisely correct, i.e. no account is taken of the accumulated error for months, years, etc.

Multiplying the daily motions by 30 and subtracting complete revolutions, we get the following monthly mean increments:

in longitude:	35;17,29,16,45,15°
in anomaly:	31;56,58,8,55,59,30°
in latitude:	36;52,49,54,28,18,30°[20]
in elongation:	5;43,20,40,8,59,30°.

Next, multiplying the daily motions by the 365 days of the Egyptian year, and subtracting complete revolutions, we get the following yearly mean increments:

in longitude:	129;22,46,13,50,32,30°
in anomaly:	88;43,7,28,41,13,55°
in latitude:	148;42,47,12,44,25,5°
in elongation:	129;37,21,28,29,23,55°.

Next, multiplying the yearly motions by 18 (this number is chosen, as we said, for convenience in tabulation), after subtracting complete revolutions we get the following mean increments over an eighteen-year period:

in longitude:	168;49,52,9,9,45°
in anomaly:	156;56,14,36,22,10,30°
in latitude:	156;50,9,49,19,31,30°
in elongation:	173;12,26,32,49,10,30°.

H281 As in the case of the sun, we will again set out three tables arranged in 45 lines, with 5 columns in each. The first column will contain the time-divisions appropriate to each table, in the first table the 18-year periods, in the second the years, again followed by the hours, in the third the months, again followed by the days. The remaining four columns will contain the degrees [and their subdivisions] corresponding to the appropriate argument: the second column, longitude, the third, anomaly, the fourth, latitude, and the fifth, elongation. The layout of the tables is as follows.

H282-93

4. {*Tables of the mean motions of the moon*}

[See pp. 182-7.]

H294

5. {*That in the simple hypothesis of the moon, too, the same phenomena are produced by both eccentric and epicyclic hypotheses*}[21]

Our next task is to demonstrate the type and size of the moon's anomaly. For the time being we shall treat this as if it were single and invariant.[22] It is apparent that this anomaly, namely the one with a period corresponding to the above period of return, is the only one which our predecessors (just about all of them)

[20] Reading $\overline{\lambda}$ for $\overline{\lambda\alpha}$ ('31') in the last place at H280,5, with D, Ar (cf. also the tables IV 4). Corrected by Manitius.

[21] See Pedersen 166-7.

[22] Reading καὶ τῆς αὐτῆς (with BD) for ταύτης ('as if this were single') at H294,6. Ar read ταύτης.

have hit upon. Later, however, we shall show that the moon also has a second anomaly, linked to its distance from the sun; this [second anomaly] reaches a maximum round about both [waxing and waning] half-moons, and goes through its period of return twice a month, [being zero] precisely at conjunction and opposition.[23] We adopt this order of procedure in our demonstration because it is impossible to determine the second [anomaly] apart from the first, which is always combined with it, whereas the first can be found apart from the second, since it is determined from lunar eclipses, at which there is no perceptible effect of the anomaly connected with [the distance from] the sun.

In this first part of our demonstrations we shall use the methods of establishing the theorem which Hipparchus, as we see, used before us.[24] We too, using three lunar eclipses, shall derive the maximum difference from mean motion and the epoch of the [moon's position] at the apogee, on the assumption that only this [first] anomaly is taken into account, and that it is produced by the epicyclic hypothesis. It is true that the same phenomena would result from the eccentric hypothesis, but we shall find the latter more suitable to represent the second anomaly, which is connected with the sun, when we come to combine both anomalies. However, the same phenomena will in all cases result from both the hypotheses we have described, whether, as in the situation described for the sun, the period of return in anomaly and the period of return in the ecliptic [i.e. in longitude] are both equal, or whether, as in the case of the moon, they are unequal, provided only that the ratios [of epicycle to deferent and eccentricity to eccentre] are taken as identical. We can see this from the following, in which we use the above-mentioned simple anomaly of the moon for our examination. H295

Since the moon completes its return with respect to the ecliptic sooner than its return with respect to this anomaly, it is clear that, in the epicyclic hypothesis, over a given period of time, the epicycle will always traverse a greater arc[25] of the circle concentric to the ecliptic than the arc of the epicycle traversed by the moon in the same time; in the eccentric hypothesis, the arc traversed by the moon on the eccentre will be similar to the arc traversed by it on the epicycle [in the epicyclic hypothesis], while the eccentre will move about the centre of the ecliptic in the same direction as the moon by an amount equal to the increment of the motion in longitude over the motion in anomaly [in the same time] (this corresponds to the increment of the arc of the deferent over the arc of the epicycle [in the epicyclic hypothesis]). In this way we can preserve the equality of the periods of both motions [i.e. in longitude and anomaly], as well as equality of the ratios, in both hypotheses. H296

With the above as a necessary basis (as is obvious from logic), let [Fig. 4.1] the circle concentric with the ecliptic be ABG on centre D and diameter AD, and let the epicycle be EZ on centre G. Let us suppose that when the epicycle was at A, the moon was at E, the apogee of the epicycle, and that in the same time as the epicycle has traversed arc AG, the moon has traversed arc EZ. Join ED, GZ.

[23] Reference to V 2-4.

[24] On Hipparchus' determination of the lunar parameters see further IV 11, Toomer[8] and Toomer[2].

[25] 'a greater arc': literally 'an arc greater than the one similar to [the arc]'.

TABLES OF THE MOON'S MEAN MOTIONS

18-Year Periods	Increment in Longitude [Epoch Position:] ♉ 11;22° °	′	″	‴	⁗	′′′′′	′′′′′′	Increment in Anomaly [Epoch Position:] 268;49° °	′	″	‴	⁗	′′′′′	′′′′′′
18	168	49	52	9	9	45	0	156	56	14	36	22	10	30
36	337	39	44	18	19	30	0	313	52	29	12	44	21	0
54	146	29	36	27	29	15	0	110	48	43	49	6	31	30
72	315	19	28	36	39	0	0	267	44	58	25	28	42	0
90	124	9	20	45	48	45	0	64	41	13	1	50	52	30
108	292	59	12	54	58	30	0	221	37	27	38	13	3	0
126	101	49	5	4	8	15	0	18	33	42	14	35	13	30
144	270	38	57	13	18	0	0	175	29	56	50	57	24	0
162	79	28	49	22	27	45	0	332	26	11	27	19	34	30
180	248	18	41	31	37	30	0	129	22	26	3	41	45	0
198	57	8	33	40	47	15	0	286	18	40	40	3	55	30
216	225	58	25	49	57	0	0	83	14	55	16	26	6	0
234	34	48	17	59	6	45	0	240	11	9	52	48	16	30
252	203	38	10	8	16	30	0	37	7	24	29	10	27	0
270	12	28	2	17	26	15	0	194	3	39	5	32	37	30
288	181	17	54	26	36	0	0	350	59	53	41	54	48	0
306	350	7	46	35	45	45	0	147	56	8	18	16	58	30
324	158	57	38	44	55	30	0	304	52	22	54	39	9	0
342	327	47	30	54	5	15	0	101	48	37	31	1	19	30
360	136	37	23	3	15	0	0	258	44	52	7	23	30	0
378	305	27	15	12	24	45	0	55	41	6	43	45	40	30
396	114	17	7	21	34	30	0	212	37	21	20	7	51	0
414	283	6	59	30	44	15	0	9	33	35	56	30	1	30
432	91	56	51	39	54	0	0	166	29	50	32	52	12	0
450	260	46	43	49	3	45	0	323	26	5	9	14	22	30
468	69	36	35	58	13	30	0	120	22	19	45	36	33	0
486	238	26	28	7	23	15	0	277	18	34	21	58	43	30
504	47	16	20	16	33	0	0	74	14	48	58	20	54	0
522	216	6	12	25	42	45	0	231	11	3	34	43	4	30
540	24	56	4	34	52	30	0	28	7	18	11	5	15	0
558	193	45	56	44	2	15	0	185	3	32	47	27	25	30
576	2	35	48	53	12	0	0	341	59	47	23	49	36	0
594	171	25	41	2	21	45	0	138	56	2	0	11	46	30
612	340	15	33	11	31	30	0	295	52	16	36	33	57	0
630	149	5	25	20	41	15	0	92	48	31	12	56	7	30
648	317	55	17	29	51	0	0	249	44	45	49	18	18	0
666	126	45	9	39	0	45	0	46	41	0	25	40	28	30
684	295	35	1	48	10	30	0	203	37	15	2	2	39	0
702	104	24	53	57	20	15	0	0	33	29	38	24	49	30
720	273	14	46	6	30	0	0	157	29	44	14	47	0	0
738	82	4	38	15	39	45	0	314	25	58	51	9	10	30
756	250	54	30	24	49	30	0	111	22	13	27	31	21	0
774	59	44	22	33	59	15	0	268	18	28	3	53	31	30
792	228	34	14	43	9	0	0	65	14	42	40	15	42	0
810	37	24	6	52	18	45	0	222	10	57	16	37	52	30

18-Year Periods	Increment in Latitude [Epoch Position:] 354;15° °	′	″	‴	⁗	′′′′′	′′′′′′	Increment in Elongation [Epoch Position] 70;37° °	′	″	‴	⁗	′′′′′	′′′′′′
18	156	50	9	49	19	31	30	173	12	26	32	49	10	30
36	313	40	19	38	39	3	0	346	24	53	5	38	21	0
54	110	30	29	27	58	34	30	159	37	19	38	27	31	30
72	267	20	39	17	18	6	0	332	49	46	11	16	42	0
90	64	10	49	6	37	37	30	146	2	12	44	5	52	30
108	221	0	58	55	57	9	0	319	14	39	16	55	3	0
126	17	51	8	45	16	40	30	132	27	5	49	44	13	30
144	174	41	18	34	36	12	0	305	39	32	22	33	24	0
162	331	31	28	23	55	43	30	118	51	58	55	22	34	30
180	128	21	38	13	15	15	0	292	4	25	28	11	45	0
198	285	11	48	2	34	46	30	105	16	52	1	0	55	30
216	82	1	57	51	54	18	0	278	29	18	33	50	6	0
234	238	52	7	41	13	49	30	91	41	45	6	39	16	30
252	35	42	17	30	33	21	0	264	54	11	39	28	27	0
270	192	32	27	19	52	52	30	78	6	38	12	17	37	30
288	349	22	37	9	12	24	0	251	19	4	45	6	48	0
306	146	12	46	58	31	55	30	64	31	31	17	55	58	30
324	303	2	56	47	51	27	0	237	43	57	50	45	9	0
342	99	53	6	37	10	58	30	50	56	24	23	34	19	30
360	256	43	16	26	30	30	0	224	8	50	56	23	30	0
378	53	33	26	15	50	1	30	37	21	17	29	12	40	30
396	210	23	36	5	9	33	0	210	33	44	2	1	51	0
414	7	13	45	54	29	4	30	23	46	10	34	51	1	30
432	164	3	55	43	48	36	0	196	58	37	7	40	12	0
450	320	54	5	33	8	7	30	10	11	3	40	29	22	30
468	117	44	15	22	27	39	0	183	23	30	13	18	33	0
486	274	34	25	11	47	10	30	356	35	56	46	7	43	30
504	71	24	35	1	6	42	0	169	48	23	18	56	54	0
522	228	14	44	50	26	13	30	343	0	49	51	46	4	30
540	25	4	54	39	45	45	0	156	13	16	24	35	15	0
558	181	55	4	29	5	16	30	329	25	42	57	24	25	30
576	338	45	14	18	24	48	0	142	38	9	30	13	36	0
594	135	35	24	7	44	19	30	315	50	36	3	2	46	30
612	292	25	33	57	3	51	0	129	3	2	35	51	57	0
630	89	15	43	46	23	22	30	302	15	29	8	41	7	30
648	246	5	53	35	42	54	0	115	27	55	41	30	18	0
666	42	56	3	25	2	25	30	288	40	22	14	19	28	30
684	199	46	13	14	21	57	0	101	52	48	47	8	39	0
702	356	36	23	3	41	28	30	275	5	15	19	57	49	30
720	153	26	32	53	1	0	0	88	17	41	52	47	0	0
738	310	16	42	42	20	31	30	261	30	8	25	36	10	30
756	107	6	52	31	40	3	0	74	42	34	58	25	21	0
774	263	57	2	20	59	34	30	247	55	1	31	14	31	30
792	60	47	12	10	19	6	0	61	7	28	4	3	42	0
810	217	37	21	59	38	37	30	234	19	54	36	52	52	30

Single Years	Increment in Longitude							Increment in Anomaly						
	°	'	''	'''	''''	'''''	''''''	°	'	''	'''	''''	'''''	''''''
1	129	22	46	13	50	32	30	88	43	7	28	41	13	55
2	258	45	32	27	41	5	0	177	26	14	57	22	27	50
3	28	8	18	41	31	37	30	266	9	22	26	3	41	45
4	157	31	4	55	22	10	0	354	52	29	54	44	55	40
5	286	53	51	9	12	42	30	83	35	37	23	26	9	35
6	56	16	37	23	3	15	0	172	18	44	52	7	23	30
7	185	39	23	36	53	47	30	261	1	52	20	48	37	25
8	315	2	9	50	44	20	0	349	44	59	49	29	51	20
9	84	24	56	4	34	52	30	78	28	7	18	11	5	15
10	213	47	42	18	25	25	0	167	11	14	46	52	19	10
11	343	10	28	32	15	57	30	255	54	22	15	33	33	5
12	112	33	14	46	6	30	0	344	37	29	44	14	47	0
13	241	56	0	59	57	2	30	73	20	37	12	56	0	55
14	11	18	47	13	47	35	0	162	3	44	41	37	14	50
15	140	41	33	27	38	7	30	250	46	52	10	18	28	45
16	270	4	19	41	28	40	0	339	29	59	38	59	42	40
17	39	27	5	55	19	12	30	68	13	7	7	40	56	35
18	168	49	52	9	9	45	0	156	56	14	36	22	10	30

Hours	Increment in Longitude							Increment in Anomaly						
	°	'	''	'''	''''	'''''	''''''	°	'	''	'''	''''	'''''	''''''
1	0	32	56	27	26	23	46	0	32	39	44	50	44	40
2	1	5	52	54	52	47	32	1	5	19	29	41	29	20
3	1	38	49	22	19	11	18	1	37	59	14	32	14	0
4	2	11	45	49	45	35	5	2	10	38	59	22	58	40
5	2	44	42	17	11	58	51	2	43	18	44	13	43	20
6	3	17	38	44	38	22	37	3	15	58	29	4	28	0
7	3	50	35	12	4	46	23	3	48	38	13	55	12	40
8	4	23	31	39	31	10	10	4	21	17	58	45	57	20
9	4	56	28	6	57	33	56	4	53	57	43	36	42	0
10	5	29	24	34	23	57	42	5	26	37	28	27	26	40
11	6	2	21	1	50	21	28	5	59	17	13	18	11	20
12	6	35	17	29	16	45	15	6	31	56	58	8	56	0
13	7	8	13	56	43	9	1	7	4	36	42	59	40	39
14	7	41	10	24	9	32	47	7	37	16	27	50	25	19
15	8	14	6	51	35	56	33	8	9	56	12	41	9	59
16	8	47	3	19	2	20	20	8	42	35	57	31	54	39
17	9	19	59	46	28	44	6	9	15	15	42	22	39	19
18	9	52	56	13	55	7	52	9	47	55	27	13	23	59
19	10	25	52	41	21	31	38	10	20	35	12	4	8	39
20	10	58	49	8	47	55	25	10	53	14	56	54	53	19
21	11	31	45	36	14	19	11	11	25	54	41	45	37	59
22	12	4	42	3	40	42	57	11	58	34	26	36	22	39
23	12	37	38	31	7	6	43	12	31	14	11	27	7	19
24	13	10	34	58	33	30	30	13	3	53	56	17	51	59

Single Years	Increment in Latitude							Increment in Elongation						
	°	'	''	'''	''''	'''''	''''''	°	'	''	'''	''''	'''''	''''''
1	148	42	47	12	44	25	5	129	37	21	28	29	23	55
2	297	25	34	25	28	50	10	259	14	42	56	58	47	50
3	86	8	21	38	13	15	15	28	52	4	25	28	11	45
4	234	51	8	50	57	40	20	158	29	25	53	57	35	40
5	23	33	56	3	42	5	25	288	6	47	22	26	59	35
6	172	16	43	16	26	30	30	57	44	8	50	56	23	30
7	320	59	30	29	10	55	35	187	21	30	19	25	47	25
8	109	42	17	41	55	20	40	316	58	51	47	55	11	20
9	258	25	4	54	39	45	45	86	36	13	16	24	35	15
10	47	7	52	7	24	10	50	216	13	34	44	53	59	10
11	195	50	39	20	8	35	55	345	50	56	13	23	23	5
12	344	33	26	32	53	1	0	115	28	17	41	52	47	0
13	133	16	13	45	37	26	5	245	5	39	10	22	10	55
14	281	59	0	58	21	51	10	14	43	0	38	51	34	50
15	70	41	48	11	6	16	15	144	20	22	7	20	58	45
16	219	24	35	23	50	41	20	273	57	43	35	50	22	40
17	8	7	22	36	35	6	25	43	35	5	4	19	46	35
18	156	50	9	49	19	31	30	173	12	26	32	49	10	30

Hours	Increment in Latitude							Increment in Elongation						
	°	'	''	'''	''''	'''''	''''''	°	'	''	'''	''''	'''''	''''''
1	0	33	4	24	9	32	22	0	30	28	36	43	20	45
2	1	6	8	48	19	4	43	1	0	57	13	26	41	30
3	1	39	13	12	28	37	5	1	31	25	50	10	2	15
4	2	12	17	36	38	9	26	2	1	54	26	53	23	0
5	2	45	22	0	47	41	48	2	32	23	3	36	43	45
6	3	18	26	24	57	14	9	3	2	51	40	20	4	30
7	3	51	30	49	6	46	31	3	33	20	17	3	25	15
8	4	24	35	13	16	18	52	4	3	48	53	46	46	0
9	4	57	39	37	25	51	14	4	34	17	30	30	6	45
10	5	30	44	1	35	23	35	5	4	46	7	13	27	30
11	6	3	48	25	44	55	57	5	35	14	43	56	48	15
12	6	36	52	49	54	28	18	6	5	43	20	40	9	0
13	7	9	57	14	4	0	40	6	36	11	57	23	29	44
14	7	43	1	38	13	33	2	7	6	40	34	6	50	29
15	8	16	6	2	23	5	23	7	37	9	10	50	11	14
16	8	49	10	26	32	37	45	8	7	37	47	33	31	59
17	9	22	14	50	42	10	6	8	38	6	24	16	52	44
18	9	55	19	14	51	42	28	9	8	35	1	0	13	29
19	10	28	23	39	1	14	49	9	39	3	37	43	34	14
20	11	1	28	3	10	47	11	10	9	32	14	26	54	59
21	11	34	32	27	20	19	32	10	40	0	51	10	15	44
22	12	7	36	51	29	51	54	11	10	29	27	53	36	29
23	12	40	41	15	39	24	15	11	40	58	4	36	57	14
24	13	13	45	39	48	56	37	12	11	26	41	20	17	59

	Increment in Longitude							Increment in Anomaly						
Months	°	′	″	‴	′′′′	′′′′′	′′′′′′	°	′	″	‴	′′′′	′′′′′	′′′′′′
30	35	17	29	16	45	15	0	31	56	58	8	55	59	30
60	70	34	58	33	30	30	0	63	53	56	17	51	59	0
90	105	52	27	50	15	45	0	95	50	54	26	47	58	30
120	141	9	57	7	1	0	0	127	47	52	35	43	58	0
150	176	27	26	23	46	15	0	159	44	50	44	39	57	30
180	211	44	55	40	31	30	0	191	41	48	53	35	57	0
210	247	2	24	57	16	45	0	223	38	47	2	31	56	30
240	282	19	54	14	2	0	0	255	35	45	11	27	56	0
270	317	37	23	30	47	15	0	287	32	43	20	23	55	30
300	352	54	52	47	32	30	0	319	29	41	29	19	55	0
330	28	12	22	4	17	45	0	351	26	39	38	15	54	30
360	63	29	51	21	3	0	0	23	23	37	47	11	54	0

	Increment in Longitude							Increment in Anomaly						
Days	°	′	″	‴	′′′′	′′′′′	′′′′′′	°	′	″	‴	′′′′	′′′′′	′′′′′′
1	13	10	34	58	33	30	30	13	3	53	56	17	51	59
2	26	21	9	57	7	1	0	26	7	47	52	35	43	58
3	39	31	44	55	40	31	30	39	11	41	48	53	35	57
4	52	42	19	54	14	2	0	52	15	35	45	11	27	56
5	65	52	54	52	47	32	30	65	19	29	41	29	19	55
6	79	3	29	51	21	3	0	78	23	23	37	47	11	54
7	92	14	4	49	54	33	30	91	27	17	34	5	3	53
8	105	24	39	48	28	4	0	104	31	11	30	22	55	52
9	118	35	14	47	1	34	30	117	35	5	26	40	47	51
10	131	45	49	45	35	5	0	130	38	59	22	58	39	50
11	144	56	24	44	8	35	30	143	42	53	19	16	31	49
12	158	6	59	42	42	6	0	156	46	47	15	34	23	48
13	171	17	34	41	15	36	30	169	50	41	11	52	15	47
14	184	28	9	39	49	7	0	182	54	35	8	10	7	46
15	197	38	44	38	22	37	30	195	58	29	4	27	59	45
16	210	49	19	36	56	8	0	209	2	23	0	45	51	44
17	223	59	54	35	29	38	30	222	6	16	57	3	43	43
18	237	10	29	34	3	9	0	235	10	10	53	21	35	42
19	250	21	4	32	36	39	30	248	14	4	49	39	27	41
20	263	31	39	31	10	10	0	261	17	58	45	57	19	40
21	276	42	14	29	43	40	30	274	21	52	42	15	11	39
22	289	52	49	28	17	11	0	287	25	46	38	33	3	38
23	303	3	24	26	50	41	30	300	29	40	34	50	55	37
24	316	13	59	25	24	12	0	313	33	34	31	8	47	36
25	329	24	34	23	57	42	30	326	37	28	27	26	39	35
26	342	35	9	22	31	13	0	339	41	22	23	44	31	34
27	355	45	44	21	4	43	30	352	45	16	20	2	23	33
28	8	56	19	19	38	14	0	5	49	10	16	20	15	32
29	22	6	54	18	11	44	30	18	53	4	12	38	7	31
30	35	17	29	16	45	15	0	31	56	58	8	55	59	30

Months	Increment in Latitude							Increment in Elongation						
	°	′	″	‴	′′′′	′′′′′	′′′′′′	°	′	″	‴	′′′′	′′′′′	′′′′′′
30	36	52	49	54	28	18	30	5	43	20	40	8	59	30
60	73	45	39	48	56	37	0	11	26	41	20	17	59	0
90	110	38	29	43	24	55	30	17	10	2	0	26	58	30
120	147	31	19	37	53	14	0	22	53	22	40	35	58	0
150	184	24	9	32	21	32	30	28	36	43	20	44	57	30
180	221	16	59	26	49	51	0	34	20	4	0	53	57	0
210	258	9	49	21	18	9	30	40	3	24	41	2	56	30
240	295	2	39	15	46	28	0	45	46	45	21	11	56	0
270	331	55	29	10	14	46	30	51	30	6	1	20	55	30
300	8	48	19	4	43	5	0	57	13	26	41	29	55	0
330	45	41	8	59	11	23	30	62	56	47	21	38	54	30
360	82	33	58	53	39	42	0	68	40	8	1	47	54	0

Days	Increment in Latitude							Increment in Elongation						
	°	′	″	‴	′′′′	′′′′′	′′′′′′	°	′	″	‴	′′′′	′′′′′	′′′′′′
1	13	13	45	39	48	56	37	12	11	26	41	20	17	59
2	26	27	31	19	37	53	14	24	22	53	22	40	35	58
3	39	41	16	59	26	49	51	36	34	20	4	0	53	57
4	52	55	2	39	15	46	28	48	45	46	45	21	11	56
5	66	8	48	19	4	43	5	60	57	13	26	41	29	55
6	79	22	33	58	53	39	42	73	8	40	8	1	47	54
7	92	36	19	38	42	36	19	85	20	6	49	22	5	53
8	105	50	5	18	31	32	56	97	31	33	30	42	23	52
9	119	3	50	58	20	29	33	109	43	0	12	2	41	51
10	132	17	36	38	9	26	10	121	54	26	53	22	59	50
11	145	31	22	17	58	22	47	134	5	53	34	43	17	49
12	158	45	7	57	47	19	24	146	17	20	16	3	35	48
13	171	58	53	37	36	16	1	158	28	46	57	23	53	47
14	185	12	39	17	25	12	38	170	40	13	38	44	11	46
15	198	26	24	57	14	9	15	182	51	40	20	4	29	45
16	211	40	10	37	3	5	52	195	3	7	1	24	47	44
17	224	53	56	16	52	2	29	207	14	33	42	45	5	43
18	238	7	41	56	40	59	6	219	26	0	24	5	23	42
19	251	21	27	36	29	55	43	231	37	27	5	25	41	41
20	264	35	13	16	18	52	20	243	48	53	46	45	59	40
21	277	48	58	56	7	48	57	256	0	20	28	6	17	39
22	291	2	44	35	56	45	34	268	11	47	9	26	35	38
23	304	16	30	15	45	42	11	280	23	13	50	46	53	37
24	317	30	15	55	34	38	48	292	34	40	32	7	11	36
25	330	44	1	35	23	35	25	304	46	7	13	27	29	35
26	343	57	47	15	12	32	2	316	57	33	54	47	47	34
27	357	11	32	55	1	28	39	329	9	0	36	8	5	33
28	10	25	18	34	50	25	16	341	20	27	17	28	23	32
29	23	39	4	14	39	21	53	353	31	53	58	48	41	31
30	36	52	49	54	28	18	30	5	43	20	40	8	59	30

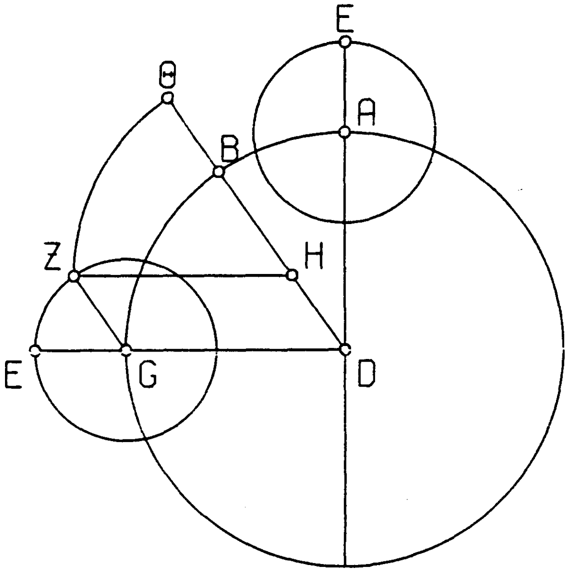

Fig. 4.1

Then, since arc AG > arc EZ,
cut off arc BG || arc EZ, and join BD.

Then it is clear that, in the same time, the eccentre will have moved through
H297 ∠ ADB, which represents the difference between the two motions, and its centre and apogee will lie along line BD.

This being so, let DH = GZ. Join ZH, and with centre H and radius HZ draw the eccentre ZΘ.

I say, that

ZH:HD = DG:GZ,

and that in this hypothesis too the moon will be at point Z, i.e.

arc ZΘ || arc EZ.

[Proof:] Since ∠ BDG = ∠ EGZ, GZ is parallel to DH.

But GZ = DH [by construction].

Therefore ZH too is equal and parallel to GD.[26]

∴ ZH:HD = DG:GZ.

H298 Furthermore, since DG is parallel to HZ,

∠ GDB = ∠ ZHΘ;

and, by hypothesis, ∠ GDB = ∠ EGZ.

∴ arc ZΘ || arc EZ.

Therefore the moon has reached point Z in the same time according to either hypothesis, since the moon itself has traversed arc EZ on the epicycle and arc ΘZ on the eccentre, which we have shown to be similar, while the epicycle

[26] Euclid I 33: straight lines joining equal and parallel lines are themselves equal and parallel.

centre has moved through arc AG, and the centre of the eccentre through arc AB, which is the increment of arc AG over arc EZ.

Q.E.D.

Moreover, even if [the members of] the ratios are unequal, and the eccentre is not the same size as the deferent, the same phenomena will result, provided the ratios are similar, as will be clear from the following.

Draw each of the hypotheses in a separate figure. Let [Fig. 4.2] the circle concentric to the ecliptic be ABG on centre D and diameter AD, and the epicycle EZ on centre G. Let the moon be at Z. Let [Fig. 4.3] the eccentre be HΘK on centre L and diameter ΘLM, with the centre of the ecliptic at M. Let H299
the moon be at K. In the first figure join DGE, GZ, DZ, and in the second figure join HM, KM, KL.

Let DG:GE = ΘL:LM.

Let us suppose that in the same time as the epicycle has moved through ∠ ADG, the moon has again moved through ∠ EGZ, the eccentre through ∠ HMΘ, and the moon, again, through ∠ ΘLK.

Therefore, because of the assumed relationship between the motions,

∠ EGZ = ∠ ΘLK,

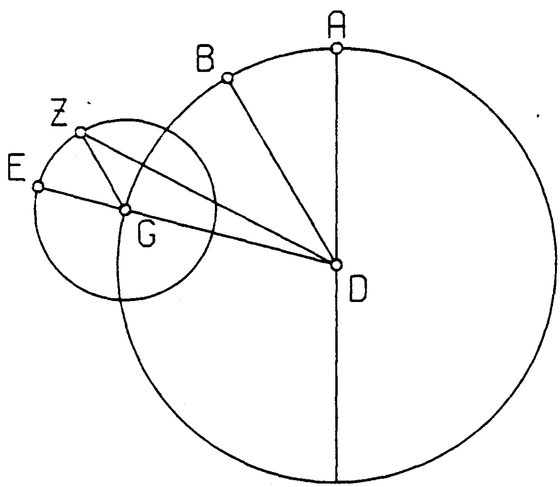

Fig. 4.2

and ∠ ADG = ∠ HMΘ + ∠ ΘLK.

This being so, I say that the moon will again appear to have traversed an equal arc in the same time according to either hypothesis, i.e.

∠ ADZ = ∠ HMK

(for at the beginning of the time-interval the moon was at the apogee and appeared along lines DA and MH, while at the end it was at points Z and K and appeared along lines ZD and MK).

[Proof:] Let arc BG again be similar to arc ΘK (or arc EZ). Join BD. H300

Then, since DG:GZ = KL:LM,

and the angles at G and L are equal,

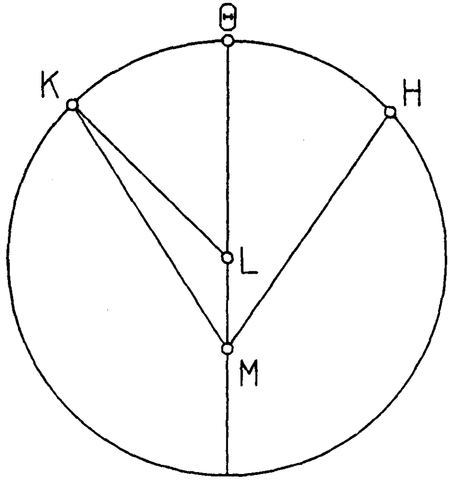

Fig. 4.3

triangle GDZ ||| triangle KLM (sides about equal angles proportional), and the angles opposite the corresponding sides are equal.

$$\therefore \angle \text{GZD} = \angle \text{LMK}.$$

$$\text{But } \angle \text{BDZ} = \angle \text{GZD},$$

for GZ is parallel to BD, since, by hypothesis, ∠ ZGE = ∠ BDG.

$$\therefore \angle \text{ZDB} = \angle \text{LMK}.$$

But, by hypothesis, ∠ ADB, the difference between the motions [in longitude and anomaly] equals ∠ HMΘ, the motion of [the centre of] the eccentre. Therefore, by addition,

$$\angle \text{ADZ} = \angle \text{KMH}.$$

Q.E.D.

6. {*Demonstration of the first, simple anomaly of the moon*}[27]

Let the preceding suffice us as preliminary theory. We shall now demonstrate
H301 the lunar anomaly in question, by means of the epicyclic hypothesis, for the reason mentioned. [For this purpose] we shall use, first, among the most ancient eclipses available to us, three [which we have selected] as being recorded in an unambiguous fashion, and, secondly, [we shall repeat the procedure] using, among contemporary eclipses, three which we ourselves have observed very accurately. In this way our results will be valid over as long a period as possible, and in particular it will be apparent that approximately the same [maximum] equation of anomaly results from both demonstrations, and that the increment in the mean motions [between the two sets of eclipses] agrees[28] with that computed from the above periods (as corrected by us).

[27] See *HAMA* 73–8, Pedersen 169–79.
[28] Reading σύμφωνος (with D, Ar) for σύμφωνος ἀεὶ ('always agrees') at H301,10.

For the purposes of demonstrating the first anomaly, considered separately, the epicyclic hypothesis which we mentioned can be described as follows. Imagine a circle in the sphere of the moon which is concentric to and lies in the same plane as the ecliptic. Inclined to this, at an angle corresponding to the amount of its [maximum] deviation in latitude, is another circle, which moves uniformly in advance (with respect to the centre of the ecliptic) with a speed equal to the difference between the motions in latitude and longitude. On this inclined circle we suppose the so-called 'epicycle' to be carried, with a uniform motion, towards the rear with respect to the heavens, corresponding to the motion in latitude. (This motion, obviously, will represent the [mean] motion in H302
longitude with respect to the ecliptic). On the epicycle itself [we suppose] the moon to move, in such a way that on the arc near the apogee its motion is in advance with respect to the heavens, at a speed corresponding to the period of return in anomaly. However, for the purposes of the present demonstration we shall suffer no ill consequences if we neglect the advance motion in latitude and the inclination of the moon's orbit, since such a small inclination has no noticeable effect on the position in longitude.[29]

First, the three ancient eclipses which are selected from those observed in Babylon.

The first is recorded as occurring in the first year of Mardokempad, Thoth [I] 29/30 in the Egyptian calendar [-720 Mar. 19/20]. The eclipse began, it says, well over an hour after moonrise, and was total.

Now since the sun was near the end of Pisces, and [therefore] the night was about 12 equinoctial hours long, the beginning of the eclipse occurred, clearly, $4\frac{1}{2}$ equinoctial hours before midnight, and mid-eclipse (since it was total) $2\frac{1}{2}$ hours before midnight.[30] Now we take as the standard meridian for all time deter-minations the meridian through Alexandria, which is about $\frac{5}{6}$ of an equinoctial H303
hour in advance [i.e. to the west] of the meridian through Babylon.[31] So at Alexandria the middle of the eclipse in question was $3\frac{1}{3}$ equinoctial hours before midnight, at which time the true position of the sun, according to the [tables] calculated above, was approximately ♓ $24\frac{1}{2}°$.

The second eclipse is recorded as occurring in the second year of the same Mardokempad, Thoth [I] 18/19 in the Egyptian calendar [-719 Mar. 8/9]. The [maximum] obscuration, it says, was 3 digits[32] from the south exactly at midnight. So, since mid-eclipse was exactly at midnight at Babylon, it must

[29] I.e. for the purposes of computing the longitude the moon's orbit is treated as if it lay in the plane of the ecliptic. The maximum resulting error (for $i \approx 5°$) is about 6′ (cf. *HAMA* 83). Ptolemy himself (VI 7 p. 297) estimates it as 5′.

[30] A total eclipse of the moon is assumed to last 4 hours from start to finish. This agrees fairly well with the duration one derives from Ptolemy's own eclipse tables (VI 8) and with the actual maximum possible duration. The duration of the eclipse in question (Oppolzer no. 741) was in fact about $3\frac{3}{4}^h$.

[31] This time difference corresponds to a longitudinal difference of $12\frac{1}{2}°$. The actual time difference is about $58\frac{1}{2}$ minutes. In the *Geography* Ptolemy amended the difference, in the right direction but by far too much, to $1\frac{1}{4}$ hours (8.20.27), corresponding to the difference between the longitudes there assigned to Alexandria ($60\frac{1}{2}°$, 4.5.9) and Babylon (79°, 5.20.6).

[32] Modern calculations give a considerably smaller eclipse: Oppolzer (no. 743) 1.6 digits, P.V. Neugebauer 1.5 digits. However Ptolemy's own tables give about $2\frac{1}{2}$ digits: see Appendix A, Example 11.

have been $\frac{5}{6}^h$ before midnight at Alexandria, at which time the true position of the sun was ♓ $13\frac{3}{4}$°.

The third eclipse is recorded as occurring in the (same) second year of Mardokempad, Phamenoth [VII] 15/16 in the Egyptian calendar [−719 Sept. 1/2]. The eclipse began, it says, after moonrise, and the [maximum] obscuration was more than half [the disk] from the north. So, since the sun was near the beginning of Virgo, the length of night at Babylon was about 11
H304 equinoctial hours, and half the night was $5\frac{1}{2}$ [equinoctial] hours. Therefore the beginning of the eclipse was about 5 equinoctial hours before midnight (since it began after moonrise), and mid-eclipse about $3\frac{1}{2}$ hours before midnight (for the total time for an eclipse of that size must have been about 3 hours).[33] So in Alexandria mid-eclipse occurred $4\frac{1}{3}$ equinoctial hours before midnight, at which time the true position of the sun was about ♍ $3\frac{1}{4}$°.

Then it is clear that the motion of the sun (which is the same as that of the moon apart from complete revolutions) is

from the middle of the first eclipse to the middle of the second: 349;15°
from the middle of the second eclipse to the middle of the third: 169;30°.

The time intervals are:

$$\text{from first to second}\ \begin{cases} 354^d 2\frac{1}{2}^h \text{ reckoned simply} \\ 345^d\ 2\frac{17}{30}^h \text{ reckoned in mean solar days} \end{cases}$$

$$\text{from second to third}\ \begin{cases} 176^d 20\frac{1}{2}^h \text{ reckoned simply} \\ 176^d 20\frac{1}{3}^h \text{ reckoned in mean solar days.} \end{cases}$$

Over such short intervals it will make no appreciable difference if one uses
H305 approximate periods [to determine the moon's mean motions].[34] The moon's mean motions are, then, (beyond complete revolutions), approximately

$$\text{in } 354^d\ 2\frac{17}{30}^h \begin{cases} 306;25° \text{ in anomaly} \\ 345;51° \text{ in longitude} \end{cases}$$

$$\text{in } 176^d\ 20\frac{1}{3}^h \begin{cases} 150;26° \text{ in anomaly} \\ 170;7° \text{ in longitude.} \end{cases}$$

Thus it is clear that the motion on the epicycle of 306;25° over the first interval has produced an increment of [349;15° − 345;51°=] 3;24° over the mean motion, and the motion [on the epicycle] of 150;26° over the second interval has produced a decrement from the mean motion of [169;30° − 170;7°=] 0;37°.

With the above as data, let [Fig. 4.4] the moon's epicycle be [circle] ABG, on

[33] At a lunar eclipse the moon is diametrically opposite the sun. Therefore moonrise coincided with sunset, which was $5\frac{1}{2}$ equinoctial hours before midnight. Ptolemy allows $\frac{1}{2}$-hour to account for 'after moonrise'. He estimates a duration of 3 hours for an eclipse of more than 6 digits (according to Oppolzer, no. 744, this eclipse had a magnitude of 6.4 digits and a duration of about $2;36^h$; P.V. Neugebauer calculates 6.1 digits and 2.4^h). Obviously this eclipse is hardly 'recorded in an unambiguous fashion' (p. 190).

[34] This is a point of methodology. Ptolemy's mean motion tables are based, not on the exact periods he took from Hipparchus, but (for the anomaly) on a correction applied to the number derived from those periods (IV 7). However, the correction is itself based in part on the parameters derived here. It is therefore important to note that the correction makes no difference over the short intervals considered here (between the first and second eclipses it is only about 1 second of arc). From IV 11 it is clear that Hipparchus had already established the principle that it was necessary to use an eclipse triple close in time, so that any long-term error in the mean motions would have a minimal effect.

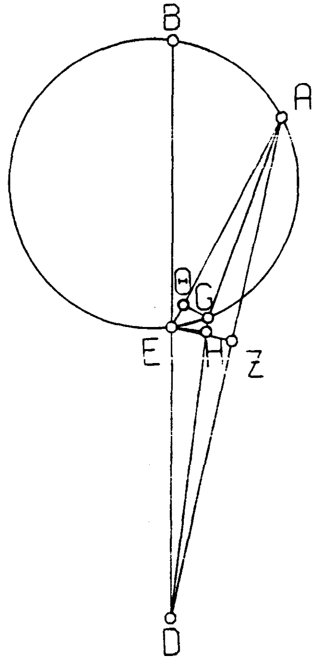

Fig. 4.4

which point A is the location of the moon at the middle of the first eclipse, B its position at the middle of the second eclipse, and G its position at the middle of the third eclipse. We must imagine the moon to move on the epicycle from B to A and from A to G in such a way that arc AGB, which is its increment in motion between the first and second eclipses, is 306;25° and produces an increment of 3;24° over the mean motion, while arc BAG, which is its increment in motion between the second and third eclipses, is 150;26°, and produces a decrement of 0;37° from the mean motion. Hence the motion from B to A is 53;35° and produces a decrement of 3;24° from the mean motion, and the motion from A to G is 96;51° and produces an increment of 2;47° over the mean motion. H306

Now the perigee of the epicycle cannot lie on arc BAG. This is clear because this arc has a subtractive effect, and is less than a semi-circle, while the greatest speed occurs at the perigee. Since, then, [the perigee] necessarily lies on arc GEB,[35] let us take the centre of the ecliptic, which is also the centre of the deferent, as point D, and draw lines DA, DEB and DG to the points representing [the positions of the moon at] the three eclipses. In order to make the sequence of the proof readily transferable for computations of this kind, whether we use the epicyclic hypothesis (as now) for our demonstration, or the

[35] For a detailed argument about the location of the observer with respect to the points on the epicycle representing the three eclipses see *HAMA* 74.

eccentric hypothesis, in which case [see Fig. 4.5] centre D is taken inside the circle, we give the following generally applicable description.

Produce one of the three straight lines drawn [DA,DB,DG] to the opposite
H307 circumference (in this case we already have DEB drawn to E from point B of the second eclipse), and draw a line joining the points of the other two eclipses (here AG). From the point where the first line produced cuts the circumference again (here E) draw lines to the other two points (here EA, EG), and [from the same point] drop perpendiculars on to the lines between the other two points and the centre of the ecliptic (here EZ on to AD and EH on to GD). From one of these

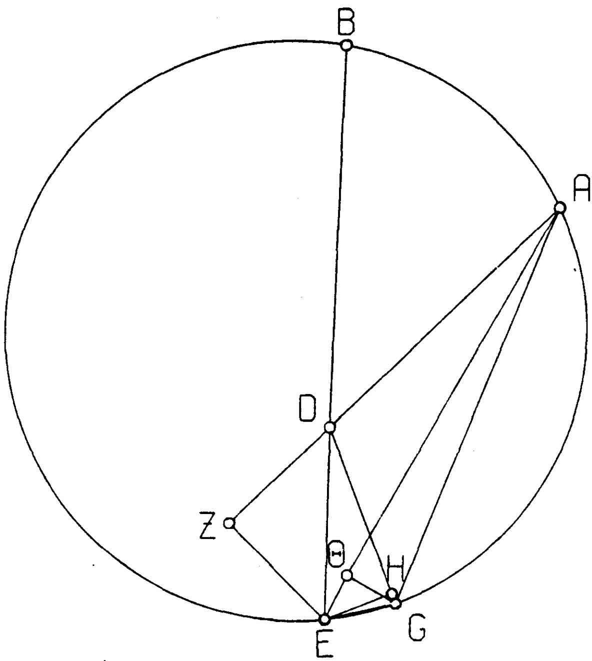

Fig. 4.5

two points (here G) drop a perpendicular on to the line drawn from the other (here A) to the extra intersection [with the circumference] (here E) resulting from [the first straight line, DB,] being produced (in this case, we drop GΘ on to AE). Whichever point we start drawing the figure from, we shall find that the same ratios result from the numbers used in the demonstration. Our choice [of starting-point] is guided merely by convenience.

So, since we found that arc BA subtends 3;24° of the ecliptic,

$$\text{the angle at its centre, } \angle\ \text{BDA} = \begin{cases} 3;24^\circ \text{ where 4 right angles } = 360^\circ \\ 6;48^{\circ\circ} \text{ where 2 right angles } = 360^{\circ\circ}. \end{cases}$$

H308 Therefore in the circle about right-angled triangle DEZ.

arc EZ = 6;48°

and EZ = 7;7,0^p where hypotenuse DE = 120^p.

Similarly, since arc BA = 53;35,
the angle [it subtends] at the circumference,
$\angle$ BEA = 53;35°° where 2 right angles = 360°°.
But, in the same units, $\angle$ BDA = 6;48°°.
Therefore, by subtraction, $\angle$ EAZ = 46;47°° in the same units.
Therefore in the circle about right-angled triangle AEZ,
arc EZ = 46;47°
and EZ = 47;38,30^p where hypotenuse EA = 120^p.
Therefore where EZ = 7;7,0^p and ED = 120^p,
AE = 17;55,32^p.
Again, since arc BAG subtends 0;37° of the ecliptic,
the angle at its centre, $\angle$ BDG = $\begin{cases} \text{0;37° where 4 right angles = 360°} \\ \text{1;14°° where 2 right angles = 360°°.} \end{cases}$
Therefore in the circle about right-angled triangle DEH,
arc EH = 1;14°
and EH = 1;17,30^p where hypotenuse DE = 120^p. H309
Similarly, since arc BAG = 150;26°,
the angle [it subtends] at the circumference,
$\angle$ BEG = 150;26°° where 2 right angles = 360°°.
But $\angle$ BDG = 1;14°° in the same units.
Therefore, by subtraction, $\angle$ EGD = 149;12°°.
Therefore in the circle about right-angled triangle GEH,
arc EH = 149;12°
and EH = 115;41,21^p[36] where hypotenuse GE = 120^p.
Therefore where EH = 1;17,30^p and DE = 120^p,
GE = 1;20,23^p,
and, as we showed, EA = 17;55,32^p in the same units.
Again since, as we showed, arc AG = 96;51°,
the angle [subtended by it] at the circumference,
$\angle$ AEG = 96;51°° where 2 right angles = 360°°.
Therefore in the circle about right-angled triangle GEΘ,
arc GΘ = 96;51°
and arc EΘ = 83;9° (complement).
So the corresponding chords
$\left.\begin{array}{r} \text{GΘ = 89;46,14}^p \\ \text{and EΘ = 79;37,55}^p \end{array}\right\}$ where hypotenuse GE = 120^p. H310
Therefore where GE = 1;20,23^p
GΘ = 1;0,8^p
and EΘ = 0;53,21^p.
And, in the same units, the whole line EA was found to be 17;55,32^p.
Therefore, by subtraction, ΘA = 17;2,11^p where GΘ = 1;0,8^p.
And the square on AΘ is 290;14,19
while the square on GΘ is 1;0,17.
But AG^2 = AΘ^2 + GΘ^2 = 291;14,36.

[36] 115;41,24 (as L) may be correct at H309,10 (computed: 115;41,28). It makes no difference to subsequent calculations whether one adopts 21, 24 or 28.

Therefore AG = 17;3,57^p where DE = 120^p and GE = 1;20,23^p.
But, where the diameter of the epicycle is 120^p, AG = 89;46,14^p
(for it subtends arc AG, which is 96;51°).
Therefore where AG = 89;46,14^p and the epicycle diameter is 120^p,
DE = 631;13,48^p
and GE = 7;2,50^p.
Therefore arc GE of the epicycle = 6;44,1°.
And, by hypothesis, arc BAG = 150;26°.
Therefore, by addition, arc BGE = 157;10,1°,
H311 so its chord, BE = 117;37,32^p where the epicycle diameter is 120^p and ED = 631;13,48^p.

Now if we had found BE equal to the diameter of the epicycle, the epicycle centre would, obviously, lie on it, and we would immediately get the ratio between the diameters [of epicycle and deferent]. Since, however, it is less than the diameter, and also arc BGE is less than a semi-circle, it is clear that the centre of the epicycle will fall outside segment BAGE.

Let it be [Fig. 4.6] in point K, and draw the line DMKL from D, the centre of the ecliptic, through K. Thus point L represents the apogee of the epicycle and M its perigee. Then

BD.DE = LD.DM;[37]

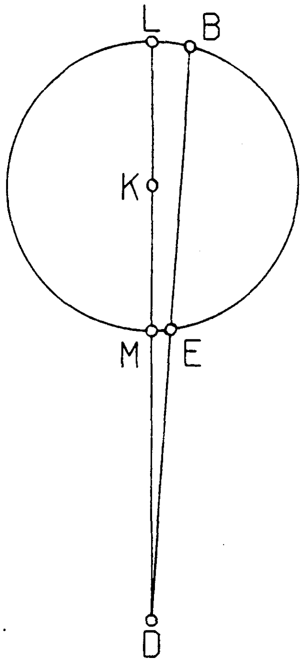

Fig. 4.6

[37] Euclid III 36: the rectangle contained by any line drawn from a point outside the circle and the segment of that line outside the circle equals the square on the tangent to the circle from that point.

and we have shown that where the epicycle diameter LKM = 120^p,

BE = 117;37,32^p and ED = 631;13,48^p.

Therefore, by addition, BD = 748;51,20^p. H312

Therefore LD.DM = BD.DE = 472700;5,32^p.

Furthermore, since LD.DM + KM^2 = DK^2,[38]

and the radius of the epicycle, KM = 60^p,

KM^2 = 3600^p,

and DK^2 = 472700;5,32^p + 3600^p = 476300;5,32^p.

Therefore DK, the radius of the deferent circle concentric to the ecliptic, is 690;8,42^p where KM, the radius of the epicycle, is 60^p.

So, where the radius of the deferent, the centre of which coincides with the observer, is 60^p, the radius of the epicycle is about 5;13^p. H313

Repeating the same figure [Fig. 4.7], drop perpendicular KNX from centre K on to BE, and join BK.

Now, where DK = 690;8,42^p,

we found that DE = 631;13,48^p

and NE = ½BE = 58;48,46^p.

Therefore, by addition, DEN = 690;2,34^p.

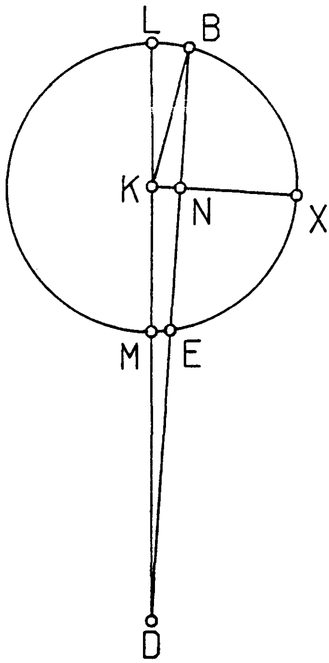

Fig. 4.7

[38] Euclid II 6: if a straight line (LM) be bisected and a straight line (DM) added to it, the rectangle contained by the whole plus the added line (LD) and the added line (DM), together with the square on the half (KM^2) is equal to the square on the line (DK) made up of the half (KM) and the added line (DM).

Therefore in the circle about right-angled triangle DNK,

$$DN = 119;58,57^p \text{ where hypotenuse } DK = 120^p,$$

$$\text{and arc } DN \approx 178;2°.$$

$$\therefore \angle DKN = \begin{cases} 178;2^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \\ 89;1° \text{ where 4 right angles} = 360°. \end{cases}$$

Therefore arc XM of the epicycle = 89;1°,

and arc LBX = 90;59° (complement),

H314 and arc XB = $\frac{1}{2}$ arc BXE = 78;35° (for arc BE was determined [p. 196] as about 157;10°).

Therefore, by subtraction, arc LB of the epicycle, which is the distance of the moon from the apogee of the epicycle at the middle of the second eclipse in question, is 12,24°.

Similarly, since, as we showed,

$$\angle DKN = 89;1° \text{ where 4 right angles} = 360°,$$

by subtraction, ∠ KDN, which represents the equation of anomaly (which is subtractive with respect to the mean motion) corresponding to the epicycle arc LB, is 0;59° (complement of ∠ DKN). Therefore the mean position of the moon at the middle of the second eclipse was ♍ 14;44°, since its true position was ♍ 13;45°, corresponding to the position of the sun in Pisces.

Let us now turn to the three eclipses which we have selected from those very carefully observed by us in Alexandria.

The first occurred in the seventeenth year of Hadrian, Pauni [X] 20/21 in the Egyptian calendar [133 May 6/7]. We computed the exact time of mid-eclipse as $\frac{3}{4}$ of an equinoctial hour before midnight. It was total.[39] At that time the true position of the sun was about ♉ 13$\frac{1}{4}$°.

H315 The second occurred in the nineteenth year of Hadrian, Choiak [IV] 2/3 in the Egyptian calendar [134 Oct. 20/21]. We computed that mid-eclipse occurred 1 equinoctial hour before midnight. [The moon] was eclipsed $\frac{5}{6}$ of its diameter from the north.[40] At that time the true position of the sun was about ♎ 25$\frac{1}{6}$°.

The third eclipse occurred in the twentieth year of Hadrian, Pharmouthi [VIII] 19/20 in the Egyptian calendar [136 Mar. 5/6]. We computed that mid-eclipse occurred 4 equinoctial hours after midnight. [The moon] was eclipsed half of its diameter from the north.[41] At that time the position of the sun was about ♓ 14$\frac{1}{12}$°.

It is clear that here too the mean motion [in longitude] of the moon, beyond complete revolutions, is equal to that of the sun, and is:

from middle of the first eclipse to middle of the second: 161;55°

from middle of the second eclipse to middle of the third: 138;55°.

The length of the first interval is:

1 Egyptian year 166 days 23$\frac{3}{4}$ equinoctial hours reckoned simply

1 Egyptian year 166 days 23$\frac{5}{8}$ equinoctial hours reckoned accurately.

[39] Oppolzer no. 2071, the circumstances of which agree well with Ptolemy's report.

[40] Oppolzer no. 2074, the circumstances of which agree extremely well with Ptolemy's report.

[41] Oppolzer no. 2075; circumstances: mid-eclipse 1;43 a.m. ≈ 3$\frac{3}{4}$ hours after midnight Alexandria, magnitude 5.5 digits.

The length of the second interval is:

1 Egyptian year 137 days 5 equinoctial hours reckoned simply H316

1 Egyptian year 137 days $5\frac{1}{2}$ equinoctial hours reckoned accurately.

The approximate mean motion of the moon (beyond complete revolutions) is:

in $1^y\ 166^d\ 23\frac{5}{8}^h$ $\begin{cases} 110;21^\circ \text{ in anomaly} \\ 169;37^\circ \text{ in longitude} \end{cases}$

and in $1^y\ 137^d\ 5\frac{1}{2}^h$ $\begin{cases} 81;36^\circ \text{ in anomaly} \\ 137;34^\circ \text{ in longitude.} \end{cases}$

Therefore, clearly, the 110;21° of motion on the epicycle over the first interval have produced a decrement from the mean motion of [161;55° − 169;37°=] 7;42°, while the 81;36° of motion on the epicycle over the second interval have produced an increment to the mean motion of [138;55° − 137;34°=] 1;21°.

With the above data, let the moon's epicycle [Fig. 4.8] be ABG. Let A be the point in which the moon was at the middle of the first eclipse, B its location at the middle of the second eclipse, and G its position at the middle of the third.

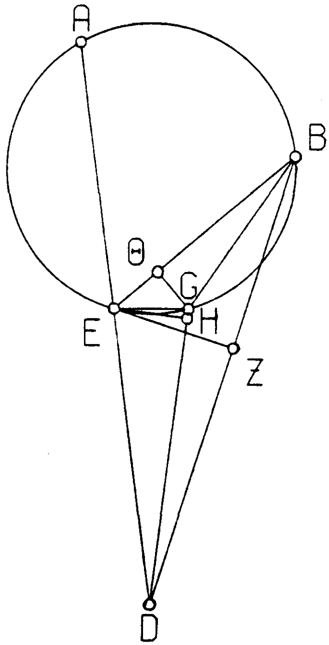

Fig. 4.8

We must, again, imagine the motion of the moon taking place from A to B and then from B to G in such a way that, as we said, arc AB, which is 110;21°, produces a decrement of 7;42° with respect to the mean motion, while arc BG, which is 81;36°, produces an increment of 1;21° with respect to the mean motion; thus the remaining arc GA is 168;3° and produces an increment to the mean motion of 6;21°, which is the difference [between 7;42° and 1;21°].

H317 It is clear that the apogee must lie on arc AB, since it can lie neither on arc BG nor on arc GA, both of which produce an additive effect and are less than a semi-circle. In the same way [as before],[42] take the centre of the ecliptic and the circle carrying the epicycle as D, and draw from it, to the points representing the 3 eclipses, lines DEA, DB, DG. Join BG and draw from point E to B and G lines EB and EG, and drop on to lines BD and DG perpendiculars EZ and EH. Also drop perpendicular GΘ from G on to BE.

Then, since arc AB subtends 7;42° on the ecliptic, the angle at the centre of the ecliptic,

$$\angle\ ADB = \begin{cases} 7;42^\circ \text{ where 4 right angles} = 360^\circ \\ 15;24^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ}. \end{cases}$$

Therefore in the circle about right-angled triangle[43] DEZ,

arc EZ = 15;24°

H318 and EZ = 16;4,42^p where hypotenuse DE = 120^p.

Similarly, since arc AB = 110;21°,

the angle [subtended by it] at the circumference,

∠ AEB = 110;21$^{\circ\circ}$ where 2 right angles = 360$^{\circ\circ}$.

But ∠ ADB = 15;24$^{\circ\circ}$ in the same units.

Therefore, by subtraction, ∠ EBD = 94;57$^{\circ\circ}$.

Therefore in the circle about right-angled triangle[44] BEZ,

arc EZ = 94;57°

and EZ = 88;26,17^p where hypotenuse BE = 120^p.

Therefore where EZ = 16;4,42^p and DE = 120^p,

BE = 21;48,59^p.

Furthermore, since, as we showed, arc GEA subtends 6;21° of the ecliptic, the angle at the centre of the ecliptic also,

$$\angle\ ADG = \begin{cases} 6;21^\circ \text{ where 4 right angles} = 360^\circ \\ 12;42^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ}. \end{cases}$$

Therefore in the circle about right-angled triangle DEH,

arc EH = 12;42°

and EH = 13;16,19^p where hypotenuse DE = 120^p.

Similarly, since arc ABG = 191;57°,

the angle [subtended by it] at the circumference,

∠ AEG = 191;57$^{\circ\circ}$ where 2 right angles = 360$^{\circ\circ}$.

H319 But ∠ ADG was found to be 12;42$^{\circ\circ}$ in the same units.

Therefore, by subtraction, ∠ EGD = 179;15$^{\circ\circ}$ in the same units.

Therefore in the circle about right-angled triangle GEH,

arc EH = 179;15°

and EH = 119;59,50^p where hypotenuse GE = 120^p.

[42] Reading ὁμοίως for ὅμως ὡς μὴ ὑποκειμένου τούτου at H317,4–5. This would mean 'Nevertheless, without this as an assumption'; but the location of the apogee on arc AB *is* (and must be) assumed in Fig. 4.8. I suppose that ὁμοίως ('similarly') was corrupted to ὅμως ('however') and the rest then added as an ancient gloss.

[43] Reading ὀρθογώνιον (with D, Ar) for τρίγωνον at H317,25. So too at H319,4 and 319,14.

[44] Reading BEZ ὀρθογώνιον (with D, Ar) for BEZ at H318,8.

Therefore where EH = $13;16,19^p$ and DE[45] = 120^p,
GE = $13;16,20^p$.

And, as we showed, BE = $21;48,59^p$ in the same units.

Furthermore, since arc BG = 81;36°,
the angle [subtended by it] at the circumference,
$\angle$ BEG = $81;36^{\circ\circ}$ where 2 right angles = $360^{\circ\circ}$.
Therefore in the circle about right-angled triangle GEΘ,
arc GΘ = 81;36°
and arc EΘ = 98;24° (supplement).
Therefore the corresponding chords

$$\left.\begin{array}{r} G\Theta = 78;24,37^p \\ \text{and } E\Theta = 90;50,22^p \end{array}\right\} \text{where hypotenuse } EG = 120^p.$$

Therefore where GE = $13;16,20^p$,
GΘ = $8;40,20^p$ and EΘ = $10;2,49^p$.
And the whole line EB was found to be $21;48,59^p$ in the same units.
Therefore, by subtraction [of EΘ from EB],
ΘB = $11;46,10^p$ where GΘ = $8;40,20^p$.
And $\Theta B^2 = 138;31,11^p$, $G\Theta^2 = 75;12,27^p$, H320
and $BG^2 = \Theta B^2 + G\Theta^2 = 213;43,38^p$.
Therefore BG = $14;37,10^p$ where DE = 120^p and GE = $13;16,20^p$.
But where the diameter of the epicycle is 120^p,
BG = $78;24,37^p$ (chord of arc BG, which is 81;36°).
Therefore where BG = $78;24,37^p$ and the epicycle diameter is 120^p,
DE = $643;36,39^p$ and GE = $71;11,4^p$.
Therefore arc GE of the epicycle = 72;46,10°.
And, by hypothesis, arc GEA = 168;3°.
Therefore, by subtraction, arc EA = 95;16,50°
and therefore its chord AE = $88;40,17^p$
where the epicycle diameter is 120^p and where ED = $643;36,39^p$.

Furthermore, since arc EA was shown to be less than a semi-circle, the centre of the epicycle will, obviously, fall outside segment EA. Take the centre as point K [Fig. 4.9], and draw line DMKL, so that, again, point L represents the apogee and point M the perigee. Then
AD.DE = LD.DM, H321
and we have shown that, where the epicycle diameter LKM = 120^p,
AE = $88;40,17^p$ and ED = $643;36,39^p$
(thus, by addition, AD = $732;16,56^p$).
$\therefore$ LD.DM = AD.DE = 471304;46,17.
Again, since
$LD.DM + KM^2 = DK^2$,
and KM, the radius of the epicycle, is 60^p,
if we add the 3600^p (of KM^2)[46] to the above $471304;46,17^p$,
we find $DK^2 = 474904;46,17^p$.

[45] Reading ἡ δὲ ΔΕ $\overline{\rho\kappa}$ for ἡ δὲ ΔΕ ἐδείχθη $\overline{\rho\kappa}$ (all mss.) at H319,7. The latter would mean 'where DE, as was shown, equals 120^p', which is nonsense, since this is assumed, not proven. D,Ar have the same nonsensical ἐδείχθη at H318,11.

[46] Reading τοῦ ἐπικύκλου τῶν αὐτῶν ἐστιν $\bar{\xi}$, εἂν τὰ $\overline{\gamma\chi}$ τοῦ τετραγώνου (with D,Ar) for τοῦ

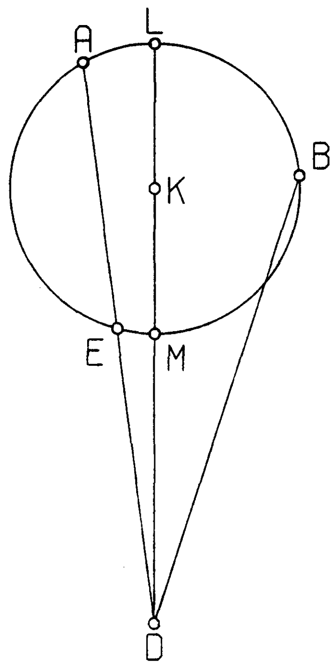

Fig. 4.9

H322 Therefore the radius of the deferent, concentric with the ecliptic,

DK = $689;8^p$ where the radius of the epicycle, KM = 60^p.

Therefore where the line joining the centres of ecliptic and epicycle is 60^p, the radius of the epicycle is $5;14^p$.

This ratio is very nearly the same as that derived just above from the more ancient eclipses.

So, in the same figure [Fig. 4.10] drop perpendicular KNX from centre K on to DEA, and join AK.

Then, as we showed, where DK = $689;8^p$, DE = $643;36,39^p$;

and NE = $\frac{1}{2}$AE = $44;20,8^p$ in the same units.

Therefore, by addition, DEN = $687;56,47^p$.

Therefore, where hypotenuse DK = 120^p, DN = $119;47,36^p$,

and in the circle about right-angled triangle DKN,

arc DN ≈ 173;17°.

$$\therefore \angle \text{DKN} = \begin{cases} 173;17^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \\ 86;38,30^{\circ} \text{ where 4 right angles} = 360^{\circ}. \end{cases}$$

ἐπικύκλου ἑξήκοντα ποιεῖ τὸ ἀπ' αὐτῆς $\overline{\gamma\chi}$, ἐὰν τὰ $\overline{\gamma\chi}$ at H321,14-15. Heiberg excises ἑξήκοντα from the latter, but it is still very clumsy.

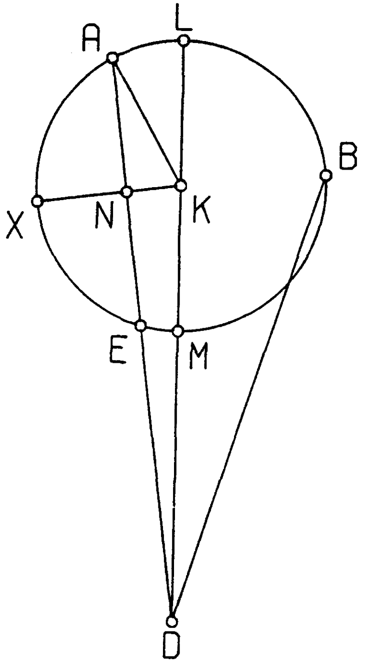

Fig. 4.10

∴ arc MEX of the epicycle = 86;38,30°,

and arc LAX = 93;21,30° (supplement), H323

and arc AX = $\frac{1}{2}$ arc AE ≈ 47;38,30°.

Therefore, by subtraction, arc AL = 45;43°.
But, by hypothesis, the whole arc AB = 110;21°.
Therefore, by subtraction, arc LB = 64;38°.
This is the distance of the moon from the apogee at the middle of the second eclipse determined above.

Similarly, as we showed,

∠ DKN ≈ 86;38°,

so ∠ KDN = 3;22° (complement),

and, by hypothesis, ∠ ADB = 7;42°.

Therefore, by subtraction, ∠ LDB = 4;20°.

This angle subtends the arc of the ecliptic representing the equation of anomaly (which is subtractive with respect to the mean motion) resulting from arc LB of the epicycle.

Therefore the mean position of the moon at the middle of the second eclipse H324
was ♈ 29;30°, since its true position was ♈ 25;10°, corresponding to the position of the sun in Libra.

7. {*On the correction of the mean positions of the moon in longitude and anomaly*}[47]

Now we have shown that the mean position of the moon at the middle of the second of the [three] ancient eclipses was:

in longitude: ♍ 14;44°
in anomaly: 12;24° from the apogee of the epicycle;

and at the second of the three eclipses in our time:

in longitude: ♈ 29;30°
in anomaly: 64;38° from the apogee.

So it is clear that in the interval between the above two eclipses the mean motion of the moon, beyond complete revolutions, was:

in longitude: 224;46°
in anomaly: 52;14°.

Now the time between Mardokempad 2, Thoth 18/19, $\frac{5}{6}$ hour before midnight, and Hadrian 19, Choiak 2/3, 1 hour before midnight is

854 Egyptian years 73^d $23\frac{5}{6}$ equinoctial hours reckoned simply
854 Egyptian years 73^p $23\frac{1}{3}$ equinoctial hours reckoned accurately (in mean solar days).

H325 In days this is 311783 days $23\frac{1}{3}$ equinoctial hours.
In this interval we find that the increment over complete revolutions, according to the daily motions derived above from the uncorrected hypotheses, is:

in longitude: 224;46°
in anomaly: 52;31°.[48]

Thus, as we said [p. 179], we find that the increment in longitude is identical with what we derived from the above observations, but the increment in anomaly is 17 minutes too great. Hence, before constructing the [mean motion] tables, we corrected the daily motion in anomaly by dividing these 17 minutes by the above total in days, and subtracting the resulting correction for 1 day (of 0;0,0,0,11,46,39°) from the uncorrected mean daily motion in anomaly. The corrected motion is 13;3,53,56,17,51,59°, which is the basis of the other entries, derived by accumulation, in the tables.

8. {*On the epoch of the mean motions of the moon in longitude and anomaly*}

In order to establish the epochs of these [mean motions] for the same first year of Nabonassar, Thoth 1 in the Egyptian calendar, noon, we took the time-interval
H326 from that moment to the middle of the second eclipse of the first trio (which is the nearer [to the epoch]). This, as we said, took place in the second year of Mardokempad, Thoth 18/19 in the Egyptian calendar, $\frac{5}{6}$th of an equinoctial hour before midnight. This interval is computed as 27 Egyptian years, 17 days

[47] On chs 7 and 8 see *HAMA* 78–9, Pedersen 180–2.
[48] If one computes accurately with Ptolemy's mean daily motions (p. 179) one finds 224, 47, 15° (cf. *HAMA* 79) and 52;32,18° respectively, i.e. in each case one minute more (not utterly negligible in this context). I suspect that Ptolemy computed, not for $23;20^h$, but for $23;18^h$, i.e. his correction for the equation of time was not precisely $-\frac{1}{2}^h$, but −32 mins. (accurate computation gives $-28\frac{1}{2}$ mins.)

and $11\frac{1}{6}$ hours both by the simple and (approximately) by the accurate reckoning.[49] To this interval corresponds (beyond complete revolutions)

123;22° in longitude, and
103;35° in anomaly.

Subtracting each of these values from the corresponding one at the middle of the second eclipse [♍ 14;44° and 12;24°, p. 198], we find for the mean positions of the moon in the first year of Nabonassar, Thoth 1 in the Egyptian calendar, noon:

in longitude:	♉ 11;22°	
in anomaly:	268;49°	from the apogee of the epicycle
in elongation:	70;37°	(for, as we showed, the [mean] position of the sun at the same moment was ♓ 0;45°).

9. {*On the correction of the mean positions in latitude of the moon, and their epochs*}[50]

By the above methods we have established the periodic motions and epochs [of the moon] in longitude and anomaly. Concerning the corresponding amounts for its latitude, we were formerly in error, because we too adopted Hipparchus' assumptions that [the diameter of] the moon goes approximately 650 times into its own orbit, and $2\frac{1}{2}$ times into [the diameter of] the earth's shadow, when it is at mean distance in the syzygies. For once these quantities and the size of the inclination of the moon's orbit are given, the limits of individual lunar eclipses are given. So we took [pairs of] eclipses separated by a known interval, computed (from the magnitude of the obscuration at mid-eclipse) the true distance [of the moon] from whichever of the two nodes [the eclipse was near] along its inclined circle in [argument of] latitude, determined the mean position [in latitude] from the true by applying the equation of anomaly as already determined, and thus found the mean position in latitude at the middle of each eclipse, and hence the motion in latitude (as increment over complete revolutions) during that interval.[51] H327

But now, using more elegant methods which do not require any of the previous assumptions for the solution of the problem, we have found that the motion in latitude computed by the above method is faulty. Furthermore, from

[49] The equation of time between era Nabonassar (−746 Feb. 26) and the eclipse in question (−719 Mar. 18) is in fact about −3 mins. This would make the mean motions 1 minute less in each case than Ptolemy's figures.

[50] See *HAMA* 80–2. Pedersen 181 is inadequate.

[51] Hipparchus' method was first explained by Schmidt, 'Maanens Middelbevaegelse'. Cf. *HAMA* 313. Norman T. Hamilton has discovered the relevance of this passage to the value of the moon's mean motion and position in latitude given in the Canobic Inscription, (*Op. Min.* 151–2, cf. *HAMA* 914), and shown that these were derived by application of the method outlined here to the two eclipses Nabonassar 28 I 18/19 (IV 6, H303) and Nabonassar 882 IV 2/3 (IV 6, H315). The first of these had already been used by Hipparchus (cf. VI 9, H526), who had found (by this method) that the moon was 9° past the node. Applying Hipparchus' mean motion in latitude to the interval between the eclipses, Ptolemy found that the moon should have been 5° past the node at the second eclipse. However, from the observed magnitude he computed that it must rather be 6° past the node, and thus 'corrected' Hipparchus' mean motion by adding 1°, to be distributed over the intervening 311784 days. Cf. IV 7. This produces exactly the value found in the Canobic Inscription.

the motion in latitude computed from our new method without those assumptions, we have proven that those very assumptions concerning sizes and distance are false, and have corrected them. We have done something similar
H328 with the hypotheses for Saturn and Mercury, changing some of our earlier, somewhat incorrect, assumptions because we later got more accurate observations. For those who approach this science in a true spirit of enquiry and love of truth ought to use any new methods they discover, which give more accurate results, to correct not merely the ancient theories, but their own too, if they need it. They should not think it disgraceful, when the goal they profess to pursue is so great and divine, even if their theories are corrected and made more accurate by others beside themselves. As for those topics [corrections to the theories of Saturn and Mercury], we will explain how we deal with them at the proper places in the later part of our treatise.[52] For the time being, to preserve the proper order of procedure, we will turn to the demonstration of the position in latitude, which is by the following method.

First, then, to correct the actual mean motion in latitude, we looked for [pairs of] lunar eclipses (among those securely recorded) separated by as great an interval as possible, at both of which

[1] the size of obscuration was equal,
[2] the eclipses took place near the same node,
[3] the eclipse was from the same side (either both from the north or both from the south) and
[4] the moon was at about the same distance [from the earth].

H329 If these conditions are fulfilled the moon's centre must be the same distance from the same node, and on the same side, at both eclipses, and hence its true motion in latitude during the interval between the observations contains an integer number of revolutions in latitude.

The first eclipse we used is the one observed in Babylon in the thirty-first year of Darius I, Tybi [V] 3/4 in the Egyptian calendar, [−490 Apr. 25/26] at the middle of the sixth hour [of night]. It is reported that at this eclipse the moon was obscured 2 digits from the south.[53]

The second eclipse we used is the one observed in Alexandria in the ninth year of Hadrian, Pachon [IX] 17/18 in the Egyptian calendar [125 Apr. 5/6], $3\frac{3}{5}$ equinoctial hours before midnight. At this eclipse too the moon was obscured $\frac{1}{6}$th of its diameter from the south.[54]

The position of the moon in latitude was near the descending node at each

[52] There is nothing in the discussions of Mercury and Saturn (Bks. IX and XI) which gives a clue to the changes which Ptolemy mentions, but Hamilton's discovery about the lunar latitude theory (see n.51) makes it plausible that Ptolemy is referring to the different parameters for Mercury and Saturn found in the Canobic Inscription. These are: for Saturn, an eccentricity of $3;15^p$ instead of $3;25^p$, ascending node 353;30° from Regulus instead of 327;30°; for Mercury, an eccentricity of $2;30^p$ instead of $3\text{-}9^p$, inclination of deferent 0;40° instead of 0;45°, inclination of epicycle 7° instead of 6;15°, slant of epicycle 2;30° instead of 7° (cf. *HAMA* 908–17).

[53] Oppolzer no. 1107: time $19;55^h$ (≈ 10 p.m. Alexandria), magnitude 1.1 digits. P.V. Neugebauer calculates ca. 22.7^h Babylon (≈ 10;15 p.m. Alexandria), 1.7 digits.

[54] Oppolzer no. 2058: time $18;57^h$ (≈ 9 p.m. Alexandria), magnitude 2 digits. Note that although this eclipse was observed in Alexandria, Ptolemy does not say that he himself was the observer. We may conjecture that it was observed by the Theon who 'transmitted' the planetary observations recorded at IX 9, X 1 and X 2 (pp. 456, 469, 471) to Ptolemy.

eclipse (such conclusions can be drawn even from quite crude hypotheses).[55] The distance [of the moon] was about the same [at both eclipses], and a little closer to the perigee than the mean distance. This too can be shown from our previous determination of the anomaly. Now, when the moon is eclipsed from H330
the south, its centre is north of the ecliptic. So it is clear that at both eclipses the moon's centre was an equal amount in advance of the descending node. In the first eclipse the distance of the moon from the apogee of the epicycle was 100;19°. (For the time of mid-eclipse was $\frac{1}{2}$-hour before midnight at Babylon, and [hence] $1\frac{1}{3}$ equinoctial hours before midnight at Alexandria;[56] from the Nabonassar epoch the time comes to

$$256 \text{ years } 122 \text{ days} \begin{cases} 10\frac{2}{3} \text{ hours reckoned simply} \\ 10\frac{1}{4} \text{ hours reckoned in true solar days.)} \end{cases}$$

Therefore the true position was 5° less than the mean.[57] In the second eclipse the moon was 251;53° from the apogee of the epicycle. (For in this case the time from epoch to the middle of the eclipse comes to

$$871 \text{ years } 256 \text{ days} \begin{cases} 8\frac{2}{3} \text{ equinoctial hours reckoned simply} \\ 8\frac{1}{12} \text{ equinoctial hours reckoned accurately.)} \end{cases}$$

Therefore the true position was 4;53° more than the mean. Therefore, in the interval between the two eclipses, which comprises 615 Egyptian years, 133 H331
days and $21\frac{5}{6}$ equinoctial hours,[58] the true motion of the moon in latitude comprises an integer number of revolutions, while its mean motion fell short of a complete revolution by 9;53°, which is the sum of both [equations of] anomaly. But according to the mean motions derived from Hipparchus' hypotheses, as set out above, in that interval it falls short of a complete revolution by about 10;2°. Thus the mean motion in latitude is greater than one would expect from his hypotheses by 9 minutes.

We divided these 9 minutes by the total of days in the above interval (approximately 224609), and added the resulting 0;0,0,0,8,39,18° to the mean daily motion [in latitude] derived above from those hypotheses; thus we found the corrected mean motion of 13;13,45,39,48,56,37°, which we again used as the basis for the other accumulated totals in the tables.

Having once, in this way, determined the mean motion in latitude, we next proceeded to establish its epoch position. For this purpose we looked for another pair of accurately observed eclipses at a known interval, in which all the same H332
conditions were fulfilled as in the previous pair (namely, for both eclipses the distance of the moon was approximately equal, and [the magnitude of] the obscuration was equal and from the same side (either from the north or from the south for both), except that here the eclipses were near opposite nodes instead of near the same node.

[55] For an example of how this can be done see *HAMA* 81 n.4.

[56] It is not clear whether Ptolemy takes the time of the observation to be given in seasonal or equinoctial hours. However, the sun is close enough to the equinox that (for $\frac{1}{2}$-hour) the difference is minimal.

[57] The simplest way to check this (and the corresponding amount at the second eclipse) is to use the equation table (IV 10) with arguments 100;19° and 251;53°.

[58] The corrections for equation of time are computed rather inaccurately, being about 4 minutes too great at both eclipses. However, these inaccuracies cancel out in the computation of the interval.

The first of these eclipses is the one which we also used for our demonstration of the anomaly [p. 191]. It occurred in the second year of Mardokempad, Thoth[I] 18/19 in the Egyptian calendar [-719 Mar. 8/9], at midnight in Babylon, and $\frac{5}{6}$ of an equinoctial hour before midnight at Alexandria; at this eclipse it is recorded that the moon was obscured 3 digits from the south.

The second, which Hipparchus too used, occurred[59] in the twentieth year of that Darius who succeeded Kambyses, Epiphi [XI] 28/29 in the Egyptian calendar [−501 Nov. 19/20], when $6\frac{1}{3}$ equinoctial hours of the night had passed; at this eclipse the moon was, again, obscured from the south $\frac{1}{4}$ of its diameter. The middle of the eclipse was $\frac{4}{5}$ of an equinoctial hour before midnight in Babylon (for the length of half the night was about $6\frac{3}{4}$ equinoctial hours on that date), and [hence] $1\frac{1}{3}$ equinoctial hours before midnight in Alexandria.[60]

H333 Both of these eclipses occurred when the moon was near its greatest distance, but the first was near the ascending node, while the second was near the descending node. So here too the centre of the moon was an equal distance north of the ecliptic at [both] eclipses.

Then let [Fig. 4.11] the moon's inclined orbit be ABG on diameter AG. Let us take point A as the ascending node, G as the descending node, and B as the

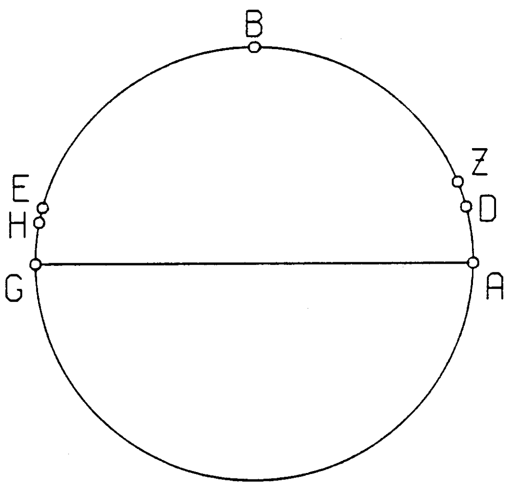

Fig. 4.11

northern limit. Cut off equal arcs, AD and GE, from nodes A and G towards the northern limit B. Then in the first eclipse the centre of the moon was at D and in the second at E.

Now the time from epoch to [the middle of] the first eclipse is 27 Egyptian years, 17 days $11\frac{1}{6}$ equinoctial hours (reckoned both simply and accurately). Hence the moon's distance from the apogee of the epicycle was 12;24°, and the

[59] Reading γενομένη with CD for γενομένῃ at H332,14.
[60] Oppolzer no. 1090: time $21;24^h$ (≈ 11:15 p.m. Alexandria), magnitude 2.1 digits.

mean position was greater than the true by 59 minutes. Likewise, the time [from epoch] to [the middle of] the second eclipse was

245 Egyptian years, 327 days $\begin{cases} 10\frac{3}{4} \text{ equinoctial hours reckoned simply} \\ 10\frac{1}{4} \text{ equinoctial hours reckoned accurately.} \end{cases}$

Hence the moon's distance from the apogee of the epicycle was 2;44°, and the mean position was greater than the true by 13 minutes. The interval between the observations contains 218 Egyptian years, 309 days $23\frac{1}{12}$ equinoctial hours, which produces, for the mean motion in latitude deduced above, an increment [over complete revolutions] of 160;4°. H334

So, because of the above, let the mean position of the centre of the moon be at Z [in Fig. 4.11] at the first eclipse and at H in the second. Then since

arc ZBH = 160;4°

and arc DZ = 0;59° and arc EH = 0;13°,

arc DE = [arc DZ + arc ZBH − arc EH =] 160;50°.

∴ (arc AD + arc EG) = 19;10° (supplement).

And, since they are equal, arc AD = arc EG = 9;35°.

That is the amount by which the true position of the moon at the first eclipse was to the rear of the ascending node, and by which the true position of the moon at the second eclipse was in advance of the descending node. Therefore, by addition,

arc AZ = [arc AD + arc DZ =] 10;34°

and, by subtraction,

arc HG = [arc EG − arc EH =] 9;22°.

Hence the mean position of the moon at the first eclipse was 10;34° to the rear of the ascending node, and [therefore] was 280;34° from the northern limit B, and at the second eclipse it was 9;22° in advance of the descending node, and [therefore] its distance from the northern limit was 80;38°. H335

Next, since the time from epoch to the middle of the first eclipse produces an increment [over complete revolutions] of [mean motion in] latitude of 286;19°, we subtract this amount from the 280;34° for the position at the first eclipse and (after adding 360°) find, for the first year of Nabonassar, Thoth 1 in the Egyptian calendar, noon: the mean position in latitude (counted from the northern limit): 354;15°.

In order to be able to check calculations concerning conjunctions and oppositions (since for those positions [of the moon] we have no need of the second anomaly which we shall demonstrate later), we shall set out a table for the individual [equations of anomaly]. We have calculated it geometrically, in the same way as we already did for the sun. In this case we used the ratio $60:5\frac{1}{4}$ [as a basis], but, as [previously], we tabulate it at intervals of 6° for the apogee quadrants, and of 3° for the perigee [quadrants]. Thus the layout of the table is identical to that for the sun: it consists of 45 lines and 3 columns; the first two columns contain the argument, in degrees of anomaly, while the third contains the equation corresponding to each argument. In calculating the longitude and the latitude, this equation has to be subtracted when the anomaly, counted from the apogee of the epicycle, is up to 180°, and added when the anomaly is more than 180°. The table is as follows. H336

H337

10. {*Table of the first, simple anomaly of the moon*}

1 Common Numbers	2	3 Equation
6	354	0 29
12	348	0 57
18	342	1 25
24	336	1 53
30	330	2 19
36	324	2 44
42	318	3 8
48	312	3 31
54	306	3 51
60	300	4 8
66	294	4 24
72	288	4 38
78	282	4 49
84	276	4 56
90	270	4 59
93	267	5 0
96	264	5 1
99	261	5 0
102	258	4 59
105	255	4 57
108	252	4 53
111	249	4 49
114	246	4 44
117	243	4 38
120	240	4 31
123	237	4 24
126	234	4 16
129	231	4 7
132	228	3 57
135	225	3 46
138	222	3 35
141	219	3 23
144	216	3 10
147	213	2 57
150	210	2 43
153	207	2 28
156	204	2 13
159	201	1 57
162	198	1 41
165	195	1 25
168	192	1 9
171	189	0 52
174	186	0 35
177	183	0 18
180	180	0 0

11. {*That the difference in the size of the lunar anomaly, according to Hipparchus, is due not to the different hypotheses employed, but to his calculations*}[61] H338

Now that we have demonstrated the above, it would be quite reasonable for someone to ask why it is that the ratio [of the eccentricity] found by Hipparchus from the lunar eclipses which he adduced for the determination of this anomaly is neither identical with the one determined by us, nor [consistent with itself, since] the first ratio he found, using the eccentric hypothesis, differs from the second, which was calculated from the epicyclic hypothesis. For in his first demonstration he derives the ratio between the radius of the eccentre and the distance between the centres of the eccentre and the ecliptic as about $3144:327\frac{2}{3}$ (which is the same as 60:6;15), while in the second he finds the ratio between the line joining the centre of the ecliptic to the centre of the epicycle, and the radius of the epicycle, as $3122\frac{1}{2}:247\frac{1}{2}$ (which is the same as 60:4;46). Now the maximum equation of anomaly for a ratio of $60:6\frac{1}{4}$ is 5;49°; for a ratio of 60:4;46 it is 4;34°, H339
while our ratio of $60:5\frac{1}{4}$ produces a maximum equation of about 5°.[62]

Such a discrepancy cannot, as some think, be due to some inconsistency between the [epicyclic and eccentric] hypotheses. Not only have we shown this by logical argument just above [IV 5], from the perfect agreement between the phenomena resulting from both hypotheses, but numerically too, if we wanted to carry out the calculations, we would find that the same ratio results from both hypotheses, provided we use the same set of data for both, and not, like Hipparchus, different sets. For in that case (if different sets of éclipses are used as basis), the discrepancy can occur [through errors] in the actual observations or in the computations of the intervals. At any rate, we will find that in the case of those eclipses [used by Hipparchus] the syzygies were observed correctly, and are in agreement with our proven theories for the mean and anomalistic motions, but the computations of the intervals (on which the demonstration of the size of the ratio depends) were not carried out as carefully as possible. We shall demonstrate both of these assertions, beginning with the first three eclipses.

He says that these three eclipses which he adduces are from the series brought H340
over from Babylon, and were observed there; that the first occurred in the archonship of Phanostratos at Athens, in the month Poseideon;[63] a small section of the moon's disk was eclipsed from the summer rising-point [i.e. the north-east] when half an hour of night was remaining. He adds that it was still eclipsed

[61] See *HAMA* 317-19.

[62] There are some inaccuracies here: $3122\frac{1}{2} : 247\frac{1}{2} \approx 60 : 4;45,21$. The maximum equation resulting from an eccentricity of 4;46 in 60 is not 4;34°, but 4;33° to the nearest minute. These inaccuracies could be eliminated by changing $3122\frac{1}{2}$ to $3112\frac{1}{2}$ (cf. p. 215 n.75), but ms. authority is unanimous at all places. Even more inaccurate is the 5;49° of the maximum equation resulting from $60 : 6\frac{1}{4}$. Correct (to the nearest minute) is 5;59°, and perhaps we should so emend it ($\overline{\nu\theta}$ for $\overline{\mu\theta}$ at H338,23).

[63] It is practically certain that this and the corresponding dates for the other two eclipses are in the astronomical Metonic calendar (see Introduction p. 12) rather than the Athenian civil calendar, for at the time when the Babylonian observations were 'brought over', the equation with the old Athenian civil calendar could hardly have been determined, and certainly was of no interest to the users of the observations.

when it set. Now this moment is in the 366th year from Nabonassar, in the Egyptian calendar (as Hipparchus himself says) Thoth 26/27 [−382 Dec. 22/23], $5\frac{1}{2}$ seasonal hours after midnight (since half an hour of night was remaining). When the sun is near the end of Sagittarius, 1 hour of night in Babylon is 18 time-degrees (for the night is $14\frac{2}{5}$ equinoctial hours long).[64] So $5\frac{1}{2}$ seasonal hours produce $6\frac{3}{5}$ equinoctial hours. Therefore the beginning of the eclipse was $18\frac{3}{5}$ equinoctial hours after noon on the 26th. And since a small section [of the disk] was obscured, the duration of the whole eclipse must have been about $1\frac{1}{2}$ hours, so the middle of the eclipse, obviously, must have been $19\frac{1}{3}$ equinoctial hours after [noon]. Therefore mid-eclipse at Alexandria was $18\frac{1}{2}$
H341 equinoctial hours after noon on the 26th.[65] The time from epoch in the first year of Nabonassar to the moment in question is

$$365 \text{ Egyptian years } 25 \text{ days} \begin{cases} 18\frac{1}{2} \text{ equinoctial hours reckoned simply} \\ 18\frac{1}{4} \text{ equinoctial hours reckoned accurately.} \end{cases}$$

At this moment, using our hypotheses as set out above, we find

the true position of the sun as ♐ 28;18°,
the mean position of the moon as ♊ 24;20°,
and its true position as ♊ 28;17°[66]

(for its distance in anomaly from the apogee of the epicycle is 227;43°).

He says that the next eclipse occurred in the archonship of Phanostratos at Athens, in the month Skirophorion, Phamenoth 24/25 in the Egyptian calendar, and that [the moon] was eclipsed from the summer rising-point [i.e. the north-east] when the first hour [of night] was well advanced. This moment is in the 366th year from Nabonassar, Phamenoth [VII] 24/25 [−381 June 18/19], about $5\frac{1}{2}$ seasonal hours before midnight. When the sun is near the end of Gemini, one hour of the night at Babylon is 12 time-degrees. Therefore the $5\frac{1}{2}$ seasonal hours produce $4\frac{2}{5}$ equinoctial hours. So the beginning of the eclipse was
H342 $7\frac{3}{5}$ equinoctial hours after noon on the 24th. And since the duration of the whole eclipse is recorded as three hours, mid-eclipse, obviously, occurred $9\frac{1}{10}$ equinoctial hours after [noon]. So in Alexandria it must have occurred about $8\frac{1}{4}$ equinoctial hours after noon on the 24th.[67] The time from epoch is

$$365 \text{ Egyptian years } 203 \text{ days} \begin{cases} 8\frac{1}{4} \text{ equinoctial hours reckoned simply} \\ 7\frac{5}{6} \text{ equinoctial hours reckoned accurately.} \end{cases}$$

For this moment we find:

true longitude of the sun: ♊ 21;46°

[64] These figures agree well enough with those derivable from the rising-time table (II 8) for Clima IV (Rhodes, $M = 14\frac{1}{2}^h$, $\varphi = 36°$), for $\lambda_\odot$ = ♐ 28;18°. In the *Geography* (5.20.6) Ptolemy assigns Babylon a latitude of 35°.

[65] Oppolzer no. 1275: time $5;5^h$ (≈ 7 a.m. Alexandria), magnitude 2.6 digits, half-duration 52 mins. P.V. Neugebauer calculates *c.* 8 a.m. Babylon (≈ 7 a.m. Alexandria), magnitude 3.0 digits, duration 1.8^h.

[66] I.e. here (and in the other five eclipses) the true moon and true sun, as calculated from Ptolemy's hypotheses, are almost exactly 180° apart, thus giving further confirmation of those hypotheses. In fact more accurate calculation gives rather worse agreement (e.g. here the discrepancy is about $4\frac{1}{2}$ minutes of arc rather than 1′), but in no case is the difference greater than could be explained by the vagueness of the time given in the eclipse report.

[67] Oppolzer no. 1276: time $18;31^h$ (≈ 8;30 p.m. Alexandria), half-duration $1;15^h$. P.V. Neugebauer calculates the beginning of the eclipse at Babylon as 19.8^h, mid-eclipse as ca. 21.1^h (≈ 8 p.m. Alexandria), duration 2.7^h.

mean longitude of the moon: ♐ 23;58°
true longitude of the moon: ♐ 21;48°

(for its distance from the apogee of the epicycle in anomaly was 27;37°).
The intervals between the first and second eclipses are:

[time:] 177^d $13\frac{3}{5}$ equinoctial hours
motion of the sun in longitude: 173;28°,

whereas Hipparchus carried out his demonstration on the basis of the intervals:

[time:] 177^d $13\frac{3}{4}$ equinoctial hours
[longitude:] $173° - \frac{1}{8}°$.

He says that the third eclipse occurred in the archonship of Euandros at Athens, in the month Poseideon I, Thoth 16/17 in the Egyptian calendar, and H343
that [the moon] was totally eclipsed, beginning from the summer rising-point [i.e. the north-east], after 4 hours [of night] had passed.[68] This moment is in the 367th year from Nabonassar, Thoth [I] 16/17 [−381 Dec. 12/13], about $2\frac{1}{2}$ hours before midnight. Now when the sun is about two-thirds through Sagittarius, one hour of night at Babylon is about 18 time-degrees. So $2\frac{1}{2}$ seasonal hours produce 3 equinoctial hours. Therefore the beginning of the eclipse was 9 equinoctial hours after noon on the 16th. And since the eclipse was total, its duration was about 4 equinoctial hours. So mid-eclipse, clearly, was about 11 hours after noon. Therefore in Alexandria mid-eclipse must have occurred $10\frac{1}{6}$ equinoctial hours after noon on the 16th.[69] The time from epoch [to this moment] is

366 Egyptian years 15 days $\begin{cases} 10\frac{1}{6} \text{ equinoctial hours reckoned simply} \\ 9\frac{5}{6} \text{ equinoctial hours reckoned accurately.} \end{cases}$

For this moment we find:

true longitude of the sun: ♐ 17;30°
mean longitude of the moon: ♊ 17;21°
true longitude of the moon: ♊ 17;28°

(for its distance from the apogee of the epicycle in anomaly was 181;12°).
The intervals from the second to the third eclipse are: H344

[in time:] 177^d 2 equinoctial hours
[in longitude:] 175:44°,

whereas Hipparchus assumed the following intervals:

[in time:] 177^d $1\frac{2}{3}$ hours
[in longitude:] $175\frac{1}{8}$°.[70]

Thus it is apparent that he committed errors in his computations of the intervals of $\frac{1}{6}$th and $\frac{1}{3}$rd of an equinoctial hour in time, and about $\frac{3}{5}$ of a degree [in

[68] Ptolemy interprets this below to mean $2\frac{1}{2}$ seasonal hours before midnight, i.e. after $3\frac{1}{2}$ seasonal hours of night (he thus arrives at a time for the beginning of the eclipse at Babylon, 9 p.m., which agrees fairly well with modern calculations: P. V. Neugebauer gives 21.3^h). But δ̄ ὡρῶν παρεληλυθυιῶν can only mean 'after 4 hours had passed'. Hence Manitius suggests emending to τῆς δ' ὥρας προεληλυθυίας ('when the fourth hour was well advanced'), comparing τῆς πρώτης ὥρας προεληλυθυίας at H341, 13–14, which is interpreted (p. 212) to mean 'half a seasonal hour after sunset'. A less violent emendation would be γ̄ for δ̄ ('when 3 hours had passed'), cf. μιᾶς ὥρας ἱκανῶς παρελθούσης at H302,16–17, 'when one hour was well past', which is interpreted as '$1\frac{1}{2}$ seasonal hours (after moonrise)'. But the whole ms. tradition is unanimous for '4'.

[69] Oppolzer no. 1277; time $20;4^h$ (≈ 10 p.m. Alexandria), half-duration $1;50^h$.

[70] Reading ρ̄ο̄ε̄ καὶ η' (with D,Ar) for ρ̄ο̄ε̄ η̄ (175;8°) at H344,5.

longitude] in each interval. Errors of this amount can produce a considerable discrepancy in the size of the ratio [derived].

We will pass to the second set of three eclipses he set out, which he says were observed in Alexandria. He says that the first of these occurred in the 54th year of the Second Kallippic Cycle, Mesore [XII] 16 in the Egyptian calendar [-200 Sept. 22]. In this eclipse the moon began to be obscured half an hour before it rose, and its full light was restored in the middle of the third hour [of night]. Therefore mid-eclipse occurred at the beginning of the second hour, 5 seasonal hours before midnight, and also 5 equinoctial hours, since the sun
H345 was near the end of Virgo. So mid-eclipse at Alexandria occurred 7 equinoctial hours after noon on the 16th.[71] And the time from epoch in the first year of Nabonassar is

546 Egyptian years 345 days $\begin{cases} \text{7 equinoctial hours reckoned simply} \\ \text{6½ equinoctial hours reckoned accurately.} \end{cases}$

For this moment we find:

true longitude of the sun:	♍ 26;6°
mean longitude of the moon:	♓ 22°
true longitude of the moon:	♓ 26;7°

(for its distance in anomaly from the apogee of the epicycle was 300;13°).

He says that the next eclipse occurred in the 55th[72] year of the same cycle, Mechir [VI] 9 in the Egyptian calendar [-199 Mar. 19], that it began when 5⅓ hours of night had passed, and was total. So the beginning of the eclipse was 11⅓ equinoctial hours after noon on the 9th (since the sun was near the end of Pisces), and mid-eclipse was 13⅓ equinoctial hours after [noon], (since the whole moon was eclipsed).[73] The time from epoch to this moment is
H346 547 Egyptian years 158 days 13⅓ equinoctial hours, whether reckoned simply or accurately.

For this moment we find:

true longitude of the sun:	♓ 26;17°
mean longitude of the moon:	♎ 1;7°
true longitude of the moon:	♍ 26;16°

(for its distance in anomaly from the apogee was 109;28°).

The intervals from first to second eclipse are:

[in time:] 178^d 6⅚ equinoctial hours

[in longitude]: 180;11°,

[71] Oppolzer no. 1545: time $17;2^h$ (≈ 7 p.m. Alexandria), half-duration $1;29^h$.

[72] Ideler, *Untersuchungen* 216–17, emended '55th' to '54th' here (H345,12) and was consequently forced to excise αὐτῷ ('the same') in the year designation of the third eclipse at H346,13. His argument was that the year begins at the summer solstice in the Kallippic calendar (see Introduction p. 12). Since year 1 of Cycle I begins at the summer solstice of −329, year 54 of Cycle II goes from June -200 to June -199, and thus includes this eclipse of March -199. However, the two passages H345,12 and 346,13 confirm one another, and we must allow the possibility that Hipparchus, who was using the Egyptian calendar within the framework of the Kallippic cycle, began the year, not at the summer solstice, but at Thoth 1. Thus in his reckoning year 55 of Cycle II would run from Oct. of -200 to Oct. of -199, and would include both the second and third eclipses. It is true that this kind of reckoning cannot be applied to the Kallippic years of the equinoxes listed in III 1, but that was in another work of Hipparchus, and there is no mention of the Egyptian calendar there. See also V 3 p. 224 with n.13.

[73] Oppolzer no. 1546: time $23;7^h$ (≈ 1 a.m. Alexandria), half-duration $1;48^h$.

whereas Hipparchus carried out his demonstration on the basis of the following intervals:

[in time:] 178^d 6 equinoctial hours

[in longitude:] 180;20°.

He says that the third eclipse occurred in the same (55th) year of the Second Cycle, on Mesore [XII] 5 in the Egyptian calendar [−199 Sept. 11] and that it began when $6\frac{1}{3}$ hours of the night had passed, and was total. He also says that mid-eclipse occurred at about $8\frac{1}{3}$ hours of night, that is $2\frac{1}{3}$ seasonal hours after midnight. Now when the sun is near the middle of Virgo, one hour of the night in Alexandria is $14\frac{2}{5}$ time-degrees. So $2\frac{1}{3}$ seasonal hours produce about $2\frac{1}{4}$ equinoctial hours. So mid-eclipse was $14\frac{1}{4}$ equinoctial hours after noon on the 5th.[74] The time from epoch to this moment is H347

547 Egyptian years 334 days $\begin{cases} 14\frac{1}{4} \text{ equinoctial hours reckoned simply} \\ 13\frac{3}{4} \text{ equinoctial hours reckoned accurately.} \end{cases}$

For this moment we find:

true position of the sun:	♍ 15;12°
mean position of the moon:	♓ 10;24°
true position of the moon:	♓ 15;13°

(for its distance in anomaly from the apogee of the epicycle was 249;9°).

The interval from second to third eclipse is:

[in time:] 176^d $\frac{2}{5}$ equinoctial hour

[in longitude:] 168;55°,

whereas Hipparchus assumed the following intervals:

[in time:] 176^d $1\frac{1}{3}$ equinoctial hours

[in longitude:] 168;33°.

Here too, then, it is apparent that he committed errors of about $\frac{1}{6}$° and $\frac{1}{3}$° [in longitude], and about $\frac{5}{6}$ and[75] ($\frac{5}{6} + \frac{1}{10}$) equinoctial hours [in time]. These errors too can result in a considerable discrepancy in the ratio calculated for the [particular] hypothesis.

[74] Oppolzer no. 1547: time Sept. 12 $0;28^h$ (≈ 2;30 a.m. Alexandria), half-duration $1;50^h$. Note that for Hipparchus the whole eclipse took place on Mesore 5, although it did not begin until after midnight (what Ptolemy would call 'the midnight which lies towards the sixth'). See Introduction p. 12.

[75] Reading ἡμίσει καὶ τρίτῳ καὶ ἡμίσει καὶ τρίτῳ καὶ δεκάτῳ for ἡμίσει καὶ τρίτῳ καὶ δεκάτῳ ('$\frac{1}{2}$ and $\frac{1}{3}$ and $\frac{1}{10}$') at H347,16–17. The difference between Ptolemy's and Hipparchus' time intervals are: I–II: $6\frac{5}{6}^h - 6^h = \frac{5}{6}^h$; II–III: $1\frac{1}{3}^h - \frac{2}{5}^h = \frac{14}{15}^h = (\frac{5}{6} + \frac{1}{10})^h$. The emendation is certain and simple, but appears never to have been made. (In the Arabic tradition, T, Q, occurs the almost correct variant '$\frac{1}{2} + \frac{1}{3}$ and $\frac{1}{2} + \frac{1}{3} + \frac{1}{12}$'.) Manitus noticed the discrepancy, but was led astray by his misunderstanding at H347, 13–14 of μιᾶς τρίτου ὥρας, which he took to mean 'a third of one hour'. Thus he supposed the difference between Ptolemy's and Hipparchus' intervals (II–III) to be ($\frac{2}{5} - \frac{1}{3}$) = 4 minutes ≈ $\frac{1}{12}$ hour, and emended Heiberg's δεκάτῳ to δωδεκάτῳ (the reading of D). I carelessly followed his interpretation and emendation in Toomer[2], in which I used Hipparchus' intervals to recompute the ratios for the eccentric and epicyclic models. The result was that, while I found fairly good agreement with the ratio 3144:327$\frac{2}{3}$ for the eccentric model, using the first triple of eclipses, I could derive a value close to the ratio 3122$\frac{1}{2}$:247$\frac{1}{2}$ for the epicyclic model and the second eclipse triple only by attributing a computational error to Hipparchus. Now, however, using the correct time interval of $1\frac{1}{3}^h$ for II–III, I find much better agreement with the above ratio, as I shall show in detail elsewhere. (If the ratio were 3112$\frac{1}{2}$:247$\frac{1}{2}$, agreement would be almost perfect, and this also provides a better fit with the equivalences given by Ptolemy.) These calculations not only vindicate Hipparchus' computational abilities, but cast doubt on my claim that he was operating with a chord table with base R = 3438.

H348 Thus we have plainly displayed the reason for the above discrepancy, and it is clear that we can have even more confidence than before in the correctness of the ratio we deduced for the anomaly at lunar syzygies, since we have found these very same eclipses agreeing closely with our hypotheses.

Book V

1. {*On the construction of an 'astrolabe' instrument*}[1]

As far as concerns the [moon's] syzygies with the sun at conjunction and opposition, and the eclipses which occur at such syzygies, we find that the hypothesis set out above for the first, simple anomaly is sufficient, even if we employ it just as it is, without any change. But for particular positions [of the moon] at other sun-moon configurations one will find that it is no longer adequate, since as we said [p. 181], we have discovered that there is a second H351
lunar anomaly, related to its distance from the sun. This anomaly is reduced to the first [i.e. becomes zero] at both syzygies, and reaches a maximum at both quadratures. We were led to awareness of and belief in this [second anomaly] by the observations of lunar positions recorded by Hipparchus,[2] and also by our own observations, which were made by means of an instrument which we constructed for this purpose. The makeup of the instrument is as follows.

We took two rings of an appropriate size, with their surfaces precisely turned on the lathe so as to be squared off [i.e. with rectangular cross-sections], equal and similar to each other in all dimensions. We joined them together at diametrically opposite points, so that they were fixed at right angles to each other, and their corresponding surfaces coincided: thus one of them [Fig. F,3] represented the ecliptic, and the other [Fig. F,4] the meridian through the poles of the ecliptic and the equator [i.e. a colure]. On the latter, using the side of the [inscribed] square [as measure], we marked the points representing the poles of the ecliptic, and pierced each point with a cylindrical peg [Fig. F,e,e] projecting beyond both outer and inner surfaces. On the outer [projections] we pivoted another ring [Fig. F,5] the concave [inner] surface of which fitted H352
closely on the convex [outer] surface of the two joined rings, in such a way that it could move freely about the above-mentioned poles of the ecliptic in the

[1] On the instrument described in this chapter the only good discussion is that of Rome[4], to which the reader is referred for all details of its construction and use. My Fig. F is based on the drawing there. The numbers and letters designating the rings and other parts of the instrument also follow Rome's notation. In modern terms, it is an 'armillary sphere'. The adjective 'astrolabe' applied to it and to its parts simply means 'for taking the [the position of] the stars', and has nothing to do with the instrument to which the name 'astrolabe' is now usually applied (on which see *HAMA* II 868–79). The latter was called the 'small astrolabe' by Theon of Alexandria: see Rome[1] I 4 n.0; by Ptolemy it was apparently called 'horoscopic instrument' (see *HAMA* II 866).

[2] Examples of these are preserved at V 3 p. 224 and V 5 pp. 227 and 230. It is notable that these are the latest three known observations of Hipparchus. The obvious conclusion is that towards the end of his career he suspected that the 'simple' lunar hypothesis was inadequate for positions outside the syzygies, and was making observations to check this.

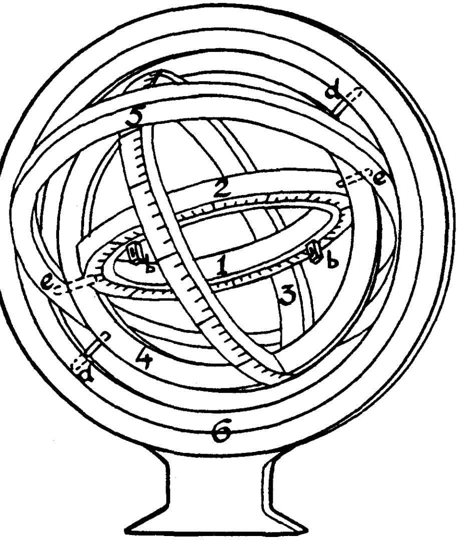

Fig. F

longitudinal direction. Similarly we pivoted another ring [Fig. F,2] on the inner [projections]; this too fitted the two [joined] rings closely, its convex surface to their concave, and, like the outer ring, moved freely in longitude about the same poles. We marked on this inner ring, and also on the ring representing the ecliptic, the divisions indicating the standard 360 degrees of the circumference, and as small subdivisions of a degree as was practical. Then we fitted snugly inside the inner of the two [movable] rings another thin ring [Fig. F,1] with sighting-holes [Fig. F,b,b] projecting from it at diametrically opposite points. [This ring was constructed] so that it could move laterally in the plane of the ring it was fitted into, towards either of the above-mentioned poles, in order to allow observation of the variation in latitude.

Having completed the above construction, we marked off from both poles of the ecliptic, on the ring representing the circle through both poles [Fig. F,4], an arc equal to the distance between the poles of ecliptic and equator (as determined above). At the ends of these arcs (which were, again, diametrically
H353 opposite) we again inserted pivots [Fig. F,d,d], attaching them to a meridian ring [Fig. F,6] similar to that[3] described at the beginning of this treatise [pp. 61–2] for making observations of the arc of the meridian between the solsticial points. This meridian ring was set up in the same position as the earlier one, perpendicular to the plane of the horizon and at an elevation of the pole appropriate for the place in question, and also parallel to the plane of the actual meridian [at that place]. Thus the inner rings [Fig. F,4 etc.] were set up so as to

[3] Reading τῷ ἐν ἀρχῇ τῆς συντάξεως ἀποδεδειγμένῳ (with D,Ar) for τῶν ἐν ἀρχῇ τῆς συντάξεως ὑποδεδειγμένων (which is untranslatable) at H353,1–2.

revolve about the poles of the equator, from east to west, following the first motion of the universe.

Once we had set up the instrument in the way described, whenever we had a situation in which both sun and moon could be observed above the earth at the same time, we set the outer astrolabe ring [Fig. F,5] to the graduation [on the ecliptic ring, fig. F,3] marking, as nearly as possible, the position of the sun at that moment. Then we rotated the ring through the poles [Fig. F,4] until the intersection [of outer astrolabe ring and ecliptic ring] marking the sun's position was exactly facing the sun, and thus both the ecliptic ring [Fig. F,3] and the [ring] which goes through the poles of the ecliptic [Fig. F,5] cast its shadow exactly on itself.[4] Or, if we were using a star as sighting [i.e. orienting] object, we set the outer [astrolabe] ring to the position assumed for that star on the ecliptic-ring, [and then rotated the ring Fig. F,4 to such a position] that when we applied one eye to one face of the outer ring [Fig. F,5] the star appeared fastened, so to speak, to both [nearer and farther] surfaces of that face,[5] and thus H354
was sighted in the plane through them. Then we rotated the other, inner astrolabe ring [Fig. F,2] towards the moon (or any other object we desired) so that the moon (or any other desired object) was sighted through both sighting-holes on the inmost ring at the same time as the sun (or the other sighting-star) was being sighted [as described above].

In this way we read off the position [of the moon or any other desired object] in longitude on the ecliptic, from the graduation occupied by the inner [astrolabe] ring [Fig. F,2] on the ring representing the ecliptic [Fig. F,3], and its deviation to north or south [of the ecliptic] along the circle through the poles of the ecliptic, from the graduations of the inner astrolabe ring [Fig. F,2]; the latter is given by the distance between the mid-point of the upper[6] sighting-hole on the inmost rotating ring [Fig. F,1] and the line drawn through the centre of the ecliptic ring.

[4] According to Ptolemy's instructions, one has to *compute* the solar longitude, set the outer astrolabe ring (Fig. F, 5) to that position on the ecliptic ring (Fig. F, 3), and then, keeping the two in that position relative to each other, swing both until one can sight the sun along the outer astrolabe ring. Both rings should then shade themselves. Theoretically, even without knowing the sun's position, one could set up the instrument by sighting the sun along the outer astrolabe ring and then moving the ecliptic ring relative to the latter until it shaded itself. Cf. p. 224 n.11.

[5] Reading ὥσπερ κεκολλημένος ἀμφοτέραις αὐτῆς ταῖς ἐπιφανείαις for καὶ διὰ τῆς ἀπεναντίον καὶ παραλλήλου τοῦ κύκλου πλευρᾶς ὥσπερ κεκολλημένος ἀμφοτέραις αὐτῶν ταῖς ἐπιφανείαις at H353.24-354.1. The latter would mean 'when we applied one eye to the [nearer] face of the outer ring and [looked] along the opposite, parallel face of the ring, the star appeared fastened, so to speak, to the surfaces of both those faces'. The words καὶ διὰ . . . πλευρᾶς are a foolish explanatory interpolation by someone who misinterpreted ἀμφοτέραις ταῖς ἐπιφανείαις to mean 'the opposite faces' of the ring instead of 'the two parts of the same face nearer to and farther from the eye'; then αὐτῆς (referring to τῇ ἑτέρᾳ τῶν πλευρῶν) was changed to αὐτῶν (referring to both πλευραί), or possibly αὐτῶν was simply interpolated. Quite apart from the technical problem, the text as printed by Heiberg is extraordinarily clumsy. The interpolation is quite early, since it is also in the Arabic tradition. Pappus' commentary to the passage betrays no hint that he read the interpolation, but is not sufficiently close to the Almagest to allow us to say that he did not.

[6] 'upper': literally 'above the earth'. Since the centre of all the rings represents the centre of the earth, the sight nearer the observer's eye is notionally 'below the earth', the other 'above the earth'.

2. {*On the hypothesis for the double anomaly of the moon*}[7]

When this type of observation was made without further analysis, it was found,
H355 both from the observations recorded by Hipparchus and from our own, that the distance of the moon from the sun was sometimes in agreement with that calculated from the above [simple] hypothesis, and sometimes in disagreement, the discrepancy being at some times small and at other times great. But when we paid more attention to the circumstances of the anomaly in question, and examined it more carefully over a continuous period, we discovered that at conjunction and opposition the discrepancy [between observation and calculation] is either imperceptible or small, the difference being of a size explicable by lunar parallax; at both quadratures, however, while the discrepancy is very small or nothing when the moon is at apogee or perigee of the epicycle, it reaches a maximum when the moon is near its mean speed and [thus] the equation of the first anomaly is also a maximum; furthermore, at either quadrature, when the first anomaly is subtractive the moon's observed position is at an even smaller longitude than that calculated by subtracting the equation of the first anomaly, but when the first anomaly is additive its true position is even greater [than that calculated by adding the equation of the first anomaly], and the size of this discrepancy is closely related to the size of the equation of the first anomaly. From these circumstances alone we could see that we must suppose the moon's epicycle to be carried on an eccentric circle, being farthest
H356 from the earth at conjunction and opposition, and nearest to the earth at both quadratures. This will come about if we modify the first hypothesis along somewhat the following lines.

Imagine the circle (in the inclined plane of the moon) concentric with the ecliptic moving in advance, as before [p. 191], (to represent the [motion in] latitude) about the poles of the ecliptic with a speed equal to the increment of the motion in latitude over the motion in longitude. Imagine, again, the moon traversing the so-called epicycle (moving in advance on its apogee arc) with a speed corresponding to the return of the first anomaly. Now, in this inclined plane, we suppose two motions to take place, in opposite directions, both uniform with respect to the centre of the eliptic: one of these carries the centre of the epicycle towards the rear through the signs with the speed of the motion in latitude, while the other carries the centre and apogee of the eccentre, which we assume located in the same [inclined] plane, (the centre of the epicycle will at all times be located on this eccentre), in advance through [i.e. in the reverse order of] the signs) by an amount corresponding to the difference between the motion in latitude and the double elongation (the elongation being the amount by which the moon's mean motion in longitude exceeds the sun's mean motion).
H357 Thus, to give an example, in one day the centre of the epicycle traverses about 13;14° in motion of latitude towards the rear through the signs, but appears to have traversed 13;11° in longitude on the ecliptic, since the whole inclined circle [of the moon] traverses the difference of 0;3° in the opposite direction, [i.e.] in advance; [meanwhile] the apogee of the eccentre, in turn, travels 11;9°

[7] On chs. 2-4 see *HAMA* 84-8, Pedersen 184-9.

in the opposite direction, (again in advance): this is the amount by which the double elongation, 24;23°, exceeds the motion in latitude, 13;14°. The combination of both of these motions, which take place in opposite directions, as we said, about the centre of the ecliptic, will produce the result that the radius carrying the centre of the epicycle and the radius carrying the centre of the eccentre will be separated by an arc which is the sum of 13;14° and 11;9°, and is twice the amount of the elongation (which is approximately 12;11½°). Hence the epicycle will traverse the eccentre twice during a mean [synodic] month. We assume that it returns to the apogee of the eccentre at mean conjunction and opposition.

In order to illustrate the details of the hypothesis, imagine [Fig. 5.1] the circle in the moon's inclined plane concentric with the ecliptic as ABGD on centre E H358
and diameter AEG. Let the apogee of the eccentre, the centre of the epicycle, the northern limit, the beginning of Aries and the mean sun [all] be located at

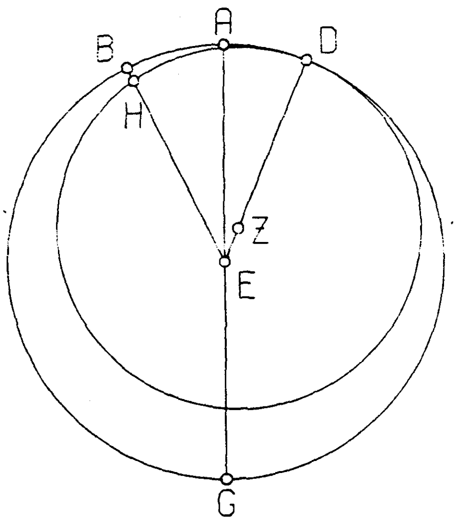

Fig. 5.1

point A at the same moment. Then I say that in the course of one day the whole [inclined] plane moves in advance from A towards D about centre E, by about 3′: thus the northern limit (which is [still represented by] A) reaches ♓ 29;57°. The two opposite motions are carried out by the radius corresponding to EA [moving] uniformly about E, the centre of the ecliptic. Thus I say that in the course of one day the radius through the centre of the eccentre corresponding to EA rotates uniformly in advance [i.e. in the reverse order] of the signs to the position ED, carrying the apogee of the eccentre to D,[8] and making arc AD

[8] Omitting καὶ γράφειν περὶ τὸ Ζ κέντρον τὸν ΔΗ ἔκκεντρον after Δ at *H358,20-21*. This would mean 'and describing eccentre DH about centre Z'. This is nonsense: EA does not 'describe the eccentre' (since it is not a radius of the eccentre), but merely marks the position of the apogee of the eccentre. If Ptolemy wanted to refer to the eccentre here, he would presumably have written (as

11;9°. [In the same time] the radius through the centre of the epicycle [corresponding to EA] rotates uniformly, again about E, towards the rear through the signs to the position EB, carrying the centre of the epicycle to H,
H359 and making arc AB 13;14°. Thus the apparent distance of H, the centre of the epicycle, is 13;14° (in motion of latitude) from the northern limit A, 13;11° (in longitude) from the beginning of Aries (for the northern limit A has moved to ♓ 29;57° in the same time), and 24;23° (the sum of arc AD and arc AB, and twice the mean daily elongation) from the apogee of the eccentre D. Since, in this way, the motion through B and the motion through D meet each other once in half a mean [synodic] month, it is obvious that these motions will always be diametrically opposite at intervals of a quarter and three-quarters of that period, i.e. at the mean quadratures. At those times the centre of the epicycle, located on EB, will be diametrically opposite the apogee of the eccentre, located on ED, and [thus] will be at the perigee of the eccentre.

It is also clear that under these circumstances the eccentre itself (that is, the fact that the arc DB is not similar to arc DH) will not produce any correction to
H360 the mean motion. For the uniform motion of the line EB is counted, not along arc DH of the eccentre, but along arc DB of the ecliptic, since it rotates, not about the centre of the eccentre Z, but about E. The only [correction] which will result is that due to the difference in the effect of the epicycle: as the epicycle moves towards the perigee it produces a continuous increase in the equation of anomaly (subtractive and additive alike), since the angle formed by the epicycle at the observer's eye is greater at positions [of the epicycle] nearer the perigee. On the other hand, there will, in general, be no difference from the first hypothesis when the centre of the epicycle is at the apogee A, which is the situation at the mean conjunctions and oppositions.

For if [Fig. 5.2][9] we draw epicycle MN about point A, AE:AM is the same ratio as that which we demonstrated from the eclipses. The greatest difference
H361 will be when the epicycle reaches H, the perigee of the eccentre (as XO here). This occurs at the mean quadratures. For the ratio XH:HE is greater than that at any other position, since XH, the radius of the epicycle, is always a constant length, while EH is the shortest of all lines drawn from the centre of the earth to the eccentre.

3. {*On the size of the anomaly of the moon which is related to the sun*}

In order to see what the maximum equation of anomaly is when the epicycle is at the perigee of the eccentre, we sought observations of the distance of the moon from the sun under the following conditions:

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does Is.) καὶ γραφέντος περὶ τὸ Z κέντρον τοῦ ΔH ἐκκέντρου 'and if the eccentre DH is described about centre Z'. However, it seems more likely that this is an interpolation by someone who wanted an explicit reference to the drawing of the eccentre DH on centre Z, represented in Fig. 5.1 and referred to by Ptolemy below.

[9] The figure given by Heiberg (p. 360), which is taken from the ms. tradition represented by A, is wrong in making E the centre of the circle and adding a point K above it. My figure agrees with the text and with part of the Arabic tradition (e.g. P), except that all Arabic mss. have the equivalent of Θ for O. Manitius already made the same correction, except that he unnecessarily added the point Z (unattested in the mss.) as the centre of the circle.

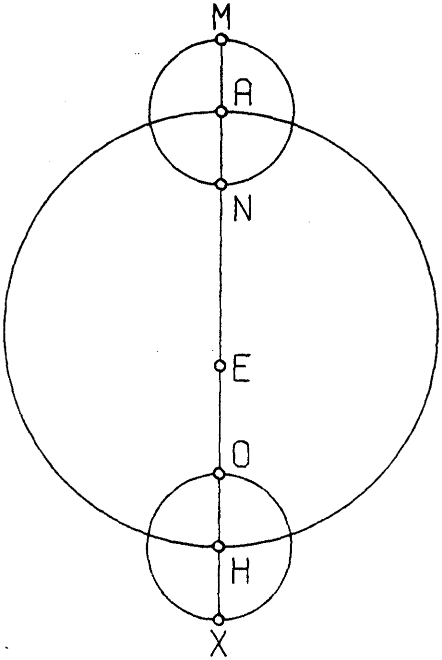

Fig. 5.2

[1] The moon's speed was about at the mean (for that is when the equation of anomaly is maximum).
[2] The mean elongation of the moon from the sun was about a quadrant (for then the epicycle was near the perigee of the eccentre).
[3] In addition to the above, the moon had no longitudinal parallax.

If these conditions are fulfilled, the apparent observed longitudinal distance is the same as the true, and thus we can safely infer the size of the second anomaly H362 which we are seeking. When we investigate on the basis of the above kind of observations, we find that, when the epicycle is closest to the earth, the greatest equation of anomaly is about 7⅔° with respect to the mean position (or 2⅔° different from [the corresponding equation of] the first anomaly).

We will illustrate the way in which this kind of determination is made from one or two observations by way of example. We sighted sun and moon in the 2nd year of Antoninus, Phamenoth [VII] 25 in the Egyptian calendar [139, Feb. 9], after sunrise, and 5¼ equinoctial hours before noon. The sun was sighted in ♒ 18⅚°, and ♐ 4 was culminating. The apparent position of the moon was ♏ 9⅔°, and that was its true position too, since when it is near the beginning of Scorpius, about 1½ hours to the west of the meridian at Alexandria, it has no noticeable parallax in longitude.[10] Now the time from epoch in the first year of

[10] I.e. at that situation the angle between ecliptic and altitude circle (derived from Table II 13) is about 90°, hence the parallax affects only the latitude, not the longitude. Interpolation in the tables

Nabonassar to the observation is

H363 885 Egyptian years 203 days 18¾ equinoctial hours (whether reckoned simply or accurately).

For this moment we find:

mean position of the sun: ♒ 16;27°

true position of the sun: ♒ 18;50° (in accordance with its sighted position according to the astrolabe).[11]

From the first hypothesis we find the mean position of the moon at that moment as ♏ 17;20° (thus its mean elongation from the sun was about a quadrant), and the moon's distance in anomaly from the apogee of the epicycle as 87;19° (which is near the position of maximum equation). Thus the true position of the moon was less than the mean by 7⅔° (instead of the 5° of the first anomaly).[12]

Again, to display the amount of the equation under similar conditions which is derived from Hipparchus' observations of such positions, we will adduce one of these. He says that he made the observation in the fifty-first year[13] of the Third Kallippic Cycle, Epiphi [XI] 16 in the Egyptian calendar [-127 Aug. 5], when ⅔ of the first hour had passed. 'The speed was [that of day] 241',[14] he says, 'and while the sun was sighted in Leo $8\frac{7}{12}$° the apparent position of the moon was Taurus 12⅓°, and its true position was approximately the same'. So the true observed distance between moon and sun was 86;15°. But when the sun is near the beginning of Leo, at Rhodes (where the observation was made), 1 hour of

H364 the day is 17⅓ time-degrees. So the 5⅓ seasonal hours (which make up the interval to [the following] noon) produce 6⅙ equinoctial hours. Therefore the

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for Clima III, ♏ 9;40°, 1½ʰ west of the meridian, gives 83;5°. Exact computation for Alexandria ($\varphi \approx 31°$) gives 83;45°. For the computations here and at the other observations of V 3 and V 5 see *HAMA* 91-2.

[11] Is this meant as a confirmation of the accuracy of the observation? This would imply that Ptolemy set up the instrument by using the shadow (cf. p. 219 n.4). It may, however, merely mean that this computation is the basis of the position to which Ptolemy set the instrument.

[12] Precise computation: mean elongation = ♒ 16;27° − ♏ 17;20° = 89;7°; equation = ♏ 9;40° − ♏ 17;20° = −7;40°; equation from first hypothesis (from Table IV 10), α(87;19°) → −4;57°. However, Ptolemy is operating with rounded numbers, quite properly here.

[13] I have, doubtfully, accepted the emendation να′ for ν′ ('fiftieth) at H363.16. The Julian date of the observation, -127 Aug. 5, is guaranteed both by the astronomical data and by Ptolemy's reckoning in the era Nabonassar. Ideler (*Historische Untersuchungen* 217–18) made the emendation because he calculated, correctly, from the known epoch of the Kallippic cycles that this must fall in the fifty-first year. In this case (cf. p. 214 n.72) using the Egyptian calendar makes no difference. However, I suspect that the error, if it is one, lies not with the scribes but with Ptolemy or even Hipparchus, and that possibly there is no error, but another method of counting which eludes us.

[14] Literally 'The true daily motion (δρόμος) was the 241st'. Hipparchus is referring to a table of the true motion of the moon over 248 days (≈ 9 anomalistic months), in which the moon was supposed to return to the same velocity. Such a table is extant on a cuneiform tablet, ACT no. 190 (III p. 131). If Hipparchus was using that table the motion on day 241 would be 13;30° or 13;31,10° (according to whether one starts at the beginning or goes in reverse from the end), i.e. close to the mean, as our passage requires. The historical interest of this passage has been missed because '241' has hitherto been interpreted as 'degrees of anomaly' (and hence 'emended', to '259' by Manitius and to μέσος, 'mean', by Halma). I think it likely that Hipparchus was the channel through which use of the 248-day lunar anomaly period was transmitted from Mesopotamia to the Greek world (e.g. Vettius Valens I 4–5, ed. Kroll 20–1, and P. Ryl. 27, on which see *HAMA* 808 ff.), and ultimately to India (the Vākya system, see *HAMA* 817 ff.) See provisionally Toomer [11] p. 108 n.12.

observation occurred $6\frac{1}{6}$ equinoctial hours before noon on the sixteenth, while ♉ 9° was culminating. Thus in this case the time from epoch to the observation is

$$\text{619 Egyptian years 314 days}\begin{cases} 17\frac{5}{6} \text{ equinoctial hours reckoned simply} \\ 17\frac{3}{4} \text{ equinoctial hours reckoned accurately.}^{15} \end{cases}$$

For this moment we find from our hypotheses (since the meridian through Rhodes is the same as that through Alexandria):[16]

mean position of the sun:	♌ 10;27°
true position of the sun:	♌ 8;20°
mean position of the moon in longitude:	♉ 4;25°

(thus the mean elongation was again nearly a quadrant)

mean distance of the moon from the apogee of the epicycle in anomaly: 257;47° (which is again near the position of the maximum equation of the anomaly due to the epicycle).

So the distance from the mean moon to the true sun is calculated as 93;55°. And the observed distance from the true moon to the true sun was 86;15°.[17] Therefore the true position of the moon was greater than the mean, again by $7\frac{2}{3}$° instead of the 5° of the first hypothesis. And it is [further] evident, that of these two observations taken near the second quadrature, ours was found to be less than the position computed from the first anomaly by $2\frac{2}{3}$°, while Hipparchus' was greater by the same amount, since the total equation of anomaly was subtractive at our observation and additive at Hipparchus'. H365

From numerous other similar observations also we find that the greatest equation of anomaly is about $7\frac{2}{3}$° when the epicycle is at the perigee of the eccentre.

4. {*On the ratio of the eccentricity of the moon's circle*}

With this as a datum, let [Fig. 5.3] the moon's eccentric circle be ABG on centre D and diameter ADG, on which E is taken as the centre of the ecliptic. Thus A is the apogee of the eccentre and G the perigee. On centre G draw the moon's epicycle ZHΘ, draw EΘB tangent to it, and join GΘ.

Then since the greatest equation of anomaly occurs when the moon is at the epicycle tangent, and we have shown that this amounts to $7\frac{2}{3}$°, the angle at the centre of the ecliptic, H366

$$\angle GE\Theta = \begin{cases} 7;40^\circ \text{ where 4 right angles} = 360^\circ \\ 15;20^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ}. \end{cases}$$

[15] As Neugebauer remarks, the equation of time for a solar longitude of ♌ 8° should be −16 mins. rather than −5 mins. For this and other inaccuracies in Ptolemy's computations see *HAMA* 92–3.

[16] In fact Rhodes is about 1.7° west of Alexandria. The notion that they lay on the same meridian was traditional: see Strabo 2.5.7, where the same meridian is supposed to pass through Meroe, Syene, Alexandria, Rhodes, the Troad, Byzantium and the Borysthenes. This is probably derived from Eratosthenes via Hipparchus.

[17] Note that Ptolemy takes only the distance observed by Hipparchus (86;15°) as accurate, and substitutes his own *calculations* of the positions of sun and moon for those observed (or calculated) by Hipparchus.

Therefore in the circle about right-angled triangle GEΘ

arc GΘ = 15;20°

and the corresponding chord

GΘ ≈ 16^p where the hypotenuse GE = 120^p.

So, where GΘ, the radius of the epicycle, is, as was shown, $5;15^p$
and EA, the distance from the centre of the ecliptic to the apogee of the eccentre, is 60^p,
EG, the distance from the centre of the ecliptic to the perigee of the eccentre, is $39;22^p$.

Therefore, by addition, diameter AG = $99;22^p$,
and DA, the radius of the eccentre = $49;41^p$
and ED, the distance between the centres of the ecliptic and the eccentre = $10;19.^p$

Thus we have demonstrated the ratio of the eccentricity.

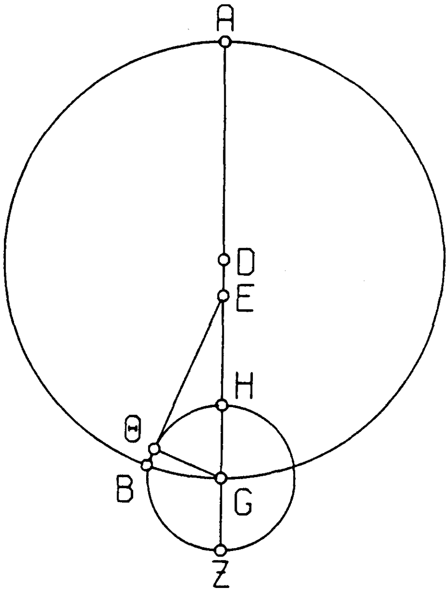

Fig. 5.3

H367 5. {*On the 'direction' of the moon's epicycle*}[18]

As far as concerns the phenomena at syzygies and at quadrature positions of the moon, the preceding discussion would provide a full explanation of the hypotheses elucidating the circles of the moon described above. But from individual observations taken at distances of the moon [from the sun] when it is sickle-shaped or gibbous (which occur when the epicycle is between the apogee

[18] See *HAMA* 88–91, Pedersen 189–95.

and the perigee of the eccentre), we find that the moon has a peculiar characteristic associated with the direction[19] in which the epicycle points. Every epicycle must, in general, possess a single, *unchanging point defining the* position of return of revolution on that epicycle. We call this point the 'mean apogee', and establish it as the beginning from which we count motion on the epicycle. Thus point Z on the previous figure [5.3] is such a point. It is defined, for the position of the epicycle at apogee or perigee of its eccentre, by the straight line drawn through all the centres [of ecliptic, eccentre and epicycle] (DEG here). Now in all other hypotheses [involving epicycle on eccentre], we see absolutely nothing in the phenomena which would count against the following H368
[model]: in other positions of the epicycle [outside apogee and perigee of the eccentre], the diameter of the epicycle through the above apogee, i.e. ZGH, always keeps the same position relative to the straight line which carries the epicycle centre round with uniform motion (here EG), and [thus] (as one would think appropriate) always points towards the centre of revolution, at which, furthermore, equal angles of uniform motion are traversed in equal times. In the case of the moon, however, the phenomena do not allow one to suppose that, for positions of the epicycle between A and G, diameter ZH points towards E, the centre of revolution, and keeps the same position relative to EG. We do indeed find that the direction in which [diameter ZH] points is a single, unchanging point on diameter AG, but that point is neither E, the centre of the ecliptic, nor D, the centre of the eccentre, but a point removed from E towards the perigee of the eccentre by an amount equal to DE. We shall show that this is so, again, by setting out, from among the numerous [relevant] observations, two which are particularly suitable for illustrating our point, since the epicycle at these observations was at distances halfway [between apogee and perigee of the eccentre], and the moon was near apogee or perigee of the epicycle; for in these H369
situations occur the greatest effects of the above direction [of the epicycle diameter].

Now Hipparchus records that he observed the sun and the moon with his instruments[20] in Rhodes in the 197th year from the death of Alexander, Pharmouthi [VIII] 11 in the Egyptian calendar [-126 May 2], at the beginning of the second hour. He says that while the sun was sighted in ♉ $7\frac{3}{4}$°, the apparent position of the centre of the moon was ♓ $21\frac{2}{3}$°, and its true position was ♓ $21\frac{1}{3} + \frac{1}{8}$° [21;27½°].[21] Therefore at the moment in question the distance of the true moon from the true sun was about 313;42°, [counting] towards the rear. Now the observation was made at the beginning of the second hour, about 5 seasonal hours (which correspond to about $5\frac{2}{3}$ equinoctial hours in Rhodes on

[19] πρόσνευσις, used by Neugebauer and Pedersen as a technical term ('prosneusis') for this element of Ptolemy's lunar theory. However, it is hardly that for Ptolemy, as he applies the word in many other contexts (see p. 43 n.38).

[20] It is usually assumed that by this is meant an armillary sphere similar to that described by Ptolemy in V 1 (and often, that Hipparchus was the inventor of that instrument). That may be true, but the vague expression here certainly does not require it, and whether the data described below do is doubtful. I consider it possible that Hipparchus used a dioptra of the type described by Heron ('Dioptra', ed. Schöne, 187 ff.).

[21] On the correction for parallax made by Hipparchus here (which is fairly accurate) see *HAMA* 92.

that date) before noon on the 11th. So the time from our epoch to the observation is

620 Egyptian years 219 days $\begin{cases} 18\frac{1}{3} \text{ equinoctial hours reckoned simply} \\ 18 \text{ equinoctial hours reckoned accurately.} \end{cases}$

For this moment we find:

mean sun in ♉ 6;41°
true sun in ♉ 7;45°

H370 mean moon $\begin{cases} \text{in ♓ 22;13° in longitude} \\ \text{at 185;30° from mean apogee of epicycle in anomaly.} \end{cases}$

Therefore the distance of the mean moon from the true sun was 314;28°.

With these data, let [Fig. 5.4] the moon's eccentric circle be ABG on centre D and diameter ADG, on which E represents the centre of the ecliptic. On centre B draw the moon's epicycle, ZHΘ. Let the sense of motion of the epicycle be towards the rear from B towards A, and the sense of motion of the moon on the epicycle be from Z towards H and [then] Θ. Join DB and EΘBZ.

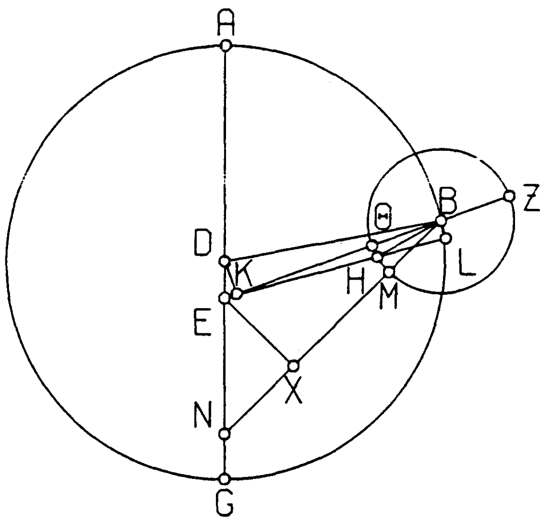

Fig. 5.4

Now in a mean [synodic] month occur two revolutions of the epicycle on the eccentre, and in the situation in question the elongation of mean moon from mean sun was 315;32°. So if we double the latter and subtract [the 360° of] a circle, we will get the elongation at that moment of the epicycle from the apogee

H371 of the eccentre, [counting] towards the rear: this is 271;4°.

$\therefore \angle$ AEB = 88;56° (remainder [when 271;4° is subtracted] from 360°).

So drop perpendicular DK from D on to EB.

$\therefore \angle \text{DEB} = \begin{cases} 88;56° \text{ where 4 right angles} = 360° \\ 177;52^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ}. \end{cases}$

Therefore in the circle about right-angled triangle DEK,

$$\text{arc DK} = 177;52^\circ$$

$$\text{and arc EK} = 2;8^\circ \text{ (supplement).}$$

Therefore the corresponding chords

$$\left.\begin{array}{r} \text{DK} = 119;59^p \\ \text{and EK} = 2;14^p \end{array}\right\} \text{ where hypotenuse DE} = 120^p.$$

Therefore where DE, the distance between the centres, is $10;19^p$

and DB, the radius of the eccentre, is $49;41^p$,

$$\text{DK} \approx 10;19^p \text{ also,}$$

$$\text{and EK} = 0;12^p.$$

$$\text{But BK}^2 = \text{DB}^2 - \text{DK}^2.$$

$$\therefore \text{BK} = 48;36^p \text{ in the same units,}$$

and, by addition, BE [= BK + EK] = $48;48^p$.

Again, since the distance of the mean moon from the true sun was found to be 314;28°, and the distance of the true moon [from the true sun] was observed to be 313:42°, the equation of anomaly is −0;46°. Now the mean position of the moon is seen along the line EB. So let the moon be located at H (since it is near the perigee), join EH and BH, and drop perpendicular BL from B on to EH produced. Then, since ∠ BEL contains the moon's equation of anomaly, H372

$$\angle \text{BEL} = \begin{cases} 0;46^\circ \text{ where 4 right angles} = 360^\circ \\ 1;32^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ}. \end{cases}$$

Therefore in the circle about right-angled triangle EBL,

$$\text{arc BL} = 1;32^\circ$$

and the corresponding chord

$$\text{BL} = 1;36^p \text{ where the hypotenuse EB} = 120^p.$$

Therefore where BE = $48;48^p$ and BH, the radius of the epicycle, is $5;15^p$,

$$\text{BL} = 0;39^p.$$

Therefore where BH, the radius of the epicycle, is 120^p,

$$\text{BL} = 14;52^p$$

and, in the circle about right-angled triangle BHL,

$$\text{arc BL} = 14;14^\circ$$

$$\therefore \angle \text{BHL} = 14;14^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ},$$ H373

$$\begin{array}{r}\text{and, by subtraction} \\ \text{[of } \angle \text{ BEL], } \angle \text{ EBH}\end{array} = \begin{cases} 12;42^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \\ 6;21^\circ \text{ where 4 right angles} = 360^\circ. \end{cases}$$

That [6;21°], then, is the size of arc HΘ of the epicycle, which comprises the distance from the moon to the true perigee [of the epicycle].

But since the distance of the moon from the mean apogee at the time of the observation was 185;30° [p. 228], it is clear that the mean perigee is in advance of the moon, i.e. of point H. Let [the mean perigee] be point M, draw line BMN, and drop perpendicular EX on to it from point E.

Then since, as was shown,

$$\text{arc ΘH} = 6;21^\circ,$$

and arc HM, the distance from the perigee, is given as 5;30°,

$$\text{by addition, arc ΘM} = 11;51^\circ.$$

$$\text{So } \angle \text{EBX} = \begin{cases} 11;51^\circ \text{ where 4 right angles} = 360^\circ \\ 23;42^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ}. \end{cases}$$

Therefore in the circle about right-angled triangle BEX,

arc EX = 23;42°

and EX = 24;39^p where hypotenuse BE = 120^p.

Therefore where BE = 48;48^p

H374 EX = 10;2^p.

Again, since [p. 228]

$$\left.\begin{aligned} \angle\ \mathrm{AEB} &= 177;52^{\circ\circ} \\ \text{and } \angle\ \mathrm{EBN} &= 23;42^{\circ\circ} \end{aligned}\right\}\ \text{where 2 right angles} = 360^\circ,$$

by subtraction, ∠ ENB = 154;10°°.

Therefore in the circle about right-angled triangle ENX,

arc EX = 154;10°

and EX = 116;58^p where hypotenuse EN = 120^p.

Therefore where EX = 10;2^p and DE, the distance between the centres, is 10;19^p,

EN = 10;18^p.

Therefore the [radius of the epicycle] through the mean perigee, BM, points in a direction such that, when produced to N, it cuts off a line EN which is very nearly equal to DE.

Similarly, in order to show that we get the same result at the opposite sides of eccentre and epicycle, we have again selected from the distances [between sun and moon] observed by Hipparchus, as already mentioned, in Rhodes, the observation he made in the same year [as the preceding one], being the 197th year from the death of Alexander, Payni [X] 17 in the Egyptian calendar
H375 [-126 July 7), at 9⅓ hours. He says that while the sun was sighted at ♋ 10 $\frac{9}{10}$° the apparent position of the moon was ♌ 29°. And this was its true position too; for at Rhodes, near the end of Leo, about one hour past the meridian, the moon has no longitudinal parallax.[22] Therefore the distance of the true moon from the true sun at the time in question was 48;6° towards the rear. Now since the observation was 3⅓ seasonal hours after noon on the 17th of Payni, which correspond to about 4 equinoctial hours in Rhodes on that date, the time from our epoch to the observation is

$$\text{620 Egyptian years 286 days}\left\{\begin{array}{l}\text{4 equinoctial hours reckoned simply}\\ 3\tfrac{2}{3}\text{ equinoctial hours reckoned accurately.}\end{array}\right.$$

For this moment we find:

mean sun at ♋ 12;5°

true sun at ♋ 10;40°

mean moon at ♌ 27;20° in longitude

(thus the distance of the mean moon from the true sun was 46;40°)

mean moon at 333;12° in anomaly from the apogee of the epicycle.[23]

With these data, let [Fig. 5.5] the moon's eccentric circle be ABG on centre D

[22] For verification of this see *HAMA* 92.

[23] For 620^{y}286^{d}3⅔^h I find: $\bar{\lambda}$ ☽ = 147;7°, $\bar{\alpha}$ ☽ = 333;1°. Since the differences from Ptolemy's positions represent the lunar motion over about 20 mins., it is obvious that he has carelessly calculated the positions for 4 hours after noon, i.e. without making the correction for the equation of time, which he had given, correctly, as about 20 mins. This error has a not inconsiderable effect on the final result, which would not agree nearly so neatly if the computation were carried out with the above figures.

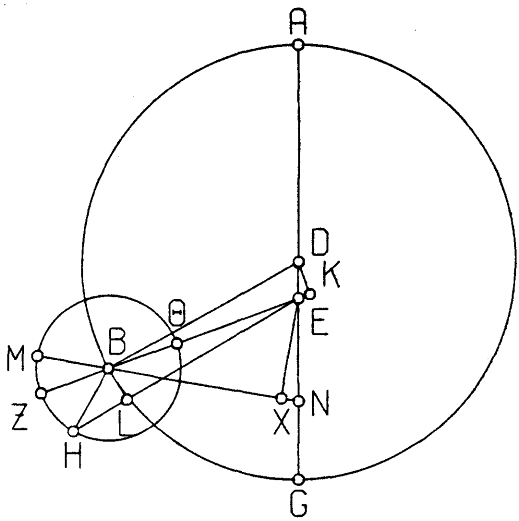

Fig. 5.5

and diameter ADG, on which the centre of the ecliptic is represented by point H376
E. About point B draw the moon's epicycle, ZHΘ, and join DB, EΘBZ.

Then since twice the mean elongation of sun and moon is 90;30°, from the theory already established

$$\angle \text{AEB} = \begin{cases} 90;30^\circ \text{ where 4 right angles} = 360^\circ \\ 181^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ}. \end{cases}$$

So if we produce BE and drop perpendicular DK on to it from D,

$$\angle \text{DEK} = 179^{\circ\circ} \text{ (supplement).}$$

Therefore in the circle about right-angled triangle DEK

$$\text{arc DK} = 179^\circ$$

$$\text{and arc EK} = 1^\circ \text{ (supplement).}$$ H377

Therefore the corresponding chords

$$\left.\begin{aligned} \text{DK} &= 119;59^p \\ \text{and EK} &= \quad 1;3^p \end{aligned}\right\} \text{ where hypotenuse DE} = 120^p.$$

Therefore where DE, the distance between the centres, is $10;19^p$
and BD, the radius of the eccenter, is $49;41^p$,

$$\text{DK} \approx 10;19^p$$

$$\text{and EK} = 0;5^p.$$

$$\text{Now since BK}^2 = \text{BD}^2 - \text{DK}^2,$$

$$\text{BK} = 48;36^p,$$

and, by subtraction [of EK], EB = $48;31^p$.

Furthermore, since the distance of mean moon from true sun was found to be 46;40°, and the distance of true moon [from true sun was observed as] 48;6°, the equation of anomaly is +1;26°. So let the position of the moon be at H (since it is near the apogee of the epicycle). Join EH, BH, and drop perpendicular BL from B on to EH.

Then since

$$\angle \mathrm{BEL} = \begin{cases} 1;26^\circ \text{ where 4 right angles} = 360^\circ \\ 2;52^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ}, \end{cases}$$

in the circle about right-angled triangle BEL,

$$\text{arc BL} = 2;52^\circ$$

H378 $$\text{and BL} = 2;59^p \text{ where hypotenuse EB} = 120^p.$$

Therefore where EB = $48;31^p$ and BH, the radius of the epicycle, is $5;15^p$,

$$\text{BL} = 1;12^p.$$

So in the circle about right-angled triangle BHL,

$$\text{BL} = 27;34^p \text{ where hypotenuse BH} = 120^p,[24]$$

$$\text{and arc BL} = 26;34^\circ.$$

$$\therefore \angle \mathrm{BHL} = 26;34^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ}.$$

and, by addition [of $\angle$ BEL = $2;52^{\circ\circ}$],

$$\angle \mathrm{ZBH} = \begin{cases} 29;26^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ}. \\ 14;43^\circ \text{ where 4 right angles} = 360^\circ. \end{cases}$$

That [$14;43^\circ$] is the size of the arc HZ of the epicycle, which comprises the distance from the moon to the true apogee.

But since [the moon's] distance from the mean apogee at the time of the observation was $333;12^\circ$, if we put the mean apogee at M, draw line MBN, and drop perpendicular EX on to it from E, then

arc HZM = $26;48^\circ$ (by subtraction [of $333;12^\circ$] from the circle),

and, by subtraction [of arc HZ = $14;43^\circ$], arc ZM = $12;5^\circ$.

$$\therefore \angle \mathrm{MBZ} = \angle \mathrm{EBX} = \begin{cases} 12;5^\circ \text{ where 4 right angles} = 360^\circ \\ 24;10^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ}. \end{cases}$$

Therefore in the circle about right-angled triangle BEX

$$\text{arc EX} = 24;10^\circ$$

H379 $$\text{and EX} = 25;7^p \text{ where hypotenuse BE} = 120^p.$$

Therefore where BE = $48;31^p$ and DE, the line between the centres, is $10;19^p$,

$$\text{EX} = 10;8^p.$$

Again, since $\angle$ AEB is given as $181^{\circ\circ}$ where 2 right angles = $360^{\circ\circ}$,
and we have shown that $\angle$ EBN = $24;10^{\circ\circ}$,
by subtraction, $\angle$ ENB = $156;50^{\circ\circ}$ in the same units,
and, in the circle about right-angled triangle ENX,

$$\text{arc EX} = 156;50^\circ$$

$$\text{and EX} = 117;33^p \text{ where hypotenuse EN} = 120^p.$$

Therefore where EX = $10;8^p$ and DE, the line between the centres, is $10;19^p$,

$$\text{EN} = 10;20^p.$$

So from this calculation too it turns out that MB, [the radius of the epicycle] through M, the mean apogee, points in a direction such that, when produced to N, it cuts off a line EN approximately equal to DE, the distance between the centres.

We also find that approximately the same ratio results by calculation from a number of other observations. Thus these observations confirm the peculiar characteristic of the direction of the epicycle in the hypothesis of the moon: the

[24] $1;12 \times 120/5;15 = 27;25,43$. Ptolemy was obviously operating, not with the value 1;12, but with 1;12,22 (which leads to 27;34,5), which is in fact what one finds from the immediately preceding calculation, $2;59 \times 48;31/120$.

[uniform] revolution of the centre of the epicycle takes place about E, the centre of the ecliptic, but the diameter of the epicycle which defines the unchanging point of the epicycle at which the mean epicyclic apogee is located points, not (as it does for the other [planets]), towards E, the centre of mean motion, but always towards N, which is removed in the opposite direction [to D from E] by an amount equal to DE, the distance between the centres. H380

6. {*How the true position of the moon can be calculated geometrically from the periodic motions*}[25]

Now that we have demonstrated the above, the appropriate sequel is to show how, for a particular position of the moon, given the amounts of the [various] mean motions, we can find from the amount of the elongation and of the moon's [motion in anomaly] on the epicycle the amount due to the equation of anomaly which should be added to or subtracted from the mean motion in longitude. If one uses [strictly] geometrical methods, the way to solve such a problem is via theorems similar to those already set out.

Let us use the last of the above figures [5.5] as an example, and take as a basis of calculation the same periodic motions in elongation and anomaly, namely

double elongation: 90;30°

anomaly counted from the mean epicyclic apogee: 333;12°. H381

[See Fig. 5.6.] We drop perpendicular NX (instead of EX) and perpendicular HL (instead of BL). Then, by the same computation as before [p. 231], since we are given

[1] The angles at centre E;

[2] hypotenuse DE and hypotenuse EN (which are equal),

$$DK = NX \approx 10;19^p$$

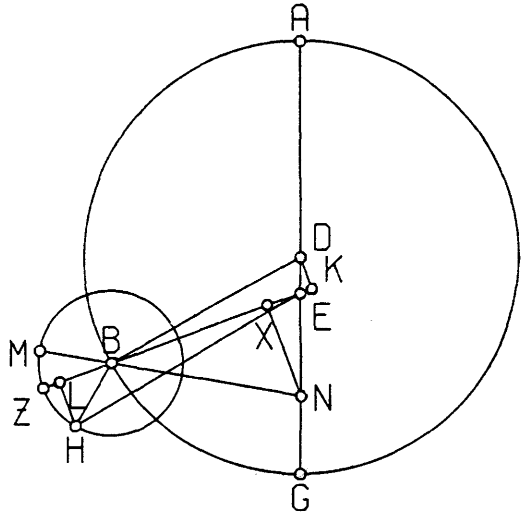

Fig. 5.6

[25] See *HAMA* 93, Pedersen 194–5.

where DB, the radius of the eccentre = $49;41^p$
and BH, the radius of the epicycle = $5;15^p$
and EK = EX = $0;5^p$.
Hence, as shown before [p. 231] BK = $48;36^p$
and similarly, [by subtraction of EK] BE = $48;31^p$
and, by subtraction [of EX] BX = $48;26^p$.
So, since $BX^2 + XN^2 = BN^2$,
BN = $49;31^p$ where NX = $10;19^p$.
Therefore, in the circle about right-angled triangle BNX,
where hypotenuse BN = 120^p
H382 NX $\approx 25^p$,
and arc NX = $24;3°$

$$\therefore \angle NBX = \angle ZBM = \begin{cases} 24;3^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \\ 12;1^\circ \text{ (approximately) where 4 right angles} = 360^\circ. \end{cases}$$

That [12;1°] is the size of the arc ZM of the epicycle.

But since the distance of point H, representing the moon, from M, the mean apogee, is one revolution minus [the mean anomaly of 333;12°], i.e. 26;48°, by subtraction [of arc ZM from arc MH], arc HZ = 14;47°.

$$\therefore \angle HBZ = \begin{cases} 14;47^\circ \text{ where 4 right angles} = 360^\circ \\ 29;34^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \end{cases}$$

and, in the circle about right-angled triangle HBL,
arc HL = 29;34°
and arc LB = 150;26° (supplement).
Therefore where hypotenuse BH = 120^p, the corresponding chords
HL = $30;37^p$ and LB = $116;2^p$.
Therefore where BH, the radius of the epicycle, is $5;15^p$
and (as was shown) BE = $48;31^p$,
HL = $1;20^p$ and LB = $5;5^p$.
H383 Therefore, by addition, EBL = $53;36^p$ where LH = $1;20^p$.
And since $EL^2 + LH^2 = EH^2$
EH $\approx 53;37^p$ in the same units.
Therefore in the circle about right-angled triangle EHL,
where hypotenuse EH = 120^p,
HL = $2;59^p$
and arc HL = 2;52°.
Therefore the equation of anomaly,

$$\angle HEL = \begin{cases} 2;52^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \\ 1;26^\circ \text{ where 4 right angles} = 360^\circ. \end{cases}$$

Q.E.D.

7. {*Construction of a table for the complete lunar anomaly*}[26]

In order again to provide a ready means of computing the individual additive or subtractive equations by setting out a table, we have supplemented the table

[26] See *HAMA* 93–5, Pedersen 195–202.

for the simple hypothesis set out above [IV 10] with columns which enable one to correct easily for the second lunar anomaly. For this purpose we again used the same geometrical methods [as explained above]. After the first two columns containing the argument, we inserted a third column containing the equation to be added to or subtracted from the anomaly in order to reduce the mean H384 motion counted from M [in Fig. 5.6], the mean apogee, to Z, the true apogee. [E.g.] above [p. 234], for the elongation of 90;30°, we showed that arc ZM is 12;1°, and thus, since the distance of the moon from M, the mean apogee, was 333;12°, we find that its distance from Z, the true apogee, was, obviously, 345;13°, which we must use as argument for the epicyclic equation correcting the mean motion in longitude. In the same way, for other elongations, taken at intervals appropriate [for the table], we calculated the corresponding amount of the equation in question. We did this by the same method [as above], (to cut a long story short), and entered the amount corresponding to each [tabulated] argument in the third column. Of the succeeding columns, the fourth will contain the equations of the epicyclic anomaly (already set out in the previous table [IV 10]), where the maximum equation reaches approximately 5;1°, corresponding to the ratio 60 : 5;15. The fifth column will contain the increments in the equations due to the second anomaly as compared with the first, in the situation where the maximum equation is $7\frac{2}{3}$°, corresponding to the H385 ratio 60 : 8.[27] Thus the fourth column is for the situation of the epicycle at the apogee of the eccentre (which occurs at the syzygies), and the fifth column is for the increments [to the equations] accruing from [the position of the epicycle][28] at the perigee of the eccentre (which occurs at the quadratures).

In order to enable one to find the proportion of these tabulated increments [in the fifth column] corresponding to a position of the epicycle in between those two locations [at apogee and perigee of the eccentre], we have added a sixth column. This contains, for each tabulated argument of elongation, the corresponding fraction (given in sixtieths) of the tabulated increment which must be added to the equation of anomaly tabulated in the fourth column. We have calculated these fractions in the following manner.

[See Fig. 5.7.] Let the moon's eccentre again be ABG on centre D and diameter ADG, on which E is taken as the centre of the ecliptic. Mark off arc AB, draw the epicycle, ZHΘK, on centre B, and draw line EBZ. Let the elongation be given, e.g., as 60°.

Hence by the same argument as before

$$\angle \text{AEB} = \text{double the given elongation} = 120°. \quad \text{H386}$$

Drop perpendicular DL from D on to BE produced, and draw HBKD. Suppose that the line from centre E to the moon, EMN, is tangent to the epicycle,

[27] The ratio is 39;22 (the distance from the earth to the perigee of the moon's eccentre, p. 226) to 5;15 (the radius of the moon's epicycle). This is approximately equal to 60 : 8.

[28] Excising ἀνωμαλίας at H385,7. Heiberg's text would have to mean 'accruing from the *anomaly* which is produced at the perigee of the eccentre, at the quadratures'. Besides being an exceedingly clumsy expression, this ruins the parallelism of the sentence. It is obvious that Ptolemy intended to contrast the two different positions of the epicycle, at apogee and perigee of the eccentre (cf. τῶν δύο τούτων θέσεων, H385,8-9). τῆς (H385,6) refers to θέσεως (understood from above; for ἀποτελεῖσθαι used with θέσις cf. H394,11-12). The interpolation of ἀνωμαλίας is the work of someone who looked for something for τῆς to refer to, but misunderstood this.

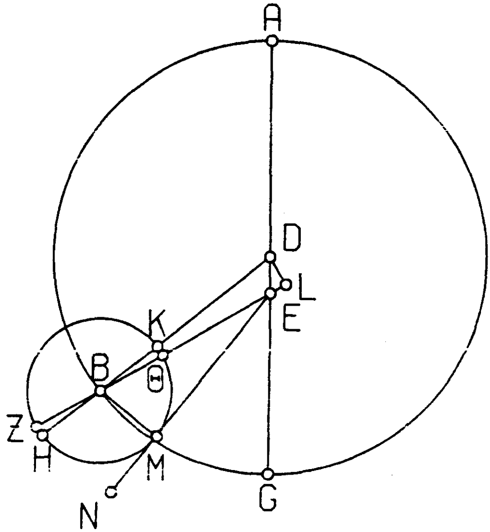

Fig. 5.7

producing a maximum equation of anomaly, and join BM. Then since

$$\angle \text{AEB} = \begin{cases} 120^\circ & \text{where 4 right angles} = 360^\circ \\ 240^{\circ\circ} & \text{where 2 right angles} = 360^{\circ\circ}. \end{cases}$$

$$\angle \text{DEL} = 120^{\circ\circ} \text{ (supplement)}.$$

Therefore in the circle about right-angled triangle DEL,

$$\text{arc DL} = 120^\circ$$
$$\text{and arc EL} = 60^\circ \text{ (supplement)}.$$

So the corresponding chords

$$\left.\begin{aligned} \text{EL} &= 60^p \\ \text{and DL} &= 103;55^p \end{aligned}\right\} \text{ where hypotenuse DE} = 120^p.$$

H387 Therefore where DE = $10;19^p$ and DB = $49;41^p$,

$$\text{EL} \approx 5;10^p$$
$$\text{and DL} = 8;56^p.$$

And, since $\text{BL}^2 = \text{BD}^2 - \text{DL}^2$,

$$\text{BEL} = 48;53^p,$$

and, by subtraction [of EL], EB = $43;43^p$,
where MB, the radius of the epicycle, is $5;15^p$.
Therefore in the circle about right-angled triangle BEM,

where hypotenuse EB = 120^p,

$$\text{BM} = 14;25^p$$
$$\text{and arc BM} = 13;48^\circ.$$

Therefore the maximum equation of anomaly,

$$\angle \text{BEM} = \begin{cases} 13;48^{\circ\circ} & \text{where 2 right angles} = 360^{\circ\circ} \\ 6;54^\circ & \text{where 4 right angles} = 360^\circ. \end{cases}$$

Thus, at this distance in elongation, the equation of anomaly differed from the 5;1° [of maximum equation] at the apogee [of the eccentre] by 1;53°. But the total difference [between maximum equation at apogee and] at perigee [of the eccentre] is 2;39°. So, where the total difference is 60, 1,53° will be 42;38. This is the amount which we will put in the sixth column corresponding to 120° of [double] elongation. H388

In exactly the same way we computed, for the other tabulated arguments, the fractions of the difference between the two maximum equations of anomaly, obtained in the above manner, and entered them, expressed in sixtieths of that difference, opposite the corresponding argument. It is obvious that the total 60 [sixtieths] correspond to the double of 90° of elongation, which is at 180° of the eccentre, the location of the perigee.

We also added a seventh column containing the position of the moon in latitude, on either side of the ecliptic, as measured along a circle through the poles of the ecliptic, i.e. the arc of the latter circle cut off between the ecliptic and the inclined circle of the moon on the same centre [as the ecliptic], for each [tabulated] position of the moon on its inclined circle. For this we have used the same procedure as we did to calculate the arcs of the circle through the poles of the equator [which are cut off] between the equator and the ecliptic [I 14]. Here, however, we took the arc between the ecliptic and the northern or southern limit of the inclined circle, as measured along the great circle through both their poles, as 5°. For, like Hipparchus, we find by calculation from the moon's most northerly and southerly apparent positions that its greatest deviation either side of the ecliptic is approximately that amount.[29] Further- H389
more, almost all circumstances of observations of the moon, whether taken with respect to the stars, or taken with instruments, fit a maximum latitudinal deviation of that amount, as will become clear from subsequent demonstrations.

The table of the complete lunar anomaly is as follows.

8. {*Table of the complete lunar anomaly*}[30] H390-1

[See p. 238.]

9. {*On the complete calculation of the moon's position*} H392

So, whenever we choose to calculate the moon's anomalistic position by means of the table set out, we take, for the moment in question at Alexandria, the mean

[29] The only details of an observation which confirm $i \approx 5°$ for the lunar orbit are at V 12 p. 247.

[30] In general the entries in this table are correct to within ±1 in the second place. However, in col. 3, arguments 123–9, 147–53 and 171–7 the error reaches −3 or −4, possibly because of interpolation between computed values. In col. 5 the first 9 values (from arguments 6 to 54 inclusive) are all too big, and the first 7 of them fit a ratio (radius of epicycle : distance of epicycle centre) of .136 (instead of .133 ≈ 5;15 : 39;22 which Ptolemy's text requires and which underlies all values from argument 60 on). This could be derived from a distance of 38;36^p or an epicycle radius of 5;21^p, neither of which has any motivation. I cannot explain this discrepancy, but it is too consistent to be the result of mere inaccurate calculation. In col. 6 the calculation to two sexagesimal places gives a quite illusory accuracy, and Ptolemy's results (for the second place) bear little relationship to what one gets with accurate calculation. However, this has a negligible effect on the accuracy of computations carried out with the table. In the Handy Tables Ptolemy quite properly tabulated only one place in this and the corresponding column in the planetary tables.

TABLE OF THE COMPLETE LUNAR ANOMALY

1	2	3	4	5	6	7	
Common Numbers		Equation for [Mean to True] Apogee	Epicyclic Equation	Increment in Epicyclic [Equation]	Sixtieths	Latitude	
6	354	0 53	0 29	0 14	0 12	4 58	Northern limit
12	348	1 46	0 57	0 28	0 24	4 54	
18	342	2 39	1 25	0 42	1 20	4 45	
24	336	3 31	1 53	0 56	2 16	4 34	
30	330	4 23	2 19	1 10	3 24	4 20	
36	324	5 15	2 44	1 23	4 32	4 3	
42	318	6 7	3 8	1 35	6 25	3 43	
48	312	6 58	3 31	1 45	8 18	3 20	
54	306	7 48	3 51	1 54	10 22	2 56	
60	300	8 36	4 8	2 3	12 26	2 30	
66	294	9 22	4 24	2 11	15 5	2 2	
72	288	10 6	4 38	2 18	17 44	1 33	
78	282	10 48	4 49	2 25	20 34	1 3	
84	276	11 27	4 56	2 31	23 24	0 32	
90	270	12 0	4 59	2 35	26 36	0 0	
93	267	12 15	5 0	2 37	28 12	0 16	
96	264	12 28	5 1	2 38	29 49	0 32	
99	261	12 39	5 0	2 39	31 25	0 48	
102	258	12 48	4 59	2 39	33 1	1 3	
105	255	12 56	4 57	2 39	34 37	1 17	
108	252	13 3	4 53	2 38	36 14	1 33	
111	249	13 6	4 49	2 38	37 50	1 48	
114	246	13 9	4 44	2 37	39 26	2 2	
117	243	13 7	4 38	2 35	41 2	2 16	
120	240	13 4	4 32	2 32	42 38	2 30	
123	237	12 59	4 25	2 28	44 3	2 43	
126	234	12 50	4 16	2 24	45 28	2 56	
129	231	12 36	4 7	2 20	46 53	3 8	
132	228	12 16	3 57	2 16	48 18	3 20	
135	225	11 54	3 46	2 11	49 32	3 32	
138	222	11 29	3 35	2 5	50 45	3 43	
141	219	11 2	3 23	1 58	51 59	3 53	
144	216	10 33	3 10	1 51	53 12	4 3	
147	213	10 0	2 57	1 43	54 3	4 11	
150	210	9 22	2 43	1 35	54 54	4 20	
153	207	8 38	2 28	1 27	55 45	4 27	
156	204	7 48	2 13	1 19	56 36	4 34	
159	201	6 56	1 57	1 11	57 15	4 40	
162	198	6 3	1 41	1 2	57 55	4 45	
165	195	5 8	1 25	0 52	58 35	4 50	
168	192	4 11	1 9	0 42	59 4	4 54	
171	189	3 12	0 52	0 31	59 26	4 56	
174	186	2 11	0 35	0 21	59 37	4 58	
177	183	1 7	0 18	0 10	59 49	4 59	Southern limit
180	180	0 0	0 0	0 0	60 0	5 0	

motions of the moon in longitude, elongation, anomaly and latitude, in the way explained.[31] Then we always, first, double the figure computed for the elongation, and (after subtracting 360°, if necessary), enter with this into the table of anomaly and take the corresponding amount in the third column. If the double elongation is less than 180° we add the amount [in the third column] to the mean anomaly, but if the double elongation is greater than 180° we subtract the amount from the mean anomaly. We enter with the resulting true anomaly into the same table, and take the corresponding equation in the fourth column and also the corresponding increment in the fifth column, and write [both] down separately. Next we enter with the doubled mean elongation into the same table, take the sixtieths corresponding to it in the sixth column, multiply the increment which we wrote down separately by that number of sixtieths, and always add the result to the previously computed equation from the fourth column. If the true anomaly is less than 180°, we subtract this sum from the mean longitude and mean [argument of] latitude, but add it to them if the true anomaly is greater than 180°. Thus we have [two] numbers: we add the one for the longitude to the position [of the mean moon] at epoch: the result will be the true position of the moon. With the one for the [argument of] latitude, counted from the northern limit, we enter into the same table: the number corresponding to it in the seventh column will be the distance of the moon's centre from the ecliptic, measured along the great circle through the poles of the ecliptic. If the argument falls within the first 15 lines, it will be to the north [of the ecliptic], but if it falls below the first 15 lines, it will be to the south. The first column of argument comprises the moon's motion from north to south, and the second column its motion from south to north. H393

10. {*That the difference at the syzygies due to the moon's eccentre is negligible*}[32] H394

Now it is likely that some people will suspect that the moon's eccentric circle might also have a considerable effect at conjunctions and oppositions and the eclipses occurring at them, since the centre of the epicycle does not always under all circumstances stand exactly at the apogee at those times, but can be removed from the apogee by an arc [of the eccentre] of considerable size, because location precisely at the apogee occurs at the mean syzygies, whereas the determination of true conjunction and opposition requires taking the anomalies of both luminaries into account. Therefore we shall try to show that this difference cannot produce any considerable error in [calculation of] the phenomena at syzygies, even if the correction due to the eccentricity is not taken into account.

[31] Ptolemy has not in fact explained how to do this, but the essence of the procedure is the same as that explained for the sun at III 8. Note here, however, that the 'mean motions' in elongation, anomaly and latitude must include the epoch positions, whereas, according to the procedure in the text, the 'mean motion in longitude' does not include the epoch position, which is added only at a later stage. For the procedure in general see *HAMA* 193–6, Pedersen 197–9 and, for a worked example, *HAMA* 96 or my Appendix A Example 9.

[32] See *HAMA* 98–9.

Let [Fig. 5.8][33] the moon's eccentric circle be ABG on centre D and diameter
H395 ADG, on which the centre of the ecliptic is taken at point E, and the point of 'direction',[34] opposite to D, as Z. Cut off arc AB from the apogee A, and draw the epicycle, HΘKL, on centre B. Join BD, HBKE and BLZ.

Now the size of the [equation of] anomaly can differ from that of the apogee situation of the epicycle (at A) in two ways:

[1] because the epicycle is removed towards the perigee, the epicycle subtends a larger angle at E;

[2] the direction in which the diameter through mean apogee and perigee [of the epicycle] points is no longer towards E but towards Z.

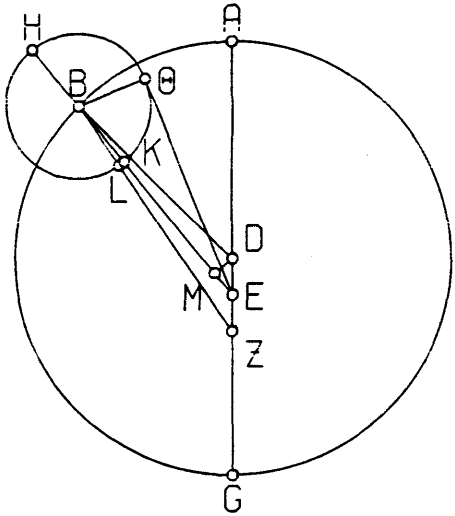

Fig. 5.8

The effect from the first factor is a maximum when the moon's equation of anomaly is a maximum, while the effect of the second factor is a maximum when the moon is near the apogee or perigee of the epicycle. Hence it is clear that when the maximum effect of the first factor occurs, the effect of the second factor will be quite negligible, since the moon's equation of anomaly hardly
H396 varies for a considerable distance either side of its situation on the tangent to the epicycle. However, [in this situation] the true syzygy can differ from the mean by the sum of the equations of the two luminaries, if one is additive and the other subtractive. On the other hand, when the maximum effect of the second factor, the difference due to the direction, occurs, then again the effect of the first factor is negligible, since the complete equation of anomaly is either zero or very small when the moon is near the apogee or perigee of the epicycle. But [in this case] the true syzygy will differ from the mean only by the sun's equation of anomaly.

[33] Fig. 5.8 is wrongly drawn in Heiberg's text, where DΘ has been connected instead of the tangent EΘ. This is an error of Heiberg's, unsupported by the mss., and corrected by Manitius.

[34] πρόσνευσις. See p. 227 n.19.

Let us suppose, then, that the sun has a maximum additive equation of 2;23°, and (first) that the moon too has a maximum (but subtractive) equation of 5;1°. Thus ∠ AEB contains twice the sum of the above, 7;24°, i.e. 14;48°. Draw EΘ from E tangent to the epicycle, and drop perpendicular BΘ on to it, and also perpendicular DM from D on to BE. Then since

$$\angle\text{ AEB} = \begin{cases} 14;48^\circ & \text{where 4 right angles} = 360^\circ \\ 29;36^{\circ\circ} & \text{where 2 right angles} = 360^{\circ\circ}, \end{cases}$$

in the circle about right-angled triangle DEM H397

arc DM = 29;36°

and arc EM = 150;24° (supplement).

Therefore the corresponding chords

$$\left.\begin{aligned} \text{DM} &= 30;39^p \\ \text{and EM} &= 116;1^p \end{aligned}\right\} \text{ where hypotenuse DE} = 120^p.$$

Therefore where DE, the distance between the centres, is $10;19^p$, and BD, the radius of the eccentre, is $49;41^p$,

DM = $2;38^p$

and EM = $9;59^p$.

And since $BM^2 = BD^2 - DM^2$,

BM = $49;37^p$,

and, by addition [of EM], BME = $59;36^p$,

where BΘ, the radius of the epicycle, is $5;15^p$.

Therefore in the circle about right-angled triangle BEΘ,

where hypotenuse EB = 120^p,

BΘ = $10;34^p$,

and arc BΘ = 10;6°.

Therefore the angle of the maximum equation of anomaly,

$$\angle\text{ BE}\Theta = \begin{cases} 10;6^{\circ\circ} & \text{where 2 right angles} = 360^{\circ\circ} \\ 5;3^\circ & \text{where 4 right angles} = 360^\circ, \end{cases}$$

instead of the 5;1° for the apogee position of the epicycle at A. H398

Therefore the difference in the equation of anomaly due to this effect was found to be 2 sixtieths of a degree, which cannot produce an error of even as much as $\frac{1}{16}$th of an hour.[35]

Next let the moon be at L, the mean perigee. Thus ∠ AEB will contain, approximately, only double the sun's [maximum] equation of anomaly, namely 4;46°. With a figure [5.9] similar [to the preceding], draw line EL, and drop perpendiculars LN (from L) and DM (from D) on to BE, and ZX from Z on to BE produced. Then, by the same kind of calculation as before, since the angle at E,

$$[\angle\text{ AEB}] = \begin{cases} 4;46^\circ & \text{where 4 right angles} = 360^\circ \\ 9;32^{\circ\circ} & \text{where 2 right angles} = 360^{\circ\circ}, \end{cases}$$

in the circles about right-angled triangles EDM and EZX,

arc DM = arc ZX = 9;32°

and arc EM = arc EX = 170;28° (supplements). H399

$$\left.\begin{aligned} \therefore \text{ Crd arc DM} = \text{Crd arc ZX} &= 9;58^p, \\ \text{and Crd arc ME} = \text{Crd arc EX} &= 119;35^p \end{aligned}\right\} \begin{aligned} &\text{where hypotenuses DE and EZ} \\ &= 120^p. \end{aligned}$$

[35] In the time of an eclipse. See p. 136 n.16.

Therefore where $DE = EZ = 10;19^p$
and DB, the radius of the eccentre, is $49;41^p$,

$$DM = ZX = 0;51^p$$

and $ME = EX = 10;17^p$.

And since $BM^2 = BD^2 - DM^2$,

$$BM \approx 49;41^p.$$

$$\therefore BE = [BM + ME =]\ 59;58^p,$$

and, by addition [of EX], $BX = 70;15^p$ where $ZX = 0;51^p$.

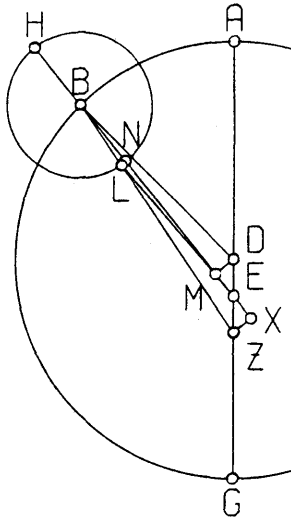

Fig. 5.9

Therefore by the same argument, hypotenuse BZ [of triangle BZX] will be approximately the same size [as BX], $70;15^p$.

And $BZ:ZX = BL:LN$ and $BZ:BX = BL:BN$.

Therefore where BL, the radius of the epicycle, is $5;15^p$,
and BE, as was shown $= 59;58^p$,

$$LN = 0;4^p \text{ and } BN \approx 5;15^p,$$

H400 and, by subtraction [of BN from BE], $NE = 54;43^p$ where $LN = 0;4^p$.

And since, from the preceding, hypotenuse EL [of triangle ELN] is not noticeably different from this amount of $54;43^p$, it follows that, where hypotenuse $EL = 120^p$,

$$LN \approx 0;8^p,$$

and, in the circle about right-angled triangle ELN,

$$\text{arc } LN = 0;8^p.$$

Therefore the difference in the moon's position due to the direction towards Z,

$$\angle BEL = \begin{cases} 0;8^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \\ 0;4^{\circ} \text{ where 4 right angles} = 360^{\circ}.\text{[36]} \end{cases}$$

[36] Ptolemy's final result is correct (to the nearest minute), but some of the intermediate results are inaccurate. E.g. just above in the computation of LN, 0;4 × 120/54;43 is much closer to 0;9 than to

Thus here too the difference in the moon's equation of anomaly is [only] 4 minutes of arc; and even this does not produce a significant error in the phenomena at the syzygies, since it cannot reach as much as $\frac{1}{8}$th of an hour, an amount one may expect to encounter frequently as a purely observational error.

We made the above argumentation, not to show that one cannot take these differences into account, very small though they be, for the computation of syzygies too, but to show that we committed no noticeable error in our previous demonstrations using lunar eclipses when we used the [simple hypothesis], and not that supplemented by introducing the eccentre.

11. {*On the moon's parallaxes*}[37] H401

With the above we have about disposed of the [elements] necessary for finding the true positions of the moon. However, in the case of the moon there is the additional problem that its apparent position does not coincide with its true position, even to the senses. For, as we said [IV 1 p. 173], the earth does not bear the ratio of a point to the distance of the moon's sphere. Hence it is both necessary and appropriate to discuss the lunar parallaxes, especially in order to deal with the theory of solar eclipses, amongst other phenomena. By means of the lunar parallaxes it will be possible, given a true position [of the moon], [i.e. its position] with respect to the centre of the earth and of the ecliptic, to determine its position as seen from the standpoint of the observer, that is from some point on the earth's surface, and, *vice versa*, to determine the true position from the apparent position. Now it is a feature of this kind of enquiry that one cannot find the amount of the parallax for individual situations unless one is first given the ratio of the distance [of the body to the earth's radius], nor can one find the ratio of the distance without the parallax for some particular situation being given. Hence for those bodies with no perceptible parallax, namely, those to [the distance of] which the earth bears the ratio of a point, it is, H402
obviously, impossible to find the ratio of the distance. But in the case of those bodies, like the moon, which do exhibit a parallax, the only appropriate procedure is, first given some particular parallax, to find the ratio of the distance. For it is possible to make an observation of a [particular] parallax of this kind by itself, but quite impossible to determine the amount of the distance [by itself].

Now Hipparchus used the sun as the main basis of his examination of this problem. For since it follows from certain other characteristics of the sun and moon (which we shall discuss subsequently) that, given the distance to one of the luminaries, the distance to the other is also given, Hipparchus tries to demonstrate the moon's distance by guessing at the sun's. First he supposes that the sun has the least perceptible parallax, in order to find its distance, and then

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0;8. It looks as if he computed to two sexagesimal fractional places, and then fudged the results somewhat in the presentation.

[37] On chs. 11 and 12 see *HAMA 100–1, Pedersen 203–4.*

he uses the solar eclipse which he adduces; at one time he assumes that the sun has no perceptible parallax, at another that it has a parallax big enough [to be observed]. As a result the ratio of the moon's distance came out different for him for each of the hypotheses he put forward; for it is altogether uncertain in the case of the sun, not only how great its parallax is, but even whether it has any parallax at all.[38]

H403 12. {*On the construction of a parallactic instrument*}[39]

We, in contrast, to avoid taking any uncertain factors into our examination of this topic, constructed an instrument to enable us to observe as accurately as possible the amount of the moon's parallax, and its zenith distance, along the great circle through the poles of the horizon and the moon.

We made two rods [Fig. G,1,2], rectangular [in cross-section], no less than 4 cubits long, so as to admit finer graduation, and with a cross-section of sufficient size that they were not distorted because of their length, but each side conformed very strictly to a straight line. Then we drew a straight line along the middle of the broader side of each rod, and affixed to one of them [Fig. G,2], at each end, centred on the line, and perpendicular [to it], two rectangular plates, of equal size and parallel to each other [Fig. G,a,b]; each plate had an aperture exactly in the centre, the aperture at the eye being small, and that towards the moon being greater, in such a way that when one eye was placed at the plate
H404 with the smaller aperture, the whole of the moon would be visible through the aperture on the other plate, which was aligned [with the first aperture]. We made a perforation of equal size through both rods at the end of the median line near the plate with the larger hole, and fitted a peg [Fig. G,c] through both perforations in such a way that the sides of the rods inscribed with the lines[40] were fastened together round the peg as a centre, but the rod with the plates could rotate freely in all directions without distortion. We wedged the rod with no plates on it [Fig. G,1] into a base [Fig. G,4]. On the median line of each rod, at the end by the base, we took a point as far as possible from the centre of the peg (the same distance from it [on both rods]), and, on the rod with the base, divided the line so defined into 60 sections, subdividing each section into as many subdivisions as possible. We also attached to the back of the same rod, at its end, [two] plates [Fig. G,d,d] having their corresponding faces aligned with

[38] This passage is supplemented by Pappus' commentary ad loc. (Rome[1] I 67–8), which extracts some details of the two procedures of Hipparchus from Books 1 and 2 respectively of the latter's 'On sizes and distances'. For details of the important historical consequences which can be drawn see Toomer[9] (showing that the solar eclipse referred to is that of −189 Mar. 14), which builds on the work of Swerdlow, 'Hipparchus'.

[39] On the instrument described in this chapter (known in the middle ages as a 'triquetrum') see Price, 'Precision Instruments' 589–90 with Fig. 344. My Fig. G is based on the text of the Almagest rather than on the figure provided by Pappus in his commentary (Rome[1] I p. 71, with a modern reconstruction; see also Rome's notes on pp. 70–5).

[40] The faces of rods 1 and 2 inscribed with the lines cannot be flush with one another, as is clear from Fig. G. Ptolemy seems to mean only that one views the inscribed faces of the two rods as radii of a circle with centre peg c.

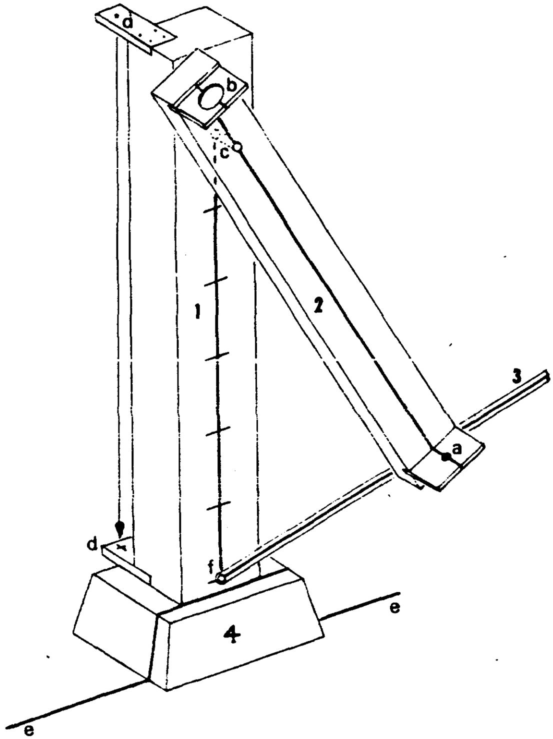

Fig. G (part I)

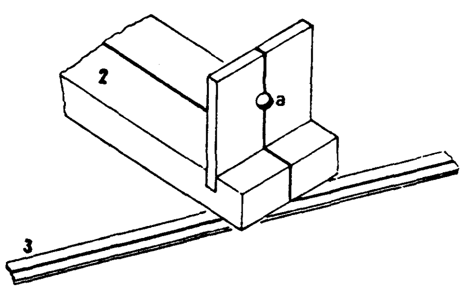

Fig. G (part II)

each other,[41] and each being equidistant in all respects from that same median line, so that when a plumb-line was suspended between them, the rod could be set up exactly perpendicular to the plane of the horizon. We also had a meridian line [Fig. G,e] ready drawn in the plane parallel to that of the horizon in an H405
unshaded place. We set the instrument upright in such a way that the sides of

[41] Excising the words πρὸς τῇ αὐτῇ γραμμῇ at H404,17–18. That would mean 'each having that face which was on the same side as the [graduated] line aligned with the other'. But this is impossible, since the plates are not to one side of the face with the graduations, but 'on the back', i.e. on the face opposite the graduated line. This is also clear from Pappus' detailed description (Rome

the rods which were held flush with each other by the peg lay in the meridian, being parallel to the meridian line, and the rod with the base was fixed exactly perpendicular, in a firm and immovable position, while the other rod could move in the plane of the meridian about the peg, responding to the pressure [of the user].[42] We also added another thin, straight rod, [Fig. G,3] attached by a small pin [Fig. G,f] at the base end of the graduated line, so that it too could be rotated, and long enough to reach the end of the line on the other rod equidistant [from the peg] when it was rotated to its maximum distance [from the base];[43] thus by rotating it at the same time as the latter, one could use it to show the straight-line distance between the ends [of the centre-lines on the two rods].

We made our observations of the moon as follows. The moon had to be located on the meridian, and near the solstices on the ecliptic, since at such situations the great circle through the poles of the horizon and the centre of the moon very nearly coincides with the great circle through the poles of the
H406 ecliptic, along which the moon's latitude is taken. Furthermore the true distance [of the moon] from the zenith can also be conveniently determined from the same situation. When the moon was precisely in the meridian, we moved the rod with the [sighting-] plates on it round to the position in which the centre of the moon, when sighted through both apertures, was in the centre of the larger aperture. We marked on the thin rod the distance between the ends of the lines on the [two] rods, then applied the distance [marked on the thin rod] to the line on the upright rod graduated into 60 sections. Thus we found the amount of that distance in those units of which the radius of the circle described by the rotation [of the rod with the sighting-plates] in the plane of the meridian contains 60. By calculating the arc corresponding to that chord, we found the angular distance of the apparent centre of the moon from the zenith, measured along the great circle through the poles of the horizon and the moon's centre, which coincided at that moment with the [great circle] through the poles of the equator and the ecliptic, [i.e.] the meridian.

In order, first, to determine the precise amount of the moon's greatest
H407 deviation in latitude, we made sightings when the moon was simultaneously

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p. 75). πρὸς τῇ αὐτῇ γραμμῇ is a stupid gloss on ἐπὶ τὰ αὐτὰ μέρη, which I have translated 'corresponding', but which literally means 'in the same direction'. The interpolation is old, since it is found in the Arabic tradition.

[42] I.e. the peg held the rods together tightly enough so that rod 2 would not move under its own weight, but loosely enough so that it could be rotated by the user.

[43] This rod has indeed to be 'thin', since it has to pass between the two rods 1 and 2, the faces of which are supposed to be flush. Pappus overcomes this difficulty by saying that rod 2 has to be hollowed out along its length to the depth of the thickness of rod 3 (Rome p. 73). There is the further difficulty that according to Ptolemy's instructions rod 3 has to be long enough to reach to the end of rod 2 at the maximum rotation, presumably 90°: hence its length should be ($\sqrt{2}$ × length of the graduated line). But since one measures the chord of the zenith distance, not directly on rod 3, but by marking it on rod 3 and then measuring it on the scale on rod 1, no zenith distance greater than 60° (the chord of which is 60^p) can be measured. Hence, presumably, Pappus (p. 73) says that rod 3 should be less than the length of the graduated line. Rome (p. 73 n.0) suggests that Ptolemy deliberately chose this limit to avoid the complications of refraction near the horizon. It seems more likely that it is simply a by-product of Ptolemy's construction, and that Pappus' shortening of the rod was done to avoid the difficulties which would result from trying to apply rod 3 to the graduated line if it were 60^p or more.

near the summer solstice and near the northern limit of its inclined circle.[44] For in the region of those points the moon's latitude remains sensibly the same over a considerable interval, and furthermore, since the moon is then very near the zenith at the parallel through Alexandria (at which we made our observations), its apparent position is approximately the same as its true position. At such situations it was found that the distance of the centre of the moon from the zenith was always about $2\frac{1}{8}°$. Hence by this method too the moon's greatest latitude either side of the ecliptic is shown to be 5°. For the zenith distance of the equator at Alexandria has been shown to be 30;58°; if we subtract from this the $2\frac{1}{8}°$ (which is the apparent distance [of the centre of the moon from the zenith]), the result [$28;50\frac{1}{2}°$] is about 5° greater than the distance from the equator to the summer solstice, which was shown to be 23;51°.

Then, in order to attack the problem of the parallaxes, we observed the moon in the same way, but this time when it was near the winter solstice, both for the H408 reason already mentioned [above] and because its distance from the zenith in that situation is the greatest of all such meridian positions, and thus provides us with a greater and more easily determinable parallax. We will set out one of a number of parallax observations which we made at such situations. By this means we shall display the method of calculation and at the same time provide a demonstration of the rest of what is to follow in the appropriate order.

13. {*Demonstration of the distances of the moon*}[45]

In the twentieth year of Hadrian, Athyr [III] 13 in the Egyptian calendar [135 Oct. 1], $5\frac{5}{6}$ equinoctial hours after noon, just before sunset, we observed the moon when it was on the meridian. The apparent distance of its centre from the *zenith, according to the instrument, was $50\frac{11}{12}°$. For the distance [measured] on* the thin rod was $51\frac{7}{12}$ of the 60 subdivisions into which the radius of revolution had been divided, and a chord of that size subtends an arc of $50\frac{11}{12}°$. Now the time from epoch in the first year of Nabonassar to the moment of the above observation is

$$882 \text{ Egyptian years 72 days} \begin{cases} 5\frac{5}{6} \text{ equinoctial hours reckoned simply} , \\ 5\frac{1}{3} \text{ equinoctial hours reckoned accurately.} \end{cases}$$ H409

For this moment we find:

mean longitude of the sun:	♎ 7;31°
true longitude of the sun:	♎ 5;28°
mean longitude of the moon:	♐ 25;44°
elongation:	78;13°

distance [in anomaly] from mean apogee of epicycle: 262;20°
distance in [argument of] latitude from the northern limit: 354;40°.

[44] Since the revolution of the node takes place once in about $18\frac{2}{3}$ years, this situation occurs $9\frac{1}{3}$ years earlier or later than the similar situation of the moon near the winter solstice, observed by Ptolemy (V 13) in Oct. 135. Therefore these observations were made either in the summer of 126, or *in the spring of 145. This is the only useful conclusion that can be drawn from the confused* discussion of Newton, 184-6.

[45] See *HAMA* 101-3, Pedersen 204-7.

Hence the complete equation of anomaly, derived from the appropriate table, was +7;26°, so that the true position of the moon at that moment was:

in longitude:	♑ 3;10°
in [argument of] latitude on the inclined circle:	2;6° from the northern limit
in latitude on the great circle through the poles of the ecliptic (which almost coincided at that moment with the meridian):[46]	4;59° north of the ecliptic.

Now ♑ 3;10° is 23;49° south of the equator on the same [meridian] circle, and the equator is, likewise, 30;58° south of the zenith at Alexandria. Therefore the true distance of the centre of the moon from the zenith was [23;49 + 30;58 − 4;59 =] 49;48°. And its apparent distance was 50;55°. Therefore the moon's
H410 parallax at the distance [of the moon from the earth] corresponding to the position in question was 1;7° along the great circle through the moon and the poles of the horizon, when its true distance from the zenith was 49;48°.

Now that we have established that, draw [Fig. 5.10] in the plane of the great circle through the poles of the horizon and the moon the following great circles, on the same centre:

that of the earth, AB;

that through the centre of the moon at the [above] observation, GD:

the great circle to which the earth bears the ratio of a point, EZHΘ.

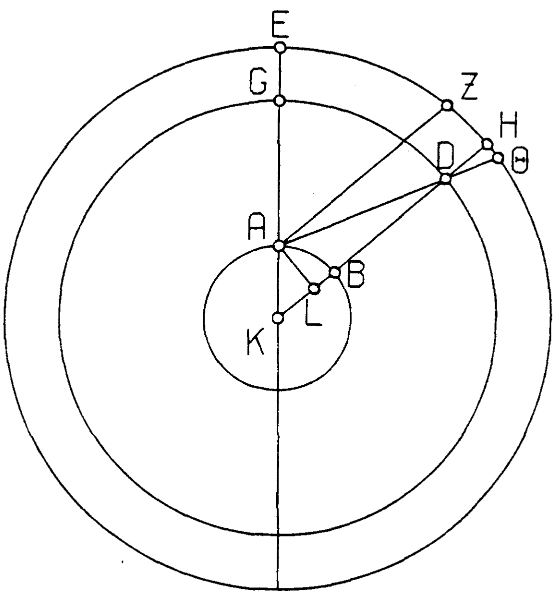

Fig. 5.10

[46] For the moon was almost at the winter solstice (cf. p. 247).

Let their common centre be K, and let the line through the points at the zenith be KAGE. Let us assume that the same distance of the moon, D, from the zenith at G is the amount already determined, 49;48°. Join KDH, ADΘ, and H411
furthermore from point A, which represents the observer's eye, draw AL as perpendicular to KB, and AZ as parallel to KH.

Then it is obvious that for an observer at point A the moon's parallax was arc HΘ. So arc HΘ is 1;7°, according to the calculation from the observation. But since arc ZΘ is negligibly greater than arc HΘ (for the whole earth bears the ratio of a point to circle EZHΘ), arc ZHΘ is very nearly the same, 1;7°. And since, again, point A is negligibly different from the centre of circle ZΘ,

$$\angle ZA\Theta = \begin{cases} 1;7^\circ & \text{where 4 right angles} = 360^\circ \\ 2;14^{\circ\circ} & \text{where 2 right angles} = 360^{\circ\circ}. \end{cases}$$

$$\text{And } \angle ADL = \angle ZA\Theta = 2;14^{\circ\circ}.$$

Therefore in the circle about right-angled triangle ADL,

$$\text{arc } AL = 2;14^\circ$$

$$\text{and Crd arc } AL = 2;21^p \text{ where hypotenuse } AD = 120^p.$$

But LD is negligibly smaller than AD.

$$\text{Therefore where } LA = 2;21^p,\ LD \approx 120^p.$$

Furthermore since, by hypothesis, arc GD = 49;48°, H412
the angle at the centre of the circle,

$$\angle GKD = \begin{cases} 49;48^\circ & \text{where 4 right angles} = 360^\circ \\ 99;36^{\circ\circ} & \text{where 2 right angles} = 360^{\circ\circ}. \end{cases}$$

Therefore in the circle about right-angled triangle ALK

$$\text{arc } AL = 99;36^\circ$$

$$\text{and arc } LK = 80;24^\circ \text{ (supplement).}$$

Therefore the corresponding chords

$$\left.\begin{aligned} AL &= 91;39^p \\ \text{and } LK &= 77;27^p \end{aligned}\right\} \text{where hypotenuse } AK = 120^p.$$

Therefore where AK, the radius of the earth, is 1^p,

$$AL = 0;46^p$$

$$\text{and } KL = 0;39^p.$$

$$\text{But where } AL = 2;21^p,\ LD, \text{ as was shown, } = 120^p.$$

$$\text{Therefore where } AL = 0;46^p,\ LD = 39;6^p.$$

And, in the same units, KL = $0;39^p$.
and the radius of the earth, KA = 1^p.
Therefore where KA, the radius of the earth, is 1^p,
by addition, KLD, which represents the distance of the moon at the observation, is $39;45^p$.[47]

Now that we have demonstrated this, let [Fig. 5.11] the moon's eccentre be ABG on centre D and diameter ADG, on which E is taken as the centre of the ecliptic, and Z as the point towards which [the mean apogee diameter of] the H413
epicycle is directed. Draw the epicycle, HΘKL, on point B, and join HBΘE, BD and BKZ. Let L represent the position of the moon at the observation in

[47] There is an accumulated error here, due to a series of small inaccuracies and roundings. More accurate would be $39;50^p$.

question, and draw perpendiculars to BE, DM from D[48] and ZN from Z.

Then since the amount of the elongation at the time of the observation was 78;13° [p. 247], it follows from the theory previously established that

$$\angle\, \mathrm{AEB} = 156;26° \text{ where 4 right angles} = 360°;$$

H414 hence its supplement, $\angle\, \mathrm{ZEN} = \angle\, \mathrm{DEM} = \begin{cases} 23;34° & \text{where 4 right angles} = 360° \\ 47;8^{\circ\circ} & \text{where 2 right angles} = 360^{\circ\circ}. \end{cases}$

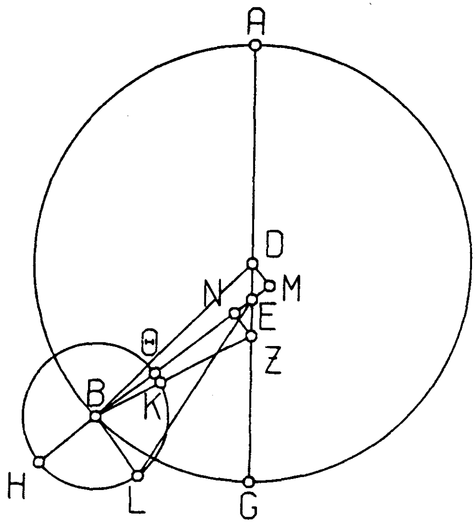

Fig. 5.11

Therefore in the circles about the corresponding right-angled triangles, [ZEN, DEM], since DE = EZ,

$$\text{arc DM} = \text{arc ZN} = 47;8°$$
$$\text{and arc EM} = \text{arc EN} = 132;52° \text{ [supplements].}$$

Therefore the corresponding chords

$$\left.\begin{array}{l} \mathrm{DM} = \mathrm{ZN} = 47;59^{p} \\ \text{and } \mathrm{EM} = \mathrm{EN} = 110;0^{p} \end{array}\right\} \text{ where hypotenuse DE} = \text{hypotenuse EZ} = 120^{p}.$$

Therefore where DE = EZ = $10;19^{p}$ and DB, the radius of the eccentre, is $49;41^{p}$,

$$\mathrm{DM} = \mathrm{ZN} = 4;8^{p}$$
$$\text{and } \mathrm{EM} = \mathrm{EN} = 9;27^{p}.$$
$$\text{And since } \mathrm{BM}^2 = \mathrm{BD}^2 - \mathrm{DM}^2,$$
$$\mathrm{BM} = 49;31^{p}.$$
$$\text{And } \mathrm{BE} = [\mathrm{BM} - \mathrm{EM} =]\ 40;4^{p},$$

and, by subtraction [of EN from BE], BN = $30;37^{p}$ where ZN = $4;8^{p}$.

$$\text{And since } \mathrm{BN}^2 + \mathrm{ZN}^2 = \mathrm{BZ}^2,$$
$$\text{hypotenuse BZ} = 30;54^{p}.$$

[48] Heiberg rightly excised ἐκβληθεῖσαν ('extended') at H413,7 as an unnecessary gloss which disturbs the sentence structure. Transferring it after BE (as Halma and Manitius) is no improvement, since the perpendicular from Z is *not* on the extension of BE.

Therefore in the circle about right-angled triangle BZN,

where hypotenuse BZ = 120^p,

ZN = $16;2^p$

and arc ZN = 15;21°.

$\therefore \angle$ ZBN = $\begin{cases} 15;21^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \\ \textit{about } 7;40^\circ \textit{ where 4 right angles} = 360^\circ. \end{cases}$ H415

That [7;40°], then, is the size of arc ΘK of the epicycle.

Next, the distance of the moon from the mean apogee of the epicycle at the moment of the observation was 262;20° [p. 247], and, obviously, its distance from K, the mean perigee, was 82;20° (by subtraction of a semi-circle).

Therefore arc KL = 82;20°

and arc ΘKL = [arc ΘK + arc KL =] 90;0°.

So ∠ ΘBL is a right angle.

$\therefore EL^2 = BL^2 + EB^2$,

and where DB, the radius of the eccentre, is $49;41^p$

and BL, the radius of the epicycle, is $5;15^p$,

EB, as we showed = $40;4^p$.

$\therefore$ EL = $40;25^p$.

Therefore the distance of the moon at the observation is $40;25^p$,

where BL, the radius of the epicycle, is $5;15^p$

and where EA, the distance from the centre of the earth to the apogee of the eccentre, is 60^p,

and where EG, the distance from the centre of the earth to the perigee of the eccentre, is $39;22^p$.

But we showed that the moon's distance at the observation, that is EL, was $39;45^p$ where the radius of the earth is one. H416

Therefore where EL, the distance of the moon at the observation, is $39;45^p$, and the earth's radius is 1^p,

EA, the mean distance at the syzygies = $59;0^p$,[49]

EG, the mean distance at the quadratures = $38;43^p$,

and the radius of the epicycle = $5;10^p$.

Q.E.D.

14. {*On the ratio of the apparent diameters of sun, moon and shadow at the syzygies*}[50]

Now that we have demonstrated the distances of the moon in the above manner, the appropriate sequel is to demonstrate those of the sun as well. This

[49] This result for the moon's mean distance agrees well with the facts (it is slightly greater than 60 earth-radii), which means that Ptolemy's parallax at syzygies (i.e. at solar eclipses) is fairly accurate. However, the process by which it is reached contains a number of errors (in the observed parallax, the latitude, the declination etc., and in the distance resulting from Ptolemy's model), which 'miraculously' cancel each other out. For details see *HAMA* 102-3. This is no accident: Ptolemy knew (approximately) what the parallax had to be at eclipses, and chose an observation which produced that amount. For a suggestion that the figure of 59 earth-radii had already been derived by Hipparchus see Toomer[9] 131.

[50] The chapter heading is placed by most Greek mss. (and by Heiberg's text) before H416, 20. I have transferred it here (before H416,9), following the Arabic mss. (cf. also D, which has it in the

too can readily be performed geometrically, if we are given, in addition to the distances of the moon at the syzygies, the sizes of the angles formed at the [observer's] eye at the syzygies by the diameters of the sun, moon and shadow.

Of the various methods used to solve the latter problem, we have rejected those claiming to measure the luminaries by measuring [the flow of] water or by the time [the bodies] take to rise at the equinox,[51] since such methods cannot
H417 provide an accurate result for the matter in hand. Instead, we too constructed the kind of dioptra which Hipparchus described, which uses a four-cubit rod,[52] and, observing with this, found that the sun's diameter always subtends approximately the same angle, there being no noticeable difference due to [the variation in] its distance, but that the moon subtends the same angle as the sun only when it is at its greatest distance from the earth (i.e. the apogee of the epicycle) at full moon, in contradiction to the hypotheses of my predecessors, [who assumed that it subtends the same angle as the sun at full moon] when it is at mean distance.[53] Furthermore, we find that the angles themselves are considerably smaller than those traditionally accepted.[54] However our computation of the latter rests, not on measurement with the dioptra, but on certain lunar eclipses. For although it was possible to determine readily from the dioptra, as constructed, when both diameters subtend the same angle (since such a determination involves no actual measurement), the *amount* [of the angle subtended] seemed utterly dubious to us, since the measurement[55] involving the positioning of the width [of the plate] which covers [the body being sighted] on

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upper margin), as a more appropriate break. Cf. Introduction p. 5. On ch. 14 see *HAMA* 103-8, Pedersen 207-9 (with the corrections Toomer [3] 140, 143, 149).

[51] According to Pappus ad loc. (Rome[1] I 87-9) 'the more ancient astronomers' used water-clocks to measure the time taken by the sun to cross the horizon, a procedure criticised by Hipparchus. He refers to a lost work of Heron, περὶ ὑδρίων ὡροσκοπείων, on which see also Proclus, *Hypotyposis* IV 73-6 (ed. Manitius p. 120-2). At H416,21 Heiberg rightly accents ὑδρομετριῶν (from the abstract ὑδρομετρία). There is no evidence for the existence of ὑδρομετρίον, 'vessel for measuring flow of water', conjectured by LSJ s.v. In the corresponding passage Proclus p. 120 line 14 we should read ὑδρολογίων. Cf. also *HAMA* 103 n. 1.

[52] There are ancient descriptions of this instrument by Pappus in his commentary ad loc. (Rome[1] I 90-2) and by Proclus, *Hypotyposis* IV 87-96 (ed. Manitius pp. 126-30). See Price, 'Precision Instruments' 591, and, for modern literature, *HAMA* 103 n.2. The essential feature is a plate (πρισμάτιον, H417,22-3) which can be moved along a graduated rod until it appears to exactly cover the object being sighted by the eye placed at one end of the rod.

[53] It was shown by Swerdlow, 'Hipparchus' 291-8, that Hipparchus was one of those who held this. An important consequence of this hypothesis is that annular solar eclipses become possible, whereas under Ptolemy's assumption they are impossible.

[54] Hipparchus (see IV 9 p. 205) assumed that the moon at mean distance subtends a six hundred and fiftieth of its circle, or about 0;33,14°; hence his figure for the sun's diameter was the same. Ptolemy (below) finds that when moon and sun have the same apparent diameter (at maximum distance) it is 0;31,20°, considerably smaller. This must be what he means here. However, his value for the lunar diameter at *mean* distance, 0;33,20°, is negligibly different from Hipparchus'.

[55] Excising πλείστης οὔσης at H417,23, to which I can attach no meaning (it cannot mean 'very laborious', as Manitius translates, nor, if it could, would it be true). The variant πλείσταις οὔσαις found in D, part of the Arabic tradition (L) and Pappus (Rome[1] I 93,21) can be translated ('involving multiple positionings'), but it is not true that sighting the moon would require more than one positioning of the plate. Unless the corruption lies deeper (e.g. πλείστης has replaced a word meaning 'delicate') one must assume that πλείσταις οὔσαις was an inept gloss intended to explain why the process was inaccurate, and that this was corrupted to the unintelligible πλείστης οὔσης by attraction to παραμετρήσεως.

the length of the rod running from the eye to the plate can be inaccurate. However, once it was determined that the moon is at its greatest distance when H418 it subtends the same angle at the eye as the sun, we computed the size of the angle it subtends from observations of lunar eclipses in which the moon was near that [greatest] distance, and thence obtained immediately the size of the angle subtended by the sun. We shall explain the method of procedure in this by means of two of the eclipses used.

In the fifth year of Nabopolassar, which is the 127th year from Nabonassar, Athyr [III] 27/28 in the Egyptian calendar [-620 Apr. 21/22], at the end of the eleventh hour in Babylon, the moon began to be eclipsed; the maximum obscuration was $\frac{1}{4}$ of the diameter from the south. Now, since the beginning of the eclipse occurred 5 seasonal hours after midnight, and mid-eclipse about 6 [seasonal hours after midnight], which correspond to $5\frac{5}{6}$ equinoctial hours at Babylon on that date (for the true position of the sun was ♈ 27;3°), it is clear that mid-eclipse, which is when the greatest part of the diameter is immersed in the shadow, occurred $5\frac{5}{6}$ equinoctial hours after midnight in Babylon, and exactly 5 [hours after midnight] at Alexandria.[56]

The time from epoch is

126 Egyptian years 86 days $\begin{cases} \text{17 equinoctial hours reckoned simply} \\ 16\frac{3}{4} \text{ equinoctial hours in mean solar days.}^{57} \end{cases}$ H419

Therefore the lunar position was as follows:

mean position in longitude: ♎ 25;32°
true position in longitude: ♎ 27;5°
distance [in anomaly] from the apogee of the epicycle: 340;7°
distance [in latitude] from the northern limit on the inclined circle: 80;40°.

Thus it is clear that when the centre of the moon near its greatest distance is $9\frac{1}{3}$° distant from the node, measured along its inclined circle, and the centre of the shadow lies on the great circle drawn through the moon's centre at right angles to the inclined circle (which is the situation at which the greatest obscuration occurs), $\frac{1}{4}$ of the moon's diameter is immersed in the shadow.

Again, in the seventh year of Kambyses, which is the 225th year from Nabonassar, Phamenoth [VII] 17/18 in the Egyptian calendar [-522 July 16/17], 1 [equinoctial] hour before midnight at Babylon, the moon was eclipsed half its diameter from the north. Thus this eclipse occurred about $1\frac{5}{6}$ equinoctial hours before midnight at Alexandria.[58] The time from epoch is

[56] Oppolzer no. 901: mid-eclipse 2;38 a.m. ($\approx 4\frac{1}{2}^h$ after midnight at Alexandria), magnitude 1.6^d. P.V. Neugebauer, *Spezieller Kanon*, gives about $5\frac{1}{3}^h$ after midnight (Babylon) for mid-eclipse, magnitude 2.1^d.

[57] The equation of time for a solar longitude of ♈ 27° is about −20 mins. rather than −15 mins.

[58] Oppolzer no. 1056: mid-eclipse $21;0^h$ ($\approx$ 11 p.m. Alexandria), magnitude 6.1^d. P.V. Neugebauer gives mid-eclipse as ca. 23.6^h Babylon, magnitude 6.1^d. The time used by Ptolemy is clearly in error (although the computed positions of sun and moon must have seemed to him to confirm it), but the source of his error is too complicated to discuss here. The best treatment is in Britton[1] 81–4. For this eclipse (alone of those preserved in Almagest) there is also an extant cuneiform report (published by Kugler, SSB I p. 71). According to A. J. Sachs this text should be translated as follows: 'Year VII, month IV, night of the fourteenth, $1\frac{2}{3}$ double hours in the night a "total" lunar eclipse took place [with only] a little remaining [uneclipsed]. The north wind blew'. Here the time agrees with modern computations (and disagrees with Ptolemy), but the magnitude disagrees with both.

224 Egyptian years 196 days $\begin{cases} 10\frac{1}{6} \text{ equinoctial hours reckoned simply} \\ 9\frac{5}{6} \text{ equinoctial hours reckoned accurately} \end{cases}$

H420 (for the position of the sun was ♋ 18;12°).

Therefore the lunar position was as follows:

mean position in longitude: ♑ 20;22°

true position in longitude: ♑ 18;14°[59]

distance [in anomaly] from the apogee of the epicycle: 28;5°[60]

distance [in latitude] from the northern limit on the inclined circle: 262;12°.

Hence it is clear that, when the centre of the moon, again near its greatest distance, is $7\frac{4}{5}$° from the node, as measured along its inclined circle, and the centre of the shadow has the same position relative to it as before, half of the moon's diameter is immersed in the shadow.

But, when the moon's centre is $9\frac{1}{3}$° from the node along the inclined circle, it is $48\frac{1}{2}$′ from the ecliptic along the great circle drawn through it at right angles to the inclined circle [the orbit]; and when it is $7\frac{4}{5}$° from the node along the inclined circle, it is $40\frac{2}{3}$′ from the ecliptic along the great circle drawn through it at right angles to the inclined circle.[61] Therefore, since the difference between [the sizes of] the two eclipses comprises $\frac{1}{4}$ of the moon's diameter, and the difference

H421 between the above distances of the moon's centre from the ecliptic (i.e. from the centre of the shadow) comprises [$48\frac{1}{2} - 40\frac{2}{3} =$] $7\frac{5}{6}$′, it is obvious that the total diameter of the moon subtends a great circle arc of [$4 \times 7\frac{5}{6} =$] $31\frac{1}{3}$′.

From the same data it is easy to see that the radius of the shadow at the same greatest distance of the moon subtends $40\frac{2}{3}$′. For when the moon's centre was that distance [$40\frac{2}{3}$′] from the centre of the shadow, it was touching the edge of the shadow's circumference, because [in that situation] half of the moon's diameter was eclipsed. This is negligibly less than $2\frac{3}{5}$ times the radius of the moon, which is $15\frac{2}{3}$′. The values we derive for the above quantities from a number of similar observations are in agreement with these;[62] hence we use them, both in other parts of the theory, concerning eclipses,[63] and in the following demonstration of the solar distance, which will be along the same lines as that followed by Hipparchus. A further presupposition [of this demonstration] is that the circles of

H422 sun, moon and earth enclosed by the cones are not noticeably less than great circles on their spheres, and the diameters too [not noticeably less than great circle diameters].[64]

[59] Possibly one should read 18;11° with D^1 (computed: 18;10).

[60] Ptolemy has made a computing error here: correct is $\bar{\alpha} = 27;54°$. Obviously, he has computed (here only) for the uncorrected time of $10\frac{1}{6}^h$. However, this has no serious consequences, since it is merely intended to show that the moon is near the apogee of the epicycle. The discrepancy in the true position (see n.59) cannot be explained by this error.

[61] On the computation of these amounts see *HAMA* 107. It seems probable that they were, properly, computed from a spherical triangle with the right angle at the moon's orbit (rather than from a plane triangle or any of the other approximations suggested there). But the computations are inaccurate: Ptolemy should have found $48\frac{3}{4}$′ and $40\frac{5}{6}$′ respectively. For similar computations with the moon at the perigee of the epicycle see VI 5 pp. 284–5.

[62] Although Ptolemy's procedure for finding the apparent diameters of moon and shadow is both elegant and theoretically correct, it suffers from serious practical disadvantages. On these, and the inaccuracies involved in his actual computations, see *HAMA* 106–8.

[63] Reference to VI 5–7 and VI 11.

[64] I.e. in Fig. 5.12 the cones from points N and X enclosing the spheres of sun (ABG), moon (EZH) and earth (KLM) have bases (the circles on AG, EH and KM) which are not sensibly less than great

15. {*On the distance of the sun and other consequences of the demonstration of that*}[65]

Now, given the above, and given that the greatest distance of the moon at the syzygies is 64;10 units where the earth's radius is 1 (for we showed [p. 251] that its mean distance is 59 of those units, and the radius of the epicyle 5;10), let us see the size of the sun's distance which results.

[See Fig. 5.12.] Let there be the following great circles of the [various] spherical bodies lying in the same plane: circle ABG of the sun's, on centre D, circle EZH of the moon's at its greatest distance, on centre Θ, circle KLM of the earth's, on centre N. Let AXG be the plane through the centres [in the tangent cone] enclosing earth and sun, and ANG the plane through the centres [in the tangent cone] enclosing sun and moon, with DΘNX as common axis. Let the straight lines through the points of tangency, which are, obviously, parallel to each other, and sensibly equal to diameters, be ADG on the sun's circle, EΘH on the moon's circle, KNM on the earth's circle, and OPR on the circle of the shadow in which H423 the moon is immersed at its greatest distance (thus ΘN equals NP, and each of them is 64;10 units where NL, the earth's radius, is 1).

Then we have to find the ratio between ND, the distance of the sun, and NL, the earth's radius.

Produce EH to [meet XG at] S.

Since we demonstrated [p. 254] that the moon's diameter at the distance in question, namely the greatest distance in the syzygies, subtends 0;31,20° of the circle drawn through the moon about the earth's centre,

$\angle$ ENH = 0;31,20° where 4 right angles = 360°. H424

and $\angle$ ΘNH = $\frac{1}{2}$ $\angle$ ENH = 0;31,20°° where 2 right angles = 360°°.

Therefore in the circle about right-angled triangle NHΘ,

arc ΘH = 0;31,20°

and arc ΘN = 179;28,40° (supplement).

Therefore the corresponding chords

$\left.\begin{array}{l}\text{H}\Theta = 0;32,48^p \\ \text{and N}\Theta \approx 120^p\end{array}\right\}$ where diameter NH = 120^p.

Therefore where NΘ = 64;10, ΘH = 0;17,33.

And NM, the radius of the earth, is 1 in the same units.

But PR:ΘH ≈ 2;36 : 1 [p. 254].

∴ PR = 0;45,38 in the same units.

∴ ΘH + PR = 1;3,11 where NM = 1.

But PR + ΘS = 2, since PR + ΘS = 2NM

(for, as we said, all [three] are parallel, and NP = NΘ).

Therefore, by subtraction [of (PR + ΘH) from (PR + ΘS)],

HS = 0;56,49 where NM = 1. H425

And NM:HS = NG:HG = ND:ΘD.

circles in those spheres: thus AG, EH and KM can be treated as diameters of the spheres. This simplifying approximation is fully justified by the magnitude of the distances of the bodies compared with their diameters.

[65] On chs. 15 and 16 see *HAMA* 109-12, Pedersen 209-13.

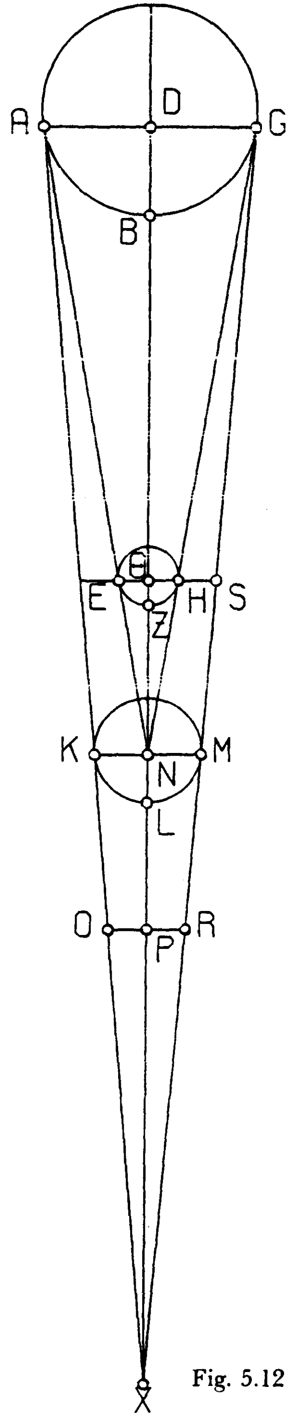

Fig. 5.12

Therefore where ND = 1, DΘ = 0;56,49, and, by subtraction, ΘN = 0;3,11.
Therefore where NΘ = 64;10 and NM = 1,
the sun's distance, ND ≈ 1210.
Similarly, as we showed, PR = 0;45,38 where NM = 1,
and NM:PR = NX:XP.
Therefore where NX = 1, XP = 0;45,38
and, by subtraction, PN = 0;14,22.
Therefore where PN = 64;10 and NM, the earth's radius, = 1,
XP ≈ 203;50,
and, by addition, XN = 268.
Therefore we have calculated that where the earth's radius is 1
the mean distance of the moon at the syzygies is 59
the distance of the sun is 1210
and the distance from the centre of the earth to the apex of the shadow cone is 268.

16. {*On the sizes of sun, moon and earth*} H426

The ratios of the volumes of the bodies are immediately derivable from the ratios of the diameters of sun, moon and earth.

For, since we have shown that, where NM, the earth's radius, is 1,
the moon's radius, ΘH = 0;17,33
and NΘ = 64;10,
and since NΘ:ΘH = ND:DG,
and ND was shown to be 1210 in the same units,
the radius of the sun, DG ≈ $5\frac{1}{2}$ in the same units.
So the diameters will have the same ratios.
Therefore where the moon's diameter is 1, the earth's diameter will be about $3\frac{2}{5}$, and the sun's $18\frac{4}{5}$.
Therefore the earth's diameter is $3\frac{2}{5}$ times the moon's
and the sun's diameter is $18\frac{4}{5}$ times the moon's
and $5\frac{1}{2}$ times the earth's. H427
And, using the same numbers,
since $1^3 = 1$,
and $3\frac{2}{5}^3 \approx 39\frac{1}{4}$,
and $18\frac{4}{5}^3 \approx 6644\frac{1}{2}$,
we conclude that, where the moon's volume is 1,
the earth's volume is $39\frac{1}{4}$ and the sun's $6644\frac{1}{2}$.
Therefore the sun's volume is about 170 times that of the earth.[66]

[66] There is no point in estimating the relative volumes of the bodies, but it was evidently traditional in Greek astronomy, for Theon of Smyrna (ed. Hiller p. 197) and Calcidius (ed. Waszink p. 143) quote from Hipparchus' work on sizes and distances the statement that the sun is 1880 times the size of the earth and the earth 27 times the size of the moon; these ratios plainly refer to relative volumes. In his *Planetary Hypotheses* (ed. Goldstein p. 9) Ptolemy gives the volumes of all the planets relative to the earth.

17. {*On the individual parallaxes of sun and moon*}[67]

With the above as basis, the next problem is to demonstrate, again briefly, how one may calculate the individual parallaxes of sun and moon from the amount of their distances. First [we deal with] the parallaxes with respect to the great circle drawn through the zenith and the body.[68]

H428 [See Fig. 5.13.] In the plane of that great circle, then, let the great circle representing the [surface of the] earth again [as in Fig. 5.10] be AB, the great circle representing the [position of the] sun or moon GD, and the great circle to which the earth bears the ratio of a point EZHΘ. Let K be the centre of all

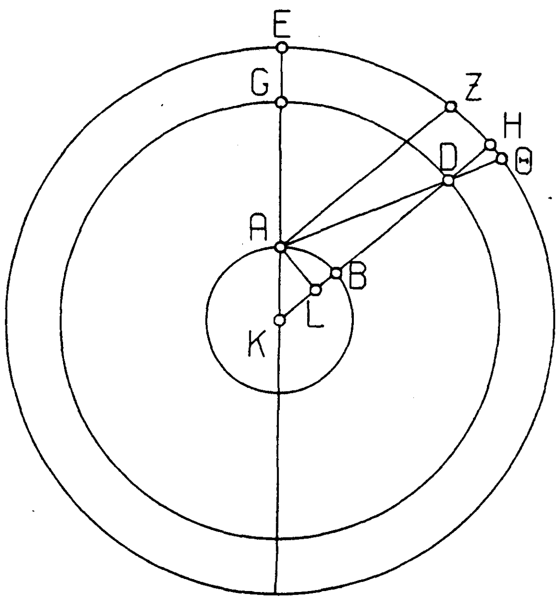

Fig. 5.13

[these circles], and KAGE the diameter through the zenith. Cut off from the zenith point G arc GD; let it be, e.g., 30°, and again draw KDH and ADΘ, from A draw AZ parallel to KH, and drop perpendicular AL on to KH.

Now neither of the luminaries always remains at the same distance. But the resulting difference in the sun's parallaxes will be very small and imperceptible, since the eccentricity of its circle is small, and its distance great. For the moon, however, the resulting difference will be very perceptible, both because of its

[67] See *HAMA* 112-15, Pedersen 213-17.
[68] In contrast to the longitudinal and latitudinal components of this 'total' parallax: these are dealt with in V 19.

motion on the epicycle and because of the motion of the epicycle on the eccentre, each of which produces quite a large difference in the distance. Therefore we shall demonstrate the solar parallaxes for a single ratio, namely 1210:1, but we shall demonstrate the lunar parallaxes for the four ratios which will be most convenient for the methods we shall subsequently develop. The four distances we have chosen are as follows: H429

The first two are

[1] when the epicycle is at the apogee of the eccentre,

[a] the distance to the apogee of the epicycle, which we concluded from our previous demonstration [p. 255] to be 64;10 earth-radii;

[b] the distance to the perigee of the epicycle, which we compute to be [59;0 − 5;10 =] 53;50 earth-radii.

The second two are

[2] when the epicycle is at the perigee of the eccentre,

[a] the distance to the apogee of the epicycle, which we concluded from our previous demonstration [p. 251] to be [38;43 + 5;10 =] 43;53 earth-radii;

[b] the distance to the perigee of the epicycle, which we compute as [38;43 − 5;10 =] 33;33 earth-radii.

Then, since arc GD = 30°, by hypothesis,

$$\angle\, GKD = \begin{cases} 30^\circ & \text{where 4 right angles} = 360^\circ \\ 60^{\circ\circ} & \text{where 2 right angles} = 360^{\circ\circ}. \end{cases}$$

Therefore in the circle about right-angled triangle AKL

arc AL = 60°,

and arc KL = 120° (supplement). H430

Therefore the corresponding chords

$$\left.\begin{array}{r} AL = 60^p \\ \text{and } KL = 103;55^p \end{array}\right\} \text{where diameter } AK = 120^p.$$

Therefore where AK = 1^p, AL = $0;30^p$ and KL = $0;52^p$.

And, in the same units,

$$KLD = \begin{cases} 1210^p \text{ for the sun's distance} & \\ 64;10^p \text{ for the moon's first limit} & [1a] \\ 53;50^p \text{ for the moon's second limit} & [1b] \\ 43;53^p \text{ for the moon's third limit} & [2a] \\ 33;33^p \text{ for the moon's fourth limit} & [2b]. \end{cases}$$

And, by subtraction, LD [= KLD − KL], which is the same as AD, since the difference is imperceptible.

$$\therefore AD = \begin{cases} 1209;8^p \text{ for the sun's distance} & \\ 63;18^p \text{ for the moon's first limit} & [1a] \\ 52;58^p \text{ for the moon's second limit} & [1b] \\ 43;1^p \text{ for the moon's third limit} & [2a] \\ 32;41^p \text{ for the moon's fourth limit} & [2b]. \end{cases}$$

Therefore, where hypotenuse AD = 120^p, then (assuming the same order, to avoid repetition)

	[Sun]	[Moon]			
		[1a]	[1b]	[2a]	[2b]
AL =	$0;2,59^p$	$0;56,52^p$	$1;7,58^p$	$1;23,41^p$	$1;50,9^p$.

Therefore in the circle about right-angled triangle DLA[69]

arc AL =	0;2,50°	0;54,18°	1;4,54°	about 1;20°	about 1;45°.
∴ ∠ ADB = ∠ ZAΘ =	0;2,50°°	0;54,18°°	1;4,54°°	1;20°°	1;45°°
			where 2 right angles = 360°°		
	0;1;25°	0;27,9°	0;32,27°	0;40°[70]	0;52,30°
			where 4 right angles = 360° .		

H431

So, since point A is negligibly different from centre K, and arc ZHΘ is negligibly greater than arc HΘ (for the whole earth has the ratio of a point to circle EZHΘ), in circle EZHΘ, the arc of the parallax

$$\text{arc H}\Theta = \begin{cases} 0;1,25^\circ \text{ for the sun's distance} \\ 0;27,9^\circ \text{ for the moon's first limit} \\ 0;32,27^\circ \text{ for the moon's second limit} \\ 0;40^\circ \text{ for the moon's third limit} \\ 0;52,30^\circ \text{ for the moon's fourth limit.} \end{cases}$$

Q.E.D.

In the same way we calculated the parallaxes for the other zenith distances (at intervals of 6° up to the 90° of the quadrant) at each limit, and constructed a table to determine the parallaxes. The table has, again, 45 lines, and 9 columns.
H432 In the first column we put the 90 degrees of the quadrant, tabulating them, obviously, at two-degree intervals; in the second column we put the minutes of solar parallax corresponding to each argument, in the third column the lunar parallax at the first limit; in the fourth column the increment in the [lunar] parallax at the second limit over the first limit; in the fifth column the [lunar] parallax at the third limit; and in the sixth the increment in the [lunar] parallax at the fourth limit over the third limit. Thus, for example, for an argument of 30° we put 0;1,25° for the sun, then 0;27,9° for the first limit of the moon; next 0;5,18°, which is the increment of the second limit over the first; then 0;40°, for the third limit, and next 0;12,30°, which is the increment of the fourth limit over the third.

We needed to provide a convenient method of calculating the parallax (corresponding to the appropriate argument) for distances [of the moon] at intermediate positions between apogee and perigee [of eccentre and epicycle] from the parallaxes tabulated at the above four limits, using minutes [of interpolation]. To this end we added the remaining three columns, to account
H433 for those differences. We calculated these columns in the following manner.

Let [Fig. 5.14] the moon's epicycle be ABGD on centre E, and let Z be the centre of the ecliptic and the earth. Join [ZE with line] AEDZ, draw ZGB, join BE, GE, and drop perpendiculars on to AD, BH from B, and GΘ from G. Let us suppose, first, that arc AB, the moon's distance from A, the true apogee [of the epicycle] as taken with respect to centre Z, is, e.g. 60°.

$$\therefore \angle \text{BEH} = \begin{cases} 60^\circ \text{ where 4 right angles} = 360^\circ \\ 120^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ}. \end{cases}$$

[69] From here on Ptolemy drastically rounds his computations for the moon's third and fourth limits. His rationale, no doubt, is that in computing solar eclipses (for which the parallax table is principally designed) the moon is by definition near the apogee of the eccentre, and hence there is no use for the third and fourth limits. Cf. p. 264 n.73.

[70] Reading o $\bar{\mu}$ (with D,Ar) for o $\bar{\mu}$ o (0;40,0) at H431,4 and at H431,13.

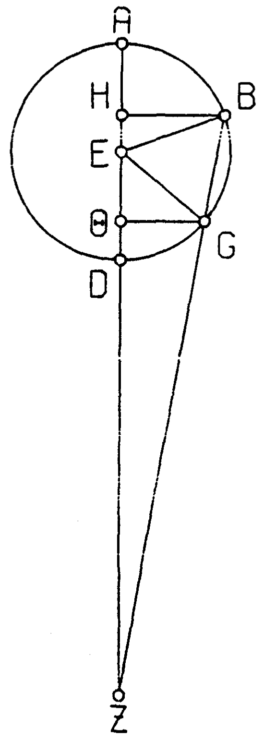

Fig. 5.14

Therefore in the circle about right-angled triangle BEH H434

arc BH = 120°

and arc EH = 60° (supplement).

Therefore the corresponding chords

$$\left.\begin{array}{r} BH = 103;55^p \\ \text{and } EH = 60^p \end{array}\right\} \text{where diameter } EB = 120^p.$$

But when centre E of the epicycle is at the apogee of the eccentre,

ZE:EB = 60 : 5;15.

Therefore, where EB = 5;15^p,

BH = 4;33^p

EH = 2;38^p

and, by addition [of EH to EZ], HEZ = 62;38^p.

And $ZB^2 = ZH^2 + HB^2$.

∴ ZB = 62;48^p, where

the distance of the first limit, ZA = 65;15^p

the distance of the second limit, ZD = 54;45^p

and the difference between the two limits, AD = 10;30^p.

Therefore the difference at B with respect to the first limit is [65;15 − 62;48 =] 2;27^p where the total difference is 10;30^p. Therefore where the total

difference is 60^p, the difference at B will be $14;0^p$. This [14;0], then, is the amount which we shall enter in the seventh column on the line [corresponding to the argument] of half of the number 60, namely 30. The reason for this is that
H435 the 90 degrees comprised in the first column of the table contain half of the 180 degrees from A to D.[71]

By the same reasoning, if we suppose arc GD to be the same size [as arc AB above], 60°, it will be shown that

$$\left.\begin{aligned} G\Theta &= 4;33^p \\ \text{and } E\Theta &= 2;38^p \end{aligned}\right\} \text{ where radius } EG = 5;15^p.$$

Hence, by subtraction [of $E\Theta$ from ZE], $Z\Theta = 57;22^p$.
By the same reasoning [as above], hypotenuse $ZG = [\sqrt{57;22^2 + 4;33^2} =]\ 57;33^p$.
We again subtract this from the $65;15^p$ of the first limit, and find that the result, $7;42^p$, is 44;0 sixtieths of the total difference. This is what we shall enter in the same [seventh] column opposite the argument 60, since arc ABG = 120°.

With the same arcs [AB and GD] as basis, let us suppose that centre E is at the perigee of the eccentre, which is the position defining the third and fourth limits. In this position

$$ZE:EB = 60:8.$$[72]

Therefore where $BE = 8^p$, and assuming both arc AB and arc GD as 60°,

H436
$$\left.\begin{aligned} BH = G\Theta &= 6;56^p \\ \text{and } EH = E\Theta &= 4;0^p \end{aligned}\right\} \text{ where } ZE = 60^p.$$

$$\therefore ZH = [ZE + EH =]\ 64^p$$
$$\text{and } Z\Theta = [ZE - EH =]\ 56^p,$$

so, by the same reasoning [as above]

$$\text{hypotenuse } ZB = [\sqrt{ZH^2 + BH^2} =]\ 64;23^p$$
$$\text{and hypotenuse } ZG = [\sqrt{Z\Theta^2 + G\Theta^2} =]\ 56;26^p,$$

where the [distance of] the third limit, $ZA = 68^p$,
and the difference between the third and fourth limits, $AD = 16^p$.
And $68^p - 64;23^p = 3;37^p$, which is 13;33 sixtieths of the total difference, 16^p. We enter this amount [13;33] in the eighth column opposite the argument 30, in the same way as before.

Also, $68^p - 56;26^p = 11;34^p$, which is 43;24 sixtieths of the total difference, 16^p. This amount we enter, similarly, in the eighth column opposite the argument 60.

That, then, is the way we shall set out the corrections computed for the motion of the moon on the epicycle. The corrections for the motion of the epicycle on the eccentre will be derived as follows.

H437 Let [Fig. 5.15] the moon's eccentre be ABGD on centre E and diameter AEG, on which Z is taken as the centre of the ecliptic. Draw BZD, and let angles AZB and GZD both, again, be taken as 60°. These situations occur at elongations of 30° (when the centre of the epicycle is at B), and 120° (when the centre of the epicycle is at D). Join BE, ED, and drop perpendicular EH from E on to BZD.

[71] The main part of Table V 18 (cols. 2 to 6) is a function of the zenith distance, which varies between 0° and 90°. The interpolation columns 7 and 8, however, are a function of the anomaly α, which varies between 0° and 180°. In order to use the same argument column for both, Ptolemy tabulates cols. 7 and 8 as a function of $\frac{1}{2}\alpha$.

[72] Cf. V 7 p. 235.

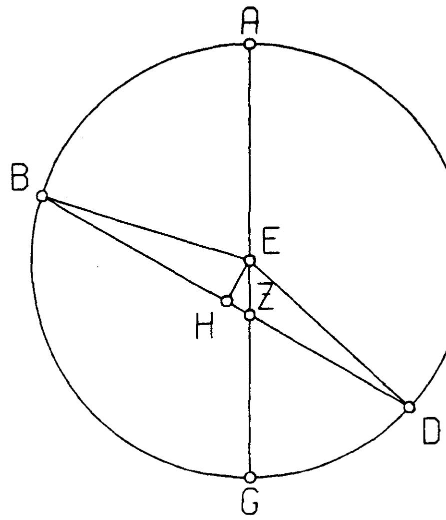

Fig. 5.15

Then, since ∠ BZA = 120°° where 2 right angles = 360°°,
in the circle about right-angled triangle EZH,

arc EH = 120°

and arc ZH = 60° (supplement).

Therefore the corresponding chords

$$\left.\begin{aligned} EH &= 103;55^p \\ \text{and } HZ &= 60^p \end{aligned}\right\} \text{where hypotenuse } EZ = 120^p.$$ H438

Therefore where the distance between the centres, EZ = $10;19^p$
and the radius of the eccentre is $49;41^p$,

EH = $8;56^p$

and ZH = $5;10^p$.

And since $BH^2 = BE^2 - EH^2$,

BH = DH = $48;53^p$ in the same units.

Therefore, by addition [of ZH to BH], ZB = $54;3^p$,
and, by subtraction [of ZH from DH], ZD = $43;43^p$
where [the distance for] the first [two] limits, ZA = 60^p
[the distance for] last [two] limits, ZG = $39;22^p$
and the difference between them = $20;38^p$.
Now $60^p - 54;3^p = 5;57^p$, which is 17;18 sixtieths of the total difference of $20;38^p$;
and $60^p - 43;43^p = 16;17^p$, which is 47;21 sixtieths of the total difference of $20;38^p$.
Therefore, obviously, we shall enter 17,18 in the ninth column opposite the argument 30° of elongation, and 47,21 opposite 120°, i.e. again opposite 60°; for, since the perigee [of the eccentre] lies at 90° [of elongation], an elongation of 60° is equivalent in distance to an elongation of 120°.

H439 In the same way we calculated the minutes [of coefficient of interpolation] for the differences over the three intervals in question for the other arcs. We performed the calculation at intervals of 12°, which corresponds to 6° in the arguments in the table, since the 180° from apogee [of the epicycle or eccentre] to perigee correspond to the 90° of [the argument column in] the table. We entered these minutes, calculated geometrically, opposite the appropriate argument. We derived the entries for the intermediate arguments by linear interpolation over the six-degree intervals: for the difference between the results so derived and [accurate] geometrical calculation is negligible over such a short interval, both for the minutes and for the actual parallaxes.

The table is as follows.

H442-3

18. {*Parallax Table*}[73]

[See p. 265.]

H444

19. {*On the determination of the parallaxes*}[74]

So, when we decide to determine the amount of the moon's parallax at any given [lunar] position, (first) with respect to the great circle drawn through the moon and the zenith, we examine its distance (in equinoctial hours) from the meridian at the latitude in question. With the distance found as argument, we enter the Table of Angles [II 13] for the appropriate latitude and zodiacal sign, and take the amount in degrees in the second column corresponding to the hour, interpolating between integer hours if necessary.[75] This gives us the distance of the moon from the zenith, measured along the great circle joining the two. With this as argument, we enter the Table of Parallaxes [V 18], determine on which line in the first column the argument is to be found, and taking the numbers corresponding to this in the four columns following the column of solar parallaxes, namely the third, fourth, fifth and sixth columns, write each one down separately. Then we take the corrected anomaly (i.e. with respect to the true apogee [of the epicycle]) at that moment: [if it is less than
H445 180°,] we take the anomaly itself, but if it is greater than 180°, we take (360° minus anomaly); we always halve the amount so obtained, and, entering with this into the same [column of] arguments, determine the number of minutes corresponding to it in both the seventh and eighth columns separately. We take the minutes found from the seventh column, multiply them into the difference

[73] As Ptolemy says (pp. 260 and 264), the entries in this table are calculated at every 6° of argument (i.e. every third entry), the intermediate values being derived by linear interpolation. Note that the values for the third and fourth limits (cols. 5 and 6), though tabulated to 3 significant places, are in fact calculated to only 2 places (for the reason see p. 260 n.69): the calculated values (for args. 6°, 12° etc.) always end in 0 or 30. They are therefore rather inaccurate.

Correction to Heiberg: H443,41, entry in col. 9 for arg. 72°, read νε κα (with D,Ar) for νε μα (55,41).

[74] See *HAMA* 114-17, Pedersen 217-19.

[75] Literally 'either *in toto*, or the amount proportional to the fraction of an hour'.

PARALLAX TABLE

1	2	3	4	5	6	7	8	9
Arguments	Sun's Parallaxes	Moon Parallaxes at First Limit	Moon Difference at Second Limit	Moon Parallaxes at Third Limit	Moon Difference at Fourth Limit	Sixtieths for Epicycle at Apogee	Sixtieths for Epicycle at Perigee	Sixtieths for Eccentre
2	0 0 7	0 1 54	0 0 23	0 3 0	0 0 50	0 14	0 11	0 15
4	0 0 13	0 3 48	0 0 45	0 6 0	0 1 40	0 28	0 22	0 30
6	0 0 19	0 5 41	0 1 7	0 9 0	0 2 30	0 42	0 33	0 45
8	0 0 25	0 7 34	0 1 29	0 11 40	0 3 20	1 22	1 7	1 33
10	0 0 31	0 9 27	0 1 51	0 14 20	0 4 10	2 2	1 41	2 21
12	0 0 37	0 11 19	0 2 12	0 17 0	0 5 0	2 42	2 15	3 9
14	0 0 42	0 13 10	0 2 33	0 19 40	0 5 50	3 35	3 13	4 22
16	0 0 48	0 15 0	0 2 54	0 22 20	0 6 40	4 28	4 11	5 35
18	0 0 53	0 16 49	0 3 15	0 25 0	0 7 30	5 21	5 9	6 48
20	0 0 58	0 18 36	0 3 36	0 27 40	0 8 20	6 39	6 25	8 25
22	0 1 4	0 20 22	0 3 57	0 30 20	0 9 10	7 57	7 41	10 2
24	0 1 9	0 22 6	0 4 18	0 33 0	0 10 0	9 15	8 57	11 39
26	0 1 14	0 23 49	0 4 39	0 35 20	0 10 50	10 50	10 29	13 32
28	0 1 20	0 25 30	0 4 59	0 37 40	0 11 40	12 25	12 1	15 25
30	0 1 25	0 27 9	0 5 18	0 40 0	0 12 30	14 0	13 33	17 18
32	0 1 30	0 28 46	0 5 37	0 42 20	0 13 20	15 52	15 22	19 23
34	0 1 35	0 30 21	0 5 55	0 44 40	0 14 10	17 44	17 11	21 28
36	0 1 40	0 31 54	0 6 13	0 47 0	0 15 0	19 36	19 0	23 33
38	0 1 44	0 33 24	0 6 30	0 49 0	0 15 40	21 36	20 59	25 40
40	0 1 49	0 34 51	0 6 47	0 51 0	0 16 20	23 36	22 58	27 47
42	0 1 54	0 36 14	0 7 4	0 53 0	0 17 0	25 36	24 57	29 54
44	0 1 58	0 37 37	0 7 20	0 55 0	0 17 40	27 40	27 1	32 0
46	0 2 3	0 38 57	0 7 35	0 57 0	0 18 20	29 44	29 5	34 6
48	0 2 8	0 40 14	0 7 49	0 59 0	0 19 0	31 48	31 9	36 12
50	0 2 12	0 41 28	0 8 3	1 0 40	0 19 40	33 52	33 14	38 9
52	0 2 16	0 42 39	0 8 16	1 2 20	0 20 20	35 56	35 19	40 6
54	0 2 20	0 43 45	0 8 29	1 4 0	0 21 0	38 0	37 24	42 3
56	0 2 23	0 44 48	0 8 42	1 5 20	0 21 20	40 0	39 24	43 49
58	0 2 26	0 45 48	0 8 53	1 6 40	0 21 40	42 0	41 24	45 35
60	0 2 29	0 46 46	0 9 3	1 8 0	0 22 0	44 0	43 24	47 21
62	0 2 32	0 47 40	0 9 13	1 9 20	0 22 20	45 50	45 13	48 49
64	0 2 34	0 48 30	0 9 22	1 10 40	0 22 40	47 40	47 2	50 17
66	0 2 36	0 49 15	0 9 31	1 12 0	0 23 0	49 30	48 51	51 45
68	0 2 38	0 49 57	0 9 39	1 13 0	0 23 10	50 56	50 24	52 57
70	0 2 40	0 50 36	0 9 46	1 14 0	0 23 20	52 22	51 57	54 9
72	0 2 42	0 51 11	0 9 53	1 15 0	0 23 30	53 48	53 30	55 21
74	0 2 44	0 51 44	0 9 59	1 15 40	0 23 40	54 57	54 41	56 12
76	0 2 46	0 52 12	0 10 4	1 16 20	0 23 50	56 6	55 52	57 3
78	0 2 47	0 52 34	0 10 8	1 17 0	0 24 0	57 15	57 3	57 54
80	0 2 48	0 52 53	0 10 11	1 17 20	0 24 10	57 57	57 47	58 26
82	0 2 49	0 53 9	0 10 14	1 17 40	0 24 20	58 39	58 31	58 58
84	0 2 50	0 53 21	0 10 16	1 18 0	0 24 30	59 21	59 15	59 30
86	0 2 50	0 53 29	0 10 16	1 18 20	0 24 40	59 34	59 30	59 40
88	0 2 51	0 53 33	0 10 17	1 18 40	0 24 50	59 47	59 45	59 50
90	0 2 51	0 53 34	0 10 17	1 19 0	0 25 0	60 0	60 0	60 0

found from the fourth column, and (always) add the result to the parallax from the third column. [Likewise] we take the minutes found from the eighth column, multiply them into the difference found from the sixth column, and again (always) add the result to the parallax from the fifth column. Thus we have obtained two parallaxes; we take the difference between these and write it down. Next we take the mean elongation of the moon from the sun, or else the mean elongation of the moon from the point opposite the [mean] sun, whichever of these two distances is the lesser,[76] and entering with this too into the arguments in the first column, take the minutes corresponding to it in the ninth and last column. We multiply these into the difference between the two parallaxes which we wrote down, and (always) add the result to the smaller (that is, the one derived from the third and fourth columns). This sum will give us the moon's parallax as measured along the great circle through the moon and the zenith.

H446 The sun's parallax for a similar situation [i.e. as measured along an altitude circle] is immediately determined, in a simple fashion, (for solar eclipses), from the number in the second column corresponding to the size of the arc from the zenith [to the sun].[77]

Now, in order to determine the parallax with respect to the ecliptic, in both longitude and latitude, at the given time, we again enter, with the same distance of the moon from the meridian in equinoctial hours [as before], into the same part of the Table of Angles [II 13], and take the number of degrees corresponding to that hour, in the third column if the moon is to the east of the meridian, or in the fourth column if it is to the west of the meridian. We examine the result, and if it is less than 90° we write down the number itself; but if it is greater than 90°, we write down its supplement, since that will be the size in degrees of the lesser of the two angles at the intersection [of ecliptic and altitude circle] in question. We double the number written down, and enter with this [doubled] number, and also with its supplement, into the Table of Chords [I 11]. The ratio of the chord of the doubled number to the chord of the supplement will give the ratio of the latitudinal parallax to the longitudinal
H447 parallax (for circular arcs of such small size are not noticeably different from straight lines). So we multiply the amounts of the chords in question by the parallax determined with respect to the altitude circle, and divide the products, each separately, by 120. The results of the division give us the separate components of the parallax. The following general rules apply.

For the latitudinal parallax, when the zenith is to the north of the point of the ecliptic then culminating, on the meridian, the [effect of the] parallax will be towards the south of it [the ecliptic]; but when the zenith is to the south of the culminating point, [the effect of] the parallax in latitude will be towards the north.

For the longitudinal parallax: the angles tabulated in the Table [II 13] represent the northernmost of the two angles cut off to the rear of the intersection

[76] I.e. (see *HAMA* 114) we take as argument η' (which cannot exceed 90°), derived from the mean elongation $\bar{\eta}$ according to the rules $0 \leq \bar{\eta} \leq 90: \eta' = \bar{\eta}$; $90 \leq \bar{\eta} \leq 180: \eta' = 180 - \bar{\eta}$; $180 \leq \bar{\eta} \leq 270$: $\eta' = \bar{\eta} - 180$; $270 \leq \bar{\eta} \leq 360 : \eta' = 360 - \bar{\eta}$.

[77] For a parallax computation see Appendix A, Example 10.

of ecliptic [and altitude circle].[78] Therefore, when the latitudinal parallax is to the north, if the angle in question is greater than a right angle, the effect of the longitudinal parallax will be in advance [i.e. in reverse order] of the signs, but if the angle is less than a right angle, the effect will be towards the rear [i.e. in the order of the signs]. However, when the latitudinal parallax is to the south, the reverse will be true: if the angle in question is greater than a right angle, the longitudinal parallax will be towards the rear [i.e. in the order] of the signs, but H448
if it is less than a right angle, the longitudinal parallax will be in advance.[79]

Our previous demonstrations concerning the sun proceeded on the assumption that it has no perceptible parallax, though we are well aware that the parallax, which, as we subsequently showed, affects the sun also, will make some difference in them.[80] However, we do not think that the resulting error in [predicting] the phenomena will be of sufficient concern to necessitate changing any of the theorems constructed without taking such a small effect into consideration. Similarly, for lunar parallaxes, we considered it sufficient to use the arcs and angles formed by the great circle through the poles of the horizon [i.e. an altitude circle] at the ecliptic, instead of those at the moon's inclined circle. For we saw that the difference which would result at syzygies in which eclipses occur is imperceptible, and to set out the latter would have been complicated to demonstrate and laborious to calculate; for the distance of the moon from the node is not fixed for a given position of the moon on the ecliptic, but undergoes multiple changes both in amount and in relative position.

In order to make clear what we mean, let [Fig. 5.16] ABG be a segment of the H449
ecliptic, AD a segment of the moon's inclined circle, point A the node, and D the centre of the moon. Draw DB at right angles to the ecliptic. Let E be the pole of the horizon, and draw through E the great circle arcs EDZ through the moon's centre, and EB through B. Let arc DH represent the moon's parallax, and through point H[81] draw HΘ at right angles to BD and HK at right angles to BZ. Thus AB represents the true distance [of the moon] in longitude from the node, and AK the apparent distance, while BD represents the true distance in latitude from the ecliptic, and KH the apparent. Furthermore an arc equal to ΘH represents the longitudinal component of parallax (with respect to the ecliptic) derived from DH, and an arc equal to DΘ represents the latitudinal component of parallax.

From the preceding theorems, [we know that] parallax DH can be found if H450
arc ED is given, and both [components of] parallax, DΘ and ΘH, if ∠ GZE is given. But what we determined previously was the arcs and angles formed at given points of the ecliptic by the altitude circle; and the only point on the ecliptic which is given in this situation is B. Hence it is clear that we are using arc EB instead of arc ED, and ∠ GBE instead of ∠ GZE.

[78] Cf. II 10 p. 105.

[79] See the last part of Appendix A, Example 10.

[80] I.e., nowhere in Bks. III to V were corrections made to the solar position to account for parallax, although in some cases it would theoretically make a difference (e.g. in observations made with the astrolabe in which both sun and moon were sighted, V 3).

[81] Reading διὰ τοῦ H (with Ar, διὰ τοῦ D) for δι' αὐτοῦ at H449,16. Suggested by Heiberg and adopted by Manitius.

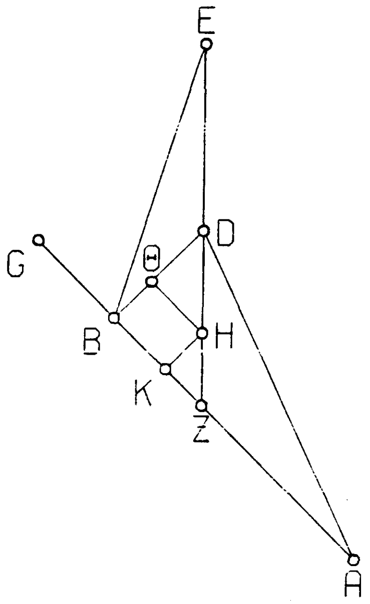

Fig. 5.16

Now Hipparchus attempted to correct this kind [of inaccuracy] too, but it is apparent that he attacked the problem in a very careless and irrational way.[82] For firstly, he does it for [just] a single value of the distance AD, instead of all [possible] values, or a number of values, as would have been appropriate in a situation where one has chosen to be nicely accurate about small [errors]. Furthermore, without realising it, he has fallen into a number of [even] stranger errors. Having also [like us] previously demonstrated [the amounts of] the arcs and angles with respect to [intersections of altitude circles with] the ecliptic, and shown that, if ED is given, DH can be found (he shows this in Bk. I of his 'On
H451 parallaxes'), in order to get ED as a given quantity, he assumes that arc EZ and ∠ EZG are given (in this way, in Bk. II, he calculates ZD and takes ED as remainder [of EZ-ZD]). However he was misled by his failure to notice that the given point of the ecliptic is not Z but B, and hence the given arc is not EZ, but EB, and the given angle not EZG but EBG. Yet it is these [arc EZ and ∠ EZG] which were the [necessary] starting-points for making even such a partial correction. For in many situations there is a quite noticeable difference between the arc ED and the arc EZ,[83] whereas the difference between BE (which really is

[82] No one has given a satisfactory explanation of the procedure of Hipparchus which Ptolemy alludes to here. Pappus devotes a section of his commentary to it (Rome[1] I 151–5), but his reconstruction of Hipparchus' method seems entirely fictitious (see *HAMA* 323–5); there are errors in Rome's text and notes ad loc.

[83] At certain situations (cf. Table II 13) the angle between altitude circle and ecliptic (∠ EZA in Fig. 5.16) can be close to 180°: then the angle betwen altitude circle and moon's orbit (∠ EDA) will also be close to 180°, and hence DZ will be a large arc, and the error of taking EZ for ED can be

given) and ED is, at most, the amount of the arc BD for any given distance [of the moon] from the node.

The logical procedure for making the correction by a [mathematically] sound method can be displayed as follows.

[First, see Fig. 5.17], let ABG be the ecliptic, and DBE at right angles to it. Let the moon be at either D or E, at a latitudinal distance from the ecliptic ABG which is a given arc, e.g. BD or BE. Then the zenith arcs and the angles are given at point B of the ecliptic, and the [corresponding arcs and angles] at D or E are to be found. H452

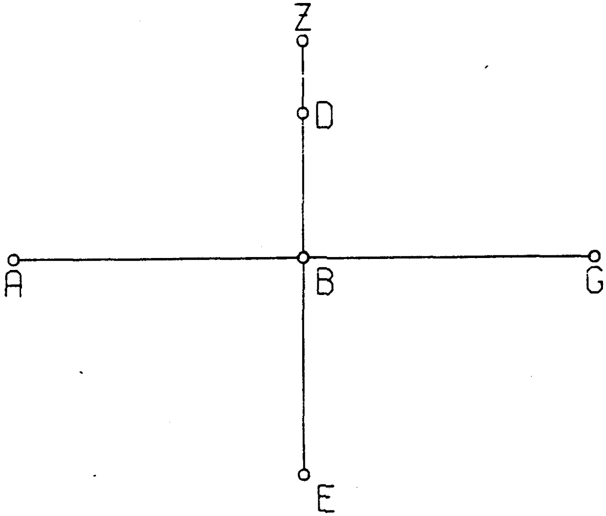

Fig. 5.17

Now if the position of the ecliptic is such that it is at right angles to the great circle drawn through point Z (which we set as the pole of the horizon) and point B, i.e. ZB, it is obvious that this great circle will coincide with arc DE, and the angles at D and E will not differ from that given at B: for [arcs] drawn through these points [from the zenith] are also at right angles to the ecliptic.

And ZD = ZB − BD

ZE = ZB + BE, where both BD and BE are given.

[Second,] let the ecliptic ABG coincide with the great circle through the zenith. Then if [see Fig. 5.18] we take A as the pole of the horizon and draw AD and AE, these [two arcs] will differ from arc AB, and angles BAD and BAE will differ from [the corresponding angle] in the previous case, which was zero.[84] H453

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considerable, whereas the error of taking EB for ED cannot exceed arc BD which (since ∠ DBA is right) cannot exceed the inclination of the moon's orbit, 5°. After this I have excised, at H451,12–13, διὰ τὸ πολὺ μᾶλλον ἐκείνων αὐτὰς μὴ δεδόσθαι, 'because the former [ED] is even farther from being given than the latter [EZ]', as an interpolation which is a (very lame) explanation of the preceding (in fact it is a consequence, not a cause). Heiberg's punctuation of this passage makes it unintelligible: remove the stop after EZΓ (line 9) and insert a comma before πολλαχῆ (line 10).

[84] Literally 'which did not exist'. The angle in question is ∠ BZD in Fig. 5.17.

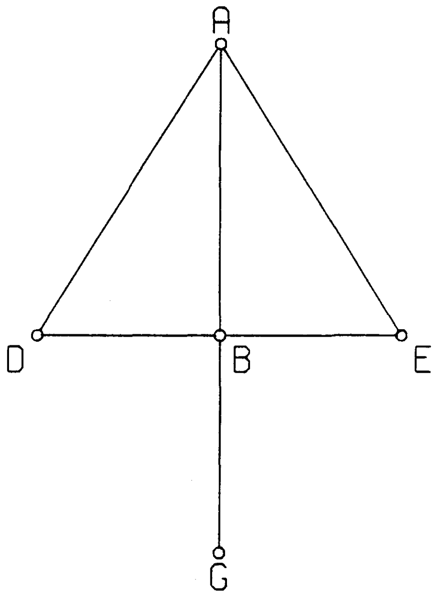

Fig. 5.18

And AD and AE are given from the quantities AB and BD, BE (we speak in terms of straight lines, since the difference [from arcs] is negligible),

$$\text{since } AB^2 + BD^2 = AD^2$$
$$AB^2 + BE^2 = AE.^2$$

And the angles BAD and BAE can thence be derived.

[Third,] let the ecliptic be inclined [to the altitude circle]. If [Fig. 5.19] we take Z as pole of the horizon and draw ZB, ZHD and ZEΘ, arc ZB and ∠ ABZ will be given, and so again, obviously, will be BD and BE. What we need to be given are arcs ZD and ZE, and angles AHZ, AΘZ. These too are given if perpendiculars DK and EL are drawn to ZB.

H454 For since ∠ ABZ is given, and ∠ ABE is always a right angle, the right-angled triangles BKD and BLE are given, and so is the ratio of ZB to the sides containing the right angle, since [the ratio of ZB] to the hypotenuses DB and BE is given. Hence there will be given ZD, the hypotenuse [of right-angled triangle ZDK, of which sides ZK and KD are given], and ZE, the hypotenuse [of right-angled triangle ZLE, of which sides ZL and LE are given], and also the angles DZK and EZL, which are the differences from the required angles. For

$$\angle AHZ = \angle ABZ + \angle DZB$$
$$\text{and } \angle A\Theta Z = \angle ABZ - \angle EZL.$$

It is clear that, for the same latitudinal distance, the greatest difference [with respect to the arcs and angles at B] will occur

[1] for the angles, when point B itself is the zenith. For if the angle [formed by the altitude circle through the moon] at B is zero, the [arcs] through D and E from the zenith form right angles with the ecliptic;

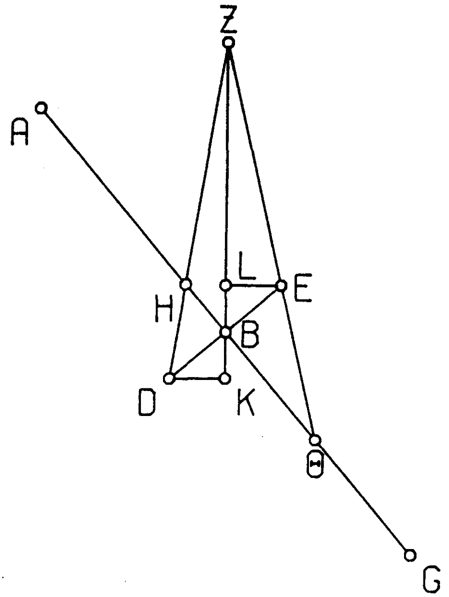

Fig. 5.19

[2] for the arcs

[a] in the same situation [i.e. when point B is in the zenith]. For when the arc [from the zenith] to B is zero, the arcs to D and E will be equal in size to the moon's latitude; also H455

[b] when the circle through the zenith is perpendicular to the ecliptic. For the difference between arc ZB and ZD or ZE will again be equal to the whole amount of the [lunar] latitude.

But in other situations, in which DE is inclined to ZB, the resultant differences between the arcs and angles will be less. Thus, when the moon's distance in latitude from the ecliptic is 5°, the greatest difference in the parallaxes [as computed at the ecliptic and at the moon's orbit] will be about 10 minutes. For the 5°, representing the greatest difference between the arcs, produces that number of minutes [when one enters Table V 18] at the least distance and the greatest difference. But when the moon is at the maximum latitude which it can attain at a solar eclipse, which is about $1\frac{1}{2}$°, the difference between the parallaxes will be the same number, [i.e.] $1\frac{1}{2}$, of minutes. And this happens rarely.[85]

[85] To verify these figures, take entries at 5° interval in Table V 18, using cols. 5 and 6 (which are chosen because they give the maximum difference). The rate of change is fastest near zero, hence: for arg. 0, 0 + 0 = 0; for arg. 5°, 0;7,30 + 0;2,5 = 0;9,35 ≈ 10′. For eclipses, which occur at conjunction, we have to take the values from cols. 3 and 4. Here, between 0° and $1\frac{1}{2}$°, we find: 0 + 0 = 0, 0;1,25 ± 0;0,18 = 0;1,43 (which is closer to $1\frac{3}{4}$′ than $1\frac{1}{2}$′). The maximum latitude of the moon at a solar eclipse is about $1\frac{1}{2}$°, the sum of the apparent radii of the bodies (each about $\frac{1}{4}$°) and the maximum parallax at conjunction (about 1°; see VI 6 p. 293). There is no reason to suspect an interpolation here, with Manitius (p. 447): he has misunderstood the passage, notably mistranslating τὰ ἴσα ἑξηκοστά, H455,15–16.

A convenient method for making the above kind of correction of the angles and arcs, if anyone wants to make it when the [differences] involved are so small, would be as follows.

H456 As a general rule, we double the amount of the angle [between altitude circle and ecliptic], and entering with this as argument into the Table of Chords take the chord corresponding to it, and also the chord corresponding to its supplement. We multiply both of the latter separately by the [moon's] latitude, in degrees, divide each of the products by 120, and record the results [separately]. As for the result derived from the first angle, we subtract it from the relevant arc from the zenith [to the ecliptic] when the moon is on the same side [of the ecliptic] as the zenith, but add it when it is on the opposite side [of the ecliptic to the zenith]. We square the result, add that to the result derived from the supplementary angle, also squared, and take the square root of the sum: this will give us the corresponding arc [ZE or ZD in Fig. 5.19] which is required.

Next we take the result which we recorded from the [second,] supplementary angle, multiply it by 120, and divide the result by the arc we found [ZE or ZD]. With the resulting [chord] we enter into the [body of the] Table of Chords [I 11], take the corresponding arc [in the column of argument], and halve it. If the corrected arc [ZE or ZD] is greater than the original [ZB] we add the result to the amount of the original angle, but if [the corrected arc is] less [than the original], we subtract it: the result will be the corrected angle.

H457 To give an example, in the previous figure [5.20] let arc ZB be 45°, ∠ABZ 30°, and both arc DB and arc BE 5° in latitude.

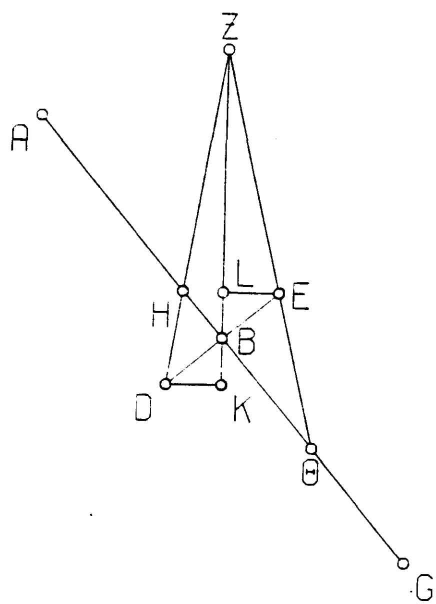

Fig. 5.20

Now Crd $(2 \times 30)^\circ$ = Crd $60^\circ = 60^p$,
and Crd $(180 - 60)^\circ$ = Crd $120^\circ \approx 104^p$,
$\therefore$ BL:LE = BK:DK[86] = 60:104, where the hypotenuse [BE or BD] = 120^p.
So we multiply each number by the 5° of the hypotenuse and divide by 120.

$\therefore$ KB = BL = 2;30°
and DK = EL = 4;20°.

First let us suppose the moon to be at E:
so we subtract the 2;30° from the 45° of arc ZB, since the moon's distance in latitude is in the same direction as the zenith (i.e. they are either both south or both north of the ecliptic).

Thus arc ZL = 42;30°.

Secondly, suppose the moon to be at point D. Then we add [2;30°] to the 45°, since the relative positions are reversed, and H458

ZK = 47;30°.

We form either $ZL^2 + EL^2 = 42;30^2 + 4;20^2$
or $ZK^2 + DK^2 = 47;30^2 + 4;20^2$,
and get either ZE ≈ 42;46°
or ZD ≈ 47;44°.

We multiply 4;20 by 120 and divide by 42;46 and 47;44 separately.

Then EL ≈ $12;8^p$ where hypotenuse ZE = 120^p
and DK ≈ $10\frac{5}{6}^p$ where hypotenuse ZD = 120^p.

The arc corresponding to the chord $12;8^p$ is about $11\frac{3}{5}°$
and the arc corresponding to the chord $10\frac{5}{6}^p$ is about $10\frac{1}{3}°$.
Taking half of these, we subtract ∠ EZL, [namely] $5\frac{4}{5}°$, from ∠ ABZ, i.e. 30°, since arc ZE is less than arc ZB. H459

Thus ∠ AΘZ = $24\frac{1}{5}°$;

and we add ∠ DZK, [namely] $5\frac{1}{6}°$, to the same [∠ ABZ, i.e.] 30°, since arc ZD is greater than arc ZB.

Thus ∠ AHZ = $35\frac{1}{6}°$.

Such is the procedure which was required.[87]

[86] Change the full stop after $\overline{\rho\delta}$ at H457,7 to a comma.

[87] Although one might expect that, as Neugebauer states (*HAMA* 116, which gives an incorrect account of Ptolemy's procedure) that this method, which treats the large spherical triangles ZBD and ZBE as plane triangles, would lead to great inaccuracy, this is not so (as I have verified by taking the worst possible cases): the reason is that the *bases* of these triangles are small (BD and BE cannot exceed 5°, the maximum lunar latitude).

Book VI

1. {*On conjunctions and oppositions of sun and moon*}[1] H461

The next subject we have to treat concerns the syzygies of sun and moon at which eclipses occur. The first topic of this, in turn, is the determination of the true conjunctions and oppositions. Now we do indeed think that the periodic and anomalistic motions which we have [already] established for each of the luminaries are sufficient for the first determination of the above; for these [motions] enable one, if he does not shrink from [the labour of] comparing the individual positions of the luminaries at every appropriate occasion,[2] to compute the places and times of the resulting syzygies, both those taken with respect to the mean motions and the true syzygies, [i.e.] taking the anomaly into account. Nevertheless, in order to provide a more convenient way of finding these [syzygies] too, by having set out in a readily available form the times and places of the mean conjunctions and oppositions, together with the position of the moon in anomaly and latitude at [these] mean times (which are the basis for the correction leading to the true syzygies and thence to the ecliptic syzygies), we constructed tables for this purpose. Their structure is as follows.

2. {*Construction of the tables of mean syzygies*} H462

First, we want to begin the epoch of the [synodic] months, like all other epochs, from the first year of Nabonassar. So we divided the mean position [of the moon] in elongation at noon, Thoth 1[3] in the Egyptian calendar in that year, which we showed above [IV 8 p. 205] to be 70;37° by the mean daily motion in elongation, and found 5;47,33^d. Therefore the previous mean conjunction preceded noon on Thoth 1 by that amount. So the next [mean conjunction] occurred about [29;31,50 − 5;47;33 =] 23;44,17^d after that noon, i.e. 0;44,17^d after noon on the 24th.

In 23;44,17^d

[1] On chs. 1 and 2 see *HAMA* 118–21, Pedersen 220–2.

[2] I.e. at every syzygy (whereas Ptolemy's tables VI 3 enable one to pick out the syzygies at which eclipses are possible with much less labour).

[3] Here (H462,5) and elsewhere in this chapter (H462,9 and 16; H463,3) most Greek mss. and Pappus' commentary give νεομηνία (literally 'new moon') to express this date. As Manitius notes (338 n. d), the word is appropriate for the first day of the month in Greek luni-solar calendars, but not in the Egyptian calendar, where the months bear no relationship to the phases of the moon. In all but the last of these places D has ᾱ ('1'), which may well have been Ptolemy's designation.

mean motion of the sun = 23;23,50°
mean motion of the moon in anomaly = 310;8,15°
mean motion of the moon in latitude = 314;2,21°.

And the mean positions at noon on Thoth 1 were:

longitude of sun: ♓ 0;45°
distance of sun from its apogee (this is convenient to have): 265;15°
anomaly of moon, counted from the apogee of the epicycle: 268;49°
H463 [argument of] latitude of moon, counted from the northern limit on [the moon's] inclined circle: 354;15°.

Therefore, at the above-mentioned moment of the [first] mean conjunction after the first day [of Thoth],

the distance of the sun and moon in mean longitude from the sun's apogee, namely ♊ 5;30°, was 288;38,50°
the distance of the moon in anomaly from the apogee [of the epicycle] was 218;57,15°
the distance of the moon in latitude from the northern limit was 308;17,21°.

So we will set out, first, a table of conjunctions, containing, again, 45 lines, and 5 columns. On the first line we will put, in the first column, year 1 of Nabonassar; in the second column, the days of Thoth, 24;44,17 (for the sixtieths [of a day] are after noon on the 24th);[4] in the third column the distance from the sun's apogee of the mean position [of sun and moon], 288;38,50°; in the fourth column the moon's distance in anomaly from the apogee [of the epicycle], 218;57,15°; and in the fifth column the [moon's] distance in [argument of] latitude from the northern limit, 308;17,21°.

Now half a mean [synodic] month comprises approximately 14;45,55^d, 14;33,12° of solar [mean] motion, 192;54,30° of lunar anomaly, and 195;20,6° of [argument of] latitude; we subtract the above amounts from the [corresponding
H464 positions] for the conjunction in question, and put the results, arranged in the same way as before, at the beginning of the second table, which has a structure similar [to the first], but will serve for the oppositions.

The entries are:

days:	9;58,22^d
distance from the sun's apogee:	274;5,38°
distance in anomaly from the moon's apogee:	26;2,45°
distance in latitude from the northern limit:	112;57,15°.

Now 25 Egyptian years less 0;2,47,5^d contain approximately an integer number of [mean synodic] months;[5] and [in 25 years] the mean motions (beyond complete revolutions) are:

sun:	353;52,34,13°
moon, anomaly:	57;21,44,1°
moon, latitude:	117;12,49,54°.

[4] Although the conjunction is only 23;44,17^d after epoch, Ptolemy tabulates 24;44,17, i.e. he is here using inclusive reckoning for dates. The convenience of this to the user became so obvious that in his Handy Tables he adopted it generally.

[5] The relationship 25 Egyptian years ≈ 309 synodic months was probably known in Egypt long before Ptolemy. For an example of its use in Egypt, and the reasons for dating its origin to the fourth century B.C., see *HAMA* II 563-64. 309 × 29;31,50,8,20^d = 2,32,4;57,12,55, which is *exactly* (not approximately, as Ptolemy implies) 0;2,47,5^d short of 25 × 365 = 2,32,5^d.

So we will increase [each line in succession of] the first columns of the two tables by 25 years, and decrease [those of] the second columns by 0;2,47,5, and increase [those of] the remaining columns, the third by 353;52,34,13°, the fourth by 57;21,44,1°, and the fifth by 117;12,49,54°.

Following this we construct a table of years, in 24 lines, and then beneath it another table, of months, in 12 lines, each having the same number of columns as the first [two tables]. In the table for months we will enter on the first line, in the first column, the first month; in the second column, the days in one [synodic] month, 29;31,50,8,20; in the third column, the [mean] motion of the sun during H465
that period, 29;6,23,1°; in the fourth column, the motion of the moon in anomaly [in one synodic month], 25;49,0,8°; and in the fifth, the motion in [argument of] latitude, 30;40,14,9°. The [line to line] increments in this table will be the same as the entries in the first line.

In the table for years we will enter on the first line, in the first column, year 1; in the second column, the number of days [beyond 365] contained in 13 synodic months, 18;53,51,48;[6] in the third column, the increment in sun's motion during that period, 18;22,59,18°; in the fourth column, the moon's motion in anomaly, 335;37,1,51°; and in the fifth column, the motion in latitude, 38;43,3,51°. The [line to line] increments in this table will sometimes be the above 13-month increments, and at other times the 12-month increments. The latter come to:

days:	354;22,1,40^{d}
sun's [mean] motion:	349;16,36,16°
moon's anomalistic motion:	309;48,1,42°
moon's latitudinal motion:	8;2,49,42°.

This [alternation between 12- and 13-month intervals] is in order that what appears in the table will be the first syzygy in each integer Egyptian year.[7]

In the actual tabular entries it will be sufficient to go only as far as the second sexagesimal [fractional] place. The layout of the tables is as follows.

3. {*Tables of conjunctions and oppositions*}[8] H466-71

[See pp. 278-80.]

4. {*How to determine the mean and true syzygies*}[9] H472

So when we want to find the mean syzygies for any given year, we calculate the number of the year in question in the era Nabonassar.[10] Then we determine what combination of 25-year periods (taken from the first or second table, as the

[6] Reading $\overline{\nu\alpha}$ for $\overline{\nu\beta}$ (18;53,52,48) at H465,10, with D,Ar. Corrected by Manitius.

[7] For an explanation of how this principle works for the choice of 12- or 13-month increment see *HAMA* 120.

[8] As Ptolemy says, these tables are computed to 3 sexagesimal fractional places, but rounded to 2 in the actual tabulation.
The eclipse limits on p. 280 are those derived later, VI 5 pp. 286-7.

[9] See *HAMA* 121-4, Pedersen 223-6.

[10] I.e. we enter with the current year. Cf. p. 276 n.4.

TABLE OF CONJUNCTIONS

1 25-year periods	2 Days of Thoth	3 Distance of Sun from its Apogee ° ′ ″	4 Anomaly of Moon from Epicyclic Apogee ° ′ ″	5 Latitude from Northern Limit ° ′ ″
1	24 44 17	288 38 50	218 57 15	308 17 21
26	24 41 30	282 31 24	276 18 59	65 30 11
51	24 38 43	276 23 58	333 40 43	182 43 1
76	24 35 56	270 16 33	31 2 27	299 55 51
101	24 33 9	264 9 7	88 24 11	57 8 41
126	24 30 22	258 1 41	145 45 55	174 21 31
151	24 27 35	251 54 15	203 7 39	291 34 20
176	24 24 47	245 46 50	260 29 23	48 47 10
201	24 22 0	239 39 24	317 51 7	166 0 0
226	24 19 13	233 31 58	15 12 51	283 12 50
251	24 16 26	227 24 32	72 34 35	40 25 40
276	24 13 39	221 17 6	129 56 19	157 38 30
301	24 10 52	215 9 41	187 18 3	274 51 20
326	24 8 5	209 2 15	244 39 47	32 4 10
351	24 5 18	202 54 49	302 1 31	149 17 0
376	24 2 31	196 47 23	359 23 15	266 29 50
401	23 59 44	190 39 57	56 44 59	23 42 39
426	23 56 57	184 32 32	114 6 43	140 55 29
451	23 54 10	178 25 6	171 28 27	258 8 19
476	23 51 22	172 17 40	228 50 11	15 21 9
501	23 48 35	166 10 14	286 11 55	132 33 59
526	23 45 48	160 2 49	343 33 39	249 46 49
551	23 43 1	153 55 23	40 55 23	6 59 39
576	23 40 14	147 47 57	98 17 7	124 12 29
601	23 37 27	141 40 31	155 38 51	241 25 19
626	23 34 40	135 33 5	213 0 35	358 38 9
651	23 31 53	129 25 40	270 22 19	115 50 58
676	23 29 6	123 18 14	327 44 3	233 3 48
701	23 26 19	117 10 48	25 5 47	350 16 38
726	23 23 32	111 3 22	82 27 31	107 29 28
751	23 20 45	104 55 57	139 49 16	224 42 18
776	23 17 57	98 48 31	197 11 0	341 55 8
801	23 15 10	92 41 5	254 32 44	99 7 58
826	23 12 23	86 33 39	311 54 28	216 20 48
851	23 9 36	80 26 13	9 16 12	333 33 38
876	23 6 49	74 18 48	66 37 56	90 46 28
901	23 4 2	68 11 22	123 59 40	207 59 17
926	23 1 15	62 3 56	181 21 24	325 12 7
951	22 58 28	55 56 30	238 43 8	82 24 57
976	22 55 41	49 49 4	296 4 52	199 37 47
1001	22 52 54	43 41 39	353 26 36	316 50 37
1026	22 50 7	37 34 13	50 48 20	74 3 27
1051	22 47 20	31 26 47	108 10 4	191 16 17
1076	22 44 32	25 19 21	165 31 48	308 29 7
1101	22 41 45	19 11 56	222 53 32	64 41 57

TABLE OF OPPOSITIONS

1 25-year periods	2 Days of Thoth	3 Distance of Sun from its Apogee ° ′ ″	4 Anomaly of Moon from Epicyclic Apogee ° ′ ″	5 Latitude from Northern Limit ° ′ ″
1	9 58 22	274 5 38	26 2 45	112 57 15
26	9 55 35	267 58 12	83 24 29	230 10 5
51	9 52 48	261 50 46	140 46 13	347 22 55
76	9 50 1	255 43 21	198 7 57	104 35 45
101	9 47 14	249 35 55	255 29 41	221 48 35
126	9 44 27	243 28 29	312 51 25	339 1 25
151	9 41 40	237 21 3	10 13 9	96 14 14
176	9 38 52	231 13 38	67 34 53	213 27 4
201	9 36 5	225 6 12	124 56 37	330 39 54
226	9 33 18	218 58 46	182 18 21	87 52 44
251	9 30 31	212 51 20	239 40 5	205 5 34
276	9 27 44	206 43 54	297 1 49	322 18 24
301	9 24 57	200 36 29	354 23 33	79 31 14
326	9 22 10	194 29 3	51 45 17	196 44 4
351	9 19 23	188 21 37	109 7 1	313 56 54
376	9 16 36	182 14 11	166 28 45	71 9 44
401	9 13 49	176 6 45	223 50 29	188 22 33
426	9 11 2	169 59 20	281 12 13	305 35 23
451	9 8 15	163 51 54	338 33 57	62 48 13
476	9 5 27	157 44 28	35 55 41	180 1 3
501	9 2 40	151 37 2	93 17 25	297 13 53
526	8 59 53	145 29 37	150 39 9	54 26 43
551	8 57 6	139 22 11	208 0 53	171 39 33
576	8 54 19	133 14 45	265 22 37	288 52 23
601	8 51 32	127 7 19	322 44 21	46 5 13
626	8 48 45	120 59 53	20 6 5	163 18 3
651	8 45 58	114 52 28	77 27 49	280 30 52
676	8 43 11	108 45 2	134 49 33	37 43 42
701	8 40 24	102 37 36	192 11 17	154 56 32
726	8 37 37	96 30 10	249 33 1	272 9 22
751	8 34 50	90 22 45	306 54 45	29 22 12
776	8 32 2	84 15 19	4 16 29	146 35 2
801	8 29 15	78 7 53	61 38 14	263 47 52
826	8 26 28	72 0 27	118 59 58	21 0 42
851	8 23 41	65 53 1	176 21 42	138 13 32
876	8 20 54	59 45 36	233 43 26	255 26 22
901	8 18 7	53 38 10	291 5 10	12 39 11
926	8 15 20	47 30 44	348 26 54	129 52 1
951	8 12 33	41 23 18	45 48 38	247 4 51
976	8 9 46	35 15 52	103 10 22	4 17 41
1001	8 6 59	29 8 27	160 32 6	121 30 31
1026	8 4 12	23 1 1	217 53 50	238 43 21
1051	8 1 25	16 53 35	275 15 34	355 56 11
1076	7 58 37	10 46 9	332 37 18	113 9 1
1101	7 55 50	4 38 44	29 59 2	230 21 51

YEARLY [AND MONTHLY] INCREMENTS for CONJUNCTION and OPPOSITION

1 Single years	2 Days	3 Sun from Apogee ° ′ ″	4 [Moon's] Anomaly ° ′ ″	5 Latitude ° ′ ″
1	18 53 52	18 22 59	335 37 2	38 43 4
2	8 15 53	7 39 36	285 25 4	46 45 54
3	27 9 45	26 2 35	261 2 5	85 28 57
4	16 31 47	15 19 11	210 50 7	93 31 47
5	5 53 49	4 35 47	160 38 9	101 34 37
6	24 47 40	22 58 47	136 15 11	140 17 41
7	14 9 42	12 15 23	86 3 12	148 20 30
8	3 31 44	1 31 59	35 51 14	156 23 20
9	22 25 36	19 54 59	11 28 16	195 6 24
10	11 47 37	9 11 35	321 16 18	203 9 14
11	1 9 39	358 28 11	271 4 19	211 12 3
12	20 3 31	16 51 10	246 41 21	249 55 7
13	9 25 32	6 7 47	196 29 23	257 57 57
14	28 19 24	24 30 46	172 6 25	296 41 1
15	17 41 26	13 47 22	121 54 26	304 43 50
16	7 3 28	3 3 59	71 42 28	312 46 40
17	25 57 19	21 26 58	47 19 30	351 29 44
18	15 19 21	10 43 34	357 7 32	359 32 34
19	4 41 23	0 0 10	306 55 33	7 35 23
20	23 35 14	18 23 10	282 32 35	46 18 27
21	12 57 16	7 39 46	232 20 37	54 21 17
22	2 19 18	356 56 22	182 8 39	62 24 7
23	21 13 10	15 19 22	157 45 41	101 7 10
24	10 35 11	4 35 58	107 33 42	109 10 0

[ECLIPSE] LIMITS OF SUN IN MEAN [LATITUDINAL] MOTION:
from 69;19° to 101;22° and from 258;38° to 290;41°
[ECLIPSE] LIMITS OF MOON IN MEAN [LATITUDINAL] MOTION:
from 74;48° to 105;12° and from 254;48° to 285;12°

Months	Days	Sun from Apogee	[Moon's] Anomaly	Latitude
1	29 31 50	29 6 23	25 49 0	30 40 14
2	59 3 40	58 12 46	51 38 0	61 20 28
3	88 35 30	87 19 9	77 27 0	92 0 42
4	118 7 21	116 25 32	103 16 1	122 40 57
5	147 39 11	145 31 55	129 5 1	153 21 11
6	177 11 1	174 38 18	154 54 1	184 1 25
7	206 42 51	203 44 41	180 43 1	214 41 39
8	236 14 41	232 51 4	206 32 1	245 21 53
9	265 46 31	261 57 27	232 21 1	276 2 7
10	295 18 21	291 3 50	258 10 1	306 42 21
11	324 50 12	320 10 13	283 59 2	337 22 36
12	354 22 2	349 16 36	309 48 2	8 2 50

case may be [i.e. for conjunction or opposition]) and single years (taken from the third table) adds up to that number of years, take the entries corresponding to those lines [in the table], and add the entries from [each] successive column separately: for conjunctions we add the entries from the first and third tables, and likewise for oppositions we add the entries from the second and third tables. The sum derived from the entries in the second column will give us the moment of syzygy, counted from the beginning of that year; e.g., if the sum is $24;44^d$, [the syzygy will be] 44 sixtieths of a day after noon on Thoth 24; or, again, if it is $34;44^d$, it will be 44 sixtieths of a day after noon on Phaophi 4. The sum derived from the entries in the third column will give us the [mean] position of the sun in degrees counted from the apogee; the fourth column, the anomaly of the moon counted from the apogee [of the epicycle]; the fifth column, the [argument of] latitude counted from the northern limit. At the same time we H473
can readily calculate the subsequent [syzygies of the year in question], either all, or some, as we choose, in logical fashion, by adding the appropriate entries in the fourth, monthly table. For practical purposes we will always convert the time measurements from sixtieths of a day into equinoctial hours. However, the time in hours resulting from the addition [of the entries] will be expressed in mean solar days, whereas the time expressed in seasonal hours is not always identical with that, but is based on true solar days. So we will correct this too, by calculating the difference due to this effect, by the method indicated above: if the amount of time-degrees corresponding to [the rising-time of] the apparent motion is greater [than the interval in mean motion], we subtract the difference from the total [of hours] derived on the basis of mean solar days, but if it is less, we add it to that total.[11]

Once we have derived, by the above procedure, the time of mean conjunction or opposition, and the position of each luminary in anomaly at that time, it will be easy to determine the time and place of the true syzygy, and also the moon's position in latitude, by comparing the anomalies of the two bodies. For by applying each anomaly in turn, we calculate the true position of sun, moon and moon's latitude, at the moment defined by the mean syzygy in question, by means of the equation thus found, and examine these positions. If we find that the bodies are still at the same longitude [for conjunction], or exactly opposite [for opposition], then the time of true syzygy will be the same [as that of mean H474
syzygy]. If not, we take the difference between the bodies in longitude, expressed in degrees, and increase it by a twelfth part of itself,[12] to account approximately for the additional motion of the sun [between mean and true syzygy]. We then determine how long, in equinoctial hours, the moon in its anomalistic [i.e. true] motion, takes to cover that interval. If the true longitude

[11] Ptolemy here echoes III 9 p. 171. There he expressed the rule in the form necessary for going from true to mean time. Here the case (and the rule) are reversed.

[12] This rule is justified by a particular example at VI 5 (p. 286); where Ptolemy, assuming the moon to move 13 times as fast as the sun, calculates that the extra distance required is $\frac{1}{13} + \frac{1}{13} \times \frac{1}{13} \approx \frac{1}{12}$ of the original. Hence Pedersen (224) assumes that Ptolemy found $\frac{1}{12}$ by summing the convergent series $\frac{1}{13} + (\frac{1}{13})^2 + \ldots$ Although the passage VI 5 supports him, one can also derive it without summing a series, as follows: if the moon starting from point A and the sun starting from point B meet at point C, and the moon's speed is 13 times the sun's, then AC = 13BC, hence AB (the original distance between them) is 12 times BC (the extra distance travelled).

of the moon [at mean syzygy] is less than the true longitude of the sun, we add the result to the time of mean syzygy, but if it [the moon's longitude] is greater, we subtract the result from the time of mean syzygy. Similarly, if the true longitude of the moon at mean syzygy is less than the sun's [true longitude], we add the interval in degrees (increased, again, by a twelfth) to both the longitude and the argument of latitude [at mean syzygy], but if it is greater we subtract it [from both]. Thus we get the time of true syzygy, and the approximate true position of the moon on its inclined circle.[13]

The method of finding the moon's true hourly motion at the syzygy for any given position is as follows. We enter the table of the moon's anomaly [IV 10] with the anomaly at the moment in question, take the corresponding equation, and then determine the size of the increment in the equation [at that point]
H475 corresponding to an increment of 1 degree in anomaly. We multiply this increment by the mean motion in anomaly in 1 hour, 0;32,40°,[14] and, if the anomaly [with which we entered the table] as argument is in the lines above the greatest equation, we subtract the product from the mean hourly motion in longitude, 0;32,56°, but if [the anomaly] is in the lines below [the greatest equation], we add the product to 0;32,56°. The result will be the moon's true motion in longitude in one equinoctial hour at that position.[15]

Now the above procedure will give us the time of true syzygy at Alexandria, since all epochs have been defined in terms of time as expressed in hours [i.e. counted from noon] with respect to the meridian through Alexandria. But it is easy to find the time of a given syzygy for any place whatever from the time of that syzygy at Alexandria.[16] From the difference in position between the two places, we determine the interval, in degrees, between the meridian through the place required and the meridian through Alexandria. If the meridian through the required place is to the east of the meridian through Alexandria, the phenomenon will appear to be observed there that amount (in time-degrees) later, but if it is to the west, that amount earlier. (Obviously, as always, 15 time-degrees represent 1 equinoctial hour.)

H476

5. {*On the ecliptic limits of sun and moon*}[17]

Now that we have explained the above methods, it would be appropriate to follow up with the considerations pertinent to the ecliptic limits for both solar and lunar eclipses. The purpose of this is that if we decide to compute, not all

[13] For a year's series of computed mean and true oppositions see *HAMA* 121, 123–4. See also Appendix A, Examples 11 and 12.

[14] Reading ο $\overline{\lambda\beta}$ $\overline{\mu}$ for ο $\overline{\lambda\beta}$ $\overline{\mu}$ ο (0;32,40,0) at H475,2, and similarly ο $\overline{\lambda\beta}$ $\overline{\nu\varsigma}$ for ο $\overline{\lambda\beta}$ $\overline{\nu\varsigma}$ ο (0;32,56,0) at H475,5–6. Supported by D,Ar.

[15] For a justification of this rule see Pedersen 226. He objects that it is approximately valid only if the lunar deferent has no eccentricity, i.e. if one uses the simple hypothesis of Bk. IV. But Ptolemy advocates its use only 'at the syzygy', and he has already shown that there is no significant difference between the two hypotheses at syzygy (V 10).

[16] Omitting the clause (H475, 15–17) δοθέντος τοῦ κατ' αὐτὴν πλήθους τῶν ἰσημερινῶν ὡρῶν τῆς ἀπὸ τοῦ μεσημβρινοῦ ἀποχῆς ('once we are given the distance of it [the syzygy] from the meridian, expressed in equinoctial hours'), a clumsy and confusing interpolation found in all mss.

[17] See *HAMA* 125–9, Pedersen 227–30.

mean syzygies [in a given year], but just those which could fall into the category concerning eclipse prognostications,[18] we may have a handy method of deciding which these are from the entry for the moon's mean position in latitude at each mean syzygy.

Now in the preceding book [V 14, p. 254] we have shown that the moon's diameter subtends an arc which is 0;31,20° of the great circle drawn about the centre of the ecliptic at the moon's greatest distance. We calculated this by means of two eclipses which occurred near the apogee of the moon's epicycle. So now too, when we propose to determine the maximum limits of ecliptic syzygies (which limits are determined by the position of the moon at the perigee of the epicycle), we shall, in this situation too, demonstrate in the same way the size of the arc subtended by the moon's diameter, by means of two eclipses [this time] from among those which have been observed near the perigee [of the epicycle]. For it is safer to demonstrate this kind of parameter from the actual phenomena.

In the seventh year of Philometor, which is the 574th from Nabonassar, on H477
Phamenoth [VII] 27/28 in the Egyptian calendar [-173 May 0/1], from the beginning of the eighth hour till the end of the tenth in Alexandria, there was an eclipse of the moon which reached a maximum obscuration of 7 digits from the north. So mid-eclipse occurred 2½ seasonal hours after midnight, which corresponds to 2⅓ equinoctial hours, since the true position of the sun was ♉ 6¼°.[19] And the time from epoch to mid-eclipse is

573 Egyptian years 206 days { 14⅓ equinoctial hours reckoned simply
{ 14 equinoctial hours reckoned in mean solar days.

At this moment the position of the centre of the moon was as follows:

mean longitude: ♏ 7;49°
true longitude: ♏ 6;16°[20]
distance [in anomaly] from the apogee of the epicycle: 163;40°
distance from the northern limit on the inclined circle: 98;20°.

Hence it is clear that when the moon's centre is 8;20° from the node (measured along the inclined circle), while the moon is near its least distance [at syzygy], and the centre of the shadow is on the great circle drawn through the moon's centre at right angles to the inclined circle (which is the position of

[18] The word used here, ἐπισημασίαι, means 'prognostication [concerning weather]' or 'significance in prognostication' at HII 204,7 and HI 536,21; 537,8; 540,7. This is a traditional meaning (e.g. Ptolemy, *Phaseis*, *Op. Min.* 11,4: 20,5), also applying to the verb ἐπισημαίνειν (ibid. 31,10; cf. *Apotelesmatica* II 14, ed. Boll-Boer 100,17). I therefore assume that meaning wherever it occurs in the Almagest, except in the phrase ἐπισημασίας ἄξιαι πόλεις, HI 188,3, where it means merely 'deserving note'. There is a good discussion of ἐπισημαίνειν and related terms in Pfeiffer, *Studien zum antiken Sternglauben* 84–93.

[19] Reading ϛ δ′ for ϛ δ̄ (6;4°) at H477,10. The reading is assured by computation ($\lambda_\odot$ = ♉ 16;13,25°) and by the position of the true moon just below. 6¼ is the reading of AD, Ar and probably all mss. (i.e. the error is Heiberg's). Corrected by Manitius.

[20] This implies an equation of −1;33°, which agrees fairly well with that derived from an anomaly of 163;40° (below : accurate would be −1;32°), if one uses the *simple* lunar hypothesis. However, if one computes with the full accuracy of the tables V 9, one finds $\lambda_☽$ = 216;23° (for at true syzygy $2\bar{\eta}$ ≈ 5½°, which produces a change in α of +50′, and hence a decrease in the equation of 4′ (precisely the maximum amount by which, according to Ptolemy in V 10 p. 243, the full hypothesis can differ from the simple at syzygy). This also affects the moon's position on its orbit, which should be 8;22° (rather than 8;20°) from the node.

greatest obscuration), ($\frac{1}{2}+\frac{1}{12}$)th of the moon's diameter is immersed in the shadow.[21]

Again, in the thirty-seventh year of the Third Kallippic Cycle, which is the
H478 607th from Nabonassar, Tybi [V] 2/3 in the Egyptian calendar [−140 Jan. 27/28], at the beginning of the fifth hour [of night] in Rhodes, the moon began to be eclipsed; the maximum obscuration was 3 digits from the south.

Here, then, the beginning of the eclipse was 2 seasonal hours before midnight, which corresponds to $2\frac{1}{3}$ equinoctial hours in Rhodes and in Alexandria, since the true position of the sun was ♒ 5;8°. And mid-eclipse, at which the greatest obscuration occurred, was about $1\frac{5}{6}$ equinoctial hours before midnight. The time from epoch to mid-eclipse is

606 Egyptian years 121 days $10\frac{1}{6}$ equinoctial hours, whether reckoned simply or in mean solar days.

At this moment the position of the centre of the moon was as follows:

mean longitude:	♌ 5;16°
true longitude:	♌ 5;8°[22]

distance [in anomaly] from the apogee of the epicycle: 178;46°
distance from the northern limit on the inclined circle: 280;36°.

Hence it is clear that when the moon's centre is 10;36° (measured along the inclined circle) from the node, while the moon is (as before) near the least distance, and the centre of the shadow is at the intersection of the ecliptic and the great circle drawn through the moon's centre at right angles to the [moon's] inclined circle, then a quarter of the moon's diameter will be immersed in the shadow.[23]

H479 But[24] when the moon's centre is $8\frac{1}{3}$° from the node on its inclined circle, it is $43\frac{1}{20}$′, measured along the great circle drawn through the poles of the inclined circle, from the ecliptic; and when it is $10\frac{3}{5}$° from the node on its inclined circle, it is $54\frac{5}{6}$′, measured along the great circle drawn through the poles of the inclined circle, from the ecliptic. Now the difference [in magnitude] between the two eclipses comprises $\frac{1}{3}$rd of the moon's diameter, and the difference in the above two distances of its centre, measured along the same great circle, from the same point of the ecliptic (i.e. the centre of the shadow) is 0;11,47°. So it is clear that the whole diameter of the moon subtends an arc of about 0;35,20° of the great circle drawn on the centre of the ecliptic at the moon's least distance [at syzygy].

Furthermore, in the second eclipse, in which $\frac{1}{4}$ of the moon's diameter was

[21] Oppolzer no 1587: mid-eclipse 23;44^h (≈ 1;45 a.m. Alexandria, which is very close to the time of true conjunction one finds from Ptolemy tables), magnitude 7.4 digits.

[22] Again (cf. p. 283 n.20) the equation implied, −0;8°, agrees well enough with that derived from the anomaly of 178;46° according to the simple hypothesis, but application of the full hypothesis produces a significant difference in the true longitude of the moon (♌ 5;13°) and its position on the orbit (10;42° from the node instead of 10;36°).

[23] That this eclipse was observed by Hipparchus, as one would expect from the date and place, is confirmed at VI 9 (p. 309). It is Oppolzer no. 1638: time 20;1^h (≈ 10 p.m. Alexandria), magnitude 3.2^d, half-duration 58 mins. Ptolemy assumes 30 mins., which is only about half of what he would derive from his own eclipse tables, VI 8. The difficulties associated with the observation and reduction of this eclipse have been much discussed: see Fotheringham [3] 579, with references to older literature, and Britton [1] 94.

[24] For the following calculations see *HAMA* 105–8, and cf. p. 254 n.61.

obscured, the moon's centre was $54\frac{5}{6}'$ from the centre of the shadow and $\frac{1}{4}$ of the moon's diameter (i.e. $8\frac{5}{6}'$) from the point at which the line joining the centres [of moon and shadow] intersects the perimeter of the shadow. Hence it is immediately obvious that, by subtraction, the radius of the shadow at the moon's least distance is 46′. This is negligibly greater than $2\frac{3}{5}$ times the moon's radius, which is $17\frac{2}{3}'$. Moreover, the sun's radius subtends 0;15,40° of the great circle drawn through the sun about the centre of the ecliptic. For, as we demonstrated [V 14], the sun covers the same amount of its circle [i.e. subtends the same angle] as the moon does when it is at its greatest distance at syzygy. Therefore, when the apparent centre of the moon is [0;17,40 + 0;15,40 =] 0;33,20° from the centre of the sun, [measured orthogonally to the moon's orbit] on either side of the ecliptic, that is the limiting position in which the moon can just be in apparent contact with the sun. H480

For example [see Fig. 6.1] let us imagine AB as an arc of the ecliptic and GD as an arc of the moon's inclined circle. These are sensibly parallel to each other, at least as far as concerns the positions [of the bodies] at the time of eclipses. We

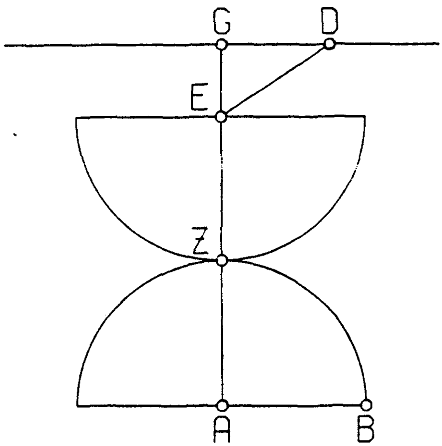

Fig. 6.1

draw the arc of the great circle through the poles of the [moon's] inclined circle, AEG, and imagine the semi-circle of the sun on centre A, and the semi-circle of the apparent moon on centre E, in such a position that it is just touching the sun at point Z. Then arc AE, which is the distance of E, the apparent centre of the moon, from A, the centre of the sun, can at times be as much as 0;33,20°, as established above. But in the regions stretching from Meroe, where the longest day is 13 equinoctial hours, up to the mouths of the Borysthenes, where the longest day is 16 equinoctial hours, the maximum northward effect of the lunar parallax for the moon at least distance in the syzygies (if we subtract the solar parallax) is about 0;8°, and the maximum southward effect, under the same conditions, is 0;58°. When its [latitudinal] parallax is 0;8° northwards, it has a maximum longitudinal parallax of about 0;30°, round about Leo and Gemini; and when its [latitudinal] parallax is 0;58° southwards, it has a maximum H481 longitudinal parallax of about 0;15°, round about Scorpius and Pisces.[25] So if we suppose that the true centre of the moon is at D, and draw line DE, which represents the total parallax, DG will (approximately) represent the parallax in longitude, and GE the parallax in latitude.

Therefore, when the moon is to the north of the sun and has a maximum southward parallax,

H482 arc DG will be 0;15°, and arc AEG [0;33,20° + 0;58° =] about 1;31°. Now the ratio between the arc from the node to G and the arc GA is about $11\frac{1}{2}$: 1 for distances between the eclipse limits: this can easily be seen from our previous demonstration of the inclination of the lunar orbit.[26] So the distance from the node to G will be 17;26°, and GD added to this makes 17;41°.

And when the moon is to the south of the sun and has its maximum northward parallax, arc DG will be 0;30°, and the whole of arc AEG, [0;33,20°+ 0;8° ≈] 0;41°. By the same kind of calculation as before, the distance from the node to G will be 7;52°, and the total distance, including arc GD, 8;22°.

Therefore, the limiting positions, in which the moon can just be in apparent contact with the sun, for the above regions of our part of the inhabited world, are when the true distance of the centre of the moon from either of the nodes on its inclined circle is 17;41° towards the north, or 8;22° towards the south.

Furthermore, since, as we showed, the maximum equation of anomaly is 2;23° for the sun and 5;1° for the moon near the syzygies, it will at times be possible for the true distance of the moon from the sun at mean syzygies to reach

H483 7;24°. But, in the time the moon takes to traverse the distance [7;24°], the sun will traverse an extra distance of about $\frac{1}{13}$th of that amount, i.e. 0;34°; and again, while the moon is traversing that extra 0;34°, the sun will traverse an extra $\frac{1}{13}$th of that, or about 0;3° (a $\frac{1}{13}$th of the latter is negligible). So if we add the sum, 0;37° (which is $\frac{1}{12}$th of the original 7;24°)[27] to the 2;23° of the solar [equation of] anomaly, we get 3°, which is, approximately, the maximum difference in longitude and [argument of] latitude between mean position [of the bodies] at mean syzygy and their true position [at true syzygy]. So the limiting positions in which the moon can just be in apparent contact with the sun, for the above regions, are when the mean distance of the centre of the moon from [either of] the nodes on its inclined circle is 20;41° to the north, or 11;22° to the south. And by the same argument, the above effect can take place in the regions in question only when the amount of the distance of the moon from the northern limit corresponding [in the fifth column of Table VI 3] to the mean syzygy falls between 69;19° and 101;22°, or between 258;38° and 290;41°.

H484 Next, to obtain the moon's ecliptic limits: since, as we showed [p. 284], the moon's radius at its least distance [at syzygy] subtends 0;17,40°, and the

[25] Ptolemy computes the maximum effect of the parallax on ecliptic limits for the region embracing the standard '7 climata' (see Introduction p. 19). There are some serious problems in his (unsupported) statements here, for which see *HAMA* 127-9.

[26] I.e. taking the inclination as 5° (V 12 p. 247), and taking the small spherical triangle formed by the latitude, the ecliptic and the moon's orbit as plane, we compute $\omega : \beta$ = Crd 110° : Crd 10° = 119;32,37 : 10;27,32 = 11.43 : 1 ≈ $11\frac{1}{2}$: 1.

[27] Cf. p. 281 n.12.

shadow's radius, being about $2\frac{3}{5}$ times that, comes to 0;45,56°,[28] it is clear that when the true distance of the moon's centre is 1;3,36° from the shadow's centre on either side of the ecliptic (as measured along the great circle drawn through the poles of the moon's inclined orbit), or about 12;12° from either of the nodes on its inclined circle (according to the ratio 1 : $11\frac{1}{2}$), that is the limiting position in which the moon can just touch the shadow. And by the same argument as was deduced above from the anomaly, the limiting position for the moon to touch the shadow will be when the distance of the mean moon's centre from the node on its inclined circle is 15;12°. Hence the [mean moon], in distance from the northern limit, must fall within the boundaries 74;48° to 105;12°, or 254;48° to 285;12°.

We will, then, include these numbers for the moon's [argument of] latitude at solar and lunar [eclipse] limits in the preceding table of syzygies, in order to provide a convenient method of determining whether [a given syzygy] could fall into the category of an eclipse. H485

6. {*On the interval of months between eclipses*}[29]

In addition to the above, it would also be useful to discuss the problem of the intervals at which, in general, it is possible for ecliptic syzygies to occur, so that, once we have determined a single example of an ecliptic syzygy, we need not apply our examination of the [ecliptic] limits to every succeeding syzygy in turn, but only to those which are separated [from the first] by an interval of months at which it is possible for an eclipse to recur.

Now it is immediately obvious that eclipses of both sun and moon can occur at 6-month intervals, since the increment in the moon's mean motion in [argument of] latitude over 6 months comes to 184;1,25°, and the arcs between the ecliptic limits [at opposite nodes], for both sun and moon, comprise less than the above amount if they are less than a semi-circle, and more than the above amount if they are greater than a semi-circle.[30]

For, in the case of the sun, the ecliptic limits cut off 20;41° (as we showed [p. 286]) to the north of both nodes on the moon's inclined circle, and 11;22° to the south. Thus[31] the arcs on which eclipses cannot occur comprise 138;38° to the north [of the nodes], and 157;16° to the south. H486

And, in the case of the moon, the ecliptic limits cut off 15;12° [above] of the circle [of the moon's orbit] from the nodes on both sides of the ecliptic. Thus each of the arcs on which eclipses cannot occur comprises 149;36°.

[28] Note that Ptolemy takes precisely $2\frac{3}{5}$ times the moon's radius, instead of the value which he had actually derived from the observations, 0;46°.

[29] See *HAMA* 129–34. Pedersen 230–1 is too summary to be useful.

[30] For what follows refer to Fig. H, and, for the increments in motion, to Table VI 3. For the moon, DA = BC = 149;36° < 184;1,25°, and AD = CB = 210;24° > 184;1,25°. For the sun, BC = 138;38° < 184;1,25°; AD = 202;44° > 184;1,25°; DA = 157;16° < 184;1,25°; and CB = 221;22° > 184;1,25°. It is necessary that *both* conditions be fulfilled for it to follow that when the (mean) moon is on one of the ecliptic arcs (AB, CD) at the beginning of the interval it will be on the other (at a distance of 184;1,25°) at the end.

[31] Omitting καὶ (with D) at H485,22.

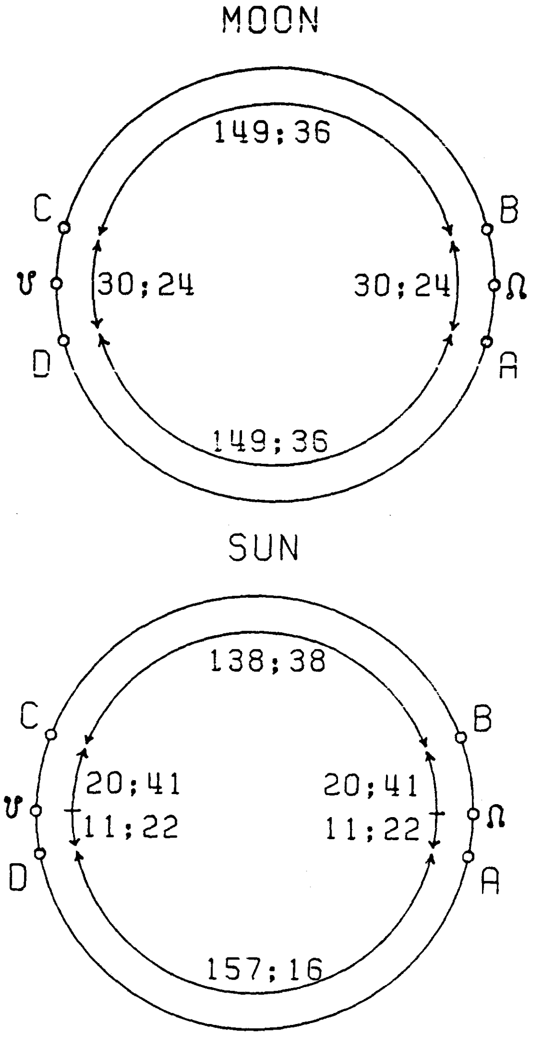

Fig. H

On the basis of the theories developed above, it is possible for eclipses of the moon to recur at a 5-month interval which is the longest possible, i.e. an interval in which the sun has the greatest possible motion and the moon the least. We can see that as follows.

In the mean 5-month interval we find the following increments in the motions:

mean motion in the longitude of both luminaries: 145;32°

motion of the moon on the epicycle in anomaly: 129;5°.

The sun's 145;32°, when its [true] motion is greatest, [i.e. distributed

symmetrically] either side of the perigee, produce an addition to the mean motion of 4;38°.[32] The 129;5° of the moon's anomaly on the epicycle, when its [true] motion is least, [i.e. distributed symmetrically] either side of the apogee, produce a decrement from the mean motion of 8;40°. Therefore over the period of 5 mean synodic months during which the sun has its greatest possible motion and the moon its least, the moon will still be in advance of the sun by the sum of both [above equations of] anomaly, i.e. 13;18°. We take $\frac{1}{12}$th of this (for the reasons explained above [p. 286]), and get about 1;6° for the additional motion of the sun before the moon overtakes it. So, since it has an additional 4;38° of H487
motion from its own anomaly, and another 1;6° from the motion needed for overtaking [the sun] at true syzygy, the greatest possible 5-month interval will be greater than the mean by 5;44° of longitude. Hence the moon's additional motion in latitude on its inclined circle will be about the same amount [5;44°] over the mean motion in latitude in 5 months, which comes to about 153;21°. Thus the true motion in latitude over the greatest possible 5-month interval comes to 159;5°.

But the ecliptic limits of the moon for the moon's mean distance enclose about 1° (either side of the ecliptic) of the great circle drawn through the poles of the moon's inclined circle; for at the moon's least distance [the corresponding amount] is 1;3,36°, and at its greatest distance 0;56,24°;[33] thus [the ecliptic limits enclose] 11;30° of the inclined circle either side of the nodes, and hence the anecliptic arc between them comprises 157;0°. This amount is 2;5° less than the 159;5° of the [moon's] inclined circle which is the increment over the greatest possible 5-month interval. From these considerations it is clear that, if H488
one takes the longest possible 5-month interval, the moon can be eclipsed at the opposition at the beginning of that interval, while it is receding from either of the nodes, and then be eclipsed again at the opposition at the end of the interval, while it is approaching the opposite node. The obscuration will take place from the same side of the ecliptic (never from opposite sides) in both eclipses.

Thus we have shown that the longest possible 5-month interval can produce two lunar eclipses. However, it is impossible for this to occur if 7 months intervene, even if we assume the shortest possible 7-month interval, namely that in which the sun has its least motion and the moon its greatest. We can see this by the same method as above.

For in the mean 7-month interval the increments in motion are as follows:

mean motion in longitude of both luminaries:	203;45°
moon's motion on the epicycle:	180;43°.

The sun's 203;45°, when its [true] motion is least, [i.e. distributed symmetrically] either side of the apogee, produce a decrement from the mean motion of 4;42°, while the 180;43° of the moon's [anomaly] on the epicycle, when its [true] motion is greatest, [i.e. distributed symmetrically] either side of the perigee, produce an addition to the mean motion of 9;58°. Therefore over the period of 7

[32] I.e. the solar equation is −2;19° at a solar anomaly of 180° − (145;32÷2)°, or 107;14°, and +2;19° at the symmetric position of 252;46°. The corresponding true longitudes are 65;30° greater, or about ♍ 20° and ♒ 20°, cf. p. 290.

[33] See pp. 287 and 254. The amount is the sum of the radii of moon and shadow. At greatest distance this is 0;15,40° + (2⅗ × 0;15,40)° = 0;56,24°.

H489 mean synodic months in which the sun has its least possible motion and the moon its greatest, the moon will be beyond the sun by the sum of both [above equations of] anomaly, 14;40°. For the same reason [as before], we take $\frac{1}{12}$th of this, [1;13°], and add it to the decrement due to the sun's anomaly, 4,42°. The result, 5,55°, gives us the approximate amount by which [the bodies'] motion in longitude over the shortest possible 7-month interval falls short of that over the mean 7-month interval. The moon's motion in latitude will fall short of that over the mean 7-month interval, 214;42°, by the same amount [5;55°]. So in the least possible 7-month interval the increment in the moon's latitudinal motion on its inclined circle will be 208;47°. But the total amount of the greatest arc between the [ecliptic] limits of the moon at mean distance, that is the arc between the limit preceding one node and the limit following the other node, is only [180° + 2 × 11;30° =] 203°. Therefore it is impossible for the moon to be eclipsed at the first opposition of a 7-month interval and then to be eclipsed again, in any way whatever, at the last opposition of that interval, even if it is the shortest possible.

We must now prove that, over the greatest possible 5-month interval, the sun too can be eclipsed twice for observers in the same place, and in all regions of our part of the inhabited world.

In the longest possible 5-month interval, the moon's increment in [argument of] latitude is, as we have shown [p. 289], 159;5°. And the arc on which solar
H490 eclipses cannot occur, for the moon's mean distance, is 167;36°: for the sun's ecliptic limits are 0;32,20° from the ecliptic, as measured along the great circle through the poles of the ecliptic, and about 6;12°, as measured along the moon's inclined circle.[34] So it is clear that, if the moon has no parallax, the event in question [solar eclipses at a 5-month interval] will be impossible, since the anecliptic arc exceeds the motion over the longest possible 5-month interval by 8;31° counted along the [moon's] inclined circle, which corresponds to about 0;45° on the [great circle] orthogonal to the ecliptic. However, at any place where the moon can attain a parallax so great that the parallax at either of the conjunctions at the two ends [of the interval], or the sum of the parallaxes at both conjunctions combined, exceeds 0;45°, it is possible for the conjunctions at both ends to produce an eclipse at that place.

Now we have shown [p. 289] that, over the period of that mean[35] 5-month interval in which the moon has its least possible motion and the sun its greatest, [which is] from two-thirds through Virgo up to two-thirds through Aquarius,[36] the moon is still in advance of the sun by the sum of both [equations of] anomaly, 13;18°. It takes the moon, in mean motion, $1^{d}\,2\frac{1}{4}^{h}$ to move (13;18° + $\frac{1}{12}$ × 13;18°).[37]

[34] The ecliptic limits of the sun are, in latitude, the sum of the radii of the sun (0;15,40°, p. 285) and the moon at mean distance (mean between 0;15,40°, p. 254, and 0;17,40°, p. 285, i.e. 0;16,40°). 0;15,40° + 0;16,40° = 0;32,20°. The corresponding distance from the node is $11\frac{1}{2}$ × 0;32,20° = 6;11,50° ≈ 6;12°. So the anecliptic arc is (180° − 2 × 6;12°) = 167;36°.

[35] It is essential to read (with D,Ar) τῆς μέσης πενταμήνου at H490,16 for τῆς μεγίστης πενταμήνου ('the greatest 5-month interval'). The meaning is 'the interval of 5 *mean* synodic months'. The change to μεγίστης was probably made by someone who compared ἐν τῇ μεγίστῃ πενταμήνῳ (H489,25), where the phrase is in order only because it refers to *true* synodic months. However, for a purely mechanical confusion between μέσον/μέγιστον compare p. 292 n.43.

[36] See p. 289 n.32.

[37] In $1^{d}\,2\frac{1}{4}^{h}$ the moon moves 14;24,42° in longitude. 13;18° + 1;6° (p. 289) = 14;24°.

Hence it is clear, since the period of the mean 5-month interval is about $147^d\ 15\frac{3}{4}^h$,[38] that the period of the longest possible 5-month interval will be $148^d\ 18^h$. Therefore the last conjunction, which takes place about two-thirds H491
through Aquarius, will be earlier [in the day] than the first conjunction, which takes place about two-thirds through Virgo, by 6 hours (which is the difference [of the above period] from an integer number of days). So we have to search for a place and time at which, if the moon is in Virgo [ca. 20°] and also, 6 hours earlier, in Aquarius [ca. 20°], its parallax exceeds the above-mentioned 0;45°, that is, either its parallax in one of those signs taken singly, or the combined parallax in both of those signs.

Now we find that the moon's northward parallax never reaches that amount (under the prescribed conditions) in any place in our part of the inhabited world. Hence it is impossible for the sun to be eclipsed twice in the longest possible 5-month interval when the moon's position is to the south of the ecliptic, that is when it is receding from the descending node at the first conjunction and approaching the ascending node at the last. However, it can achieve a southward parallax of this amount, in all regions (beginning almost at the equator, and going northwards), if one takes the combined parallax at both the above signs with a 6-hour difference. This occurs when ♍ 20° is at the setting-point at the first conjunction, and ♒ 20° in the meridian at the second conjunction. For in those situations we find the following approximate southward parallaxes, for the moon at mean distance (subtracting the solar H492
parallax):[39]

	☽ in ♍	☽ in ♒
at the equator	0;22°	0;14°
where the longest day is $12\frac{1}{2}^h$	0;27°	0;22°.

Thus already in latter region the combined parallaxes exceed the 0;45° in question by 4 minutes. And since the southward parallax increases as one takes regions farther north, it is obvious that there will be an increasing possibility, [as one goes to regions farther north,] for the sun to be eclipsed for the inhabitants of those regions twice in the longest possible 5-month interval. However, this can happen only while the moon's position is to the north of the ecliptic, that is when it is receding from the ascending node at the first eclipse and approaching the descending node at the second.

I say, furthermore, that it is possible for the sun to be eclipsed twice for observers in the same place also in the shortest 7-month interval. For, as we have shown [p. 290], the moon's motion in [argument of] latitude over the shortest 7-month interval is 208;47°. And the greatest arc of the [moon's] inclined circle intercepted between [two] ecliptic limits (which is the arc between the limit preceding one node and the limit succeeding the opposite node) is, for the sun when the moon is at mean distance, 192;24°.[40] So it is again H493
clear that, if the moon has no parallax, the event in question cannot take place, since the arc of the [moon's] inclined circle covered in the shortest 7-month

[38] Result of multiplying 29;31;50,8,20^d by 5. More accurate would be $15\frac{2}{3}^h$.

[39] The details of the computation of these are given in the commentary of Pappus (Rome [1] I 225–9), who finds 0;29° instead of 0;27°.

[40] I.e. 180° + 2×6;12°. Cf. p. 290 n.34.

interval exceeds the greatest arc cut off between the sun's ecliptic limits by 16;23°, as measured on the inclined circle, [which corresponds to] 1;25° on the circle through the poles of the ecliptic. But in any place where the moon's parallax is great enough so that the parallax at either of the conjunctions at the two ends [of the interval], or the sum of the parallaxes at both conjunctions combined, exceeds 1;25°, it is possible for the conjunctions at both ends to produce an eclipse at that place.

Now we have shown [p. 290] that, over the period of that mean 7-month interval in which the moon has its greatest [true] motion, and the sun its least, [which is] from the end of Aquarius to the middle of Virgo,[41] the moon, in true motion, has already overtaken the sun by 14;40°. The moon in mean motion traverses $(14;40 + \frac{1}{12} \times 14;40)°$ in $1^d\ 5^h$.[42] Hence, since the period of the mean 7-month interval comprises about $206^d\ 17^h$, the period of the shortest possible 7-month interval will be $205^d\ 12^h$. Therefore, the last conjunction, which takes place
H494 about the middle of Virgo, will be 12 hours later [in the day] than the first conjunction, which takes place about the end of Aquarius. So we have to search for a place and time at which the moon's parallax can exceed 1;25°, either at one of those situations singly or at both situations combined, when the two situations are separated by 12 hours, i.e. one sign is setting and the other rising (for otherwise it will be impossible for both eclipses to occur above the horizon).

Now, again, it is impossible for the moon to achieve a northward parallax of that amount for any region in our part of the inhabited world, since, even for those living directly below the equator, the [northward] parallax in latitude at the [moon's] mean[43] distance never exceeds 23 minutes. Hence it is impossible for the sun to be eclipsed twice in the shortest 7-month interval when the moon's position is to the south of the ecliptic, i.e. when it is approaching the ascending node at the first conjunction and receding from the descending node at the last conjunction. But we find that a southward parallax of that amount [i.e. greater than 1;25°] is achieved [for regions north of a latitude which is] approximately the parallel through Rhodes, when the end of Aquarius is rising and the middle of Virgo is setting. For in Rhodes, and those regions beneath the same parallel,
H495 at both of the above situations the parallax of the moon at mean distance (with the solar parallax subtracted) is about 0;46° southwards.[44] Thus already in these regions the sum of the parallaxes at both conjunctions is greater than 1;25°. And since for regions yet farther north than this parallel the southward parallax is greater, it is obvious that for the inhabitants of those regions an

[41] Cf. p. 289 n.32. Here the longitudes are given by
$$65;30° \mp \tfrac{1}{2}(203;45° - 4;42°) = \begin{cases} ♒\ 25;58\tfrac{1}{2}° \\ ♍\ 15;1\tfrac{1}{2}°. \end{cases}$$
[42] $\bar{\lambda}$ ☽ in $1^d\ 5^h$ = 15;55,17°. $\frac{13}{12} \times 14;40°$ = 15;53,20°.

[43] Reading μέσον (with Ar) for μέγιστον ('greatest distance') at H494,12. The reading is multiply guaranteed: Ptolemy uses the moon's mean distance throughout this section (cf. pp. 289, 290); taking the *greatest* distance *decreases* the parallax (which is in conflict with the argument here). Numerically, from Table V 18, for a zenith distance of 24° (the maximum zenith distance of the ecliptic at the terrestrial equator) the parallax (lunar minus solar) at mean distance is $0;22,6 + \frac{1}{2} \times 0;4,18 - 0;1,9 = 0;23,6°$ (likewise at minimum distance it is $0;22,6 + 0;4,18 - 0;1,9 = 0;25,15°$, cf. p. 294). Corrected by Manitius.

[44] A somewhat unsatisfactory numerical verification of this (using the Handy Tables) is in Pappus' commentary (Rome[1] I 232-4).

eclipse of the sun can be observed twice in the shortest 7-month interval. However, this is, again, possible only when the moon's position is north of the ecliptic, i.e. when it is approaching the descending node at the first eclipse and receding from the ascending node at the second.

It remains for us to prove that it is impossible for the sun to be eclipsed twice at one month's interval in our part of the inhabited world, either [for observers] at the same latitude or at different latitudes, even if one assumes a combination of conditions which could not in fact all hold true at the same time, but which may be lumped together in a vain attempt to provide a possibility of the event in question happening. These assumptions are, that the moon is at least distance (to make its parallax greater); that the month is the shortest possible (so that the amount by which the month's motion in latitude exceeds the distance between the sun's ecliptic limits be as small as possible);[45] and that we use, without analysis [of whether it is a possible situation], those times and zodiacal signs in H496 which the moon's apparent parallax is greatest.

Now in 1 mean synodic month the mean motions of the bodies are as follows:

increment of motion in longitude for both luminaries: 29;6°

moon's [anomaly] on the epicycle: 25;49°.

The 29;6° of the sun's motion, [when distributed symmetrically] either side of the apogee to produce its least [true] motion, result in an equation of −1;8° from the mean. And the 25;49° of the moon's motion, [when distributed symmetrically] either side of the perigee to produce its greatest [true] motion, result in an equation of +2;28° to the mean. In accordance with our previous demonstration, we take the sum of both equations of anomaly, 3;36°, and add $\frac{1}{12}$th of this, 0;18°, to the amount by which the sun was behind [i.e. 1;8°]. This gives us 1;26° for the amount by which the motion over the shortest month in longitude and [argument of] latitude is exceeded by that in 1 mean synodic month. Therefore, since the motion in latitude during one mean synodic month is 30;40°, that in the shortest month is 29;14°, which corresponds to about 2;33° on the great circle perpendicular to the ecliptic. But the total amount of [the corresponding distance at] the sun's ecliptic limits when the moon is at least distance is 1;6°,[46] which the shortest-month distance exceeds by 1;27°. H497 Therefore, if the sun is to be eclipsed twice at an interval of 1 month, it would be absolutely necessary either for the moon to have no parallax at one conjunction and more than 1;27° at the other, or, secondly, for the parallax at both conjunctions to be in the same direction and for the difference between the parallaxes to be greater than 1;27°, or, [thirdly], for the parallax at one conjunction to be towards the north and the parallax at the other to be towards the south, while their sum exceeded that amount [1;27°]. But nowhere on earth does the moon at syzygy, even at its least distance, have a latitudinal parallax of more than 1° (when the solar parallax is subtracted). Therefore it will not be possible for a solar eclipse to occur twice at the interval of the shortest month

[45] As Ptolemy implies, these two conditions cannot both hold: for the moon, to achieve greatest parallax, has to be at the perigee of the epicycle, but to produce the shortest month (see below) has to be at symmetrical positions either side of the perigee.

[46] The sum of the radii of sun and moon at least distance is 0;33,20° (p. 285). Ptolemy rounds this to 0;33° and doubles it (since we are dealing with two eclipses).

either when the moon has no parallax at one conjunction or when its parallax is in the same direction at both conjunctions. For the difference between the parallaxes cannot exceed 1°, and we need 1;27°. Hence the event in question could occur only under the condition that the two parallaxes are in opposite
H498 directions, and that the sum of both exceed 1;27°. This could happen for parts of the inhabited zones in different [parts of the earth], since it is possible for the southward parallax of the moon in the regions north of the equator, in our part of the inhabited world, and the northward parallax in the regions south of the equator, among the so-called 'antipodes', to reach as much as 1° (with subtraction of the solar parallax).[47] However, it could never happen in the same part of the inhabited world, since in both [oikoumenai] alike, for those situated directly beneath the equator, the maximum parallax of the moon, both to the north and to the south, does not exceed 25′,[48] and for those at the extreme north, or extreme south [respectively of their oikoumene] the parallax in the opposite direction does not exceed the above-mentioned 1°, so that even in this case [i.e. taking the equator and the extreme northern or southern limits] the sum of the parallaxes is still less than 1;27°. And since both opposite parallaxes become progressively much smaller in regions between the equator and the other extreme [of each oikoumene], the impossibility becomes ever greater for such regions. Therefore it is impossible for the sun to be eclipsed twice in one month for the same observers anywhere on earth, or for different observers in the same part of the inhabited world. This was what we intended to prove.

H499 7. {*Construction of eclipse tables*}[49]

By means of the above it has become clear to us which intervals between syzygies should be taken into account when we are examining for eclipses. Now, after having determined the times of mid-eclipse at these [syzygies], and computed the moon's positions at that moment, (the apparent positions at conjunctions and the true positions at oppositions), we want to have a convenient means of determining, from the moon's position in latitude, which of those syzygies will definitely produce an eclipse, and the magnitudes and times of obscuration for these eclipses. To solve this problem we have constructed tables, two for solar eclipses and two for lunar eclipses ([in each case] one for the moon's greatest distance and one for its least distance). The interval which we establish [between successive entries in the tables] is

[47] This was already shown by Hipparchus, as is clear from Pliny, N H II 57, a passage which shows that Hipparchus had anticipated Ptolemy in the investigation of the topic of eclipse intervals. Cf. *HAMA* 322. The word I have translated 'antipodes' is ἀντίχθονες ('[people in] the opposite [part of the] earth'). See LSJ s.v. 2. I have excised ἀπὸ ο κε̄ at H498,8. This would have to mean 'to be between the limits of 0;25° and 1°', which is nonsense, since the lower limit is zero. The phrase was interpolated by someone who misunderstood this use of μέχρι, and took the 25′ (senseless in this context) from just below.

[48] Cf. p. 292 n.43.

[49] See *HAMA* 134–41, Pedersen 231–5.

determined by the amount of obscuration, being $\frac{1}{12}$th of the diameter of whichever of the luminaries is eclipsed.[50]

The first table for solar eclipses, which covers the interval between the limits of eclipses at the moon's greatest distance, will be arranged on 25 lines in 4 columns. The first two columns will contain the apparent position of the moon in [argument of] latitude on the [moon's] inclined circle for each [unit of] obscuration. Since the sun's diameter is 0;31,20°, and, as was proven [p. 254], the H500
moon's diameter at its greatest distance is also 0;31,20°, it follows that when the moon's apparent centre is 0;31,20° from the sun's centre on the great circle through both their centres, (and thus is 6° from the node along its inclined circle, according to the previous ratio, 11;30 : 1), that will be the situation in which the moon just touches the sun. So in the first line of the first column we put '84°', and in the first line of the second column, '276°'; again, in the last line of the first column we put '96°', and in the last line of the second column, '264°'.

Furthermore, since the amount of the [moon's] inclined circle which corresponds to $\frac{1}{12}$th of the solar diameter is about 0;30°,[51] we increase or decrease the entries in the above-mentioned two columns by that amount, beginning from the lines at both ends and going towards the middle. On the middle line we put '90°' and '270°'.

The third column will contain the magnitude of the obscuration. On the two lines at top and bottom we put the '0' representing the touching position, on the two lines next to those '1 digit' (representing $\frac{1}{12}$th of the diameter), and so forth for the rest, with an increment [from line to line] of 1 digit up to the middle line, which will receive the entry '12 digits'. H501

The fourth column will contain the distance travelled by the centre of the moon corresponding to each [tabulated] obscuration, without however taking into account either the sun's additional motion [during the phase of the eclipse] or the moon's epiparallax [i.e. the change in the moon's parallax].

The second table for solar eclipses, which covers the interval between the limits of eclipses at the moon's least distance, will be arranged in the same way as the first, except that it will have 27 lines in 4 columns. The moon's radius at its least distance is, as we have shown [p. 284], 0;17,40° where the sun's radius is 0;15,40°. So when the moon [at least distance] is just touching the sun, its apparent centre is 0;33,20° from the sun's centre, and 6;24° from the node along its inclined circle. So[52] the entries for the apparent [argument of] latitude in the top and bottom lines are '83;36°, 276;24°', and '96;24°, 263;36°' [respectively],

[50] I.e. the intervals between successive arguments in the tables (cols. 1 and 2 in Table VI 8) is determined by taking integer values of the magnitude (col. 3), in contrast with the normal procedure, in which one takes the argument at purely arbitrary intervals. This is more of a convenience for the compiler of the tables than for the user, but it persisted in eclipse tables of the Handy Tables and in many of the mediaeval tables derived from them (see e.g. Toomer [10] no. 59 p. 88).

[51] $\frac{0;31,20}{12} \times 11\frac{1}{2} = 0;30,2 \approx 0;30.$

[52] Heiberg's text in this paragraph is in disarray. To produce a logical sequence, insert a strong stop at the end of 501,9, begin the next sentence ⟨καὶ⟩ διὰ (with Ar), remove the strong stop at the end of 501,17, and excise the γὰρ (with D,Ar) in 501,18.

and the entry for the digits on the middle line, if we use linear extrapolation, will be 12$\frac{4}{5}$ digits. For this entry there will also be a duration of totality.[53]

H502 Each of the lunar [eclipse] tables will be arranged in 45 lines and 5 columns. In the first table we will tabulate the [argument of] latitude for greatest distance of the moon. The moon's radius at its greatest distance is, as we showed [p. 254], 0;15,40°, and the radius of the shadow, 0;40,44°. So, when the moon is just touching the shadow, the moon's centre is 0;56,24° from the shadow's centre along the great circle through both centres, and 10;48° from the node along the [moon's] inclined circle. So we put, on the first line, '79;12°' [in the first column] and '280;48°' [in the second column], and on the last line '100;48°' and 259;12°'. By the same reasoning as in the first [solar table], we increase or decrease each line by 0;30°, which corresponds to $\frac{1}{12}$th of the lunar diameter for that distance.

In the second table we will tabulate the [argument of] latitude for the moon at least distance, at which, as was shown [p. 284], its radius is 0;17,40°, and the radius of the shadow 0;45,56°. Therefore, when the moon just touches the shadow, its centre is, by the same argument as before, 1;3,36° from the centre of
H503 the shadow, and 12;12° from the node along the moon's inclined circle. Hence we put, on the first line, '77;48°' and '282;12°', and, on the last line, '102;12°' and '257;48°', and again increase or decrease the entries by the amount corresponding to $\frac{1}{12}$th of the lunar diameter for that distance, [namely] 0;34°.

The third column [in each table], for the digits, will be arranged in the same way as that in the solar tables. So too will be the succeeding columns, which contain the travel of the moon for each [tabulated] obscuration, namely [the fourth column] for both immersion and emersion, and also [the fifth column] for half of totality.

We computed the travel of the moon tabulated for each obscuration geometrically, but as if [the problem were confined to] a single plane and straight lines, since such small arcs do not differ sensibly from the corresponding chords, and furthermore the moon's motion on its inclined circle is not noticeably different from its motion with respect to the ecliptic.

[I say this] in case anyone should suppose that we do not realise that, in general, the moon's motion in longitude is affected by the use of arcs of the inclined
H504 circle instead of arcs of the ecliptic, and also that it does not follow that the time of syzygy is exactly the same as the time of mid-eclipse. [To illustrate this, see Fig. 6.2], we cut off from the node A two equal arcs of the circles in question [orbit and ecliptic], AB and AG, join BG and from B draw BD perpendicular to AG. Then it is immediately obvious that, if we suppose the moon at B, when we use arc AG of the ecliptic instead of arc AD, then, since motion with respect to the ecliptic is determined by [the great circle] through the poles of the ecliptic, the difference [in longitude] due to the inclination of the lunar orbit will be GD.

[53] The interval of argument corresponding to 1 digit of eclipse magnitude is 0;30° elsewhere in the table. Since the interval here is 0;24°, the corresponding amount in digits is $\frac{4}{5}$. Accurate computation from the radii 0;17,40° and 0;15,40° gives the magnitude of the maximum solar eclipse as 12;46^d. The amount beyond 12 digits represents the 'duration of totality' (μονή), as in lunar eclipses. See also p. 305 n.63.

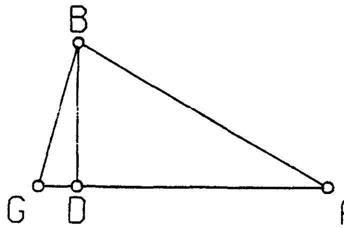

Fig. 6.2

Or again, if we imagine the sun or the centre of the shadow at B,[54] the time of syzygy will occur when the moon is at G ([we can say this] since the difference due to the two circles [ecliptic and orbit] is negligible), but the time of mid-eclipse when the moon is at D, since, again, the time of mid-eclipse is defined by the circle through the poles of the moon's orbit. And [thus] the time of syzygy will differ from the time of mid-eclipse by arc GD.

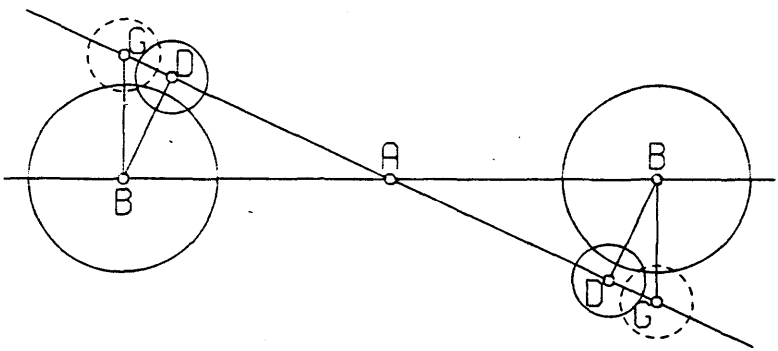

Fig. J

The reason that we did not take these arcs into account in our derivations of the individual [entries] is that the differences they cause are small and imperceptible. While it would be absurd not to recognise any of these effects, on the other hand, when one considers the resulting complication in the methods necessary to deal with each of them, deliberate neglect of effects small enough to be overlooked both in theory and observation evokes [in the reader] a strong feeling of the advantage of greater simplicity, and no regret, or little, for the resulting error in representing the phenomena. In any case, we find that the arc corresponding to GD does not, in general, exceed 0;5°. This can be demonstrated by means of the same theorem which we used [I 16] to calculate the difference between arcs of the equator and corresponding arcs of the ecliptic, as defined by a [great] circle drawn through the poles of the equator. And in eclipses [the arc corresponding to GD] does not exceed 2′. For, if we take H505

[54] I.e. the two arcs are now interchanged, AB being the ecliptic and AG the moon's orbit. Instead of using the same figure, Ptolemy should have drawn another one, in which GB is perpendicular to AB (i.e. AB ≠ AG). Compare Fig. J (taken from Manitius 452–53), which shows that the true syzygy (at G) precedes the eclipse-middle (at D) before the node, but succeeds it after the node.

arc AB = arc AG = 12°, which is the maximum amount of the moon's distance [from the node] at eclipses, then BD is about 1°. And hence AD is about 11;58°, and, by subtraction, GD is 2′, which corresponds to less than $\frac{1}{16}$th of an equinoctial hour.[55] Scrupulous accuracy about such a small amount is a sign of
H506 vain conceit rather than love of truth.

For the above reasons we have computed the travel of the moon during the obscurations in question as if the circles [of ecliptic and orbit] were sensibly identical. The method of calculation, to give one or two examples, is as follows.

Let [Fig. 6.3][56] A be the centre of the sun or the shadow, and BGD the straight line representing the arc of the moon's [inclined] circle. Let the points representing the moon's centre when it is just touching the sun or the shadow be, at the moon's approach [i.e. at first contact] B, and at its recession [i.e. at last contact] D. Join AB and AD, and drop perpendicular AG from A on to BD.

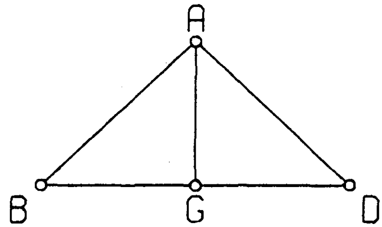

Fig. 6.3

Now it is clear that eclipse middle and greatest obscuration occur when the centre of the moon is at G, because [1] AB equals AD, and hence the distances travelled, BG and GD, are also equal, and because [2] AG is the least of all lines joining the two centres [when the moon is] on BD. It is also clear that AB and AD each comprise the sum of the radii of moon and sun or [moon and] shadow, and that each of them exceeds AG by that part of the diameter of the eclipsed body which is cut off by the obscuration.

H507 This being the case, let the obscuration be, e.g., 3 digits. First let A represent the sun's centre.

Therefore,[57] when the moon is at its greatest distance,

AB = 31;20 minutes [p. 295].

∴ AB^2 = 981;47.

And AG = 23;30 minutes, since it is less than AB by $\frac{3}{12}$ths of the sun's diameter, i.e. 7;50 minutes.

[55] Cf. *HAMA* 83 n.5, estimating a maximum error of 6′ as a result of neglecting the inclination of the lunar orbit in computing longitudes. Using the formula $\tan\lambda = \tan\omega \cos\iota$, I find, for $\iota = 5°$, the maximum difference between λ and ω as about $6\frac{1}{2}'$ for $\omega \approx 45;3°$. Using the same formula for ω = 12°, I find λ = 11;57,20°, hence GD = 0;2,40°, which still leads to less than $\frac{1}{12}$th of an hour's difference in the time of mid-eclipse. Ptolemy computes crudely BD ≈ AB/$11\frac{1}{2}$ ≈ 1, AD = $\sqrt{12^2 - 1^2}$ ≈ 11;58.

[56] Figs. 6.3 and 6.4 are elucidated by Figs. K and L respectively, in which the circles representing the sun, moon and shadow are drawn in. These are taken from Manitius, but are also very similar to the alternative diagrams found in ms. D.

[57] Reading ἐπὶ μὲν ἄρα (with D) for ἐπὶ μὲν οὖν ἄρα at H507,3.

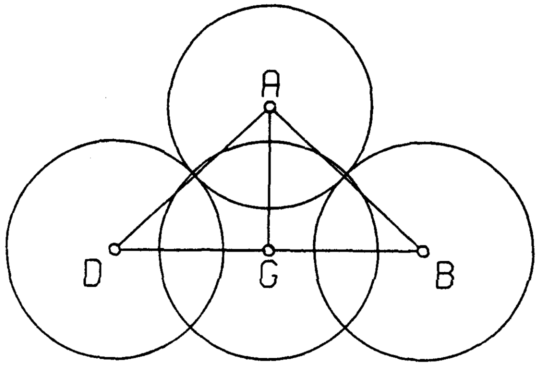

Fig. K

So $AG^2 = 552;15$.
Hence $BG^2 = 429;32$,
and $BG \approx 20;43$ minutes.

This is the amount which we will enter in the fourth column of the first table for solar [eclipses] opposite '3 digits'.

For the moon's least distance

$AB = 33;20$ minutes [p. 295].
$\therefore AB^2 = 1111;7$.
And $AG = [0;33,20° - 0;7,50° =]$ 25;30 minutes,
so $AG^2 = 650;15$.
And, by subtraction, $BG^2 = 460;52$,
and so $BG = 21;28$ minutes.

This is the amount which we will enter in the fourth column of the second table for solar [eclipses] opposite '3 digits'.

Next let A represent the centre of the shadow, and let the obscuration be the same fraction as before, $\frac{1}{4}$, [but now] of the lunar diameter.

Then, for the moon's greatest distance,

$AB = 56;24$ minutes [p. 296],
so $AB^2 = 3180;58$. H508
and $AG = 48;34$ minutes, since it is less than AB by $\frac{1}{4}$ of the lunar diameter, i.e. (for the moon's greatest distance) 7;50 minutes.
So $AG^2 = 2358;43$.
Hence, by subtraction, $BG^2 = 822;15$,
and $BG = 28;41$ minutes.

This is the amount which we will enter in the fourth column of the first table for lunar [eclipses] opposite '3 digits'. It represents the travel during immersion, which is sensibly equal to that during emersion.

For the [moon's] least distance

$AB = 63;36$ minutes [p. 296],
so $AB^2 = 4044;58$.

And $AG = 54;46$ minutes, since the difference [between AB and AG], 8;50 minutes, is, again, $\frac{1}{4}$ of the moon's diameter, [here] at least distance.

$\therefore AG^2 = 2999;23$.

So, by subtraction, $BG^2 = 1045;35$,

and $BG = 32;20$ minutes.

This is the amount which we will enter opposite '3 digits', as before, in the fourth column of the second table for lunar [eclipses].

Next, to represent those [phases of] the lunar obscurations comprising the
H509 duration of totality, let [Fig. 6.4] A be the centre of the shadow, and BGDEZ the straight line standing for the arc of the moon's inclined circle. Let B represent

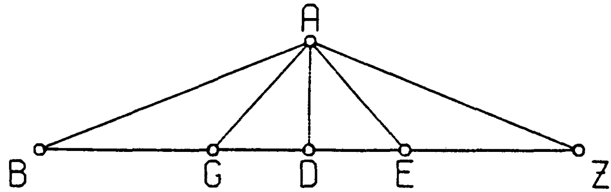

Fig. 6.4

the place of the centre of the moon when it is just externally tangent to the circle of the shadow, at approach, G the place of the centre of the moon when it is just internally tangent to the circle of the shadow at the beginning of totality, E the place of the centre of the moon when it is just internally tangent to the circle of the shadow as [the moon] recedes [at the end of totality], and Z the place of the centre of the moon when it is externally tangent to the shadow at the very end of its emersion [from obscuration]. Again drop perpendicular AD from A on to BZ. The same conclusions as before remain valid, and it is furthermore clear

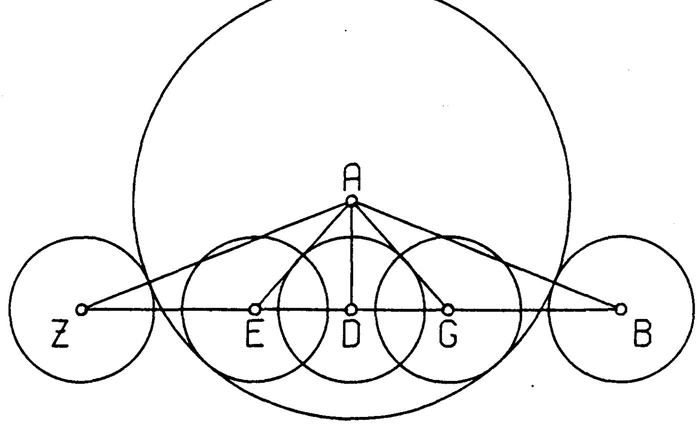

Fig. L

that AG and AE each comprise the amount by which the radius of the shadow exceeds the radius of the moon. Hence the distance GD is equal to the distance DE, and each represents half of totality, while BG, the remainder [of BD-GD], which represents the immersion, is equal to EZ, the remainder [of DZ-DE], which represents the emersion.

So let us take [for an example] an eclipse for which the entry [in the table] is '15 lunar digits', i.e. one in which D, the moon's centre [at mid-eclipse], lies $1\frac{1}{4}$ lunar diameters inside the boundary set by the limits of the eclipse. That is to say, when H510

$$(AB - AD) = (AZ - AD) = 1\tfrac{1}{4} \text{ lunar diameters}$$
$$\text{and } (AG - AD) = (AE - AD) = \tfrac{1}{4} \text{ lunar diameter.}$$

Then, for the moon's greatest distance,

as before [p. 299], $AB = 56;24$ minutes and $AB^2 = 3180;58$.

And $AG = 25;4$ minutes, since the moon's diameter at greatest distance is 31;20 minutes.

$\therefore AG^2 = 628;20$,

and, by a similar argument, $AD = [56;24 - (31;20 + 7;50) =]\ 17;14$ minutes and $AD^2 = 296;59$.

So, by subtraction [of AD^2 from AB^2], $BD^2 = 2883;59$,

and $BD = 53;42$ minutes.

And, by subtraction [of AD^2 from AG^2], $GD^2 = 331;21$,

and $GD = 18;12$ minutes.

So, by subtraction, $BG = 35;30$ minutes.

So we will put, opposite the entry '15 digits' in the first table for lunar eclipses, in the fourth column '35;30 minutes' for the immersion (which will be the same for the emersion), and, in the fifth column '18;12 minutes' for half the duration of totality.

For the moon's least distance, H511

as before [p. 299], $AB = 63;36$ minutes

and $AB^2 = 4044;58$;

$AG = 28;16$ minutes, since, as was shown, the moon's diameter at least distance is 35;20 minutes,

and $AG^2 = 799;0$.

And, by a similar argument, $AD = [63;36 - (35;20 + 8;50) =]\ 19;26$ minutes,

so $AD^2 = 377;39$.

Therefore, by subtraction, $BD^2 = 3667;19$,

and $BD = 60;34$ minutes.

And, by subtraction, $GD^2 = 421;21$

and $GD = 20;32$ minutes.

So, by subtraction, $BG = 40;2$ minutes.

Therefore we will put, opposite the entry '15 digits' in the second table for lunar eclipses, in the fourth column '40;2 minutes' for the immersion (which will again be the same for the emersion), and, in the fifth column, '20;32 minutes' for half the duration of totality.

In order to have a convenient way of obtaining the fraction of the difference [between values derived from the first and second tables] for positions of the

moon on the epicycle in between greatest and least distances ([which we do] by the method of sixtieths [of interpolation]), we have drawn up, below the above tables, another little table. This contains, as argument, the position [in anomaly] on the epicycle, and, [as function], the corresponding number of
H512 sixtieths to be applied [as interpolation coefficient] in every case to the difference [between values] derived from[58] the first and second eclipse tables. We have already computed the amounts of these sixtieths for the table of the moon's parallax [V 18]: they are set out in the seventh column [of that table], since the epicycle has to be taken at the apogee of the eccentre to represent [the situation at] syzygy.

But most of those who observe the [weather] indications derived from eclipses measure the size of the obscuration, not by the diameters of the disks [of sun and moon], but, on the whole, by [the amount of] the total surface of the disks,[59] since, when one approaches the problem naively, the eye compares the whole part of the surface which is visible with the whole of that which is invisible. For this reason we have added to the above table yet another little table with 12 lines and 3 columns. In the first column we put the digits from 1 to 12, where each digit represents $\frac{1}{12}$th of the diameter of each luminary, as in the actual eclipse tables. In the other two columns we put twelfths of the whole surface-area corresponding to these [linear digits], those for the sun in the second, and those for the moon in the third. We computed these amounts only for the sizes [of the apparent diameters] for the moon at mean distance, since very nearly the same ratio will result [at other distances], given so small a variation in the diameters. Furthermore, we assumed that the ratio of the circumference to the
H513 diameter is 3;8,30 : 1, since this ratio is about half-way between $3\frac{1}{7}$: 1 and $3\frac{10}{71}$: 1, which Archimedes used as rough [bounds].[60]

First, to represent solar eclipses, let [Fig. 6.5] the sun's disk be ABGD on centre E, and the disk of the moon at mean distance AZGH on centre Θ, intersecting the sun's disk at points A and G. Join BEΘH, and let us suppose that $\frac{1}{4}$ of the sun's diameter is eclipsed.

Thus ZD = 3 where diameter BD = 12,

and the moon's diameter, ZH ≈ 12;20 in the same units, according to the ratio

15;40 : 16;40.[61]

H514 Hence EΘ = [$\frac{1}{2}$ (12 + 12;20) − 3 =] 9;10 in the same units.

Therefore the circumferences of the disks are, according to the ratio 1 : 3;8,30,

[58] Reading γινομένων for φαινομένων ('which appear from') at H512,1. Although found in all Greek mss. and part of the Arabic tradition, the latter is without parallel in the *Almagest*, and must be replaced by a word like γινομένων (palaeographically close), or συναγομένων . Cf. e.g. H384,21-2, τῶν γινομένων διαφορῶν ἐκ τῆς δευτέρας ἀνωμαλίας, H385,5-7, τῶν συναγομένων ὑπεροχῶν ἐκ τῆς . . . ἀνωμαλίας. Is has 'allati tuḳraju', which supports my emendation.

[59] Although there is no reason to doubt Ptolemy's statement, I know of no surviving ancient eclipse magnitude which is unambiguously given in these 'area digits'.

[60] Archimedes, 'Measurement of the Circle', ed. Heiberg I 232-42, tr. Heath 91-8.

[61] The sun's radius is 0;15,40° (p. 285). The moon's radius at mean distance is the mean between 0;15,40° and 0;17,40°, i.e. 0;16,40°. But Ptolemy has made a calculating error (cf. Manitius p. 385 n. b) and Pappus, Rome [1] 1261): 12 × (16;40/15;40) ≈ 12;46, not 12;20. This affects the accuracy of every entry in the second column, but the results are so crudely rounded that it is of little importance.

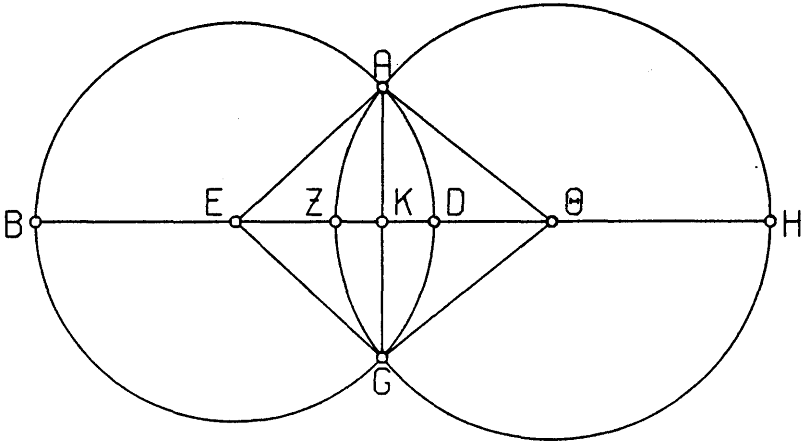

Fig. 6.5

sun's circumference: $37;42^p$
moon's circumference: $38;46^p$.

Similarly, since the product of the radius and the circumference is twice the area of the circle, the areas of the whole disks are:

sun's area: $113;6^p$
moon's area: $119;32^p$.

With the above as given quantities, let the problem be to find the area of the surface enclosed by ADGZ, where the total area of the sun's surface is 12 parts.

Join AE, AΘ, GE, GΘ, and also draw perpendicular AKG.

$$\left.\begin{array}{r}\text{Now, where } E\Theta = 9;10^p, \\ AE = EG = 6^p \\ \text{and } A\Theta = \Theta G = 6;10^p\end{array}\right\} \text{by assumption.}$$

Furthermore, the angle at K is right.

Therefore, if we divide $(\Theta A^2 - AE^2)$, or 2;2, by EΘ, we will get $(K\Theta - EK)$ as $0;13\frac{1}{3}^p$.[62]

Hence EK comes out to $4;28^p$ and KΘ to $4;42^p$.

Therefore $AK = KG \approx 4^p$.

Accordingly the area of triangle $AEG = 17;52^p$ H515
and the area of triangle $A\Theta G = 18;48^p$.

Furthermore, where diameter $BD = 12^p$ and $ZH = 12;20^p$, $AG = 8^p$;

so where diameter $BD = 120^p$, $AG = 80^p$,
and where diameter $ZH = 120^p$, $AG = 77;50^p$.

Therefore the corresponding arcs are:

arc $ADG = 83;37°$ of circle ABGD
and arc $AZG = 80;52°$ of circle AZGH.

[62] For $\Theta A^2 - AK^2 = K\Theta^2$, $AE^2 - AK^2 = EK^2$; subtracting, $\Theta A^2 - AE^2 = K\Theta^2 - EK^2 = (K\Theta + EK)(K\Theta - EK) = E\Theta(K\Theta - EK)$.
At H514,20 I read $\overline{\iota\gamma}\ \gamma'$ (with A,D², Is) for $\overline{\iota\gamma}\ \overline{\gamma}$ (13;3'). Corrected by Rome[1]I 262 n. (3), whence Neugebauer in the 2nd edn. of Manitius.

So, since the ratio of a circle to one of its arcs equals the ratio of the area of the whole circle to the area of the sector beneath that arc,

area of sector AEGD = 26;16^p where area of circle ABGD = 113;6^p, as was shown,

and, in the same units, area of sector AΘGZ = 26;51^p
(for circle AZGH was shown to be 119;32^p).

And, in the same units, we showed that
area of triangle AEG = 17;52^p
and area of triangle AΘG = 18;48^p.

Therefore, by subtraction, area of segment ADGK = 8;24^p
and area of segment AZGK = 8;3^p.

So, by addition, area of AZGD = 16;27^p where area of circle ABGD = 113;6^p.

Therefore where the area of the sun's disk equals 12^p,

H516 the area of the eclipsed part ≈ $1\frac{3}{4}^p$.

This is the amount which we will enter in the above-mentioned table in the second column on the line with '3 digits' [as argument].

Again, in the same figure [Fig. 6.5], to represent lunar eclipses, let the moon's disk be ABGD, and the shadow's disk at mean [lunar] distance AZGH, and, as before, let $\frac{1}{4}$ of the diameter of the moon be eclipsed.

Hence, where diameter BD = 12^p, the eclipsed section, ZD = 3^p.

And, according to the ratio 2;36 : 1,
the diameter of the shadow, ZH = 31;12^p.

Therefore EKΘ comes to [$\frac{1}{2}$(12 + 31;12) − 3 =] 18;36^p.

So the circumferences are as follows:

moon's disk:	37;42^p
shadow's disk:	98;1^p
and the areas are:	
moon's disk:	113;6^p
shadow's disk:	764;32^p.

Here again, where EΘ = 18;36^p,

$$\left.\begin{array}{r} AE = EG = 6^p \\ \text{and } A\Theta = \Theta G = 15;36^p \end{array}\right\} \text{by assumption.}$$

∴ (KΘ − EK) = (ΘA² − AE²)/EΘ = 11;8^p.

H517 So EK comes out to 3;44^p and KΘ to 14;52^p.

Hence AK = KG = 4;42^p.

Accordingly, the area of triangle AEG = 17;33^p
and the area of triangle AΘG = 69;52^p.

Furthermore, where diameter BD = 12^p and ZH = 31;12^p, AG = 9;24^p.

So where diameter BD = 120^p, AG = 94^p,
and where diameter ZH = 120^p, AG = 36;9^p.

Therefore the corresponding arcs are:
arc ADG = 103;8° of circle ABGD
and arc AZG = 35;4° of circle AZGH.

Therefore, by the previous argument,
area of sector AEGD = 32;24^p where, as was shown, area of circle ABGD = 113;6^p
and, in the same units, area of sector AGΘZ = 74;28^p,
since area of circle AZGH was shown to be 764;32^p.

And, as we showed, in the same units
area of triangle AEG = 17;33^p
and area of triangle AΘG = 69;52^p.
Therefore, by subtraction, area of segment ADGK = 14;51^p
and area of segment AZGK = 4;36^p.
So, by addition, the area enclosed by AZGD is 19;27^p
where the area of circle ABGD is taken as 113;6^p. H518
Therefore, where the area of the lunar disk is 12^p,
the area comprised by its eclipsed section will be about $2\frac{1}{15}^p$.
This is the amount which we will enter in the above-mentioned table in the third, lunar, column, on the line with '3 digits' [as argument].

The layout of the tables is as follows.

8. {*Eclipse tables*}[63] H519-22

[See pp. 306-8.]

9. {*Determination of lunar eclipses*}[64] H523

Having set out the above as a preliminary, we can predict lunar eclipses in the following manner.

We set down the amounts in degrees, computed for the required opposition at the time of mid-syzygy at Alexandria, of the so-called anomaly, [counted] from the apogee of the epicycle, and the [argument of] latitude, [counted] from the northern limit. Having corrected the latter by means of the equation [of anomaly], we first enter with this corrected [argument of] latitude into the tables for lunar eclipses. If it falls within the range of the numbers in the first two columns, we take the amounts corresponding to the argument of latitude in the columns for the [lunar] travel and the column for the digits [of magnitude] in both tables, and write them down separately. Then, with the anomaly as

[63] There are a number of individual errors in these tables, but it is not always certain which are due to corruption and which to Ptolemy's faulty computation. Certain scribal errors (corrected in the translation) are:
Solar eclipse, least distance col. 4, arg. 90;0. Heiberg (H519,20) prints this (following most Greek mss.) as λγ κβ ο, i.e. 33;22,0. It was originally two entries, 33;20 (correctly computed) and 2;0, where the first represents the immersion, and the second the duration of totality (μονή), computed from the difference between lunar and solar radii, 17;40′ and 15;40′. There is a reference to this on p. 296 (H501,23), but I suspect both that remark and the entry 2;0 here of being interpolations. Most Arabic mss. have just 33;20.
Lunar eclipse, least distance col. 5, args. 89;8 and 90;52, read κζ νβ for κζ μβ (27;42) at H521,27 (with D, Ar) and H521,31 (with Ar). Same col., for arg. 90;0, read κη ις for κη ς (28;6) at H521,29, with D,Ar.
Lunar eclipse, col. 3, for arg. 90;0, text has τελεία (all mss. except P, which has '21'). From the ratio shadow to moon of $2\frac{3}{5}$: 1 one finds the maximum magnitude of a lunar eclipse as 21;36 digits in all cases. From Ptolemy's interpolation method (cf. p. 296 n.53) one finds 21;36 at greatest distance and about 21;32 at least distance.
[64] See *HAMA* 138-9 (with computed examples), Pedersen 234-5, and Appendix A, Example 11.

argument, we enter into the correction table, and take the corresponding number of sixtieths. We then take this fraction of the difference between the [two sets of] digits, [derived from] the two tables, which we wrote down, and also of the difference between the [two sets of] minutes of travel, and add the results to the amounts derived from the first table. If, however, it happens that the argument of latitude falls within the range of the second table only, we take [as
H524 final result] the appropriate fraction (determined by the number of sixtieths found [from the correction table]) of the digits and minutes [of travel] corresponding [to the argument of latitude] in the second table alone. The number of digits which we find as a result of the above correction will give us the magnitude of the obscuration, in twelfths of the lunar diameter, at mid-eclipse.

As for the minutes [of travel] resulting from the same correction, we always increase them by $\frac{1}{12}$th, to allow for the sun's additional motion [during the phase of the eclipse], and divide the result by the moon's anomalistic [i.e. true] hourly

TABLE FOR SOLAR ECLIPSES

GREATEST DISTANCE				LEAST DISTANCE				
1	2	3	4	1	2	3	4	
Arguments of Latitude		Digits	Minutes of Immersion	Arguments of Latitude		Digits	Minutes of Immersion	
				83 36	276 24	0	0 0	
84 0	276 0	0	0 0	84 6	275 54	1	12 57	
84 30	275 30	1	12 32	84 36	275 24	2	17 54	
85 0	275 0	2	17 19	85 6	274 54	3	21 28	
85 30	274 30	3	20 43	85 36	274 24	4	24 14	
86 0	274 0	4	23 27	86 6	273 54	5	26 27	
86 30	273 30	5	25 38	86 36	273 24	6	28 16	
87 0	273 0	6	27 8	87 6	272 54	7	29 45	
87 30	272 30	7	28 29	87 36	272 24	8	30 55	
88 0	272 0	8	29 32	88 6	271 54	9	31 51	
88 30	271 30	9	30 20	88 36	271 24	10	32 33	
89 0	271 0	10	30 54	89 6	270 54	11	33 1	
89 30	270 30	11	31 13	89 36	270 24	12	33 16	
90 0	270 0	12	31 20	90 0	270 0	$12\frac{4}{5}$	33 20	2 0
90 30	269 30	11	31 13	90 24	269 36	12	33 16	
91 0	269 0	10	30 54	90 54	269 6	11	33 1	
91 30	268 30	9	30 20	91 24	268 36	10	32 33	
92 0	268 0	8	29 32	91 54	268 6	9	31 51	
92 30	267 30	7	28 29	92 24	267 36	8	30 55	
93 0	267 0	6	27 8	92 54	267 6	7	29 45	
93 30	266 30	5	25 38	93 24	266 36	6	28 16	
94 0	266 0	4	23 27	93 54	266 6	5	26 27	
94 30	265 30	3	20 43	94 24	265 36	4	24 14	
95 0	265 0	2	17 19	94 54	265 6	3	21 28	
95 30	264 30	1	12 32	95 24	264 36	2	17 54	
96 0	264 0	0	0 0	95 54	264 6	1	12 57	
				96 24	263 36	0	0 0	

LUNAR ECLIPSES

GREATEST DISTANCE					LEAST DISTANCE				
1	2	3	4	5	1	2	3	4	5
Arguments of Latitude		Digits	Minutes of Immersion	Half Totality	Arguments of Latitude		Digits	Minutes of Immersion	Half Totality
79 12	280 48	0	0 0		77 48	282 12	0	0 0	
79 42	280 18	1	16 59		78 22	281 38	1	19 9	
80 12	279 48	2	23 43		78 56	281 4	2	26 45	
80 42	279 18	3	28 41		79 30	280 30	3	32 20	
81 12	278 48	4	32 42		80 4	279 56	4	36 53	
81 42	278 18	5	36 6		80 38	279 22	5	40 42	
82 12	277 48	6	39 1		81 12	278 48	6	43 59	
82 42	277 18	7	41 34		81 46	278 14	7	46 53	
83 12	276 48	8	43 50		82 20	277 40	8	49 25	
83 42	276 18	9	45 48		82 54	277 6	9	51 40	
84 12	275 48	10	47 35		83 28	276 32	10	53 39	
84 42	275 18	11	49 9		84 2	275 58	11	55 25	
85 12	274 48	12	50 31		84 36	275 24	12	56 59	
85 42	274 18	13	40 35	11 9	85 10	274 50	13	45 47	12 34
86 12	273 48	14	37 28	15 20	85 44	274 16	14	42 15	17 17
86 42	273 18	15	35 30	18 12	86 18	273 42	15	40 2	20 32
87 12	272 48	16	34 6	20 22	86 52	273 8	16	38 28	22 58
87 42	272 18	17	33 7	22 0	87 26	272 34	17	37 20	24 49
88 12	271 48	18	32 23	23 14	88 0	272 0	18	36 37	26 1
88 42	271 18	19	31 51	24 8	88 34	271 26	19	35 55	27 13
89 12	270 48	20	31 32	24 43	89 8	270 52	20	35 34	27 52
89 42	270 18	21	31 22	25 1	89 42	270 18	21	35 22	28 12
90 0	270 0	entire	31 20	25 4	90 0	270 0	entire	35 20	28 16
90 18	269 42	21	31 22	25 1	90 18	269 42	21	35 22	28 12
90 48	269 12	20	31 32	24 43	90 52	269 8	20	35 34	27 52
91 18	268 42	19	31 51	24 8	91 26	268 34	19	35 55	27 13
91 48	268 12	18	32 23	23 14	92 0	268 0	18	36 37	26 1
92 18	267 42	17	33 7	22 0	92 34	267 26	17	37 20	24 49
92 48	267 12	16	34 6	20 22	93 8	266 52	16	38 28	22 58
93 18	266 42	15	35 30	18 12	93 42	266 18	15	40 2	20 32
93 48	266 12	14	37 28	15 20	94 16	265 44	14	42 15	17 17
94 18	265 42	13	40 35	11 9	94 50	265 10	13	45 47	12 34
94 48	265 12	12	50 31		95 24	264 36	12	56 59	
95 18	264 42	11	49 9		95 58	264 2	11	55 25	
95 48	264 12	10	47 35		96 32	263 28	10	53 39	
96 18	263 42	9	45 48		97 6	262 54	9	51 40	
96 48	263 12	8	43 50		97 40	262 20	8	49 25	
97 18	262 42	7	41 34		98 14	261 46	7	46 53	
97 48	262 12	6	39 1		98 48	261 12	6	43 59	
98 18	261 42	5	36 6		99 22	260 38	5	40 42	
98 48	261 12	4	32 42		99 56	260 4	4	36 53	
99 18	260 42	3	28 41		100 30	259 30	3	32 20	
99 48	260 12	2	23 43		101 4	258 56	2	26 45	
100 18	259 42	1	16 59		101 38	258 22	1	19 9	
100 48	259 12	0	0 0		102 12	257 48	0	0 0	

Table of Correction			Table for Magnitudes of Solar and Lunar [Eclipses]		
1 Common Numbers (Anomaly)	2 Common Numbers (Anomaly)	3 Sixtieths	[Linear] Digits	[Area] Digits of Sun	[Area] Digits of Moon
6	354	0′ 21	1	$0\frac{1}{3}$	$0\frac{1}{2}$
12	348	0 42	2	1	$1\frac{1}{6}$
18	342	1 42	3	$1\frac{3}{4}$	$2\frac{1}{13}$
24	336	2 42	4	$2\frac{2}{3}$	$3\frac{1}{6}$
30	330	4 1	5	$3\frac{2}{3}$	$4\frac{1}{3}$
36	324	5 21	6	$4\frac{2}{3}$	$5\frac{1}{2}$
42	318	7 18	7	$5\frac{5}{6}$	$6\frac{3}{4}$
48	312	9 15	8	7	8
54	306	11 37	9	$8\frac{1}{3}$	$9\frac{1}{6}$
60	300	14 0	10	$9\frac{2}{3}$	$10\frac{1}{3}$
66	294	16 48	11	$10\frac{5}{6}$	$11\frac{1}{3}$
72	288	19 36	12	12	12
78	282	22 36			
84	276	25 36			
90	270	28 42			
96	264	31 48			
102	258	34 54			
108	252	38 0			
114	246	41 0			
120	240	44 0			
126	234	46 45			
132	228	49 30			
138	222	51 39			
144	216	53 48			
150	210	55 32			
156	204	57 15			
162	198	58 18			
168	192	59 21			
174	186	59 41			
180	180	60 0			

motion at that point.[65] The results of the division will give us the duration of each phase of the eclipse in equinoctial hours: the result derived from the fourth column will give the duration of the immersion (and also that of the emersion likewise); and the result derived from the fifth column will give the duration of half of the totality. The times of entry and exit at beginning and end [of the various phases] can be derived immediately by adding or subtracting the individual durations to or from the time of the middle of totality, that is, approximately, the time of true opposition. We can also immediately find the area digits by entering with the digits of the diameter into the final small table

[65] This will already have been determined in the computation of the time of the true syzygy (cf. p. 282).

and taking the corresponding amount in the third column (and similarly for solar eclipses by taking the corresponding amount in the second column).

Now reason informs us that the time interval from the beginning of an eclipse to its middle is not always equal to the time interval from mid-eclipse to the end, because of solar and lunar anomaly, the effect of which is that equal distances are covered [by the bodies] in unequal times. However, as far as the senses are concerned, no noticeable error with respect to the phenomena would result from supposing these intervals equal in time. For, even when [the luminaries] are near mean speed, where the change [in speed] resulting from an [equal] increment [in the argument] is greater [than elsewhere], the motion over the number of hours represented by the whole duration of [even] the maximum possible eclipse does not exhibit the least noticeable difference [in duration] due to the change [in speed]. H525

Furthermore, we can [now] see, by examining the matter on the above basis, that we were quite right to reject as erroneous the period for the moon's [return in] latitude which Hipparchus demonstrated. [As we saw, p. 207,] the increment [in argument of latitude] between the [two] eclipses which he set out appeared smaller according to his hypothesis, whereas according to our calculations it was found to be greater.[66]

To demonstrate his thesis [of the period of return in latitude], he chose two eclipses with an interval between them of 7160 [synodic] months, in both of which it happened that a quarter of the moon's diameter was eclipsed, at the H526
same distance from the ascending node. The first of these was observed in the second year of Mardokempad and the second in the thirty-seventh year of the Third Kallippic Cycle.[67] In order to demonstrate the return [in latitude], he makes the assumption that each eclipse exhibits the same position in mean argument of latitude,[68] on the grounds that the first eclipse occurred when the moon was at the apogee of the epicycle, and the second when it was at the perigee, and hence, he thought, the anomaly had no effect. However, his first mistake is in this very point, since there indeed was a considerable effect from the anomaly: the mean motion was greater than the true at both eclipses, [and] not by an equal amount, but by about 1° in the first eclipse, and $\frac{1}{8}$° in the second eclipse. Thus, in this respect, the period in latitude [between the two eclipses] falls short of an integer number of returns by $\frac{7}{8}$° of the moon's orbit. Furthermore, he failed to take into account the effect of the lunar distance on the size of the obscuration, although the difference [due to this effect] was the greatest possible between [precisely] these eclipses, since the first occurred when the moon was at its greatest distance, and the second when it was at its least. For H527

[66] The increment in argument of latitude over the $211438^d\ 23^h$ between the two eclipses mentioned below is, according to Hipparchus' value for the mean motion, only about 3′ beyond complete revolutions, but about 12′ according to Ptolemy's value.

[67] These are the eclipses of −719 Mar. 8 and −140 Jan. 27, both of which have been used before: see IV 6 p. 191, IV 9 p. 208, and VI 5 p. 284, q.v. for details of the anomaly. See also, for the first eclipse, Appendix A, Example 11.

[68] Literally 'the same position in latitude is comprised at each of the eclipses, from uniform [motion] (ἐξ ὁμαλοῦ)'. On the assumption that the moon was precisely at apogee and perigee of the epicycle, then (in Hipparchus' simple lunar hypothesis) the true position of the moon coincides with the mean.

the same obscuration, of $\frac{1}{4}$ [of the diameter], must necessarily result at a lesser distance from the ascending node at the first eclipse, and a greater distance at the second. We have shown that the difference between these distances comes to $1\frac{1}{5}°$.[69] Hence, in this respect, the period of latitude exceeds an integer number of returns by that amount [$1\frac{1}{5}°$]. Thus, with respect to the absolute error, the return in latitude would have been out by about two degrees (the sum of the [above] two errors), if it happened that the effect of both had been subtractive or additive. However, since one had the effect of falling short of a return and the other of exceeding a return, by a chance stroke of good luck (perhaps Hipparchus too noticed that these effects counterbalance each other somewhat) it turns out that the [motion in latitude] exceeds an [exact] return by only the difference between the [two] errors, [or] a third of a degree.

10. {*Determination of solar eclipses*}[70]

Correct prediction of lunar eclipses can be achieved merely by the above, if the computations are carried out accurately in the way described. Solar eclipses,
H528 however, with which we deal next, are more complicated to predict because of lunar parallax. We will do it as follows.

We determine the number of equinoctial hours by which the time of true syzygy at Alexandria precedes or follows noon. Then, if the geographical position in question, [i.e.] that of the required place, is different [from that], i.e. if it does not lie beneath the same meridian as Alexandria, we add or subtract the difference in longitude between the two meridians, expressed in equinoctial hours, and [thus] decide how many hours before or after noon the true syzygy occurred at that place too. Then we determine, first, the time of apparent syzygy (which will be approximately the same as mid-eclipse) at the required geographical location, by applying the method of computing parallaxes which we explained previously [V 19], [as follows].

We enter the Table of Angles [II 13] and the Table of Parallaxes [V 18], using [as arguments] the appropriate latitude, distance in hours from the meridian, point on the ecliptic where the conjunction occurred, and also distance of the moon. We thus find, first, the moon's parallax along the great circle drawn through the zenith and the moon's centre. We always subtract from this that solar parallax which is on the same line, and from the result determine, in the way
H529 indicated, the component of parallax in longitude by itself, which is computed by means of the angle we found [from the table] between the ecliptic and the great circle through the zenith. We always add to this [longitudinal parallax] the increment of 'epiparallax' corresponding to the number of equinoctial hours represented by the longitudinal parallax. This epiparallax is determined as follows. We take the difference (as determined from the same table) between the parallax corresponding to the original zenith distance and the parallax

[69] From Table VI 8, moon, entries for magnitude 3 digits: greatest distance, ω = 80;42°, least distance, ω = 79;30°; difference 1;12°.

[70] See Appendix A, Example 12.

corresponding to the zenith distance after the passage of the number of equinoctial hours [represented by the longitudinal parallax]. We take the longitudinal component of this by itself, plus an additional amount (if it is significant) which is the same fraction of the latter as the latter is of the original [longitudinal] parallax.[71] To the total parallax in longitude, computed in this way, we add $\frac{1}{12}$th of itself, to account for the additional motion of the sun, and convert the total to equinoctial hours by dividing it by the moon's true hourly motion at the conjunction. If the longitudinal parallax we found is towards the rear [i.e. in the order] of the signs (we explained previously [p. 267] how to determine this), we subtract the amount in degrees which we had converted into equinoctial hours from the moon's position, as previously determined, at H530
the moment of true conjunction, in longitude, latitude and anomaly (each separately): this gives us the [corresponding] true positions of the moon at the moment of apparent conjunction, while the number of hours itself [resulting from the above computation] tells us by how much the apparent conjunction precedes the true. But if the longitudinal parallax we found is in advance [i.e. in the reverse order] of the signs, contrariwise, we add the amount in degrees to the position, as previously determined, at the moment of true conjunction, in longitude, latitude and anomaly (each separately); and the number of hours will give us the amount by which the apparent conjunction is later than the true.

Next, using the same methods, we determine from the distance in equinoctial hours of the apparent conjunction from the meridian, first, what the moon's parallax is measured along the great circle through the moon and the zenith. From the result we subtract the solar parallax for the same argument, and use this result to determine, as before, (by means of the angle formed between the circles [of ecliptic and altitude] at that moment), the latitudinal parallax [i.e. the parallax] along a circle orthogonal to the ecliptic. We convert the result to a distance along [the moon's] inclined circle, i.e. we multiply it by 12.[72] If the effect of the latitudinal parallax is northwards with respect to the ecliptic, we H531
add the result to the previously determined true position in [argument of] latitude at the moment of apparent conjunction when the moon is near the ascending node, but subtract it when the moon is near the descending node. Contrariwise, if the effect of the latitudinal parallax is southwards with respect to the ecliptic, we subtract the distance derived from the parallax from the previously determined position in [argument of] latitude at the moment of apparent conjunction, when the moon is near the ascending node, but add it when the moon is near the descending node.

We thus obtain the amount of apparent [argument of] latitude at the moment of apparent conjunction. With this as argument, we enter the solar eclipse tables, and if our argument falls within the range of the numbers in the

[71] I.e. suppose the original longitudinal parallax to be l_1: this gives us a correction to the time of conjunction (for the method of computing which see below), and hence a new zenith distance, which will lead to a new longitudinal parallax l_2. Ptolemy's rule is: form $l_2 - l_1 = e$. Then the 'epiparallax' e' is given by $e' = e + e\,(e/l_1)$, and the final longitudinal parallax by $l = l_1 + e' = l_1 + (l_2 - l_1) + (l_2 - l_1)^2/l_1$.

[72] From Ptolemy's earlier practice (e.g. VI 5 p. 286 with n.26) one would expect '11½', and this is indeed found in the Arabic tradition (Q, Ger). However, the crudity of the approximation to 1/sin 5° is almost negligible when one considers that the latitudinal parallax is usually small.

first two columns, we can say that there will be a solar eclipse, and that its middle coincides approximately with the moment defining apparent conjunction. So we set down separately the amounts of the [magnitude in] digits and the minutes of immersion and emersion corresponding to the argument of latitude, as derived from each of the two tables, then enter, with the distance of the moon in anomaly from the apogee [of the epicycle] at the apparent conjunction, into the table of correction, take the corresponding number of minutes, and take the corresponding fraction of the difference between each [pair of] results we wrote
H532 down. In every case we add the result to the number derived from the first table. The digits found by this procedure give us, again, the amount, in twelfths of the sun's diameter, which will be obscured at approximately mid-eclipse. We increase the minutes of travel [found by this procedure] for both [stretches, i.e. immersion and emersion] by $\frac{1}{12}$th, to account for the sun's additional motion, and convert the result into equinoctial hours [by dividing] by the moon's true [hourly] motion. Thus we have the length of both immersion and emersion: this, however, is on the assumption that the [change in] parallax has no effect on these time-intervals.

Now there is in fact a noticeable inequality in these intervals, due, not to the anomalistic motion of the luminaries,[73] but to the moon's parallax. The effect of this is to make each of the two intervals [immersion and emersion], separately, always greater than the amount derived by the above method, and, generally, unequal to each other. We shall not neglect to take this into account, even if it is small. This phenomenon is due to the fact that the effect of the parallax on the moon's apparent motion is always to produce the appearance of motion which would be in advance (if one were to disregard the moon's proper motion towards the rear). For suppose, first, that the moon's apparent position is before [i.e. to the east of] the meridian: then, as it gradually rises higher [above the horizon], its eastward parallax becomes continually smaller than at the
H533 moment preceding, and thus its motion towards the rear appears slower. Or suppose, secondly, that its apparent position is after [i.e. to the west of] the meridian: then, again, as it gradually descends [towards the horizon], its westward parallax becomes continually greater than at the moment preceding, and thus, as before, its motion towards the rear appears slower. For this reason the intervals in question are always greater than those derived by the simple procedure described. Furthermore, the difference between successive parallaxes [at equal intervals of time] becomes greater as one approaches the meridian: hence those intervals [of immersion or emersion] which are nearer the meridian must necessarily become more drawn-out. For this reason, the only situation in which the time of immersion is approximately equal to the time of emersion is when mid-eclipse occurs precisely at noon, for then the appearance of motion in advance resulting from the parallax is about equal on both sides [of mid-eclipse]. But when mid-eclipse occurs before noon, then the interval of emersion is closer to the meridian and [thus] longer, while if mid-eclipse occurs after noon, then the interval of immersion is closer to the meridian and longer.

So in order to correct the time-intervals for this effect, we [first] determine, in

[73] I.e. to the fact that the true speed of both sun and moon does not remain constant over the course of the eclipse. Cf. p. 309.

the way explained, the uncorrected length of each of the intervals in question, H534
and the zenith distance at mid-eclipse. Suppose, for example, that each interval is 1 equinoctial hour, and the zenith distance 75°. In the Parallax Table [V 18] we look for the minutes of parallax corresponding to the argument 75° (for, e.g., the moon's greatest distance, for which one takes the entries in the third column). We find, corresponding to 75°, 52′. Since, by hypothesis, the time-intervals of both immersion and emersion, in the mean, is 1 equinoctial hour, or 15 time-degrees, we subtract these 15° from the 75° of the zenith distance, and find the minutes of parallax in the same column corresponding to the resulting 60°, [namely], 47′. Hence the displacement in advance resulting from the parallax at the (average)[74] position nearer the meridian comes to 5′. We also add the [15°] to the 75°, and find the minutes of total parallax corresponding to the resulting 90° in the same column, 53½′. Thus here the displacement in advance resulting from the [parallax at] the position nearer the horizon is 1½′. We take the longitudinal components of these increments we have found, and convert each [separately] into a fraction of an equinoctial hour by means of the H535
moon's true motion, as described, and then add each result to the appropriate mean interval, calculated simply, of immersion or emersion; that is, we add the greater to the interval bounded by the position nearer to the meridian, and the lesser to the interval bounded by the position nearer the horizon. It is obvious that the difference between the two intervals in the above example is 3½′, or about ⅑th of an equinoctial hour, which is the time taken by the moon in mean motion to traverse that distance.[75]

There remains only the readily accomplished task, if we wish, of converting the time in equinoctial hours at each interval into the seasonal hours particular [to the given latitude and date], by the method explained in the earlier part of our treatise [II 9].

11. {*On the angles of inclination at eclipses*}[76]

The next topic is the examination of the inclinations[77] which are formed at eclipses. This kind of investigation is based both on the inclination of the

[74] μέσην. If not an interpolation, this must mean, taking the position obtained by applying the 15° of the motion of the heavens in 1 hour directly to the zenith distance. In fact 15° is the *maximum* possible change in the zenith distance in 1 equinoctial hour. Cf. n.75.

[75] Ptolemy's procedure here is, to say the least, crude. Instead of computing the actual zenith distances of the bodies at beginning and end of the eclipse, he simply applies the 15° of one hour's motion of the heavens to the zenith distance at mid-eclipse. Finding the total parallax from the zenith distance, he applies it as if it were the longitudinal parallax. The procedure is perhaps explicable as illustrating the *maximum* possible effect of this factor: the longest possible solar eclipse is about 2 hours; to get the maximum parallactic difference between the two intervals we have to take the zenith distance as great as possible. Allowing 15° hourly motion (cf. n.74), 75° is the maximum zenith distance which permits the whole eclipse to be visible. The total parallax is the maximum possible value of the longitudinal parallax. To be consistent, however, Ptolemy should have taken the moon at least distance (for which the difference between parallaxes is greater), i.e. col. 3 + col. 4 in V 18. This would have given him corrections of 6′ and 2′, with a difference of 4′ (still only ⅛th of an hour).

[76] On Chs. 11–13 see *HAMA* 141–4.

[77] Or 'directions', προσνεύσεις. For other uses of this word see p. 43 n.38 and p. 227 n.19. The purpose of computing these angles was presumably weather prediction: see *HAMA* II 999.

eclipsed part [of the body] to the ecliptic and on the inclination of the ecliptic itself to the horizon. Both of these angles, during the course of every eclipse
H536 phase, undergo great changes as a result of the shift in position [of the bodies], in a way which could not be controlled if one wanted to undertake the task of computing the inclinations throughout the whole of the duration [of the eclipse], a superfluous task, since predictions on such a scale are not in the least necessary or useful. For, since the situation of the ecliptic relative to the horizon is determined from the position on the horizon occupied by its rising or setting points, the angle formed by the ecliptic at the horizon must necessarily change continuously during the course of an eclipse, as those points on the ecliptic which are rising or setting change continuously. Similarly, since the inclination of the eclipsed part [of the body] to the ecliptic is determined from the great circle drawn through the two centres, [i.e.] the centres of moon and shadow or the centres of moon and sun, it is, again, a necessary consequence of the motion of the moon's centre during the course of an eclipse that the circle through the two centres occupy a continuously varying position relative to the ecliptic, and [hence] that the angle formed at their intersection vary continuously. Therefore [the need for] this kind of examination will be satisfied if it is carried out only for those points in [the progress of] the eclipse which have some significance, and only roughly for the inclinations with respect to the horizon. [To achieve this kind of accuracy] people who actually observe the eclipse as it occurs could,
H537 merely by eye, estimate the important inclinations by looking at the relative positions in both cases [at eclipse and horizon], since, as we said, a rough notion [of the amount] is sufficient in such matters. Nevertheless, not to pass over this topic altogether, we shall try to set out some ways of achieving the kind of result desired as conveniently as possible.

The points in [the progress of] the eclipse which we too take into consideration as deserving to be thought significant are:

[1] the point of the start of obscuration, which coincides with the very beginning of the whole eclipse;

[2] the point of the completion of obscuration, which coincides with the beginning of the phase of totality;

[3] the point of greatest obscuration, which coincides with the middle of totality;[78]

[4] the point of the start of emersion, which coincides with the end of the whole total phase;

[5] the point of the completion of emersion, which coincides with the end of the whole eclipse.

The inclinations [with respect to the horizon] which we take into consideration as being more reasonable and more significant are those bounded by the meridian and also those bounded by the rising and setting points of the ecliptic at the equinoxes and at summer and winter solstices. As for the points bounding

[78] Reading ἥτις ἐν τῷ μέσῳ χρόνῳ τῆς μονῆς γινεται (with D,Ar) for ἥτις ἐν τῷ μέσῳ χρόνῳ τῆς ἐκλείψεως ἄνευ τῆς μονῆς γίνεται at H537, 12-13. The latter would mean 'which coincides with the middle of the eclipse [for those eclipses] in which there is no total phase'. The interpolation is presumably the remains of a feeble attempt to list all possible cases.

the various 'wind-directions',[79] they may be understood in many different ways by many people; nevertheless, if desired, they can be indicated by means of the angles we set out along the horizon.

Considering the intersections of meridian with horizon, let us make the H538
following definitions:

the northern intersection is the 'northpoint';

the southern intersection is the 'southpoint'.

Considering the rising and setting [points of the ecliptic, let us make the following definitions]:

the intersections of the beginning of Aries or Libra with the horizon are known as 'equinoctial rising' and 'equinoctial setting'; these are always the same distance, [i.e.] a quadrant, from the point where the meridian intersects [the horizon];

the intersections of the beginning of Cancer [are known] as 'summer rising' and 'summer setting', and the intersections of the beginning of Capricorn as 'winter rising' and 'winter setting'.

The distances [from the meridian intersection] of these last [four] points vary according to the latitude in question. The inclinations are sufficiently characterised by saying that they are at one of the above situations or between some pair of them.

To enable one to determine the position of the ecliptic relative to the horizon for any given situation, we computed, by the method indicated in the first books of our treatise,[80] the distance along the horizon, at rising and setting, of the beginning of each zodiacal sign from the points where the equator intersects [the horizon, computing them] on either side of it [i.e. north or south]. We did this for each of those latitudes from Meroe to Borysthenes for which we [earlier] tabulated the angles [II 13]. To provide a means of readily surveying these,[81] instead of a table, we drew a diagram [Fig 6.7] consisting of 8 concentric circles, conceived as lying in the plane of the horizon, to contain the [various] distances and nomenclature for the 7 climata. Then we drew two lines, at right angles to H539
each other, through all the circles: a horizontal one representing the intersection of the planes of horizon and equator, and another, vertical one representing the intersection of the planes of horizon and meridian. On the innermost[82] circle we wrote, at the ends of the horizontal line 'equinoctial rising' and 'equinoctial setting', and at the ends of the vertical line 'north' and 'south'. Similarly we drew [four] straight lines through all the circles at equal

[79] Greek astronomy sometimes adopted the popular way of indicating the points of the compass by wind-names. These do not occur in the Almagest, except for ἀπηλιώτης and λίψ in VIII 4 to designate the general directions 'east' and 'west', and in the diagram Fig. 6.7, where they are a later interpolation in the mss., not mentioned in the text (see below n.82). On the systems of wind-names (which do indeed vary) see Rehm, *Griechische Windrosen*.

[80] II 2 p. 77.

[81] κατὰ τὸ εὐθεώρητον. One would rather expect διὰ τὸ εὐθεώρητον, which is implied by Isḥāq's translation.

[82] In the figures in the Greek mss. these designations are on the *outermost* circle; hence Heiberg (at H539,7; cf. ibid. p. VI) emended ἐντός, the reading of all mss., to ἐκτός ('outermost'). But in the Arabic tradition they do appear, all or in part, on the inmost circle, and it seems likely that they were transferred to the outermost circle when the names of the winds were (after Ptolemy) added in the inmost circle (cf. above n.79).

inclinations either side of the equator [i.e. the horizontal line], and wrote along these, in the seven interlinear spaces, the horizon distance of the solsticial point from the equator which we found for each latitude (in units where one quadrant contains 90°). At the ends where these lines meet the inmost circle we wrote, for the southern ones, 'winter rising' and 'winter setting', and for the northern ones, 'summer rising' and 'summer setting'. To indicate the signs in between [equinoxes and solstices] we inserted two more lines in each of the four segments, and [wrote] along these the horizon distance from the equator of [the beginning of] the appropriate zodiacal sign, adding the name of each sign on the outermost circle. We also wrote, along the meridian line, for [each] parallel, its
H540 name, the length [of the longest day] in hours, and the elevation of the pole. In writing in [the data for all of the above], we began with the largest, outermost circle for the northernmost data, [and so on].[83]

In order to have tabulated the apparent inclinations of the actual phases to the ecliptic, i.e. the angles formed between the ecliptic and the great circle joining the centres in question at each of the significant points mentioned above, we computed these too, for [successive] positions of the moon corresponding to a difference of 1 digit in obscuration. However, we did this only for lunar positions at mean distance (since that is sufficient), and under the assumption that those arcs of the ecliptic and the moon's inclined circle which we consider for the obscurations are sensibly parallel to each other.

For example, let [Fig. 6.6] line AB represent the arc of the ecliptic, with A as the centre of the sun or the shadow, and let line GDE represent the moon's inclined circle, with G as the point at which the moon's centre is at eclipse middle, and D as the point at which the centre is when it is just totally eclipsed
H541 or just about to begin emerging from totality (i.e. when the moon is internally tangent to the circle of the shadow). Let E be the point at which the moon's

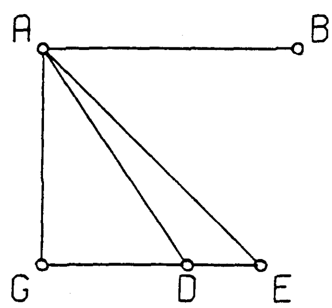

Fig. 6.6

[83] On this figure see *HAMA* 38–9. As Ptolemy drew it, it is, as he says, a schematic representation of a table. But it closely resembles a representation in polar coordinates. If it were truly such, however, all the straight lines except the horizontal and vertical ones would become curves (see *HAMA* p. 1216 Fig. 32). I have omitted the wind-names found in the Greek and some Arabic mss., and in Heiberg's figure. Cf. p. 315 n.82. The figure is on p. 320.

Correction to Heiberg: for the latitude of Clima VI read με α (with AD, Is) for με λδ (45;34°). Corrected by Heiberg ad loc.

centre is when either sun or moon is just beginning to be eclipsed or has just completed emersion (i.e. when the circles are externally tangent). Join AG, AD, AE.

It is obvious that angles BAG and AGE, which correspond to the time of mid-eclipse, are right angles to the senses, and that ∠ BAE represents the angles at the beginning and end of the eclipse, while ∠ BAD represents the angles at the end of [the partial phase of] the eclipse and at the beginning of emersion. And it is immediately clear that AE represents the sum of the radii of both circles, and AD their difference.[84]

Then let us take as an example an eclipse in which half the sun's diameter is obscured at mid-eclipse. Let A be the sun's centre. Then in all cases (since we assume the moon at mean distance) AE comes to [0;15,40° + 0;16,40° =] 0;32,20°, and AG. which is less than that by half the sun's diameter, comes to 0;16,40°.

Therefore, since AG = 16;40^p where hypotenuse EA = 32;20^p H542
(according to the magnitude of obscuration assumed),

where hypotenuse AE = 120^p

AG = 61;51^p,

and, in the circle about right-angled triangle AGE

arc AG = 62;2°.

$$\therefore \angle \text{AEG} = \angle \text{BAE} = \begin{cases} 62;2^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \\ 31;1^{\circ} \text{ where 4 right angles} = 360^{\circ}. \end{cases}$$

Again, to take the case of a lunar eclipse, let A be the centre of the shadow. Then, since, as before, we assume the moon at mean distance, AE will always be the same amount, namely [0;43,20° + 0;16,40°=] 60 minutes, and AD, likewise, will always be [0;43,20° − 0;16,40° =] 26;40 minutes. Let the moon be eclipsed in a situation such that the magnitude is 18 digits. Thus AG is again less than AD by half the diameter [of the moon][85] and, by subtraction [of 16;40′ from 26;40′], AG comes to 10;0 minutes.

Then, where hypotenuse AE = 120^p, AG = 20;0^p,
and, in the circle about right-angled triangle AGE,

arc AG = 19;12°.

$$\therefore \angle \text{AEG} = \angle \text{BAE} = \begin{cases} 19;12^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \\ 9;36^{\circ} \text{ where 4 right angles} = 360^{\circ}. \end{cases}$$ H543

Similarly, where hypotenuse AD = 120^p, AG = 45^p,
and, in the circle about right-angled triangle AGD,

arc AG = 44;2°.

$$\therefore \angle \text{ADG} = \angle \text{BAD} = \begin{cases} 44;2^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \\ 22;1^{\circ} \text{ where 4 right angles} = 360^{\circ}. \end{cases}$$

In the same way we computed the sizes of the angles for the other [integer] digits [of magnitude], [always taking] that angle which was less than a right angle, in units where one right angle equals 90° (corresponding to the graduation of the quadrant of the horizon). We constructed a table with 22 lines

[84] Cf. *HAMA* Fig. 124 p. 1244.

[85] See Fig. M (copied from the figure on p. 409 of Manitius). Since the eclipse has a magnitude of 18 digits, by definition XY = 6^d = radius of moon. Therefore AX = AY − XY = radius of shadow minus radius of moon = AD. Therefore AG = AX − XG = AD minus radius of moon.

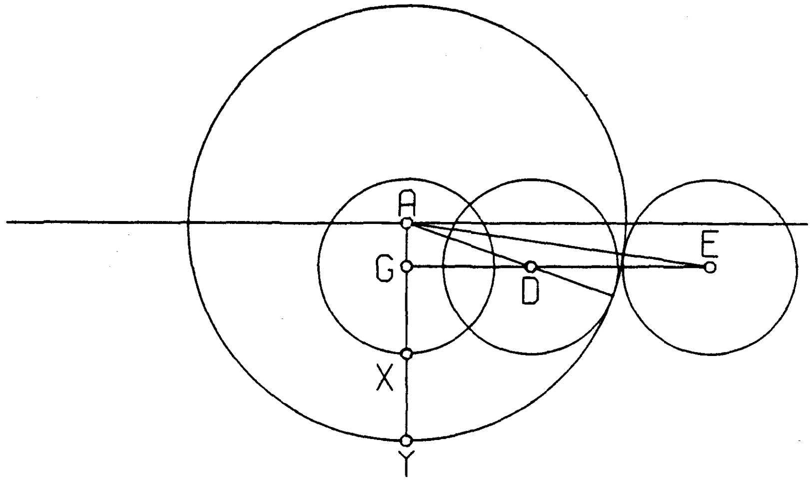

Fig. M

and 4 columns. The first column contains the digits of actual obscuration, measured along the diameter, found for mid-eclipse; the second contains the angles occurring at solar eclipses at the moment of the beginning of the eclipse and the moment of the end of emersion; the third contains the angles occurring at lunar eclipses at the moments of the beginning of the eclipse and of the end of emersion; and the fourth also contains the angles occurring at lunar eclipses, at the moment of the end of [the partial phase of] the eclipse and the moment of the beginning of emersion. The layout of table and circle [diagram] are as follows.

H544 12. {*Display of diagrams for the inclinations*}[86]

[See pp. 319,320.]

H545 13. {*Determination of the inclinations*}

Thus, as a preliminary, we determine, by the method explained [VI 9–10], the time of each significant point [in the eclipse] listed above, and, from the times, those points on the ecliptic which are rising and setting at those moments, and, from the diagram [Fig. 6.7], the situation [of ecliptic] with respect to the horizon. Then, when the centre of the moon (the apparent centre at solar eclipses and the true centre at lunar eclipses) is exactly on the ecliptic, we get the inclination for a solar eclipse at the beginning of the eclipse, and the inclination for

[86] Corrections to Heiberg:
Arg. 4 digits, col. 3, read νδ λδ for νδ κζ (54;27°) at H544,13. All mss. have the incorrect reading, but it is obviously repeated in error from the line above.
Arg. 14 digits, col. 4, read νβ κα for νβ κδ (52;24°), with D,Ar, at H544,23.

1 Digits	2 Sun Beginning of Eclipse and End of Emersion	3 Moon Beginning of Eclipse and End of Emersion	4 [Moon] End of Partial Phase and Beginning of Emersion
0	90° 0	90° 0	
1	66 50	72 30	
2	56 59	65 10	
3	49 16	59 27	
4	42 36	54 34	
5	36 35	50 14	
6	31 1	46 15	
7	25 46	42 31	
8	20 44	39 2	
9	15 51	35 42	
10	11 6	32 29	
11	6 25	29 23	
12	1 47	26 23	90° 0
13		23 28	63 37
14		20 36	52 21
15		17 48	43 26
16		15 1	35 41
17		12 18	28 38
18		9 36	22 1
19		6 55	15 43
20		4 15	9 36
21		1 36	3 35

a lunar eclipse at the end of the partial phase and also at the end of emersion, from the situation on the horizon of the point of the ecliptic setting at the moment in question; we get the inclination for a solar eclipse at the end of the eclipse, and the inclination for a lunar eclipse at the beginning of the eclipse and the beginning of emersion [i.e. end of totality], from the [horizon situation] of the rising-point of the ecliptic. When the moon's centre is not exactly on the ecliptic, we take from the table the angles corresponding to the relevant magnitude [of the eclipse] in digits, and apply those angles to the intersection of horizon and ecliptic. If the moon's centre is north of the ecliptic, we set off the angle to the north of the setting-point for eclipse-beginning in solar eclipses and for the end of the partial phase in lunar eclipses; we set it off to the north of the H546 rising-point for the end of emersion in solar eclipses and the beginning of emersion in lunar eclipses; furthermore we set it off to the south of the rising-point for eclipse-beginning in lunar eclipses, and to the south of the setting-point for eclipse-end in lunar eclipses. If the moon's centre is south of the ecliptic, we set the angle off to the south of the setting-point for eclipse-beginning in solar eclipses and for end of the partial phase in lunar eclipses; to the south of the rising-point for the eclipse-end in solar eclipses and for the beginning of emersion in lunar eclipses; to the north of the rising-point for

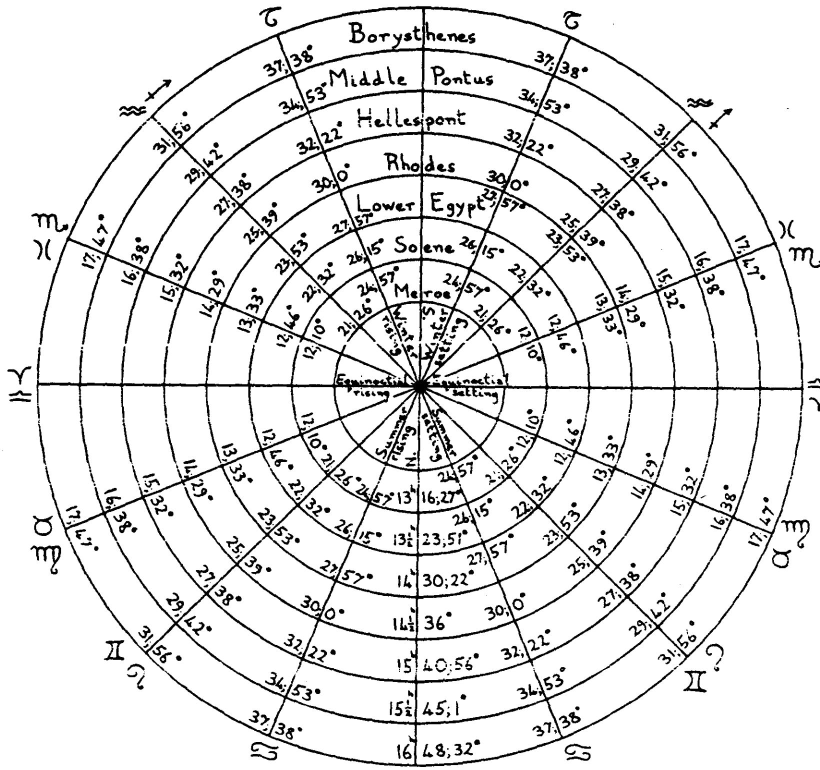

Fig. 6.7

eclipse-beginning in lunar eclipses; and to the north of the setting-point for eclipse-end in lunar eclipses. The result of this procedure will give us the point on the horizon towards which (speaking roughly, as we said), are inclined those points of the luminaries comprising the significant [moments of the phases], namely the beginning and end of eclipse and of total phase.[87]

[87] Literally 'the beginnings and ends of the eclipse and emersion', i.e. beginning of eclipse, end of partial phase (= beginning of totality), beginning of emersion (= end of totality), end of emersion.

Book VII

1. {*That the fixed stars always maintain the same position relative to each other*}[1] H2

In the preceding part of this treatise, Syrus, we discussed the phenomena associated with *sphaera recta* and *sphaera obliqua*, and also the details of the hypotheses for the motions of sun and moon and the combinations of positions which are seen to result from them. Now, to deal with the next part of the theory, we shall begin discussing the stars, and first, in accordance with the logical order, the so-called fixed stars.

First of all we must make the following introductory point. Concerning the terminology we use, in as much as the stars themselves patently maintain the formations [of their constellations] unchanged and their distances from each other the same, we are quite right to call them 'fixed'; but in as much as their sphere, taken as a whole, to which they are attached, as it were, as they are carried around, also [like the other spheres] has a regular motion of its own towards the rear and the east with respect to the first [daily] motion,[2] it would not be appropriate to call this [sphere] too 'fixed'. For we find that both these statements are true, at least on the [observational] basis afforded by the amount of time [preceding us]: even before this Hipparchus conceived of both these notions on the basis of the phenomena available to him, but under conditions which forced him, as far as concerns the effect over a long period, to conjecture rather than to predict, since he had found very few observations of fixed stars H3 before his own time, in fact practically none besides those recorded by Aristyllos and Timocharis, and even these were neither free from uncertainty nor carefully worked out; but we too come to the same conclusions by comparing present phenomena with those of that time, but with more assurance, both because our examination is conducted [with material taken] from a longer time-interval, and because the fixed-star observations recorded by Hipparchus, which are our chief source for comparisons, have been handed down to us in a thoroughly satisfactory form.

First, then, no change has taken place in the relative positions of the stars even up to the present time. On the contrary, the configurations observed in

[1] On. chs. 1 and 2 see Pedersen 237–45.

[2] Note that the motion which in modern terminology is 'precession of the equinoxes' (i.e. a motion in the direction of decreasing longitudes of the tropical points with respect to the fixed stars) is described by Ptolemy as a motion of the fixed stars with respect to the tropical points in the direction of increasing longitudes. This accords with his taking the tropical points as the primary reference points (III 1 p. 132). Hipparchus, however, seems at times to have adopted the modern convention, to judge from the title of his work 'On the displacement of the solsticial and equinoctial points' (III 1 p. 132 and VII 2 pp. 327 and 329).

Hipparchus' time are seen to be absolutely identical now too. This is true not only of the positions of the stars in the zodiac relative to each other, or of the stars outside the zodiac relative to other stars outside the zodiac (which would [still] be the case if only stars in the vicinity of the zodiac had a rearward motion, as Hipparchus proposes in the first hypothesis he puts forward); but it is also true of the positions of stars in the zodiac relative to those outside it, even those at considerable distances. This can easily be seen by anyone who is willing to make an inspection of the matter and examine, in the spirit of love of truth,
H4 whether present phenomena agree with those recorded for Hipparchus' time.

In any case, to provide a convenient test of the matter, we too will adduce here a few of his observations, [namely] those which are most suitable for easy comprehension and also for giving an overview of the whole method of comparison, by showing that the configurations formed by stars outside the zodiac, both with each other and with stars in the zodiac, have been preserved unchanged.[3]

Stars in Cancer. [Hipparchus] records that the star in the southern claw of Cancer [α Cnc], the bright star which is in advance of the latter and of the head of Hydra [β Cnc], and the bright star in Procyon [α CMi] lie almost on a straight line.[4] For the one in the middle lies $1\frac{1}{2}$ digits[5] to the north and east of the[6] straight line joining the two end ones, and the distances [from it to each of them] are equal.

Stars in Leo. [He records] that the easternmost two [μ, ε Leo] of the four stars in the head of Leo [μ, ε, κ, λ], and the star in the place where the neck joins [the head] of Hydra [ω Hya], lie on a straight line.[7] Also, that the line drawn through the tail of Leo [β] and the star in the end of the tail of Ursa Major [η
H5 UMa] cuts off the bright star under the tail of Ursa Major [α CVn] 1 digit to the west [i.e. passes 1 digit to the east of it].[8] Similarly, [he records] that the line through the star under the tail of Ursa Major and the tail of Leo passes through the more advanced of the stars in Coma [Berenices].[9]

[3] In the following lists I give in brackets the modern designation of the stars in question, when the identification is reasonably certain, and, in footnotes, the equivalent in Ptolemy's catalogue. Several of the stars mentioned by Hipparchus are not recorded in that catalogue, and his descriptions of those that are often differ from Ptolemy's. In Ptolemy's own alignments which follow, the descriptions also vary somewhat from the catalogue. The alignments are discussed in detail by Manitius, 'Fixsternbeobachtungen'.

[4] Catalogue XXV 6 and 9 and XXXIX 2. Like Manitius, I do not understand 'to the north and east'. In the given situation, the only possible deviation is to the north-west or the south-east. I calculate that in Hipparchus' time it was about 5′ to the north and west.

[5] The 'digit' (δάκτυλος) and 'cubit' (πῆχυς, see p. 323) as astronomical measurements were taken by Hipparchus from Babylonian astronomy (in the Almagest they are found only in the Babylonian observations IX 7, pp. 452-3, and XI 7, p. 541, and in passages derived from Hipparchus). The cubit in Babylonian astronomy can represent either $2\frac{1}{2}°$ or 2° (the latter normal in the Hellenistic period: see *HAMA* II 591–93). Strabo, 2.1.18, quotes data from Hipparchus in which the 2° norm is certain. It is also found in Hipparchus' commentary on Aratus, where Vogt, 'Wiederherstellung', col. 30, argued for the $2\frac{1}{2}°$ norm. In the passage below, a 2° cubit produces a smaller error in the estimated distance (inaccurate in either case). The 'digit' in Babylonian astronomy is $\frac{1}{24}$th of the 2° cubit or $\frac{1}{30}$th of the $2\frac{1}{2}°$ cubit, 5′ in either case.

[6] Reading τῆς for τὴν (misprint in Heiberg) at H4, 14.

[7] Catalogue XXVI 3 and 4 and XLI 6.

[8] Catalogue XXVI 27, II 27 and II 28. By my calculation, the line passed more like half a degree to the east of α CVn.

[9] The latter are probably catalogue XXVI 33 and 34, doubtfully identified as 15 and 7 Com.

Stars in Virgo. [He records] that between the northern foot of Virgo [μ Vir] and the right foot of Bootes [ζ Boo][10] lie two stars; the southern one of these [109 Boo], which is equally bright as the [right] foot of Bootes, lies to the east of the line joining the feet, while the northern one [31 Boo], which is half-bright, lies on a straight line with the feet. Furthermore, of these two stars, the half-bright one is preceded by two bright stars, which form, together with the half-bright one, an isosceles triangle of which the half-bright one is the apex.[11] These [two bright stars] lie on a straight line with Arcturus [α Boo] and the southern foot of Virgo [λ Vir].[12] Also, that between Spica [α Vir] and the second star from the end of the tail in Hydra [γ Hya][13] lie three stars, all on one straight line [57,63,69 Vir].[14] The middle one of these [63] lies on a straight line with Spica and the second star from the end of the tail in Hydra.

Stars in Libra. [He records] that the star [μ Ser] which is very nearly on a straight line towards the north with the [two] bright stars in the claws [α, β Lib] is bright and triple: for on both sides of it lie single small stars [36,30 Ser].[15]

Stars in Scorpius. [He records] that the straight line drawn through the rearmost of the stars in the sting of Scorpius [λ Sco] and through the right knee of Ophiuchus [η Oph] bisects the interval between the two advance stars in the right foot of Ophiuchus [36,θ Oph][16] and that the fifth and seventh joints [in the tail of Scorpius, θ, κ Sco] lie on a straight line with the bright star in the middle of Ara [α Ara].[17] Furthermore, that the northernmost star [σ] of the two in the base of Ara [σ, θ][18] lies between and almost on a straight line with the fifth joint and the star in the middle of Ara, being almost equidistant from both. H6

Stars in Sagittarius. [He records] that to the east and south of the Circle under Sagittarius [i.e. of Corona Australis] lie two bright stars [α, β Sgr], quite some distance (about 3 cubits) from each other.[19] The southernmost and brighter of these [β], which is on the foot of Sagittarius, lies very nearly on a straight line with the midmost [α CrA] of the three bright stars in the Circle (which lie furthest towards the east in that [constellation]) [γ, α, β CrA], and with the rearmost [ζ Sgr] of the [two] bright stars [ζ, σ Sgr] at opposite angles of the Quadrilateral [in Sagittarius, ζ, τ, σ, φ]: the two intervals [between these three stars] are equal. The northernmost [of the two stars to the east of the Circle, α Sgr] lies to the east of this straight line, but is on a straight line with the [two] bright stars [ζ, σ] at opposite angles of the Quadrilateral.[20]

[10] Catalogue XXVII 26 and V 19.
[11] Manitius identifies these two stars as nos. 43 and 46 of Bootes in the catalogue of Heis (Köln, 1872). I have not tracked these down in a more recent catalogue, since any identification seems utterly uncertain.
[12] Catalogue V 23 and XXVII 25.
[13] Catalogue XXVII 14 and XLI 24.
[14] This seems preferable to Manitius' identification (61, 63, 69).
[15] The first three are catalogue XIV 11 and XXVIII 1 and 3. My identification of the 'triple star' is far more likely than Manitius' α Ser plus λ, 29 Ser.
[16] Catalogue XXIX 20 and XIII 12, 14 and 15.
[17] Catalogue XXIX 17 and 19 and XLVI 3.
[18] Catalogue XLVI 1 and 2.
[19] Catalogue XXX 24 and 23. On the cubit see p. 322 n.5.
[20] The equivalents in Ptolemy's catalogue are: α, β Sgr: XXX 24, 23; γ, α, β CrA: XLVII 8, 7, 6; ζ, τ, σ, φ Sgr: XXX 22, 21, 6, 7 (*not* described as a quadrilateral).

Stars in Aquarius. [He records] that the two stars close together in the head of Pegasus [θ, ν Peg] and the rear shoulder of Aquarius [α Aqr] are almost on a
H7 straight line,[21] to which the line from the advance shoulder of Aquarius [β Aqr] to the star in the cheek of Pegasus [ε Peg] is parallel.[22] Also, that the advance shoulder of Aquarius [β], the bright star [ζ Peg] of the two in the neck of Pegasus [ζ, ξ], and the star in the navel of Pegasus [α And] lie on a straight line, with equal intervals between them.[23] Furthermore, that the line through the muzzle [ε] of Pegasus and the easternmost [η Aqr] of the four stars in the vessel [of Aquarius, η, ζ, π, γ][24] bisects, almost at right angles, the line through the two stars [θ, ν] close together in the head of Pegasus.

Stars in Pisces. [He records] that the star [β Psc] in the snout of the southernmost fish [of Pisces], the bright star in the shoulders of Pegasus [α Peg], and the bright star in the chest of Pegasus [β Peg] lie on a straight line.[25]

Stars in Aries. [He records] that the advance star [β Tri] in the base of Triangulum lies 1 digit to the east of the straight line drawn through the star in the muzzle of Aries [α Ari] and the left foot of Andromeda [γ And].[26] Also, that the most advanced of the stars in the head of Aries [β, γ Ari] and the midpoint of the base of Triangulum [i.e. halfway between β and γ Tri] lie on a straight line.[27]

Stars in Taurus. [He records] that the [two] easternmost stars of the Hyades [α, ε Tau] and that star [π^1 Ori] in the pelt held in Orion's left hand which is sixth, counted from the south, lie on a straight line.[28] And that the line drawn through
H8 the advance eye of Taurus [ε Tau] and the seventh star from the south in the pelt [o^2 Ori] cuts off the bright star in the Hyades [α Tau] 1 digit to the north.[29]

Stars in Gemini. [He records] that the heads of Gemini [α, β Gem] lie on a straight line with a certain star [ζ Cnc] which lies to the rear of the rearmost head by a distance three times that between the heads, and that the same star also lies on a straight line with the [two] southernmost [θ, δ Cnc] of the four stars [θ, δ, γ, η] round the nebula [Praesepe].[30]

In these alignments, and similar alignments which enable us to carry out

[21] Catalogue XIX 15 and 16 and XXXII 2.

[22] Catalogue XXXII 4 and XIX 17.

[23] Catalogue XIX 11, 12 and 1.

[24] Catalogue XXXII 12, 11, 10, 9.

[25] Catalogue XXXIII 1 and XIX 4 and 3.

[26] Catalogue XXI 2, XXII 14 and XX 15. Using the coordinates for these 3 stars computed by Peters-Knobel (pp. 81–2) for the time of Hipparchus, I find β Tri well over a degree to the east of the line connecting α Ari and γ And. There is no doubt about the identification of the stars.

[27] Catalogue XXII 2 and 1 and XXI 2 and 4. I have dubiously adopted Manitius' identifications here. However, it seems possible that by 'the midpoint of the base of the triangle' Hipparchus may have been referring to the star δ Tri. This lies approximately on a straight line with λ and β Ari. While γ Ari is 'more advanced' than either of these, Hipparchus may, like Ptolemy, have put that 'on the horn' rather than 'in the head'. λ Ari is not included in Ptolemy's catalogue.

[28] Catalogue XXIII 14 and 15 and XXXV 20. Ptolemy counts the stars in the pelt from the opposite direction, the north.

[29] Catalogue XXIII 15, XXXV 19 and XXIII 14. Manitius identifies the first star with δ Tau, but not only is this discrepant from Ptolemy's catalogue, but it produces a deviation from the line of about 1° to the north, whereas, if one takes the line from ε Tau to o^2 Ori, α Tau lies about 8′ to the north, in good agreement with the equivalence, 1 digit = 5′.

[30] Catalogue XXIV 1 and 2; XXIV 25; XXV 3, 5, 4, 2; and XXV 1.

comparisons practically throughout the sphere [of the fixed stars], we see that no change has occurred up to the present time. Yet very noticeable changes would have occurred in the 260 or so years between [Hipparchus and now] if the stars near the ecliptic were the only ones to perform an eastward motion.

But, in order to provide those who come after us with a means of comparison over a longer interval [than was possible for us], from an even larger number of alignments of the above kind, we shall add the most easily recognisable from among those which we have observed but which were not previously recorded. We begin from the

Stars in Aries. The two northernmost [α, β Ari] of the three stars in the head of Aries [α, β, γ] and the bright star in the southern knee of Perseus [ε Per] and the H9
star called Capella [α Aur] lie on a straight line.[31]

[*Stars in Taurus.*] The line drawn through the star called Capella [α Aur] and the bright star in the Hyades [α Tau] cuts off[32] the star in the advance leg of Auriga [ι Aur] a little to the east.[33] Also, the star called Capella [α Aur], the star which is common to the rearmost foot of Auriga and the tip of the northern horn of Taurus [β Tau], and the star in the advance shoulder of Orion [γ Ori] lie on a straight line.[34]

[*Stars in Gemini.*] Furthermore, the [two] bright stars in the heads of Gemini [α, β Gem] and the bright star in the neck of Hydra [θ Hya] lie very nearly on a straight line.[35]

[*Stars in Cancer.*] Furthermore, the two stars close together in the front leg of Ursa Major [ι, κ UMa], the star on the tip of the northern claw of Cancer [ι Cnc], and the northernmost of the [two] 'Aselli' [γ Cnc] lie on a straight line.[36] Similarly, the southern Asellus [δ Cnc], the bright star in Procyon [α CMi], and the bright star between them (which is in advance of the head of Hydra) [β Cnc]), lie almost on a straight line.[37]

[*Stars in Leo.*] Furthermore, the straight line drawn from the midmost star [γ Leo] of the [three] bright stars in the neck of Leo [ζ, γ, η] to the bright star in Hydra [α Hya] cuts off the star on the heart of Leo [α Leo] a little to the east.[38] The [line] from the bright star in the rump of Leo [δ Leo] to the bright star [γ UMa] in the back of the thigh of Ursa Major (which is the southernmost star on H10
the rear side of the quadrilateral), cuts off, a little to the west, the two stars which are close together in the rear paw of Ursa Major [ν, ξ UMa].[39]

[*Stars in Virgo.*] Furthermore, the line from the star in the back of the thigh of

[31] Catalogue XXII 14, 2, 1: XI 23: and XII 3.

[32] Reading ἀπολαμβάνει (with DG) for λαμβάνει at H9, 4. Corrected by Manitius and by Heiberg himself (*Op. Min.* p. XIV).

[33] Catalogue XII 3, XXIII 14 and XII 10.

[34] Catalogue XII 3, XII 11 and XXXV 3.

[35] Catalogue XXIV 1 and 2 and XLI 7.

[36] Catalogue II 12 and 13 and XXV 7 and 4. The identifications are certain, but the line through ι and κ UMa passes far to the east of γ and ι Cnc, both now and (according to the coordinates of Peters-Knobel) in Ptolemy's time. I have not computed whether modern proper motions suffice to account for this discrepancy. If Ptolemy had written 'the northernmost of the two stars close together' the alignment would be more plausible.

[37] Catalogue XXV 5, XXXIX 2 and XXV 9.

[38] Catalogue XXVI 5, 6, 7 (ζ, γ, η Leonis); XLI 12; and XXVI 8.

[39] Catalogue XXVI 20, II 19, II 23 and 24.

Virgo [ζ Vir] to the second star from the tip of Hydra's tail [γ Hya] cuts off the star called Spica [α Vir] a little to the west. The line from Spica to the star in the head of Bootes [β Boo] cuts off Arcturus [α Boo] a little to the east. Spica and the [two] stars on the wings of Corvus [δ, γ Crv] lie on a straight line. Spica, the star in the back of Virgo's thigh [ζ Vir], and the northernmost, bright star [η Boo] of the three in the advance knee of Bootes [η, τ, υ] lie on a straight line.[40]

[*Stars in Libra.*] Furthermore, the [two] bright stars in the claws [α, β Lib] and the star on the tip of Hydra's tail [π Hya] are very nearly on a straight line. The bright star in the southern claw [α Lib], Arcturus [α Boo], and the midmost [ζ UMa] of the three stars in the tail of Ursa Major [ε, ζ, η] lie on a straight line. The bright star in the northern claw [β Lib], Arcturus [α Boo], and the star in the back of the thigh of Ursa Major [γ UMa] lie on a straight line.[41]

[*Stars in Scorpius.*] Furthermore, the star on the rear shin of Ophiuchus [ξ Oph], the star in the fifth tail-joint of Scorpius [θ Sco], and the more advanced [υ] of the two stars close together in its sting [λ, υ] lie on a straight line. The most advanced [σ] of the three stars in the breast of Scorpius [σ, α, τ], and the two
H11 stars in the knees of Ophiuchus [η, ζ Oph], form an isosceles triangle, the apex of which is the most advanced of the three stars in the breast.[42]

[*Stars in Sagittarius.*] Furthermore, the star on the front, southern hock of Sagittarius (which is of second magnitude) [β Sgr], the star on the arrow-head [γ Sgr], and the star in the rear knee of Ophiuchus [η Oph] lie on a straight line.. The star [α Sgr] in the knee of the same [front] leg of Sagittarius (which lies near Corona [Australis]), the star on the arrow-head [γ Sgr], and the star in the advance knee of Ophiuchus [ζ Oph] lie on a straight line.[43]

[*Stars in Capricorn.*] Furthermore, the line drawn from the bright star in Lyra [α Lyr] to the stars[44] in the horns of Capricorn [α, β, ν, ξ Cap] cuts off the bright star in Aquila [α Aql] a little to the east. The line from the bright star in Aquila to the first-magnitude star in the mouth of Piscis Austrinus [α PsA] bisects, approximately, the interval between the two bright stars on the tail of Capricorn [γ, δ Cap].[45]

[*Stars in Aquarius.*] Furthermore, the line from the first-magnitude star in the mouth of Piscis Austrinus [α PsA] to the star in the muzzle of Pegasus [ε Peg] cuts off the bright star in the rear shoulder of Aquarius [α Aqr], a little to the east.[46]

[*Stars in Pisces.*] Furthermore, the stars in the mouths of Piscis Austrinus [α

[40] Catalogue XXVII 15 (ζ Vir), XLI 24 (γ Hya), XXVII 14 (α Vir), V 6 and 23 (β, α Boo), XLIII 5 and 4 (δ, γ Cor); and V 20, 21, 22 (η, τ, υ Boo).

[41] Catalogue XXVIII 1 and 3 (α, β Lib); XLI 25 (π Hya); V 23 (α Boo); II 25, 26, 27 (ε, ζ, η UMa); and II 19 (γ UMa).

[42] Catalogue XIII 13 (ξ Oph); XXIX 17, 20, 21 (θ, λ, υ Sco); XXIX 7, 8, 9 (σ, α, τ Sco) and XIII 12 and 19 (η, ζ Oph).

[43] Catalogue XXX 23 and 1 (β, γ Sgr); XIII 12 (η Oph); XXX 24 (α Sgr); and XIII 19 (ζ Oph).

[44] Reading τοὺς, with D, Ar (other Greek mss. τοῦ) for Heiberg's emendation τὸν 'the star' at H11, 10. Corrected by Manitius, who supposes the stars to be α and β Cap. But these would not give the correct alignment, and in the catalogue Ptolemy puts both these stars on the same horn. I therefore suppose that he is referring to the general direction from Vega of the group of stars.

[45] Catalogue VIII 1 (α Lyr); XXXI 1, 2, 3, 4 (α, ν, β, ξ Cap); XVI 3 (α Aql); XLVIII 1 (α PsA); and XXXI 23, 24 (γ, δ Cap).

[46] Catalogue XLVIII 1, XIX 17 and XXXII 2.

PsA] and the southern fish [of Pisces, β Psc] and the [two] advance stars of the quadrilateral in Pegasus [α, β Peg] lie on a straight line.[47]

If one were to match the above alignments too against the diagrams forming the constellations on Hipparchus' celestial globe,[48] he would find that the positions of the [relevant stars] on the globe resulting from the observations made at that time [of Hipparchus], according to what he recorded, are very nearly the same as at present. H12

2. {*That the sphere of the fixed stars, too, performs a rearward motion along the ecliptic*}

From these considerations, and others like these, we can be assured that absolutely all the so-called fixed stars maintain one and the same position relative [to each other], and share one and the same motion. But the sphere of the fixed stars also performs a motion of its own in the opposite direction to the revolution of the universe, that is, [the motion of] the great circle through both poles, that of the equator and that of the ecliptic.[49] We can see this mainly from the fact that the same stars do not maintain the same distances with respect to the solsticial and equinoctial points in our times as they had in former times: rather, the distance [of a given star] towards the rear with respect to [one of] those same points is found to be greater in proportion as the time [of observation] is later.

For Hipparchus too, in his work 'On the displacement of the solsticial and equinoctial points', adducing lunar eclipses from among those accurately observed by himself, and from those observed earlier by Timocharis, computes that the distance by which Spica is in advance of the autumnal [equinoctial] point is about 6° in his own time, but was about 8° in Timocharis' time.[50] For his H13
final conclusion is expressed as follows: 'If, then, Spica, for example, was formerly 8°, in zodiacal longitude, in advance of the autumnal [equinoctial] point, but is now 6° in advance', and so forth. Furthermore he shows that in the case of almost all the other fixed stars for which he carried out the comparison, the rearward motion was of the same amount. And we also, comparing the distances of fixed stars from the solsticial and equinoctial points as they appear in our time with those observed and recorded by Hipparchus, find that their motion towards the rear with respect to the ecliptic is, proportionally, similar to the above amount. We conducted this type of investigation by means of the instrument which we constructed previously [see V 1] for the observations of

[47] Catalogue XLVIII 1, XXXIII 1, and XIX 4 and 3. The 'quadrilateral' in Pegasus (not mentioned in the catalogue) is formed by the stars α Peg, β Peg, α And and γ Peg.

[48] I interpret this to mean that Hipparchus published a description of the constellations to be drawn on a celestial globe (literally 'solid sphere', στερεὰ σφαῖρα, cf. VIII 3). What relationship, if any, this had to Hipparchus' putative 'Catalogue' is obscure. On the general problem see *HAMA* 284–92.

[49] Reference back to I 8 pp. 46–7. This makes it obvious that we must delete εἰς τὰ ἑπόμενα (omitted by al-Ḥajjāj) at H12,12: it is senseless to talk about a motion 'towards the rear' with respect to a circle which is itself in motion. The motive for the interpolation was to gloss 'in the opposite direction'.

[50] Cf. III 1 p. 135 with n.14 for the lunar eclipses involved.

individual moon-sun distances. [In this case] we set one of the astrolabe rings to the apparent position of the moon (computed for the moment of observation), then adjusted the other astrolabe ring to align it with the star being sighted, so that both moon and star would be sighted simultaneously in the proper positions. Thus we obtained the position of every one of the bright stars from its distance from the moon.[51]

To [illustrate this procedure] by a single example. In the second year of
H14 Antoninus, on Pharmouthi [VIII] 9 in the Egyptian calendar [139 Feb. 23], when the sun was just about to set in Alexandria, and the last degree of Taurus was culminating, i.e. $5\frac{1}{2}$ equinoctial hours after noon on the ninth, we observed the apparent distance of the moon from the sun (which was sighted at about ♓ 3°) as $92\frac{1}{8}$°. Half an hour later, the sun now having set, and the [first] quarter of Gemini [i.e. ♊ 7;30°] culminating, the apparent moon was sighted in the same position [with respect to the astrolabe ring], and the star on the heart of Leo [α Leo, Regulus]] had an apparent distance from the moon, [as measured] by means of the other astrolabe [ring], of $57\frac{1}{5}$° towards the rear along the ecliptic.

Now at the first [observation] the true position of the sun was very nearly ♓ $3\frac{1}{20}$°. Hence the apparent position of the moon, since it was $92\frac{1}{8}$° towards the rear [of the sun], was approximately ♊ $5\frac{1}{6}$°, which is also the position it ought to occupy according to our hypotheses. Half an hour later the moon should have moved about $\frac{1}{4}$° towards the rear, and have a parallax in advance, relative to the first situation, of about $\frac{1}{12}$°. Therefore the apparent position of the moon half an
H15 hour later was ♊ $5\frac{1}{3}$°. Hence the star on the heart, since its apparent distance from the moon was $57\frac{1}{6}$° to the rear, had a position of ♌ $2\frac{1}{2}$°, and its distance from the summer solstice was $32\frac{1}{2}$°.[52]

But in the 50th year of the Third Kallippic Cycle [−128/7], as Hipparchus records from his own observations, [that star] had a distance to the rear of the summer solstice of $29\frac{5}{6}$°. Therefore the star on the heart of Leo has moved $2\frac{2}{3}$° towards the rear along the ecliptic in the 265 or so years from the observation of Hipparchus to the beginning [of the reign] of Antoninus [137/8], which was when we made the majority of our observations of the positions of the fixed stars. From this we find that 1° rearward motion takes place in approximately 100 years, as Hipparchus too seems to have suspected, according to the following quotation from his work 'On the length of the year': 'For if the solstices and equinoxes were moving, from that cause, not less than $\frac{1}{100}$th of a
H16 degree in advance [i.e. in the reverse order] of the signs, in the 300 years they should have moved not less than 3°'.[53]

In the same way we took sightings of Spica and the brightest among those stars near the ecliptic, from the moon, and then [having done that], were in a

[51] See V 1, with notes, for a detailed explanation of the use of the instrument. Ptolemy's procedure explains why the mean error in the longitudes of his star catalogue, about 1°, is the same as the mean error of his lunar and solar positions, derived from his faulty equinox (see III 1 p. 138 with n.21).

[52] This observation is discussed in some detail by Pedersen, 240–5, with a computation of the parallax. Unfortunately he has made errors, notably in the angle between ecliptic and hour-circle in the first observation (see Toomer [3] p. 143).

[53] The '300 years' is a reference to the interval between the solstice observation of Meton (−431, cf. III 1 p. 138) and Hipparchus' own time. This was obviously one of the comparisons which Hipparchus made.

better position to use those stars to take sightings of the rest. We [thus] find that their distances relative to each other are, again, very nearly the same as those observed by Hipparchus, but their individual distances from the solsticial or equinoctial points are in each case about $2\frac{2}{3}°$ farther to the rear than those derivable from what Hipparchus recorded.

3. {*That the rearward motion of the sphere of the fixed stars, too, takes place about the poles of the ecliptic*}[54]

From the above it has become clear to us that the sphere of the fixed stars, too, performs a rearward motion along the ecliptic, of approximately the amount indicated. Our next task is to determine the type of this motion, that is to say, whether it takes place about the poles of the equator or about the poles of the inclined circle of the ecliptic. Since great circles drawn through the poles of either one of the above [equator or ecliptic] cut off unequal arcs on the other, [the answer to] the above [question] would become apparent merely from the motion in longitude, were it not for the fact that the motion in longitude over the H17
time available [for comparison of observations] is so extremely small that the difference due to the above effect would be, as yet, imperceptible. The easiest way to detect [the answer] is through [comparison of] the positions [of the stars] in latitude[55] in ancient times and now. For it is obvious that whichever of the two circles, equator and ecliptic, it is from which they can be shown to maintain a constant distance in latitude, that is the circle about the poles of which the motion of their sphere will take place.

Now Hipparchus agrees with [the idea of] the motion taking place about the poles of the ecliptic. For in 'On the displacement of the solsticial and equinoctial points' he deduces from the observations of Timocharis and himself that Spica (again) has maintained the same distance in latitude, not with respect to the equator but with respect to the ecliptic, being 2° south of the ecliptic at both earlier and later periods. That is why in 'On the length of the year' he assumes only the motion which takes place about the poles of the ecliptic, although he is H18
still dubious, as he himself declares, both because the observations of the school of Timocharis are not trustworthy, having been made very crudely, and because the difference in time between [Timocharis and himself] is not sufficient to provide a secure result. We, however, find the [latitudinal distances with respect to the ecliptic] preserved over the much longer interval [down to our times], and that for practically all fixed stars. We can therefore with good reason consider the motion about the poles of the ecliptic as now more firmly established. For when we observe the latitudinal distance of any star with respect to the ecliptic, as measured along the great circle through the poles of the ecliptic, we find that it is practically the same as that computed from the

[54] See Pedersen 246–9.

[55] 'latitude' is ambiguous here and below. It means 'direction orthogonal to the circle in question', i.e. 'latitude' (in the modern sense) with respect to the ecliptic, and 'declination' with respect to the equator. Cf. Introduction p. 21 and p. 63 n.74.

records of Hipparchus,[56] or if there is a discrepancy, it is of very small size, such as can be accounted for by small observational errors. But when we consider the distances [of the stars] from the equator, as measured along great circles through the poles of the equator, we find [1] that those observed by us do not agree with those recorded in the same way by Hipparchus, and [2] that the latter do not agree with those recorded even earlier by Timocharis and his associates; rather, the constancy of their latitudes with respect to the ecliptic is confirmed even more by these very observations, since the distances from the
H19 equator of the stars located on the hemisphere from the winter solstice through the spring equinox to the summer solstice are found to be ever more northerly compared to those [of the same stars] in earlier periods, while for stars located on the opposite hemisphere they are ever more southerly. Furthermore the differences [between earlier and later observations] are greater for stars near the equinoctial points, and less for stars near the solstices, and these differences are just about the same as the amount by which that section of the ecliptic to the rear [of the earliest longitude of any particular star] defined by the corresponding motion in longitude [during the period in question] produces a displacement to the north or south of the equator.

In order to illustrate this point for a few easily recognisable stars we will set out, for each of the two hemispheres mentioned, their vertical distances from the equator, as measured along the great circle through the poles of the equator, as recorded by the school of Timocharis, as recorded by Hipparchus, and also as determined in the same fashion by ourselves.[57] [See p. 331.]

In the case of all the above stars, which are located (to speak of their longitudinal position) on that one of the above-defined hemispheres which contains the spring equinox, the vertical distances from the equator which are later in time are all more northerly than the earlier, and for those stars very near the solsticial points [the difference] is very small, while for those near the equinoxes[58] it is quite considerable: this accords with a rearward motion about the poles of the ecliptic, for if one takes successive sections of this semi-circle [of the ecliptic] going towards the rear, each is more northerly than the one in advance of it, and the difference [between successive equal sections] is again greater near the equinoxes and less near the solstices.

[See p. 332.]

[56] ταῖς κατὰ τὸν Ἵππαρχον ἀναγεγραμμέναις καὶ συναγομέναις, literally 'those recorded and computed according to Hipparchus'. I take this to mean that Hipparchus recorded certain stellar positions (mainly declinations), from which Ptolemy computed the latitudes. All the evidence (including this passage) is in favour of the hypothesis that Hipparchus did not record stellar positions in latitude and longitude (except for a few special cases like that of Spica mentioned above, for the specific purpose of determining the precession). Otherwise it is impossible to explain why Ptolemy went through the cumbersome process of comparing declinations (pp. 331–2), instead of simply comparing latitudes observed by Hipparchus and himself.

[57] These stars are listed in Ptolemy's catalogue as follows, 1, XVI 3;2, not listed, but cf. XXIII 30–2; 3, XXIII 14; 4,XII 3; 5,XXXV 3; 6,XXXV 2;7, XXXVIII 1; 8, XXIV 1; 9,XXIV 2. I have followed Manitius in arranging Ptolemy's continuous text in tabular form.

[58] *Sic* (plural, although only the spring equinox is involved). The inaccuracy is probably Ptolemy's, caused by his thinking of the general situation (differences large near either equinox, small near either solstice).

		North or south of equator	As recorded by [Aristyllos or] Timocharis	As recorded by Hipparchus	As found by us
	[1] The bright star in Aquila	north	$5\frac{4}{5}°$	$5\frac{4}{5}°$	$5\frac{5}{6}°$
	[2] The middle of the Pleiades	north	$14\frac{1}{2}°$	$15\frac{1}{6}°$	$16\frac{1}{4}°$
H20	[3] The bright star in the Hyades	north	$8\frac{3}{4}°$	$9\frac{3}{4}°$	$11°$
	[4] The brightest star in Auriga, called Capella	north	$40°$ (Aristyllos)	$40\frac{2}{3}°$	$41\frac{1}{6}°$
	[5] The star in the advance shoulder of Orion	north	$1\frac{1}{5}°$	$1\frac{4}{5}°$	$2\frac{1}{2}°$
	[6] The star in the rear shoulder of Orion	north	$3\frac{5}{6}°$	$4\frac{1}{3}°$	$5\frac{1}{4}°$
	[7] The bright star in the mouth of Canis Major	south	$16\frac{1}{3}°$	$16°$	$15\frac{3}{4}°$
	[8] The more advanced of the [two] bright stars in the heads of Gemini	north	$33°$ (Aristyllos)	$33\frac{1}{6}°$	$33\frac{2}{5}°$
H21	[9] The rearmost [of the bright stars in the heads of Gemini]	north	$30°$ (Aristyllos)	$30°$	$30\frac{1}{6}°$

In the opposite hemisphere:[59]

		North or south of equator	As recorded by [Aristyllos or] Timocharis	As recorded by Hipparchus	As found by us
	[1] The star on the heart of Leo	north	21⅓°	20⅔°	19⅚°
H22	[2] The star called Spica	north	1⅕°	only ⅗° north	½° south
	Of the 3 stars in the tail of Ursa Major:				
	[3] the one at the tip	north	61½° (Aristyllos)	60¾°	59⅔°
	[4] the second from the end, in the middle of the tail	north	67¼° (Aristyllos)	66½°	65°
	[5] the third from the end, about where the tail joins [the body]	north	68½° (Aristyllos)	67⅕°[60]	66¼°
	[6] Arcturus	north	31½°	31°	29⅚°
	Of the bright stars in the claws of Scorpius [i.e. in Libra]:				
	[7] the one in the tip of the southern claw	south	5°	5⅗°	7⅙°
H23	[8] the one in the tip of the northern claw	north	1⅕°	only ⅖° north	1° south
	[9] The bright star in the chest of Scorpius, called Antares	south	18⅓°	19°	20¼°

[59] These stars are listed in Ptolemy's catalogue as follows: 1, XXVI 8; 2,XXVII 14; 3,II 27; 4, II 26; 5, II 25; 6,V 23; 7,XXVIII 1; 8,XXVIII 3; 9,XXIX 8.
[60] D² and Ar have 67⅗, which may be correct.

Thus in the case of all these stars, the reverse [of the above] is true, as one would logically expect: the later vertical distances from the equator are more southerly than the earlier, in proportion [to the time intervals and locations].

Furthermore one can conclude from these data that the rearward motion in longitude of the sphere of the fixed stars is, as we said previously [p. 328], 1° in about 100 years, or $2\frac{2}{3}^\circ$ in the 265 years between Hipparchus' and our observations. It is particularly [easy to do this] from the differences in declination found for those stars near the equinoctial points.

For the middle of the Pleiades, which was found to be $15\frac{1}{6}^\circ$ north of the equator in Hipparchus' time, and $16\frac{1}{4}^\circ$ in our time, has [thus] moved $1\frac{1}{12}^\circ$ northward in the interval between us: this is nearly the same as the difference in declination from the equator between [both ends of] the $2\frac{2}{3}^\circ$ of the ecliptic near H24
the end of Aries which represents the rearward motion in longitude over that interval.[61] And the star called Capella, which was found to be $40\frac{2}{5}^\circ$ north of the equator in Hipparchus' time, and $41\frac{1}{6}^\circ$ in our time, has [thus] moved northward $\frac{4}{5}^\circ$: this is, again, the same as the difference in declination from the equator of [the ends of] the [intervening] $2\frac{2}{3}^\circ$ of the ecliptic near the middle of Taurus. Also, the star on the advance shoulder of Orion, which was found to be $1\frac{4}{5}^\circ$ north of the equator in Hipparchus' time, and $2\frac{1}{2}^\circ$ in our time, has [thus] moved northward about $\frac{2}{3}^\circ$, which is nearly the same as the difference in declination from the equator of [the ends of] the [intervening] $2\frac{2}{3}^\circ$ of the ecliptic two-thirds through Taurus.[62]

The situation is similar on the opposite hemisphere. Spica, which was found to be $\frac{3}{5}^\circ$ north of the equator in Hipparchus' time, but $\frac{1}{2}^\circ$ south in our time, has [thus] moved southwards $1\frac{1}{10}^\circ$, which is, again, the amount of the difference in declination from the equator of the [ends of the] $2\frac{2}{3}^\circ$ of the ecliptic near the end of Virgo. And the star in the tip of the tail of Ursa Major, which was found to be H25
$60\frac{3}{4}^\circ$ north of the equator in Hipparchus' time, but $59\frac{2}{3}^\circ$ in our time, has [thus] moved southwards $1\frac{1}{12}^\circ$, which is the amount of the difference in declination from the equator of the $2\frac{2}{3}^\circ$ of the ecliptic near the beginning of the sign of Libra.[63] Also, Arcturus, which was found to be 31° north of the equator in Hipparchus' time, but $29\frac{5}{6}^\circ$ in our time, has [thus] moved southward $1\frac{1}{6}^\circ$, which is, likewise, approximately the amount of the difference in declination from the equator of the $2\frac{2}{3}^\circ$ of the ecliptic near the beginning of Libra.

[61] From Table I 15.

$$\left.\begin{array}{l}\delta\,(30^\circ) = 11;39.59\\ \delta\,(32\frac{2}{3}^\circ) = 12;36,29\end{array}\right\}\Delta = 0:57.30^\circ.$$

which is considerably less than Ptolemy's $1\frac{1}{12}^\circ$. Perhaps he has carelessly computed $\delta\,(30^\circ) \approx 11;40^\circ$, $2\frac{2}{3}/30 \times 11;40^\circ \approx 1;2^\circ$.

[62] In the catalogue these two stars have very nearly the same longitude. Capella being placed in ♉ 25 (XII 3) and the star in Orion in ♉ 24 (XXXV 3). Yet here they are placed 'in the middle of Taurus' and 'two-thirds through Taurus' respectively, and this is the basis of Ptolemy's calculations. For, from table I 15, the difference in declination of $2\frac{2}{3}^\circ$ near 45° is about $49'$, and near 55° is about $41'$. Thus the statement regarding Capella seems to rest on an error.

[63] *Sic!* The longitude of the star in question is ♌ $29\frac{5}{6}$ in the catalogue (II 27), so one would expect 'the beginning of Virgo' here. But the mss. are unanimous, and I hesitate to emend, both because of the other gross inaccuracies in this passage, and because a difference in declination of $1\frac{1}{12}^\circ$ is too great for the beginning of Virgo (from Table I 15 one finds about $57'$ for an argument of 30°). However, Ptolemy gives the same amount, $1\frac{1}{12}^\circ$, for the 'end of Aries' (above, with n.61).

The point in question will become even clearer to us from the following observations.[64]

[Firstly] Timocharis, who observed at Alexandria, records the following. In the 47th year of the First Kallippic 76-year period, on the eighth of Anthesterion,[65] which is Athyr 29 in the Egyptian calendar, towards the end of the third hour [of night], the southern half of the moon was seen to cover exactly
H26 either the rearmost third or [the rearmost] half[66] of the Pleiades. That moment is in the 465th year from Nabonassar, Athyr [III] 29/30 in the Egyptian calendar [-282 Jan. 29/30], 3 seasonal hours before midnight, or $3\frac{1}{3}$ equinoctial hours (since the sun was in about ♒ 7°). The interval reckoned in mean solar days comes to about the same number of equinoctial hours [$3\frac{1}{3}$] before midnight. At that moment, according to the hypotheses we demonstrated previously, the position of the moon was as follows:

true longitude: ♉ 0;20°
(i.e. distance from the spring equinox: 30;20°)
[latitude]: 3;45° north of the ecliptic

$$\left.\begin{array}{l}\text{apparent longitude} \\ \\ \text{apparent [latitude]}\end{array}\right\} \begin{array}{l}29;20^\circ \\ \text{in Alexandria} \\ 3;35^\circ \text{ north of the ecliptic}\end{array}$$[67]

(for the culminating point was $\frac{2}{3}$rds through Gemini).

Therefore at that time the rearmost end of the Pleiades was about $29\frac{1}{2}$° towards the rear from the spring equinox (for the moon's centre was still in advance of it), and was about $3\frac{2}{3}$° north of the ecliptic (for, again, it was a little north of the moon's centre).

H27 [Secondly] Agrippa, who observed in Bithynia, records that in the twelfth year of Domitian, on the seventh of Metroos according to the calendar of that region,[68] at the beginning of the third hour of night, the moon occulted the rearmost, southern part of the Pleiades with its southern horn. That moment is in the 840th year from Nabonassar, Tybi [V] 2/3 in the Egyptian calendar [92, Nov. 29/30], 4 seasonal hours before midnight, or 5 equinoctial hours (since the sun was in about ♐ 6°).[69] Therefore, reduced to the meridian of

[64] There are numerous difficulties connected with the following observations of occultations, Ptolemy's interpretations of them, and his calculations. To deal with them here would require too lengthy a discussion. Although they have been much discussed (e.g. by Schjellerup, 'Recherches' III, Fotheringham [1] and Fotheringham [2]), the only satisfactory treatment is in Britton [1], 107–28, to which the reader interested in Ptolemy's (often strange) interpretation of the data is referred. However, Britton does not consider the aspect of the errors resulting from Ptolemy's miscomputations on the basis of his own theory. The more gross of these are noted below. These only reinforce Britton's conclusion that the observations could not have been selected at random.

[65] These and similar dates (pp. 335, 336 and 337) attributed to Timocharis must be dates in the artificial Metonic/Kallippic calendar. See Introduction p. 12.

[66] It is most unclear what is meant here. Were there discrepancies in Timocharis' report (or in the mss. of it available to Ptolemy)? Or does this represent variations in the *Almagest* ms. tradition? The translation of al-Ḥajjāj has 'a half' only.

[67] Computed from Ptolemy's tables: λ ☉ ♒ 7;8°, λ ☾ 30;11°, β ☾ +3;45°. Apparent longitude and latitude at Alexandria 29;0° and +3;38°.

[68] Metroos is the month of the Bithynian calendar. See Introduction p. 14. Agrippa is unknown apart from this passage.

[69] This implies that the longest day was about that of Clima V (Hellespont), which is approximately correct for Bithynia. But Ptolemy's correction of −20 mins. for reduction to the

Alexandria, the observation occurred $5\frac{1}{3}$ equinoctial hours before midnight, or $5\frac{3}{4}$ hours with respect to mean solar days. At this moment the positions of the centre of the moon were as follows:

true longitude: ♉ 3;7°

[latitude]: $4\frac{5}{6}$° north of the ecliptic

apparent longitude ♉ 3;15° } in Bithynia

apparent [latitude] 4° north of the ecliptic

(for the culminating point was two-thirds through Pisces).[70]

Therefore at that time the rearmost section of the Pleiades was, in longitude, $33\frac{1}{4}$° towards the rear from the spring equinox, and, [in latitude], $3\frac{2}{3}$° north of the ecliptic.[71]

Hence it is clear that the rearmost part of the Pleiades was, both then and H28
now, the same distance in latitude, $3\frac{2}{3}$°, north of the ecliptic, as measured along the great circle through the poles of the ecliptic, while in longitude it has moved 3;45° towards the rear from the spring equinox (since it was $29\frac{1}{2}$° from the equinox at the first observation and $33\frac{1}{4}$° at the second) in the interval of 375 years comprised between the two observations.[72] Therefore in 100 years the rearmost part of the Pleiades has moved 1° towards the rear.

Again, [firstly] Timocharis, who observed at Alexandria, records that in the 36th year of the First Kallippic Cycle, on Elaphebolion 15, which is Tybi 5, at the beginning of the third hour, the moon covered Spica with the middle of that edge of its disk which is towards the equinoctial rising-point [i.e. the east], and that Spica, in passing through, cut off exactly the northern third of [the moon's] diameter.

This moment is in the 454th year from Nabonassar, Tybi [V] 5/6 in the Egyptian calendar [-293 Mar. 9/10], 4 seasonal hours before midnight, which is also 4 equinoctial hours approximately, since the sun was in about ♓ 15°; and reckoning with respect to mean solar days leads to about the same number of hours before [midnight]. At that moment the positions of the moon's centre H29
were as follows:

true longitude: ♍ 21;21°

(i.e. distance from the summer solstice was 81;21° towards the rear)

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meridian of Alexandria implies that Agrippa was observing at a place 5° to the east: in fact no place in Bithynia was more than 3° to the east of Alexandria; moreover, in the *Geography* (8.17.3-7) Ptolemy puts all the cities in Bithynia *west* of Alexandria.

[70] There are some gross errors here. Computed (for 6;15p.m. Alexandria): λ☾ = 32;13° (0;54° less than the text!), β☾ = +4;53°. One might think that Ptolemy computed for 8 p.m., i.e. took 'at the beginning of the third hour' as if it were *equinoctial* hours at *Alexandria*, were it not that the culminating point he gives is approximately correct (for 7 p.m. local time Bithynia I find ♓ 18;5°). His parallax corrections are also inaccurate (I find pλ = +0;19°, pβ = −0;38°, and hence, for the apparent position of the moon, λ = 32;32°, β = +4;15°. One need hardly say that this error is disastrous for the 'verification' of Ptolemy's precession constant.

[71] As Manitius points out (p. 402), in his catalogue (XXIII 32) Ptolemy assigns a latitude of $+3\frac{1}{3}$ to the rearmost end of the Pleiades. But the discrepancy can easily be explained by the fact that he is referring, not to a specific star, but to part of the general mass.

[72] From Nabonassar 465 to Nabonassar 840.

true [latitude]: $1\frac{5}{6}°$ south of ecliptic
apparent longitude: $82\frac{1}{12}°$ from the summer solstice[73]
apparent [latitude]: about 2° south of the ecliptic
(for the middle of Cancer was culminating).

Therefore, from the above, [we conclude that] Spica was at that moment $82\frac{1}{3}°$ in longitude from the summer solstice, and just about 2° south of the ecliptic.

Likewise, [secondly] in the 48th year of the same [First Kallippic] Cycle, he says that on the sixth day from the end of the last third of Pyanepsion,[74] which is Thoth 7, when as much as half an hour of the tenth hour had gone by, and the moon had risen above the horizon, Spica appeared exactly touching the northern point on [the moon].

This moment is in the 466th year from Nabonassar, Thoth [I] 7/8 in the Egyptian calendar [-282 Nov. 8/9]; [the hour is], according to Timocharis himself, $3\frac{1}{2}$ seasonal hours after midnight, or approximately $3\frac{1}{8}$ equinoctial
H30 hours,[75] since the sun was near the middle of Scorpius; but, according to logical reasoning, [it must have been] $2\frac{1}{2}$ hours after midnight. For that is the time when ♊ $22\frac{1}{2}°$ is culminating, and ♍ $22\frac{1}{2}°$ (approximately) is rising:[76] and that [♍ $22\frac{1}{2}°$] was the longitude of the moon at that moment when, as he says, it was rising. Reckoning with respect to mean solar days, we find that only 2 equinoctial hours had passed since midnight. At this time the positions of the centre of the moon were as follows:

true [longitude]: distance from the summer solstice: 81;30°
true [latitude]: $2\frac{1}{6}°$ south of the ecliptic
apparent longitude: $82\frac{1}{2}°$ [from the summer solstice]
apparent [latitude]: $2\frac{1}{4}°$ south [of the ecliptic].[77]

Therefore, according to this observation too, Spica was the same distance of about 2° south of the ecliptic, and was $82\frac{1}{2}°$ from the summer solstice. So in the 12 years between the two observations it moved about $\frac{1}{6}°$ towards the rear from the summer solstice.

[Thirdly] the geometer Menelaus says that the following observation was made [by him] in Rome. In the first year of Trajan, Mechir 15/16, when the tenth hour [of night] was completed, Spica had been occulted by the moon (for it could not be seen), but towards the end of the eleventh hour it was seen in

[73] Reading $\overline{\pi\beta}$ ιβ΄ (with A¹BCD) for $\overline{\pi\beta}$ $\overline{\iota\beta}$ (82;12°, the reading of Ar) at H29,7. In the circumstances of the observation this seems more likely to lead to the position of $82\frac{1}{3}°$ which Ptolemy deduces for Spica (below). It is also closer to my computation (λ☾, apparent, 172;7°), though this is no argument. Corrected by Manitius.

[74] τῇ ϛ′ φθίνοντος, i.e. the 25th of the month. For this way of counting days see Introduction p. 13. The true Attic form of the month name is Πυανοψιών, but the spelling with epsilon is found outside Attica (see LSJ s.v.), and is probably that used by Timocharis himself.

[75] Since the length of 1 seasonal night-hour was 16;38°, the length of $3\frac{1}{2}$ hours was 58;13°, or about $3\frac{7}{8}$ equinoctial hours. Hence I considered emending the text at H29,21 to $\overline{\delta}$ λειπούσας η′ $(4 - \frac{1}{8})$. However, it seems more probable that Ptolemy simply made the error of computing day-hours instead of night-hours, which does indeed lead to $3\frac{1}{8}$ equinoctial hours. The error has no consequences, since Ptolemy takes a quite different time.

[76] For calculations of these see Appendix A Examples 4 and 5.

[77] Calculated (cf. Appendix A Examples 9 and 10): λ☾ = 171;39°, β☾ = −2;7°. Apparent positions: λ = 173;1°, β = −2;20°.

advance of the moon's centre, equidistant from the [two] horns by an amount less than the moon's diameter. H31

This moment is in the 845th year from Nabonassar, Mechir [VI] 15/16 in the Egyptian calendar [98 Jan. 10/11], 4 seasonal hours after midnight when the moon's centre was approximately covering Spica, which corresponds to 5 equinoctial hours, since the sun was in about ♑ 20°; when reduced to the meridian through Alexandria this is $6\frac{1}{3}$ equinoctial hours,[78] and [this], with respect to mean solar days, is $6\frac{1}{4}$ hours (or a little more). At this moment the positions of the centre of the moon were as follows:

true [longitude]:	$85\frac{3}{4}$° from the summer solstice
true [latitude]:	about $1\frac{1}{3}$° south of the ecliptic
apparent longitude:	$86\frac{1}{4}$° from [the summer solstice]
apparent [latitude]:	2° south [of the ecliptic]

(for the culminating point was about a quarter of the way through Libra).[79] Therefore that was the position of Spica too at that moment.[80]

It is clear that Spica was, again, the same amount south of the ecliptic, namely 2°, both in Timocharis' time and in our time, and that its movement towards the rear in longitude is

3;55° in the 391 years from the observation in the 36th year [of the First Kallippic Cycle to the observation of Menelaus], and

3;45° in the 379 years[81] from the observation in the 48th year. H32

Hence from these data too we conclude that the motion of Spica towards the rear in 100 years is about 1°.

Again, Timocharis, who observed in Alexandria, says that in the 36th year of the First Kallippic Cycle, on Poseideon 25, which is Phaophi 16, at the beginning of the tenth hour, the moon appeared to occult the northernmost of the stars in the forehead of Scorpius very precisely with its northern rim.

This moment is in the 454th year from Nabonassar, Phaophi [II] 16/17 in the Egyptian calendar [-294 Dec. 20/21], 3 seasonal hours after midnight, or $3\frac{2}{5}$ equinoctial hours, since the sun was in about ♐ 26°. Reduced to mean solar days this is $3\frac{1}{6}$ hours. At this moment the position of the centre of the moon was as follows:

in true [longitude]:	$31\frac{1}{4}$° from the autumnal equinox [towards the rear]
[in true latitude]:	$1\frac{1}{5}$° north of the ecliptic[82]

[78] I.e. the longitudinal difference between Rome and Alexandria is taken as about 20°. In fact it is about $17\frac{1}{2}$°. In the *Geography* the error is even more exaggerated. There (8.5.3 Nobbe) Ptolemy states that Rome is $1\frac{5}{8}^h$ to the west of Alexandria, in accordance with the assigned longitudes of $36\frac{2}{3}$° and $60\frac{1}{2}$° (ibid. 3.1.61 and 4.5.9). Heron, *Dioptra*, took the difference as 2 hours (Neugebauer [3], 22).

[79] Here too my computations show significant discrepancies: λ ☾ 175;27°, β ☾ −1;19,30°. Apparent positions at Rome, λ 175;39°, β −2;10°. Ptolemy's parallaxes, +30′ in longitude and −40′ in latitude, imply a total parallax of 50′, which is approximately correct, and an angle between altitude circle and ecliptic of *c.* 140°, which is impossible at the situation in question (moon roughly $\frac{1}{2}^h$ west of meridian, as his culminating point shows). Could he have taken the *eastern* angle in error?

[80] In the catalogue (XXVII 14) Spica has coordinates of ♍ $26\frac{2}{3}$° and −2°, in agreement with the data here (allowing for a movement of 25′ in longitude in about forty years).

[81] Reading $\overline{\text{τοθ}}$ (with D,Ar) for $\overline{\text{τοε}}$ ('375') at H32,1. Corrected by Manitius, and by Heiberg, *Op. Min.* p. XIV.

[82] Computed: λ ☾ 211;23°, β ☾ +1;17°.

in apparent longitude:[83] 32° [from the autumnal equinox]
in apparent [latitude]: $1\frac{1}{12}$° north of the ecliptic[84]
(for the culminating point was the middle of Leo).

Therefore at that moment the northernmost of the stars in the forehead of
H33 Scorpius was the same amount, 32°, from the autumnal equinox in longitude, and about $1\frac{1}{3}$° north of the ecliptic [in latitude].

Similarly, Menelaus, who observed in Rome, says that in the first year of Trajan, Mechir 18/19, towards the end of the eleventh hour, the southern horn of the moon appeared on a straight line with the middle and the southernmost of the stars in the forehead of Scorpius, and its centre was to the rear of that straight line, and was the same distance from the middle star as the middle star was from the southernmost; it appeared to have occulted the northernmost of the stars in the forehead, since [this star] was nowhere to be seen.

This moment is, again, in the 845th year from Nabonassar, Mechir [VI] 18/19 in the Egyptian calendar [98 Jan. 13/14], 5 seasonal hours after midnight, or $6\frac{1}{6}$ equinoctial hours, since the sun was in about ♑ 23°. Reduced to the meridian of Alexandria this is $7\frac{1}{2}$ equinoctial hours, and it is about the same with respect to mean solar days. At this moment the position of the centre of the moon was as follows:

true [longitude]: $35\frac{1}{3}$° from the autumnal equinox [towards the rear]
true [latitude]: $2\frac{1}{6}$° north of the ecliptic[85]
apparent longitude: 35;55° [from the autumnal equinox]
apparent [latitude]: $1\frac{1}{3}$° north [of the ecliptic]
(for the culminating point was the end of Libra).[86]

Therefore the northernmost of the stars in the forehead of Scorpius had approximately the same position at the moment.

H34 Hence it is clear that for this star too its distance in latitude from the ecliptic has been observed to be the same in former times and in our times, while its position in longitude has moved away from the autumnal equinox towards the rear by an amount of 3;55° in the time between the observations, which comprise 391 years, from which it follows that in 100 years the motion of the star towards the rear amounts to 1°.

[83] Reading ἀπέχον (with D,Ar) for ἐπέχον here (H32,18) and at the similar place H33,20. Corrected by Manitius.

[84] Reading $\bar{\alpha}$ ιβ´ (with Ar) for $\bar{\alpha}$ $\overline{\iota\beta}$ (1;12°) at H32,19. This gives better agreement with the observational data if a latitude of $1\frac{1}{3}$° is to be deduced (below). Corrected by Manitius. Computed apparent position: λ☾ 212;30°, β☾ +1;1°.

[85] Computed: λ☾ 215;21°, β☾ +2;5°.

[86] Neugebauer has displayed all the computations leading up to this in various places in *HAMA* I, culminating in his remarks on pp. 117–18 about the impossibility of assigning a specific cause to the error in the final result. He also suggests (117 n.7) that one should read 2;6° and 1;3° for the true and apparent latitude. Although these numbers agree better with the calculation, $1\frac{1}{3}$° is certainly the correct reading, for it agrees with the latitude found from Timocharis' observation, and also with that assigned to this star in the catalogue (XXIX 1).

4. {*On the method used to record* [*the positions of*] *the fixed stars*}

Thus, from our observations and comparisons of the above stars, from similar observations and comparisons of the other bright stars, and from the fact that we found the distances of the other stars with respect to the [bright stars] which we had established to be in agreement [with the results of our predecessors], we have confirmed that the sphere of the fixed stars, too, has a movement towards the rear with respect to the solsticial and equinoctial points of the amount determined (in so far as the time [for which observations are] available allows); furthermore, [we have confirmed] that this motion of theirs takes place about the poles of the ecliptic, and not those of the equator (i.e. the poles of the first motion). So we thought it appropriate, in making our observations and records of each of the above fixed stars, and of the others too, to give their positions, as H35 observed in our time, in terms of longitude and latitude, not with respect to the equator, but with respect to the ecliptic, [i.e.] as determined by the great circle drawn through the poles of the ecliptic and each individual star. In this way, in accordance with the hypothesis of their motion established above, their positions in latitude with respect to the ecliptic must necessarily remain the same, while their positions in longitude must always traverse equal arcs towards the rear in equal times.

Hence, again using the same instrument [as we did for the moon, V 1], (because the astrolabe rings in it are constructed to rotate about the poles of the ecliptic), we observed as many stars as we could sight down to the sixth magnitude. [We proceeded as follows.] We always arranged the first of the above-mentioned astrolabe rings [Fig. F,5] [to sight] one of the bright stars whose position we had previously determined by means of the moon, setting the ring to the proper graduation on the ecliptic [ring (Fig. F,3) for that star], then set the other ring [Fig. F,2], which was graduated along its entire length and could also be rotated in latitude towards the poles of the ecliptic,[87] to the required star, so that at the same time as the control star was sighted [in its proper position], this star too was sighted through the hole on its own ring. For H36 when these conditions were met, we could readily obtain both coordinates of the required star at the same time by means of its astrolabe ring [Fig. F,2]: the position in longitude was defined by the intersection of that ring and the ecliptic [ring], and the position in latitude by the arc of the astrolabe ring cut off between the same intersection and the upper[88] sighting-hole.

In order to display the arrangement of stars on the solid globe[89] according to the above method, we have set it out below in the form of a table in four sections. For each star (taken by constellation), we give, in the first section, its description as a part of the constellation;[90] in the second section, its position in longitude, as

[87] If the text is sound, Ptolemy is speaking carelessly here. As is clear from the description at V 1, ring no. 2 is indeed graduated, but cannot perform a latitudinal movement; that is done by ring no. 1, which fits inside no. 2 and has the sighting-holes attached to it.

[88] Literally 'above the earth'. Cf. p. 219 n.6.

[89] For a description of this instrument see VIII 3.

[90] Literally 'the shapes' (τὰς μορφώσεις), i.e. its position as a part of the mythological figure (animal, anthropomorphic or inanimate) which was delineated on the globe and (notionally) in the heavens.

derived from observation, for the beginning of the reign of Antoninus[91] ([the position is given] within a sign of the zodiac, the beginning of each quadrant of the zodiac being, as before, established at [one of] the solsticial or equinoctial points); in the third section we give its distance from the ecliptic in latitude, to the north or south as the case may be for the particular star; and in the fourth, the class to which it belongs in magnitude. The latitudinal distances will remain always unchanged, and the positions in longitude can provide a ready means of determining the [corresponding] longitude at other points in time, if we [calculate] the distance in degrees between the epoch and the time in question on the basis of a motion of 1° in 100 years, [and] subtract it from the epoch
H37 position for earlier times, but add it to the epoch position[92] for later times.

For the same reasons, our indications [of relative positions] in the descriptions must also be understood to accord with the above kind of hypothesis about the arrangement of the stars, and with the definition [of position] by [circles drawn] through the poles of the ecliptic. Thus, when we speak of a star as 'in advance of' or 'to the rear of' another, we mean that it occupies the relative position in question as defined by the ecliptic position [of the two stars, 'in advance of'] referring to the section of the ecliptic which is in advance, and ['to the rear'] referring to the section of the ecliptic which is towards the rear;[93] and by 'more to the south' or 'more to the north', we mean nearer to the pole of the ecliptic (southern or northern as the case may be). Furthermore, the descriptions which we have applied to the individual stars as parts of the constellation are not in every case the same as those of our predecessors (just as their descriptions differ from their predecessors'): in many cases our descriptions are different because they seemed to be more natural and to give a better proportioned outline to the figures described. Thus, for instance, those stars which Hipparchus places 'on the shoulders of Virgo' we describe as 'on her sides',[94] since their distance from the stars in her head appears greater than their distance from the stars in her hands, and that situation fits [a location] 'on her side', but is totally inappropriate to [a location] 'on her shoulders'. However, one has a ready means of identifying those stars which are described differently [by others]; this can be done immediately simply by comparing the recorded positions.

The layout of the catalogue is as follows.

[91] I.e. according to the Canon Basileon (see Introduction p.11), Thoth 1 of Nabonassar 885 (= 137 July 20).

[92] Reading ταῖς τῆς ἐποχῆς ἐπὶ τοῦ μεταγενεστέρου (with D,Ar) at H37,2 for ταῖς τοῦ μεταγενεστέρου. Corrected by Manitius.

[93] Although this is in general true, there appear to be exceptions. See Introduction p. 20, p.344 n.110 (on catalogue III 15–18) and p. 377 n.35 (on catalogue XXXII 23–4).

[94] Thus δ Vir is described by Hipparchus (*Comm. in Arat.* 2.5.5., ed. Manitius p.190,10) as 'the northern shoulder of Virgo', and by Ptolemy (catalogue XXVII 10) as 'the star in the right side under the girdle'.

H38

5 {*Tabular layout of the constellations in the northern hemisphere*}[95]

[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
	[I] Constellation of Ursa Minor				
1	The star on the end of the tail	♊ $0\frac{1}{6}$	+66	3	α UMi
2	The one next to it on the tail	♊ $2\frac{1}{2}$	+70	4	δ UMi
3	The one next to that, before the place where the tail joins [the body]	*♊ $10\frac{1}{6}$[96]	$+74\frac{1}{2}$	4	ε UMi
4	The southernmost of the stars in the advance side of the rectangle	♊ $29\frac{2}{3}$	$+75\frac{2}{3}$	4	ζ UMi
5	The northernmost of [those in] the same side	♋ $3\frac{2}{3}$	$+77\frac{2}{3}$	4	η UMi
6	The southern star in the rear side	*♋ $17\frac{1}{2}$[97]	$+72\frac{5}{6}$	2	β UMi
7	The northern one in the same side	♋ $26\frac{1}{6}$	$+74\frac{5}{6}$	2	γ UMi
	{7 stars, 2 of the second magnitude, 1 of the third, 4 of the fourth}				
	Nearby star outside the constellation:				
8	The star lying on a straight line with the stars in the rear side [of the rectangle] and south of them	♋ 13	$+71\frac{1}{6}$	4	5 UMi
	{1 star of the fourth magnitude}				

[95] On the principles on which my translation of the star catalogue is arranged see Introduction pp. 14–17. Here I note only that I have followed Manitius in adding, as the first and sixth columns, running numbers within each constellation, and the identification of the star; and that an asterisk next to any element indicates that there is some uncertainty about its correctness. For an idea of the arrangement in the Greek mss. see Peters and Knobel pls. II–IV.

Abbreviations:

S (plus number)	the list of variants in the various Arabic versions according to ibn aṣ-Ṣalāḥ, 83–96 of Kunitzsch's edn.
P–K	Peters and Knobel, *Ptolemy's Catalogue of Stars*
BSC	Yale Catalogue of Bright Stars
CGal	Cumulus Galacticus (Galactic cluster)
CGlo	Cumulus Globaris (Globular cluster).

[96] Reading ι ς′, with B. According to S 1, $10\frac{1}{6}$ was in the Syriac and al-Ḥasan versions. Heiberg (H39,6) prints ις (16), which is also the reading of the rest of the Arabic tradition.

[97] The reading $17\frac{1}{2}$ is found in some Greek mss. (not recorded by Heiberg) and the Arabic tradition (L,T,E,F1), and is adopted by P–K.

H40

[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
	[II] Constellation of Ursa Major				
1	The star on the end of the snout	♊ $25\frac{1}{3}$	$+39\frac{5}{6}$	4	ο UMa
2	The more advanced of the stars in the two eyes	♊ $25\frac{5}{6}$	+43	5	2(A) UMa
3	The one to the rear	♊ $26\frac{1}{3}$	+43	5	π^2 UMa
4	The more advanced of the two stars in the forehead	♊ $26\frac{1}{6}$	$+47\frac{1}{6}$	5	ρ UMa
5	The one to the rear	*♊ $26\frac{2}{3}$[98]	+47	5	σ^2 UMa
6	The star on the tip of the advance ear	♊ $28\frac{1}{6}$	$+50\frac{1}{2}$	5	24(d) UMa
7	The more advanced of the two stars in the neck	♋ $0\frac{1}{2}$	$+43\frac{5}{6}$	4	τ UMa
8	The one to the rear	♋ $2\frac{1}{2}$	$+44\frac{1}{3}$	4	23(h) UMa
9	The northernmost of the two stars in the chest	♋ 9	+42	4	υ UMa
10	The southernmost of them	♋ 11	*+44[99]	<4	φ UMa
11	The star on the left knee	♋ $10\frac{2}{3}$	+35	3	θ UMa
12	The northernmost of the [two] in the front left paw[100]	♋ $5\frac{1}{2}$	$+29\frac{1}{3}$	3	ι UMa
13	The southernmost of them	♋ $6\frac{1}{3}$	$+28\frac{1}{3}$	3	κ UMa
14	The star above the right knee	♋ $5\frac{2}{3}$	+36	4	18(e) UMa
15	The star below the right knee	♋ $5\frac{5}{6}$	+33[101]	4	15(f) UMa
16–19	The stars in the quadrilateral:				
16	*the one on the back*	♋ $17\frac{2}{3}$	+49	2	α UMa
17	the one on the flank	♋ $22\frac{1}{6}$	$+44\frac{1}{2}$	2	β UMa
18	the one on the place where the tail joins [the body]	♌ $3\frac{1}{6}$	+51	3	δ UMa
19	the remaining one, on the left hind thigh	♌ 3	$+46\frac{1}{2}$	2	γ UMa
20	The more advanced of the [two stars] in the left hind paw	♋ $22\frac{2}{3}$	$+29\frac{1}{3}$	3	λ UMa
21	The one to the rear of it	♋ $24\frac{1}{6}$	$+28\frac{1}{4}$	3	μ UMa

[98] P–K adopt $27\frac{2}{3}$ on very poor authority.

[99] The Greek and Arabic ms. tradition is solid for 44, which is much too great. According to S 2 both the Isḥāq and Thābit versions had 41 (the latter is not borne out by extant mss.) Independently of each other ibn aṣ-Ṣalāḥ (p. 48) and Peters (p. 96 no. 18) correct to 37 and $37\frac{1}{4}$, but the corrections are palaeographically improbable.

[100] ἀκρόπους. Following Hellenistic practice, Ptolemy normally uses πούς and χείρ to denote 'leg' and 'arm'. (But not always, cf. e.g. XIII 14 and 24, or XIV 11 and 12). Hence for 'foot' and 'hand' he has to use terms like ἀκρόπους (ἀκροπόδιον) and ἀκροχείριον. Translators have often misrepresented the latter by expressions such as 'tip of the foot' and 'end of the hand'.

[101] The variant $30\frac{1}{3}$ is found in part of both Greek (A¹BD) and Arabic traditions (S 3).

[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
22	The star on the left knee-bend	♌ $1\frac{2}{3}$	$+35\frac{1}{4}$	>4	ψ UMa
23	The northernmost of the [two stars] in the right hind paw	♌ $9\frac{5}{6}$	$+25\frac{5}{6}$	3	ν UMa
24	The southernmost of them	♌ $10\frac{1}{3}$[102]	+25	3	ξ UMa
25	The first of the three stars on the tail next to the place where it joins [the body]	♌ $12\frac{1}{6}$	$+53\frac{1}{2}$	2	ε UMa
26	The middle one	♌ 18	$+55\frac{2}{3}$	2	ζ UMa
27	The third, on the end of the tail	♌ $29\frac{5}{6}$	+54	2	η UMa
	[27 stars, 6 of the second magnitude, 8 of the third, 8 of the fourth, 5 of the fifth]				
	Stars under [Ursa Major] outside the constellation:				
28	The star under the tail, at some distance towards the south	♌ $27\frac{5}{6}$	$+39\frac{3}{4}$	3	α CVn
29	The rather faint star in advance of it	♌ $20\frac{1}{6}$[103]	$+41\frac{1}{3}$	5	β CVn
30	The southernmost of the [two] stars between the front legs of Ursa [Major] and the head of Leo	♋ 15	$+17\frac{1}{4}$	4	40 Lyn
31	The one north of it	♋ $13\frac{1}{3}$	$+19\frac{1}{6}$	4	38 Lyn
32	The rearmost of the remaining three faint stars	♋ $16\frac{1}{6}$	+20	f.	*10 LMi
33	The one in advance of this	*♋ $12\frac{1}{6}$	*$+22\frac{1}{2}$[104]	f.	*BSC 3809[105]
34	The one in advance again of the latter	♋ $11\frac{1}{6}$	*$+20\frac{1}{3}$[106]	f.	*BSC 3612[107]
35	The star between the front legs [of Ursa Major] and Gemini	♋ 0	$+22\frac{1}{4}$	f.	31 Lyn
	[8 stars outside the constellation, 1 of the third magnitude, 2 of the fourth, 1 of the fifth, 4 faint]				

[102] The variant 13 occurs in part of the Arabic tradition according to S 4.

[103] The variant 26 occurs in part of the Greek tradition (C) and, according to S 6, part of the Arabic.

[104] Reading κβ ∠′ (with BC) at H43,14 for κβ ∠′ $\overline{\Gamma o}$ ($22+\frac{1}{2}+\frac{1}{3}$), which is impossible. Corrected by Manitius. Part of the Arabic tradition (L,T,F) has $22\frac{3}{4}$, which is adopted by P–K.

[105] Identification highly uncertain. Mine is that of P–K (*Piazzi* IX 115), who however also emend the longitude to $15\frac{1}{6}$.

[106] Reading κ γ′ at H43,15 for κγ (23). The reading adopted is that of the Greek ms. B and part of the Arabic tradition (see S 7).

[107] The identification corresponds to that of P–K (*Piazzi* VIII 245).

	[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
	6	The one under that elbow, which also touches it	♓ 10	+74	4	θ Cep
	7	The star in the chest	♓ 28½	+65½	5	ξ Cep
	8	The star on the left arm	♈ 7½	+62½	>4	ι Cep
H48	9	The southernmost of the 3 stars on the tiara[115]	♓ 16⅓	+60¼	5	ε Cep
	10	The middle one of the three	♓ 17⅓	+61¼[116]	4	ζ Cep
	11	The northernmost of the three	♓ 19	+61⅓	5	λ Cep
		{11 stars, 1 of the third magnitude, 7 of the fourth, 3 of the fifth}				
		Stars around Cepheus outside the constellation:				
	12	The one in advance of the tiara	♓ 13⅔	+64	5	μ Cep
	13	The one to the rear of the tiara	♓ 21⅓	+59½	4	δ Cep
		{2 stars outside the constellation, 1 of the fourth magnitude, 1 of the fifth}				
		[V] Constellation of Bootes				
	1	The most advanced of the three in the left arm	♍ 2⅓	+58⅔	5	κ Boo
	2	The middle and southernmost of the three	♍ 4⅙	+58⅓	5	ι Boo
	3	The rearmost of the three	♍ 5⅓	+60⅙	5	θ Boo
	4	The star on the left elbow	♍ 9⅔	+54⅔	5	λ Boo
	5	The star on the left shoulder	♍ 19⅔	+49	3	γ Boo
	6	The star on the head	♍ 26⅔	+53⅚	>4	β Boo
	7	The star on the right shoulder	♎ 5⅔	+48⅔	>4	δ Boo
	8	The one to the north of these,[117] on the staff[118]	♎ 5⅔	+53¼	4	μ Boo

[115] Cepheus was represented wearing the tiara, the high head-dress of the Persian king, because in many versions of the myth (involving Perseus, Andromeda and her father Cepheus) he was said to be an oriental ruler. See Boll-Gundel, 'Sternbilder' cols. 884–5, with illustration from Vat. Gr. 1087.

[116] The variant 64¼ occurs in the earlier Arabic tradition according to S 9.

[117] The star is to the north only of no. 7, not of no. 6. Hence Manitius emends αὐτῶν at H48,18 to αὐτοῦ, 'of this'. However, it seems probable that Ptolemy was careless, being misled by the fact that the *declination* of no. 8 is greater than that of both the other stars.

[118] κολλόροβον, a kind of curved stick traditionally applied to the object held by Bootes, and also to that wielded by Orion (XXXV 11). Variously translated as 'shepherd's staff' or 'club'. The former would be more appropriate to the herdsman Bootes, the latter more plausible for the hunter Orion. However, the object carried by Bootes is called by Ptolemy (no. 10) a club (ῥόπαλον), and that is what is represented on the Farnese globe (Thiele Pl. VI top). The object in Thiele Fig. 22 p. 96 resembles a shepherd's crook.

	[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
	9	The one farther to the north again of this, on the tip of the staff	♎ 5	+57½	4	ν Boo
H50	10	The northernmost of the two stars below the shoulder, in the club	♎ 7⅔	+46½	>4	*η CrB
	11	The southernmost of them	♎ 8½	+45½	5	ο CrB
	12	The star on the end of the right arm	♎ 8⅙	+41⅓	5	*45(c) Boo
	13	The more advanced of the two stars in the wrist	♎ 6⅔	+41⅔	5	*ψ Boo
	14	The rearmost of them	♎ 7	+42½	5	*46(b) Boo
	15	The star on the end of the handle of the staff	♎ 7⅓	+40⅓	5	*ω Boo
	16	The star on the right thigh, in the apron[119]	♎ 0	+40¼	3	ε Boo
	17	The rearmost of the two stars in the belt	♍ 25⅔	+41⅔	4	σ Boo
	18	The more advanced of them	♍ 25	+42⅙	>4	ρ Boo
	19	The star on the right heel	♎ 5⅓	+28	3	ζ Boo
	20	The northernmost of the 3 stars in the left lower leg	♍ 21⅓	+28	3	η Boo
	21	The middle one of the three	♍ 20½	+26½	4	τ Boo
	22	The southernmost of them	♍ 21⅓	+25	4	υ Boo
		{22 stars, 4 of the third magnitude, 9 of the fourth, 9 of the fifth}				
		Star under [Bootes] outside the constellation:				
	23	The star between the thighs, called 'Arcturus', reddish	♍ 27	+31½	1	α Boo
		{1 star of the first magnitude}				
H52		[VI] Constellation of Corona Borealis				
	1	The bright star in the crown	♎ 14⅔	+44½	>2	α CrB
	2	The star most in advance of all	♎ 11⅔	+46½	>4	β CrB
	3	The one to the rear and to the north of this	♎ 11⅚	+48	5	θ CrB
	4	The one to the rear and north again of this	♎ 13⅔	+50½	6	π CrB
	5	The one to the rear of the bright star from the south[120]	♎ 17⅙	+44¾	4	γ CrB
	6	The one to the rear again of the latter, close by	♎ 19⅙	+44⅚	4	δ CrB

[119] περίζωμα, a kind of girdle. In the representations I have seen (e.g. Thiele, as in n. 118) Bootes wears an ἐξωμίς, a tunic which leaves one shoulder bare.

[120] The latitude of this star (+44¾°), if the text is correct, is in fact more *northerly* than that of no. 1 (44½°). Perhaps Ptolemy merely means to contrast it with the more northerly star no. 4 (also 'to the rear' of no. 1). It seems unlikely that he describes it as 'to the south' because no. 5 has a lesser declination than no. 1.

	[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
		[III] Constellation of Draco				
	1	The star on the tongue	♎ 26⅔	+76½	4	μ Dra
H44	2	The star in the mouth	♏ 11⅚	+78½	>4	ν Dra
	3	The star above the eye	♏ 13⅙	+75⅔	3	β Dra
	4	The star on the jaw[108]	♏ 27⅓	+80⅓	4	ξ Dra
	5	The star above the head	♏ 29⅔	+75½	3	γ Dra
	6	The northernmost of the 3 stars in a straight line in the first bend of the neck	♐ 24⅔	+82⅓	4	39(b) Dra
	7	The southernmost of these	♑ 2⅓	+78¼	4	46(c) Dra
	8	The middle one	♐ 28⅚	+80⅓	4	45 (d) Dra
	9	The star to the rear and due east of the latter	♑ 19½	+81½	4	ο Dra
	10	The southern star of the [two] forming the advance side of the quadrilateral in the next bend	♓ 8	+81⅔	4	π Dra
	11	The more northerly star of the advance side	♓ 20½	+83	4	δ Dra
	12	The northern star of the rear side [of the quadrilateral]	♈ 7⅔	+78⅚	4	ε Dra
	13	The southern star of the rear side	♓ 22⅚	+77⅚	4	ρ Dra
	14	The southern star of [those forming] the triangle in the next bend	♈ 10⅔	+80½	5	σ Dra
	15	The more advanced of the other two stars of the triangle	♈ 21⅔	*+81⅓[109]	5	υ Dra
	16	The one to the rear	♈ 26⅙	+80¼	5	τ Dra
	17	The most advanced[110] of the three stars in the next triangle, which is in advance [of the last]	♊ 13⅓	+84½	4	ψ Dra

[108] γένυς, which could also be translated 'cheek'.

[109] P–K adopt 81⅔, from the Arabic (all mss. which I have examined).

[110] Reading προηγούμενος (with DL E Ger) for ἑπόμενος (other Greek mss., FT), 'rearmost', at H44,19. Although no. 17 has a greater ecliptic longitude than no. 18, and thus would normally be 'to the rear' of it, for stars with extreme northern latitudes, their declinations may be greater than that of the pole of the ecliptic (90° − ε), in which case the normal rule may not apply. Indeed, on Ptolemy's star globe the equatorial coordinates of nos. 15–18 would be

	α	δ
15	291.9°	67.9°
16	294.7°	68.7°
17	274.9°	71.4°
18	282.6°	70.7°

Thus 16 is 'to the rear of' 15, but 17 is 'in advance of' 18, and 17 and 18 are 'in advance of' 15 and 16, in agreement with the text I adopt. Corrected by Manitius.

	[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
	18	The southernmost of the other two forming the triangle	♉ 20⅓	*+83½[111]	4	χ Dra
	19	The northernmost of the other two	♉ 11⅚	+84⅚	4	φ Dra
	20	The rearmost of the two small stars to the west of the triangle	♋ 28⅔	+87½	6	27(f) Dra
H46	21	The one in advance	♋ 21⅔	+86⅚	6	ω Dra
	22	The southernmost of the next 3 stars in a straight line	♍ 9	+81¼	5	18(g) Dra
	23	The middle one of the three	♍ 9⅓	+83[112]	5	19(h) Dra
	24	The northernmost of them	♍ 8⅓	+84⅚	3	ζ Dra
	25	The northernmost of the next 2 to the west	♍ 10	+78	3	η Dra
	26	The southernmost of these	♍ 13[113]	+74⅔	>4	θ Dra
	27	The star to the west of these, in the bend by the tail	♍ 12⅔	+70	3	ι Dra
	28	The advance star of the 2 quite some distance from the latter	♌ 7⅓	+64⅔	4	10(i) Dra
	29	The rear star of these [two]	♌ 11⅙	+65½	3	α Dra
	30	The star close by these, by the tail	♋ 19⅙	+61¼	3	κ Dra
	31	The remaining star, on the tip of the tail	♋ 13⅙	+56¼	3	λ Dra
		[31 stars, 8 of the third magnitude, 16 of the fourth, 5 of the fifth, 2 of the sixth][114]				
		[IV] Constellation of Cepheus				
	1	The star on the right leg	♉ 5	+75⅔	4	κ Cep
	2	The one on the left leg	♉ 3	+64¼	4	γ Cep
	3	The star under the belt on the right side	♈ 7⅓	+71⅙	4	β Cep
	4	The star over the right shoulder, which touches it	♓ 16⅔	+69	3	α Cep
	5	The star over the right elbow, which touches it	♓ 9⅓	+72	4	η Cep

[111] Reading πγ ∠΄ (with B, Ar) for πζ ∠΄ (87½) at H45,20. Corrected by P–K. 83½ fits both Ptolemy's description and the actual location of χ Dra much better.

[112] Reading πγ (with Ar) for π γ΄ (80⅓) at H47,4. 80⅓ must be wrong, since Ptolemy's description ensures that the latitude of no. 23 lies between that of 22 (81¼) and that of 24 (84⅚). Corrected by Manitius and P–K.

[113] Reading ιγ (with Ar, adopted by P–K) for ι γ΄ (10⅓) at H47,7. According to S 8 the Arabic tradition of no.27 is unanimous for 13½°; the context makes it clear that he has mistakenly attributed the coordinates of no. 26 to no. 27. '13½' is probably a scribal error in ibn aṣ-Ṣalāḥ for '13'.

[114] Deleting ὁμοῦ λα ('31 altogether'), with D, at H46,13.

[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
3	The southernmost of them	♐ 20½	+61	>4	ζ Lyr
4	The one to the rear of these, in between the points where the horns [of the lyre][131] are attached	♐ 23⅔	+60	4	*δ Lyr
5	The northernmost of the 2 stars close together in the region to the east of the shell	♑ 2	+61⅓	4	η Lyr
6	The southernmost of them	♑ 1⅔	+60⅓	4	θ Lyr
7	The northernmost of the two advance stars in the bridge[132]	♐ 21	+56⅙	3	β Lyr
8	The southernmost of them	♐ 20⅚	+55	<4	*ν Lyr
H58 9	The northernmost of the two rear stars in the bridge	♐ 24⅙	+55⅓	3	γ Lyr
10	The southernmost of them	*♐ 24⅙[133]	+54¾	<4	λ Lyr
	{10 stars, 1 of the first magnitude, 2 of the third, 7 of the fourth}				
	[IX] Constellation of Cygnus[134]				
1	The star on the beak	♑ 4½	*+49	3	β Cyg
2	The one to the rear of this, on the head	♑ 9	+50½	5	φ Cyg
3	The star in the middle of the neck	♑ 16⅓	+54½	>4	η Cyg
4	The star in the breast	♑ 28½	+57⅓	3	γ Cyg
5	The bright star in the tail	♒ 9⅙	+60	2	α Cyg
6	The star in the bend of the right wing	*♑ 19⅓	+64⅔	3	δ Cyg
7	The southernmost of the 3 in the right wing-feathers	♑ 22½	+69⅔	4	θ Cyg
8	The middle one of the three	♑ 21⅙	+71½	>4	ι Cyg
9	The northernmost of them, on the tip of the wing-feathers	♑ 16⅔	+74	>4	κ Cyg
10	The star on the bend of the left wing	♒ 0⅚	+49⅓	3	ε Cyg
11	The star north of this,[135] in the middle of the same wing	♒ 3⅚	+52⅙	>4	λ Cyg

[131] Conceivably a reference to the version of the myth in which Hermes used the horns of the cattle he stole from Apollo to make this part of the lyre (scholion on Germanicus, ed. Breysig 84). Cf. the depiction in Vat. Gr. 1087, reproduced in Boll-Gundel col. 904, and Thiele Fig. 38 p. 114.

[132] ζύγωμα, the 'cross-bar' of the lyre.

[133] Reading κδ ς´ (with D) at H59,3. Heiberg has κδ (24), which is the reading of Ar. But all other Greek mss. have κα (21).

[134] ὄρνις, literally 'bird'. It is not identified with a swan (Cygnus) or any particular bird in the earlier Greek tradition (e.g. Aratus 278), but the extant pictorial representations (e.g. Thiele Fig. 39 p. 114) mostly resemble a swan. For the origin of the appellation 'swan' see Gundel, art. 'Kyknos'. RE 11.2, 2442-3.

[135] Reading αὐτοῦ (with Is) for αὐτῶν ('of these') at H58,16. The change is necessary, since the star is north only of no. 10.

[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
12	The star in the tip of the feathers of the left wing	♒ 6⅔	+44	3	ζ Cyg
13	The star on the left leg	♒ 10	+55⅙	>4	ν Cyg
14	The star on the left knee	♒ 14½	+57	>4	ξ Cyg
H60 15	The more advanced of the 2 stars in the right leg	♒ 1⅙	+64	4	o^1 Cyg
16	The one to the rear	♒ 2⅔	+64½	4	o^2 Cyg
17	The nebulous star[136] on the right knee	♒ 12⅙	+64¾	5	ω Cyg
	{17 stars, 1 of the second magnitude, 5 of the third, 9 of the fourth, 2 of the fifth}				
	Stars around [Cygnus] outside the constellation				
18	The southernmost of the 2 stars under the left wing	♒ 10⅔[137]	+49⅔	>4	τ Cyg
19	The northernmost of them	♒ 13⅚	+51⅔	>4	σ Cyg
	{2 stars of the fourth magnitude}				
	[X] Constellation of Cassiopeia				
1	The star on the head	♈ 7⅚	+45⅓	>4	ζ Cas
2	The star in the breast	♈ 10⅚	+46¾	3	α Cas
3	The one north of that, on the belt	♈ 13	+47⅚	4	η Cas
4	The star over the throne, just over the thighs	♈ 16⅔	+49	>3	γ Cas
5	The star in the knees	♈ 20⅔	+45½	3	δ Cas
6	The star on the lower leg	♈ 27	+47¾	4	ε Cas
7	The star on the end of the leg	*♉ 1⅓	+47⅓	4	ι Cas
8	The star on the left upper arm	♈ 14⅔	+44⅓	4	*θ Cas
9	The star below the left elbow	♈ 17⅔	+45	5	*φ Cas
H62 10	The star on the right fore-arm	♈ 2⅓	+50	6	σ Cas
11	The star above the foot of the throne	♈ 15	+52⅔	<4	κ Cas
12	The star on the middle of the back of the throne	♈ 7⅚	+51⅔	3	β Cas
13	The star on the top of the throne-back	♈ 3⅔	+51⅔	6	ρ Cas
	{13 stars, 4 of the third magnitude, 6 of the fourth, 1 of the fifth, 2 of the sixth}				

[136] ω Cyg is not a nebula, but a multiple star system.

[137] The variant 13⅔ occurs in both Greek (D) and the later Arabic traditions (see S 17).

[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
7	The one to the rear again of these	♎ 21⅓	+46⅙	4	ε CrB
8	The star to the rear of all [the others] in the crown	♎ 21⅔	+49⅓	4	ι CrB
	[8 stars, 1 of the second magnitude, 5 of the fourth, 1 of the fifth, 1 of the sixth]				
	[VII] Constellation of Hercules[121]				
1	The star on the head	♏ 17⅔	+37½	3	α Her
2	The star on the right shoulder by the armpit	♏ 3⅔	+43	3	β Her
3	The star on the right upper arm	♏ 1⅔	+40⅙	3	γ Her
4	The star on the right elbow	♎ 28	+37⅙	4	κ Her
5	The star on the left shoulder	♏ 16⅔	+48	3	δ Her
6	The star on the left upper arm	♏ 22	+49½	>4	λ Her
7	The star on the left elbow	♏ 27⅔	+52	>4	μ Her
H54 8	The rearmost of the 3 stars in the left wrist	♐ 5½	+52⅚	>4	ο Her
9	The northernmost of the other 2	♐ 1⅓	+54	>4	ν Her
10	The southernmost of them	♐ 1½	+53	4	ξ Her
11	The star in the right side	♏ 3⅚[122]	*+53⅙[123]	3	ζ Her
12	The star in the left side	♏ 10⅙[124]	+53½	*5[125]	ε Her
13	The one north of the latter, on the left buttock	♏ 10	+56½	5	59(d) Her
14	The one on the place where the thigh joins the same [buttock]	♏ 11⅙	+58½	3	61(c) Her
15	The most advanced of the 3 in the left thigh	♏ 14	+59⅚	4	π Her

[121] Literally 'the [figure] on its knees'. Cf. Aratus 63-7. The figure is not identified with any mythological personage in the earlier Greek tradition, or by Germanicus or Ptolemy. For various late identifications with Hercules and other figures see pseudo-Eratosthenes, ed. Robert, 62-6, Avienius, *Aratea* 175-94, and Boll-Gundel, 'Sternbilder' cols. 900-3.

[122] The variant 6⅔ occurs in the Greek tradition (A¹BC, written '6+½ + ⅙'!), and, according to S 10, the earlier Arabic tradition.

[123] Reading νγ ς′, with Is (confirmed by S 10), found as a variant in L, for ν ϛ̄′ (50⅔) at H55,5 (D and al-Ḥajjāj have 56⅙, derived from the correct reading by a common scribal error). P-K also adopt 53⅙ (from aṣ-Ṣūfī).

[124] The variant 16 occurs in the tradition of both Greek (A¹D) and Arabic (see S 11).

[125] D,Ar have the magnitude >4, in better agreement with modern estimates of the magnitude of ε Her (3.9). As Manitius (p. 401) says, adopting this would upset the partial and complete totals of 4th and 5th magnitude stars. But since these are probably later accretions, they indicate only that this was counted as a 5th magnitude star in the late Alexandrine tradition.

[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
16	The one to the rear of this	♏ 15⅓	*+60⅓[126]	4	69(e) Her
17	The one yet further to the rear of this	♏ 16⅓	+61¼	>4	ρ Her
18	The star on the left knee	♐ 0⅚	+61	4	θ Her
19	The star on the left shin	♏ 22⅙	+69⅓	4	ι Her
20	The most advanced of the 3 stars in the left foot	♏ 15⅓	+70¼[127]	6	74(x) Her
21	The middle one of the three	♏ 16⅚	+71¼	6	77(y) Her
22	The rearmost of them	♏ 19⅔	*+72¼[128]	6	82(z) Her
23	The star on the place where the right thigh joins [the buttock]	♏ 0⅔	*+60¼[129]	>4	η Her
24	The star north of it in the same thigh	♎ 25⅓	+63	4	σ Her
H56 25	The star on the right knee	♎ 15⅔	+65½	>4	τ Her
26	The southernmost of the 2 stars under the right knee	♎ 13⅔	+63⅔	4	φ Her
27	The northernmost of them	♎ 10⅙	+64¼	4	υ Her
28	The star in the right lower leg	♎ 11⅙	+60	4	χ Her
	The star on the end of the right leg is the same as the one on the tip of the staff [of Bootes, V 9]				
	[Not counting the latter, 28 stars, 6 of the third magnitude, 17 of the fourth, 2 of the fifth, 3 of the sixth]				
	Star outside this constellation:				
29	The star south of the one in the right upper arm	♏ 2⅔	+38⅙	5	*ω Her
	[1 star of the fifth magnitude]				
	[VIII] Constellation of Lyra				
1	The bright star on the shell,[130] called Lyra	♐ 17⅓	+62	1	α Lyr
2	The northernmost of the 2 stars lying near the latter, close together	♐ 20⅓	+62⅔	>4	ε Lyr

[126] Most Greek mss. have ξγ (63). Heiberg adopted 60⅓ from Bode's conjecture. It is in fact the reading of most of the Arabic tradition, according to S 12, and is found in all Arabic mss. examined by Kunitzsch.

[127] The variant 74 is found in the earlier Arabic tradition according to S 13.

[128] D,Ar have 72, adopted by P-K.

[129] All Greek mss. have ξδ (64). Heiberg adopted 60¼ by conjecture, but it is in fact the reading of almost all the later Arabic tradition (see S 14).

[130] The shell of the tortoise from which, in Greek myth, the infant Hermes constructed the first lyre. See e.g. the Homeric Hymn to Hermes 33, Aratus 268-9, and (for other ancient references) Boll-Gundel cols. 904-5. The modern name for the star is Vega.

	[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
	14	The small star over the left foot	*♉ $20\frac{2}{3}$[146]	*+$10\frac{1}{3}$[147]	6	*14 Aur[148]
		[14 stars, 1 of the first magnitude, 1 of the second, 2 of the third, 7 of the fourth, 2 of the fifth, 1 of the sixth]				
		[XIII] Constellation of Ophiuchus				
	1	The star on the head	♏ $24\frac{5}{6}$	+36	>3	α Oph
H68	2	The more advanced of the 2 stars on the right shoulder	♏ 28	+$27\frac{1}{4}$	>4	β Oph
	3	The rearmost of them	♏ 29	+$26\frac{1}{2}$	4	γ Oph
	4	The more advanced of the 2 stars on the left shoulder	♏ $13\frac{1}{3}$	+33	4	ι Oph
	5	The rearmost of them	♏ $14\frac{2}{3}$	+$31\frac{5}{6}$	4	κ Oph
	6	The star on the left elbow	♏ $8\frac{1}{3}$	*+$24\frac{1}{2}$[149]	4	λ Oph
	7	The more advanced of the 2 stars in the left hand	♏ 5	+17	3	δ Oph
	8	The rearmost of them	♏ 6	+$16\frac{1}{2}$	3	ε Oph
	9	The star on the right elbow	♏ $26\frac{2}{3}$	+15	4	μ Oph
	10	The more advanced of the 2 stars in the right hand	♐ $2\frac{1}{3}$	+$13\frac{2}{3}$	<4	ν Oph
	11	The rearmost of them	♐ $3\frac{1}{3}$	+$14\frac{1}{3}$	4	τ Oph
	12	The star on the right knee	♏ $21\frac{1}{6}$	+$7\frac{1}{2}$	3	η Oph
	13	The star on the right lower leg	*♏ $23\frac{2}{3}$[150]	+$2\frac{1}{4}$	>4	*ξ Oph
	14	The most advanced of the 4 stars on the right foot	♏ 23	*−$2\frac{1}{4}$	4	*36(A) Oph
	15	The one to the rear of this	♏ $24\frac{1}{3}$	*−$1\frac{1}{2}$	>4	*θ Oph
	16	The one to the rear again of that	♏ 25	*−$0\frac{1}{3}$	4	*44(b) Oph
	17	The last and rearmost of the 4	♏ $25\frac{5}{6}$	*−$0\frac{1}{4}$	5	*51(c) Oph[151]

[146] P–K adopt the reading 23 from the late Greek ms. Par. 2394. There is no good authority for it.

[147] Reading ι γ´ (with A[1] and part of the Arabic tradition, see S 20) for ις (16) at H67,19. The related variant ιγ (13) is also found, in D and the later Arabic tradition (ibid.). P–K adopt $10\frac{1}{2}$.

[148] The identification is very uncertain and depends on the coordinates adopted. Kunitzsch (ibn aṣ-Ṣalāḥ 86 n.d) suggests 5 Aur, adopting the coordinates ♉ $20\frac{2}{3}$, + 16. I retain that of P–K. 14 Aur, which is supported by the location with respect to the Milky Way, described in VIII 2 p. 402 (this virtually excludes Manitius' identification, 2 Aur).

[149] $24\frac{1}{2}$ is the reading of DL, adopted by Heiberg. Most Greek mss. have $33\frac{5}{6}$. P–K adopt $23\frac{3}{4}$, claiming that it is the reading of some Greek and one Arabic ms. (it appears to be that of T).

[150] Reading κγ ⌈ο (with A[1]DAr) for κς ⌈ο ($26\frac{2}{3}$) at H69,13. The same correction was made by Manitius and P–K.

[151] The uncertainty connected with nos. 14 to 17 is whether the latitudes are south or north (for details of the variations see P–K p. 186 nos. 247–50). Consequently the identifications are uncertain (*pace* P–K, n. on p. 99).

	[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
	18	The star to the rear of these, which touches the heel	♏ $27\frac{1}{6}$	+1	5	*51 Oph
	19	The star in the left knee	♏ $12\frac{1}{6}$	+$11\frac{5}{6}$	3	ζ Oph
	20	The northernmost of the 3 stars in a straight line in the left lower leg	♏ $11\frac{2}{3}$	+$5\frac{1}{3}$	>5	φ Oph
	21	The middle one of these	♏ $10\frac{2}{3}$	+$3\frac{1}{6}$	5	χ Oph
	22	The southernmost of the three	♏ $9\frac{5}{6}$	*+$1\frac{2}{3}$	>5	ψ Oph
H70	23	The star on the left heel	♏ $12\frac{1}{3}$	+$0\frac{2}{3}$	5	ω Oph
	24	The star touching the hollow of the left foot	♏ $10\frac{1}{3}$	−$0\frac{3}{4}$	4	ρ Oph
		[24 stars, 5 of the third magnitude, 13 of the fourth, 6 of the fifth]				
		Stars around Ophiuchus outside the constellation:				
	25	The northernmost of the 3 to the east of the right shoulder	♐ 2	+$28\frac{1}{6}$	4	66(n) Oph
	26	The middle one of the three	♐ $2\frac{2}{3}$	+$26\frac{1}{3}$	4	67 Oph
	27	The southernmost of them	♐ 3[152]	+25	4	68 Oph
	28	The star to the rear of these 3, approximately over the middle one	♐ $3\frac{2}{3}$	+27	4	70 Oph
	29	The lone star north of [these] 4 [nos 25–28]	♐ $4\frac{2}{3}$	+33	4	72 Oph
		[5 stars of the fourth magnitude]				
		[XIV] Constellation of Serpens[153]				
	1-5	Stars on the quadrilateral in the heads:				
	1	the one on the end of the jaw	♎ $18\frac{5}{6}$	+38	4	ι Ser
	2	the one touching the nostrils	♎ $21\frac{2}{3}$	+40	4	ρ Ser
	3	the one in the temple	*♎ $24\frac{1}{3}$[154]	+36	3	γ Ser
	4	the one where the neck joins [the head]	♎ 22	+$34\frac{1}{4}$	3	β Ser
	5	the one in the middle of the quadrilateral, in the mouth	♎ $21\frac{1}{3}$	+$37\frac{1}{4}$	4	κ Ser
	6	The star outside the head, to the north of it	♎ $23\frac{1}{6}$[155]	+$42\frac{1}{2}$	4	π Ser
	7	The one after the first bend in the neck	♎ $21\frac{2}{3}$	+$29\frac{1}{4}$	3	δ Ser

[152] The later Arabic tradition is solid for the variant $0\frac{1}{3}$ (see S 20).

[153] Literally 'of the snake of the snake-holder [Ophiuchus]'. This is to distinguish it from Draco and Hydra (the big snake and the water-snake).

[154] The Greek tradition is uniform for $21\frac{1}{3}$. Heiberg adopted $24\frac{1}{3}$ as an emendation by Bode. However, it is well-attested in the Arabic tradition: see S 22.

[155] Reading κγ ς´ (with BCD and the later Arabic tradition, see S 23) for κς ς´ ($26\frac{1}{6}$) at H71,18.

	[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
		[XI] Constellation of Perseus				
	1	The nebulous mass on the right hand	♈ 26⅔	+40½	neb.	CGal 884 + 869[138]
	2	The star on the right elbow	♉ 1⅙	+37½	4	η Per
	3	The star on the right shoulder	♉ 2⅔	+34½	<3	γ Per
	4	The star on the left shoulder	♈ 27½	+32⅓	4	θ Per
	5	The star on the head	♉ 0⅔	+34½	4	τ Per
	6	The star on the place between the shoulders[139]	♉ 1½	+31⅙	4	ι Per
	7	The bright star in the right side	♉ 4⅚	+30	2	α Per
	8	The most advanced of the 3 stars next to the one in the side	♉ 5⅓	+27⅚	4	σ Per
	9	The middle one of the three	♉ 7	+27⅔	4	ψ Per
	10	The rearmost of them	♉ 7⅔	+27⅓	3	δ Per
	11	The star on the left elbow	♉ 0½	+27	4	κ Per
	12-15	Stars in the Gorgon-head:[140]				
	12	the bright one	♈ 29⅔	+23	2	β Per
H64	13	the one to the rear of this	♈ 29⅙	+21	4	ω Per
	14	the one in advance of the bright star	♈ 27⅔	+21	4	ρ Per
	15	the remaining one, yet again in advance of this	♈ 26⅚	+22¼	4	π Per
	16	The star in the right knee	♉ 14⅚	*+28[141]	4	72(b) Per
	17	The one in advance of this, over the knee	♉ 13	+28⅙	4	λ Per
	18	The more advanced of the 2 stars above the bend in the knee	♉ 12⅓	+25	4	48 Per
	19	The rearmost of them, just over the bend in the knee	♉ 14	+26¼	4	μ Per
	20	The star on the right calf	♉ 14⅙	+24½	5	53(d) Per
	21	The star on the right ankle	♉ 16⅓	+18¾	*5	58(e) Per
	22	The star in the left thigh	♉ 6⅚	+21⅚	>4	ν Per
	23	The star on the left knee	♉ 8⅔	+19¼	3	ε Per
	24	The star on the left lower leg	♉ 8⅓	+14¾	4	ξ Per
	25	The star on the left heel	♉ 4⅙	+12	<3	ο Per

[138] Manitius identifies this as h Per, P–K as χ Per. These are, respectively, the Galactic Clusters 869 and 884, which appear as a single hazy patch to the naked eye (see Burnham III 1438).
[139] μετάφρενον. See p. 356 n.159. Here Perseus *may* be envisioned as partly turned to the side, so that some of his back is visible.
[140] The head of Medusa, carried in Perseus' left hand (see the depiction in Boll-Gundel col. 914).
[141] 28 is the reading of all Greek mss., 28⅙ that of some Arabic mss. (L,E,F), adopted by P–K.

	[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
	26	The one to the rear of this, on the left foot	♉ 6⅓	+11	>3	ζ Per
		[26 stars, 2 of the second magnitude, 5 of the third, 16 of the fourth, 2 of the fifth, [1] nebulous]				
		Stars around Perseus outside the constellation:				
	27	The star to the east of the one on the left knee [no. 23]	♉ 11⅚	+18	5	52(f) Per
	28	The star to the north of the one[142] in the right knee [no. 16]	♉ 15	+31	5	*BSC 1314
H66	29	The star in advance of those in the Gorgon-head [nos. 12-15]	♈ 24⅔	+20⅔	f.	16 Per
		[3 stars, 2 of the fifth magnitude, 1 faint]				
		[XII] Constellation of Auriga				
	1	The southernmost of the two on the head	♊ 2½	+30	4	δ Aur
	2	The northernmost [of these], over the head	♊ 2¼	+31⅚	4	ξ Aur
	3	The star on the left shoulder, called Capella	♉ 25	+22½	1	α Aur
	4	The star on the right shoulder	♊ 2⅚	+20	2	β Aur
	5	The star on the right elbow	♊ 1⅙	+15¼	4	ν Aur
	6	The star on the right wrist	♊ 2⅚	+13¼	>4	θ Aur
	7	The star on the left elbow	♉ 22	+20⅔	>4	ε Aur
	8	The rearmost of the two stars on the left wrist, which are called 'Haedi'	♉ 22⅙	+18	>4	η Aur
	9	The more advanced of these	♉ 22	+18	4	ζ Aur
	10	The star on the left ankle	♉ 19⅚	+10⅙[143]	<3	ι Aur
	11	The star on the right ankle, which is [applied in] common to the horn [of Taurus][144]	♉ 25⅔	+5	>3	β Tau
	12	The one to the north of the latter, in the lower hem [of the garment][145]	♉ 26	+8½	5	χ Aur
	13	The one north again of this, on the buttock	♉ 26⅓	+12⅙	5	φ Aur

[142] Reading τοῦ ἐν (with D,E,T,Ger) for τῶν ἐν ('those in') at H64,19.
[143] The variant 16 is found in the later Arabic tradition according to S 19.
[144] See XXIII 21. The magnitude there is given as 3. The star is also known as γ Aurigae, but today is included in the constellation Taurus.
[145] περιπόδιον. Auriga ('the charioteer') is depicted as wearing a long tunic reaching to the feet, like the well-known bronze Delphic charioteer (see e.g. Richter, *Handbook of Greek Art*, Fig. 113 p. 85).

[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
8	The southernmost of the 3 stars between the tail and the rhombus	♑ 17½	*+30¼[163]	6	η Del
9	The more advanced of the other 2 to the north	♑ 17⅔	+31⅚	6	ζ Del
10	The remaining, rearmost one	♑ 19	+31½	6	θ Del
	{10 stars, 5 of the third magnitude, 2 of the fourth, 3 of the sixth}				
	[XVIII] Constellation of Equuleus[164]				
1	The more advanced of the 2 stars in the head	♑ 26⅓	+20½	f.	α Equ
2	The rearmost of them	♑ 28	+20⅚	f.	β Equ
3	The more advanced of the two stars in the mouth	♑ 26⅓	+25½	f.	γ Equ
4	The rearmost of them	♑ 27⅔	+25	f.	δ Equ
	{4 stars, [all] faint}				
	[XIX] Constellation of Pegasus[165]				
1	The star on the navel, which is [applied in] common to the head of Andromeda	♓ 17⅚	+26	<2	α And[166]
H78 2	The star on the rump and the wing-tip	♓ 12⅙	+12½	<2	γ Peg
3	The star on the right shoulder and the place where the leg joins [it]	♓ 2⅙	+31	<2	β Peg
4	The star on the place between the shoulders and the shoulder-part of the wing	♒ 26⅔	+19⅔	<2	α Peg
5	The northernmost of the two stars in the body under the wing	♓ 4½	+25½	4	τ Peg
6	The southernmost of them	♓ 5	+25	4	υ Peg
7	The northernmost of the two stars in the right knee	♒ 29	+35	3	η Peg
8	The southernmost of them	♒ 28½	+34½	5	ο Peg

[163] The variant 34 occurs in the Greek (C; '31' in D) and Arabic traditions (see S 26).
[164] Literally 'bust' or 'figurehead' (προτομή) 'of a horse'. In fact only the head appears to have been represented. There are no ancient illustrations (see Boll-Gundel 927–8), and indeed most ancient authorities ignore this constellation. The designation is confusing, since Pegasus too is represented as only the forepart of a horse.
[165] Literally 'the horse'; but the references to its wings make it clear that it is depicted as Pegasus. The identification was made as early as Aratus (216–24).
[166] The star is also known as δ Pegasi, but in modern times is defined as being in Andromeda.

[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
9	The more advanced of the two stars close together in the chest	♒ 26⅙	+29	4	λ Peg
10	The rearmost of them	♒ 27	+29½	4	μ Peg
11	The more advanced of the 2 stars close together in the neck	♒ 18⅚	+18	3	ζ Peg
12	The rearmost of them	♒ 20½	+19	4	ξ Peg
13	The southernmost of the two stars on the mane	♒ 21⅓	+15	5	ρ Peg
14	The northernmost of them	♒ 20½	+16	5	σ Peg
15	The northernmost of the two stars close together on the head	*♒ 9⅓[167]	+16½	3	θ Peg
16	The southernmost of them	♒ 8	+16	4	ν Peg
17	The star in the muzzle	♒ 5⅓	+22½	>3	ε Peg
18	The star in the right hock	♒ 23⅓	+41⅙	>4	π Peg
19	The star on the left knee	♒ 17⅓	+34¼	>4	ι Peg
H80 20	The star in the left hock	♒ 12⅓	+36⅚	>4	κ Peg
	{20 stars, 4 of the second magnitude, 4 of the third, 9 of the fourth, 3 of the fifth}				
	[XX] Constellation of Andromeda				
1	The star in the place between the shoulders	♓ 25⅓	+24½	3	δ And
2	The star in the right shoulder	♓ 26⅓	+27	4	π And
3	The star in the left shoulder	♓ 24⅓	+23	4	ε And
4	The southernmost of the 3 stars on the right upper arm	♓ 23⅔	+32	4	σ And
5	The northernmost of them	♓ 24⅔	+33½	4	θ And
6	The middle one of the three	♓ 25	+32⅓	5	ρ And
7	The southernmost of the 3 stars on the right hand	♓ 19⅔	+41	4	ι And
8	The middle one of these	♓ 20⅔	+42	4	κ And
9	The northernmost of the three	♓ 22⅙	+44	4	λ And
10	The star on the left upper arm	♓ 24⅙	+17½	4	ζ And
11	The star on the left elbow	♓ 25⅔	+15⅚	4	η And
12	The southernmost of the 3 stars over the girdle	♈ 3⅚	+26⅓	3	β And
13	The middle one of these	♈ 1⅚	+30	4	μ And
14	The northernmost of the three	♈ 2	+32½	4	ν And

[167] Most Greek mss ($\mathrm{A^1BC}$) and Is have 9⅙. Heiberg adopts the reading of D,L.

	[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
	8	The northernmost of the 3 following this	♎ $24\frac{5}{6}$	$+26\frac{1}{2}$	4	λ Ser
	9	The middle one of the three	♎ $24\frac{1}{3}$	$+25\frac{1}{3}$	3	α Ser
	10	The southernmost of them	♎ $26\frac{1}{3}$	+24	3	ε Ser
H72	11	The star after the next bend, which is in advance of the left hand of Ophiuchus	♎ $28\frac{5}{6}$	$+16\frac{1}{2}$	4	μ Ser
	12	The star to the rear of those in the hand [of Ophiuchus, nos. XIII 7-8]	♏ $8\frac{1}{6}$	*$+13\frac{1}{4}$[156]	5	υ Oph
	13	The one after the back of the right thigh of Ophiuchus	♏ $23\frac{2}{3}$	$+10\frac{1}{2}$	4	ν Ser
	14	The southernmost of the 2 to the rear of the latter	♏ 27	$+8\frac{1}{2}$	>4	ξ Ser
	15	The northernmost of them	♏ $27\frac{5}{6}$	$+10\frac{5}{6}$	4	ο Ser
	16	The one after the right hand [of Ophiuchus], on the bend in the tail	♐ $3\frac{2}{3}$	+20	4	ζ Ser
	17	The one to the rear of this, likewise on the tail	♐ $8\frac{2}{3}$	$+21\frac{1}{6}$	>4	η Ser
	18	The star on the tip of the tail	♐ $18\frac{1}{3}$	+27	4	θ Ser
		[18 stars, 5 of the third magnitude, 12 of the fourth, 1 of the fifth]				
		[XV] Constellation of Sagitta				
	1	The lone star on the arrow-head	♑ $10\frac{1}{6}$[157]	$+39\frac{1}{3}$	4	γ Sge
	2	The rearmost of the three stars in the shaft	♑ $6\frac{2}{3}$	$+39\frac{1}{6}$	6	ζ Sge
	3	The middle one	♑ $5\frac{5}{6}$	$+39\frac{1}{2}$	5	δ Sge
	4	The most advanced of the three	♑ $4\frac{2}{3}$	+39	5	α Sge
	5	The star on the end of the notch	♑ $3\frac{1}{3}$	*$+38\frac{2}{3}$[158]	5	β Sge
		[5 stars, 1 of the fourth magnitude, 3 of the fifth, 1 of the sixth]				
		[XVI] Constellation of Aquila				
	1	The star in the middle of the head	♑ $7\frac{1}{6}$	$+26\frac{5}{6}$	4	τ Aql
	2	The one in advance of this, on the neck	♑ $4\frac{5}{6}$	$+27\frac{1}{6}$	3	β Aql
	3	The bright star on the place between the shoulders, called Aquila[159]	♑ $3\frac{5}{6}$	$+29\frac{1}{6}$	>2	α Aql

[156] $13\frac{1}{4}$ is Heiberg's emendation (following Bode, who in fact conjectured '13'). All mss. have $16\frac{1}{4}$. See the discussion of P–K, pp. 99–100.
[157] The variant 16 is found in the Greek ms. D and in the later Arabic tradition (see S 24).
[158] This is the reading of D, adopted by Heiberg, where most Greek mss. have $37\frac{2}{3}$. The Arabic tradition varies between $38\frac{1}{3}$ and $38\frac{2}{3}$ (see S 25).
[159] The phrase 'place between the shoulders' is my translation of μετάφρενον. This seems more accurate than LSJ's 'broad of the back', which is certainly impossible here because of the iconography. It is clear from the orientation ('left', 'right' and 'head') that one is supposed to see the *underside* of the bird (in agreement with the depiction on the Farnese globe, Thiele Pl. III bottom, where one is looking at the *outside*, cf. Introduction p. 15). Therefore one can have at best only a glimpse of the back. The modern name of this star is Altair.

	[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
H74	4	The one close to this towards the north	♑ $4\frac{2}{3}$	+30	<3	*ο Aql
	5	The more advanced of the 2 in the left shoulder	♑ $3\frac{1}{6}$	$+31\frac{1}{2}$	3	γ Aql
	6	The rearmost of them	♑ 6	$+31\frac{1}{2}$	5	φ Aql
	7	The more advanced of the two in the right shoulder	♐ $29\frac{2}{3}$	$+28\frac{2}{3}$	5	μ Aql
	8	The rearmost of them	♑ $1\frac{1}{6}$	$+26\frac{2}{3}$	>5	σ Aql
	9	The star some distance under the tail of Aquila, touching the Milky Way	♐ $22\frac{1}{6}$	$+36\frac{2}{3}$	3	ζ Aql
		[9 stars, 1 of the second magnitude, 4 of the third, 1 of the fourth, 3 of the fifth]				
		The stars around Aquila, to which the name 'Antinous' is given[160]				
	10	The more advanced of the 2 stars south of the head of Aquila	♑ $3\frac{2}{3}$	$+21\frac{2}{3}$	3	η Aql
	11	The rearmost of them	♑ $8\frac{5}{6}$	$+19\frac{1}{6}$	3	θ Aql
	12	The star to the south and west of the right shoulder of Aquila	♐ 26	+25	>4	δ Aql
	13	The one to the south of this	♐ $28\frac{1}{2}$	+20	3	ι Aql
	14	The one to the south again of the latter	♐ $29\frac{2}{3}$	$+15\frac{1}{2}$	5	*κ Aql
	15	The star most in advance of all	*♐ $21\frac{1}{6}$	$+18\frac{1}{6}$	3	λ Aql
		[6 stars, 4 of the third magnitude, 1 of the fourth, 1 of the fifth]				
		[XVII] Constellation of Delphinus				
	1	The most advanced of the 3 stars in the tail	♑ $17\frac{2}{3}$	$+29\frac{1}{6}$	<3	ε Del
	2	The northernmost of the other 2	♑ $18\frac{2}{3}$	+29	<4	ι Del
H76	3	The southernmost of them	♑ $18\frac{2}{3}$	$+27\frac{1}{4}$	4	κ Del
	4-7	The stars in the rhomboid[161] quadrilateral:				
	4	the southernmost one on the advance side	♑ $18\frac{1}{2}$	+32	<3	β Del
	5	the northernmost one on the advance side	*♑ $20\frac{1}{6}$[162]	*$+33\frac{5}{6}$	<3	α Del
	6	the southernmost one on the rear side of the rhombus	♑ $21\frac{1}{3}$	+32	<3	δ Del
	7	the northernmost one on the rear side	♑ $23\frac{1}{6}$	$+33\frac{1}{6}$	<3	γ Del

[160] Antinous was the emperor Hadrian's favourite, who died by drowning in the Nile in A.D. 130. This 'catasterism' is confirmation of the statement in Dio Cassius (69,11,4) that Hadrian claimed to have himself seen the star into which the soul of Antinous was transformed. Could Ptolemy have had anything to do with this identification? It turned out to be ephemeral.
[161] I.e. with only two of its four sides parallel.
[162] All Greek mss. have κς (26). Heiberg adopted κ ς′ ($20\frac{1}{6}$) as an emendation of Bode; but it is in fact found in all Arabic mss. I have examined.

	[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
	3	The one close again to the latter	•♈ $24\frac{1}{3}$[175]	$-8\frac{1}{2}$	4	ξ Tau
	4	The southernmost of the 4	♈ $24\frac{1}{3}$	$-9\frac{1}{4}$	4	ο Tau
	5	The one to the rear of these, on the right shoulder-blade	♈ $29\frac{2}{3}$	$-9\frac{1}{2}$	5	30(e) Tau
	6	The star in the chest	♉ $3\frac{2}{3}$	-8	3	λ Tau
	7	The star on the right knee	♉ $6\frac{2}{3}$	$-12\frac{2}{3}$	4	μ Tau
	8	The star on the right hock	♉ 3[176]	$-14\frac{5}{6}$	4	ν Tau
	9	The star on the left knee	♉ $12\frac{1}{6}$	-10	4	90(c¹) Tau
	10	The star on the left lower leg	•♉ 13[177]	-13[178]	4	88(d) Tau
	11-15	The stars in the face, called 'the Hyades':				
	11	the one on the nostrils	♉ 9	$-5\frac{3}{4}$	<3	γ Tau
	12	the one between this and the northern eye	♉ $10\frac{1}{3}$	$-4\frac{1}{4}$[179]	<3	δ¹ Tau
	13	the one between it [no. 11] and the southern eye	♉ $10\frac{5}{6}$	$-5\frac{5}{6}$	<3	θ¹ Tau
H88	14	the bright star of the Hyades, the reddish one on the southern eye	♉ $12\frac{2}{3}$	$-5\frac{1}{6}$	1	α Tau
	15	the remaining one, on the northern eye	•♉ $11\frac{5}{6}$	-3	<3	ε Tau
	16	The star on the place where the southern horn and the ear join [the head]	•♉ $17\frac{1}{6}$[180]	-4	4	97(i) Tau
	17	The southernmost of the 2 stars on the southern horn	♉ $20\frac{1}{3}$	-5	5	104(m) Tau
	18	The northernmost of these	♉ 20	$-3\frac{1}{2}$	5	106(l) Tau
	19	The star on the tip of the southern horn	♉ $27\frac{2}{3}$	$-2\frac{1}{2}$	3	ζ Tau
	20	The star on the place where the northern horn joins [the head]	♉ $15\frac{2}{3}$	•$-0\frac{1}{4}$[181]	4	τ Tau
	21	The star on the tip of the northern horn, which is the same as the one on the right foot of Auriga [XII no. 11]	♉ $25\frac{2}{3}$	+5	3[182]	β Tau

[175] P–K adopt $24\frac{2}{3}$, the reading of Ar, which is no doubt the origin of the corruption $21\frac{2}{3}$ in D.
[176] The variant $0\frac{1}{3}$ is found in part of the Arabic tradition according to S 28.
[177] Manitius (p. 401) changes to $10\frac{1}{3}$ (ι γ´ for ιγ), with no ms. authority.
[178] The variant $10\frac{1}{3}$ occurs in the later Arabic tradition (see S 29).
[179] The variant $1\frac{1}{4}$ is found in the earlier Arabic tradition according to S 30.
[180] Reading ιζ ς´ (with D,Ar, adopted by P–K) for ιζ ∠´ ($17\frac{1}{2}$) at H89,4.
[181] The variant 4 is found in some Greek mss. (BC) and in the whole of the Arabic tradition according to S 32. $0\frac{1}{4}$ is undoubtedly correct, but the latitude might be north instead of south (see P–K on no. 399 p. 101, Manitius pp. 401-2).
[182] In Auriga (XII,11) the magnitude is given as > 3.

	[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
	22	The northernmost of the 2 stars close together in the northern ear	♉ 12	$+0\frac{1}{2}$[183]	5	υ Tau
	23	The southernmost of them	♉ $11\frac{2}{3}$	$+0\frac{1}{4}$[184]	5	κ Tau
	24	The more advanced of the 2 small stars in the neck	♉ 7	$+0\frac{2}{3}$	5	37(A¹) Tau
	25	The rearmost of them	♉ 9	•-1[185]	6	ω Tau
	26-29	The quadrilateral in the neck:				
	26	the southernmost star on the advance side	•♉ 8	+5	5	44(p) Tau
	27	the northernmost star on the advance side	•♉ $8\frac{1}{2}$[186]	•$+7\frac{1}{3}$[187]	5	•ψ Tau
	28	the southernmost star on the rear side	♉ 12	+3	5	χ Tau
	29	the northernmost one on the rear side	♉ $11\frac{2}{3}$	+5	5	φ Tau
H90	30-33	The Pleiades:				
	30	the northern end of the advance side	♉ $2\frac{1}{6}$	$+4\frac{1}{2}$	5	•19 Tau
	31	the southern end of the advance side	♉ $2\frac{1}{2}$	$+3\frac{2}{3}$	5	•23 Tau
	32	the rearmost and narrowest end of the Pleiades	♉ $3\frac{2}{3}$	$+3\frac{1}{3}$[188]	5	•27 Tau[189]
	33	the small star outside[190] the Pleiades, towards the north	♉ $3\frac{2}{3}$	+5	•4[191]	•BSC 1188
		{32 stars,[192] 1 of the first magnitude, 6 of the third, 11 of the fourth, 13 of the fifth, 1 of the sixth}				

[183] The variant 6 occurs in the earlier Arabic tradition according to S 33.
[184] The variant 4 occurs in the Greek (B) and in the Arabic tradition (see S 34).
[185] The latitude is north in the Greek mss. A¹D and in almost the whole Arabic tradition according to S 35.
[186] On the longitudes of nos. 26 and 27 see Manitius p. 402, who interchanges them.
[187] D,Ar have the variant $7\frac{1}{6}$, adopted by P–K.
[188] As Manitius notes (p. 402) the rearmost part of the Pleiades is said to have the latitude $+3\frac{2}{3}$° at VII 3 p. 335. See n.71 there for the explanation of the discrepancy.
[189] The identifications of nos. 30-2 are those of P–K. However, I do not believe that Ptolemy was referring to specific stars, but rather to points on the general outline of the group. Nevertheless, the stars named are conveniently placed to serve as reference points.
[190] Reading ἐκτὸς (with D,Ar, cf. Kunitzsch, *Der Almagest* no. 293 p. 270) for ἕκτος ('the sixth small star') at H90,5. Corrected by Manitius.
[191] If the identification adopted here (which is that of P–K, Piazzi III 170) is correct, the magnitude of this star is 5.38, which casts doubt on the reading '4', particularly since Ptolemy emphasises that this is a small star. The reading 4 is confirmed by the sub-total for Taurus, but since that is probably an interpolation it proves only that the reading is ancient. Cf. p. 348 n.125.
[192] '32' is correct, since no. 21 is not counted, having already been recorded as part of Auriga.

	[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
	15	The star over the left foot	♈ 16⅚	+28	3	γ And
H82	16	The star in the right foot	♈ 17⅙	+37⅓	<4	φ Per
	17	The one south of the latter	♈ 15⅙	+35⅔	>4	51 And[168]
	18	The northernmost of the 2 stars on the left knee-bend	♈ 12⅓	+29	4	υ And
	19	The southernmost of them	♈ 12	+28	4	τ And
	20	The star on the right knee	♈ 10⅙	+35½	5	φ And
	21	The northernmost of the two stars in the lower hem [of the garment]	♈ 12⅔	+34½	5	*49(A) And
	22	The southernmost of them	♈ 14⅙	+32½	5	*χ And
	23	The star in advance of the three in the right hand, outside [of it]	♓ 11⅔	+44	3	o And
		[23 stars, 4 of the third magnitude, 15 of the fourth, 4 of the fifth]				
		[XXI] Constellation of Triangulum				
	1	The star in the apex of the triangle	♈ 11	+16½	3	α Tri
	2	The most advanced of the 3 on the base	♈ 16	+20⅔	3	β Tri
	3	The middle one of these	♈ 16⅓	+19⅔	4	δ Tri
	4	The rearmost of the three	♈ 16⅚	+19	3	γ Tri
		[4 stars, 3 of the third magnitude, 1 of the fourth]				
		[Total for the northern segment: 360 stars, 3 of the first magnitude, 18 of the second, 81 of the third, 177 of the fourth, 58 of the fifth, 13 of the sixth, 9 faint, 1 nebulous]				
		[Constellations in the zodiac]				
H84		[XXII] Constellation of Aries				
	1	The more advanced of the 2 stars on the horn	♈ 6⅔	+7⅓	<3	γ Ari
	2	The rearmost of them	♈ 7⅔	+8⅓	3	β Ari
	3	The northernmost of the 2 stars on the muzzle	♈ 11	+7⅔	5	η Ari
	4	The southernmost of them	♈ 11½	+6	5	θ Ari

[168] Also known as υ Persei, but within the constellation Andromeda according to the modern boundaries.

	[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
	5	The star on the neck	*♈ 6½[169]	+5½	5	ι Ari
	6	The star on the rump	♈ 17⅔	+6	6	ν Ari
	7	The star on the place where the tail joins [the body]	♈ 21⅓	+4⅚	5	ε Ari
	8	The most advanced of the 3 stars in the tail	♈ 23⅚	+1⅔	4	δ Ari
	9	The middle one of the three	♈ 25⅓	+2½	4	ζ Ari
	10	The rearmost of them	♈ 27	+1⅚	4	τ Ari
	11	The star in the back of the thigh[170]	♈ 19⅔	*+1½[171]	5	ρ Ari
	12	The star under the knee-bend	♈ 18	−1½	5	σ Ari
	13	The star on the hind hoof	♈ 15	−5¼	>4	*μ Cet
		13 stars, 2 of the third magnitude, 4 of the fourth, 6 of the fifth, 1 of the sixth]				
		Stars around Aries outside the constellation:				
	14	The star over the head, which Hipparchus [calls] 'the one on the muzzle'[172]	♈ 10⅔	*+10[173]	>3	α Ari
	15–18	The 4 stars over the rump:				
	15	the rearmost, which is brighter [than the others]	♈ 21⅔	+10⅙	4	41(c) Ari
H86	16	the northernmost of the other 3, fainter stars	♈ 21⅓	+12⅔	5	39 Ari
	17	the middle one of these three	♈ 19⅔	+11⅙	5	35 Ari
	18	the southernmost of them	♈ 19⅙	+10⅔	5	33 Ari
		[5 stars, 1 of the third magnitude, 1 of the fourth, 3 of the fifth]				
		[XXIII] Constellation of Taurus				
	1	The northernmost of the 4 stars in the cut-off[174]	♈ 26⅓	−6	4	5(f) Tau
	2	The one close by this	♈ 26	−7¼	4	4(s) Tau

[169] Manitius (see his note p. 401) changes to 6⅚ without ms. authority.
[170] B has ὀπισθίῳ μήρῳ ('the hind thigh'), which is also possible. However, the Arabic translations are based on ὀπισθομήρῳ, the reading adopted by Heiberg (see Kunitzsch, *Der Almagest* no. 261 p. 264).
[171] The variant 1⅙ is found in D,Ar, and is adopted by P–K.
[172] VII 1 p. 324 (which has, however, 'in (ἐν) the muzzle'), and Hipparchus, *Comm. in Arat.* I.6.9 (ed. Manitius 58,22–3).
[173] Reading ι (with D,Ar, adopted by P–K) for ι ∠´ (10½) at H85,18.
[174] ἀποτομή. Only the front half of the bull is represented. See e.g. Thiele Pl. IV, and compare the similar phrase for Argo (XL 32).

	[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
		[XXV] Constellation of Cancer				
	1	The middle of the nebulous mass in the chest, called Praesepe[207]	♋ 10⅓	*+0⅓[208]	neb.	CGal 2632 (Messier 44)
	2-5	The quadrilateral containing the nebula [no. 1]:				
	2	the northernmost of the two stars in advance	♋ 7⅓	+1¼	<4	η Cnc
	3	the southernmost of the two stars in advance	♋ 8	-1⅙	<4	θ Cnc
H96	4	the northernmost of the rear 2 stars on the quadrilateral, which are called 'Aselli'[209]	♋ 10⅓	+ 2⅔	>4	γ Cnc
	5	the southernmost of these two	♋ 11⅓	-0⅙	>4	δ Cnc
	6	The star on the southern claw	♋ 16½	-5½	4	α Cnc
	7	The star on the northern claw	♋ 8⅓	+11⅚	4	ι Cnc
	8	The star on the northern back leg	♋ 2⅔	+1	5	μ Cnc
	9	The star on the southern back leg	♋ 7⅙	*-7½[210]	>4	β Cnc
		[9 stars, 7 of the fourth magnitude, 1 of the fifth, 1 nebulous]				
		Stars around Cancer outside the constellation:				
	10	The star over the joint in the southern claw	*♋ 19⅔[211]	-2⅓	<4	*π Cnc
	11	The star to the rear of the tip of the southern claw	♋ 21⅙	-5⅔	<4	κ Cnc
	12	The more advanced of the two stars over the nebula and to the rear of it	♋ 14	*+7¼[212]	5	*ν Cnc
	13	The rearmost of these [two]	♋ 17	*+4⅚	5	*ξ Cnc
		[4 stars, 2 of the fourth magnitude, 2 of the fifth]				

[207] φάτνη ('manger'). Manitius and P–K identify this as ε Cnc, which is indeed in the middle of the galactic cluster, but Ptolemy is clearly not referring to an individual star.

[208] The variants 3 (B) and 0⅔, i.e. Γο (Ar) are found. The latter is adopted by P–K.

[209] ὄνοι ('asses').

[210] On the large error in latitude see P–K no. 457 p. 102.

[211] The variant 19⅙ is found in some Greek mss. (BC) and in the earlier Arabic tradition. According to S 40 the Isḥāq translation and Thābit's revision of it had 15⅙. Extant Arabic mss. (except for aṭ-Ṭūsī's revision, which has 15⅙) exhibit 19⅔. If we accept the latter, the most probable identification is π Cnc (adopted by Manitius). P–K adopt 15⅔(!) and identify the star as $o^1 + o^2$ Cnc.

[212] Following P–K and Manitius (who does it without comment), I have dubiously transposed the latitudes of nos. 12 and 13, which then fit the actual positions of ν and ξ Cnc fairly well. There is no ms. authority for this.

	[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
		[XXVI] Constellation of Leo				
	1	The star on the tip of the nostrils	♋ 18⅓	+10	4	κ Leo
	2	The star in the gaping jaws	♋ 21⅙	+7½	4	λ Leo
	3	The northernmost of the two stars in the head	♋ 24⅓	+12	3	μ Leo
H98	4	The southernmost of these	♋ 24⅙	+9½	>3	ε Leo
	5	The northernmost of the 3 stars in the neck	♌ 0⅙[213]	+11	3	ζ Leo
	6	The one close to this, the middle one of the three	♌ 2⅙	+8½	2	γ Leo
	7	The southernmost of them	♌ 0⅔	+4½	3	η Leo
	8	The star on the heart, called 'Regulus'	♌ 2½	+0⅙	1	α Leo
	9	The one south of this, about on the chest	♌ 3½	-1⅚	4	31(A) Leo
	10	The star a little in advance of the star on the heart [no. 8]	♌ 0	-0¼	5	ν Leo
	11	The star on the right knee	♋ 27⅓	0	5	ψ Leo
	12	The star on the right front claw-clutch[214]	♋ 24⅙	-3⅔	*5[215]	ξ Leo
	13	The star on the left front claw-clutch	♋ 27⅓	-4⅙	4	ο Leo
	14	The star on the left [front] knee	♌ 2½	-4¼	4	π Leo
	15	The star on the left armpit	♌ 9⅙	-0⅙	4	ρ Leo
	16	The most advanced of the three stars in the belly	♌ 7	+4	6	46(i) Leo
	17	The northernmost of the other, rearmost 2	♌ 10⅓[216]	+5⅓	6	52(k) Leo
	18	The southernmost of these [two]	♌ 12⅙	+2⅓	6	53(l) Leo
	19	The more advanced of the two stars on the rump	♌ 11⅓	+12¼	*6[217]	60(b) Leo
	20	The rearmost of them	♌ 14⅙	+13⅔	<2	δ Leo
	21	The northernmost of the 2 stars in the buttocks	*♌ 14⅓	+11⅙	5	*81 Leo[218]
H100	22	The southernmost of them	♌ 16⅓	+9⅔	3	θ Leo

[213] The variant 4⅙ occurs in the early Arabic tradition according to S 41.

[214] δράξ, literally 'grasping hand'. The lion is represented with claws out and hooked, as in Thiele Fig. 26 p. 99.

[215] All mss. except D give magnitude 6 here. Heiberg adopts 5 to reach agreement with the sub-total for the constellation. P–K adopt 6 here and 5 at no. 19 (from the Arabic), perhaps rightly.

[216] The variant 13 occurs in the Arabic tradition (see S 42).

[217] Cf. n. 215 on no. 12. All Greek mss. have 6, but the Arabic tradition is unanimous for 5. If correctly identified as 60 Leonis, this star has, by modern definition and measurement, magnitude 4.4.

[218] The identification is extremely uncertain: see P–K on no. 482, pp. 102-3. 81 Leonis is possible only if the longitude is emended to be greater than that of no. 22, for which there is no authority.

[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
	Stars around Taurus outside the constellation:				
34	The star under the right foot and[193] the shoulder-blade	♈ 25	−17⅓	4	10 Tau
35	The most advanced of the 3 stars over the southern horn	♉ 20[194]	−2	5	ι Tau
36	The middle one of the three	•♉ 24[195]	−1¾	5	109(n) Tau
37	The rearmost of them	♉ 26	−2	5	114(o) Tau
38	The northernmost of the 2 stars under the tip of the southern horn	♉ 29	−6⅓	5	•126 Tau
39	The southernmost of them	♉ 29	−7⅔	5	•129 Tau
40–44	The 5 stars under and to the rear of the northern horn:				
40	the most advanced	♉ 27	+0⅔	5	•121 Tau
41	the one to the rear of this	♉ 29	+1	5	•125 Tau
42	the one to the rear again of the latter	♊ 1	+1⅓	5	•132 Tau
43	the northernmost of the remaining, rearmost 2	♊ 2⅓	+3⅓	5	•136 Tau
44	the southernmost of these two	♊ 3⅓	+1¼	5	•139 Tau
	{11 stars, 1 of the fourth magnitude, 10 of the fifth}				
H92	[XXIV] Constellation of Gemini				
1	The star on the head of the advance twin	♊ 23⅓	•+9½[196]	2	α Gem
2	The reddish star on the head of the rear twin	♊ 26⅔	+6¼	2	β Gem
3	The star in the left forearm of the advance twin	♊ 16⅔	+10	4	θ Gem
4	The star in the same [left] upper arm	♊ 18⅔	+7⅓	4	τ Gem
5	The one to the rear of that, just over the place between the shoulders	♊ 22	+5½	4	ι Gem
6	The one to the rear of this, on the right shoulder of the same [advance] twin	♊ 24	+4⅚	4	υ Gem
7	The star on the rear shoulder of the rear twin	♊ 26⅔	+2⅔	4	κ Gem
8	The star on the right side of the advance twin	♊ 21⅔	+2⅔	5	57(A) Gem
9	The star on the left side of the rear twin	•♊ 23⅙[197]	•+0⅓[198]	5	•58 Gem

[193] Tallgren (see Kunitzsch, *Der Almagest* no. 295 p. 270) suggested emending καὶ at H90,8 to κατὰ ('opposite the shoulder-blade'), following the translation of al-Ḥajjāj. This may be correct, but the rest of the Arabic tradition is based on καὶ (see Kunitzsch, ibn aṣ-Ṣalāḥ p. 59 n.91).
[194] The variant 16 is found in the earlier Arabic tradition according to S 37.
[195] Reading κδ (with A¹D and the later Arabic tradition, see S 38) for κα (21) at H91,10. Corrected by Manitius.
[196] D,Ar have the variant 9⅔, adopted by P–K.
[197] Most Greek mss. have 26⅙. Heiberg's text is the reading of D,Ar. The identification of this star is very uncertain.
[198] Most mss., both Greek and Arabic, have 3. γ´ (⅓) appears in C and as a variant in A¹, and is adopted by Heiberg and P–K.

[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
10	The star on the left knee of the advance twin	♊ 13	+1½	3	ε Gem
11	The star under[199] the left knee of the rear twin	•♊ 18¼[200]	−2½	3	ζ Gem
12	The star in the left groin of the rear twin	♊ 21⅔	−0½	3	δ Gem
13	The star over the bend in the right knee[201] of the same [rear] twin	•♊ 21⅔[202]	•−6[203]	3	λ Gem
14	The star on the forward foot[204] of the advance twin	♊ 6½	−1½	>4	η Gem
15	The one to the rear of this on the same foot	•♊ 8½[205]	−1¼	>4	μ Gem
16	The star on the right foot of the advance twin	♊ 10⅙	−3½	>4	ν Gem
17	The star on the left foot of the rear twin	♊ 12	−7½	3	γ Gem
H94 18	The star on the right foot of the rear twin	♊ 14⅔	−10½	4	ξ Gem
	{18 stars, 2 of the second magnitude, 5 of the third, 9 of the fourth, 2 of the fifth}				
	Stars around Gemini outside the constellation:				
19	The star in advance of the forward foot of the advance twin	♊ 4⅙	−0⅔	4	1(H) Gem
20	The bright star in advance of the advance knee	♊ 6½	+5⅚	>4	κ Aur
21	The star in advance of the left knee of the rear twin	♊ 15⅙	−2¼	5	36(d) Gem
22	The northernmost of the three stars in a straight line to the rear of the right arm of the rear twin	♊ 28⅓	−1⅓	5	•85 Gem
23	The middle one of the three	♊ 26⅓	−3⅓	5	•81(g) Gem
24	The southernmost of them, near the forearm of the [right] arm	♊ 26	−4½	5	•74(f) Gem
25	The bright star to the rear of the above-mentioned 3	•♋ 0⅔[206]	−2⅔	4	•ζ Cnc
	{7 stars, 3 of the fourth magnitude, 4 of the fifth}				

[199] D has ὑπὲρ ('over'), and this was the reading behind the Arabic ('fawqa').
[200] P–K emend to 18⅙ (the reading of Ger; the rest of the Arabic tradition has 18¼), on the grounds that the fraction ¼ is not used in the longitudes (the only other examples are XXVII 17 and 29 and XXXIII 36). But in X 1 (p. 469) Ptolemy gives the longitude of this star as 18¼°, and that position is confirmed by his subsequent computations.
[201] ἀγκύλη. This would normally mean 'elbow', and is so translated by Manitius. But the position of the star on the figure shows that it must be on the leg, and therefore we must refer it to the bend in the leg (as in animal figures, e.g. Aries, XXII 12).
[202] The variant 21⅓ is found in D,Ar.
[203] This is the reading of D,Ar ('5', F). Most Greek mss. have ∠ ς´ (½ + ⅙), which is very strange, as ⅔ is normally written as ⳼.
[204] 'forward foot': πρόπους, also used e.g. as the spur of a mountain. The twin is depicted with one foot (or leg) advanced before the other. πρόπους was used as a name for this particular star, see Hipparchus, *Comm. in Arat.* 3.4.12 (ed. Manitius 268,28).
[205] 8⅙ D,Ar, adopted by P–K.
[206] D has 3, adopted by Manitius. P–K adopt 5⅔ (from Ger). All the Arabic mss. I have seen have 0⅔.

[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
20	The star on the left knee	♎ 1⅔	−1½[239]	5	86 Vir
21	The star in the back of the right thigh	♍ 28	+8½	5	*90(p) Vir
22	The middle star of the 3 in the garment-hem round the feet	*♎ 6⅔	+7½[240]	4	ι Vir
23	The southernmost of them	♎ 7⅓	+2⅔	4	κ Vir
24	The northernmost of the three	♎ 8⅓	+11⅔	4	φ Vir
25	The star on the left, southern foot	♎ 10	+0½	4	λ Vir
26	The star on the right, northern foot	♎ 12⅔	+9⅚	4	μ Vir
	{26 stars, 1 of the first magnitude, 6 of the third, 7 of the fourth, 10 of the fifth, 2 of the sixth}				
	Stars around Virgo outside the constellation:				
27	The most advanced of the three in a straight line under the left forearm	♍ 14⅔	−3½	5	χ Vir
28	The middle one of these	♍ 19	−3½	5	ψ Vir
29	The rearmost of the 3	*♍ 22¼	−3⅓	5	49 Vir
30	The most advanced of the 3 stars almost on a straight line under Spica	♍ 27⅙	*−7⅙[241]	6	53 Vir
31	The middle one of these, which is a double star	♍ 28⅙	−8⅓	5	*61 + 63 Vir[242]
32	The rearmost of the three	♎ 5	−7⅚	6	89 Vir
	{6 stars, 4 of the fifth magnitude, 2 of the sixth}				

[239] In part of the Arabic tradition the latitude is northerly (see S 52).
[240] Longitude: 6⅔ is the reading of D,Ar. Most Greek mss. have 6⅓. Latitude: following P–K, I read ζ ∠′ (with all mss. except D) for ζ ς′ (7⅙) at H105,7.
[241] This is the reading of the Greek mss. P–K adopt 7⅓, found as a variant in the Arabic (L,E,T[2], Ger).
[242] For this identification of the 'double star' (which it is not), see P–K no. 527 on p. 104. It is extremely dubious.

VIII 1. Constellation XXVIII : Libra

1 {*Tabular layout of the constellations in the southern hemisphere*}

	[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
		[XXVIII] Constellation of Libra[1]				
	1-2	Stars on the tip of the southern claw:				
	1	the bright one	♎ 18	+0⅔	2	α Lib
	2	the star to the north of this and fainter than it	♎ 17	+2½	5	μ Lib
	3-4	Stars on the tip of the northern claw:				
	3	the bright one	♎ 22⅙	+8⅚	2	β Lib
	4	the faint star in advance of this	♎ 17⅔	+8½	5	δ Lib
	5	The star in the middle of the southern claw	♎ 24	−1⅔	4	ι Lib
H108	6	The one in advance of this on the same claw	♎ 21⅓	+1¼	4	ν Lib
	7	The star in the middle of the northern claw	♎ 27⅚	+4¾	4	γ Lib
	8	The one to the rear of this on the same claw	♏ 3	+3½	<4	θ Lib
		{8 stars, 2 of the second magnitude, 4 of the fourth, 2 of the fifth}				
		Stars around Libra outside the constellation:				
	9	The most advanced of the 3 stars north of the northern claw	♎ 26⅙	+9	5	37 Lib
	10	The southernmost of the rearmost 2 [of these]	♏ 3⅔	+6⅔	<4	48 Lib
	11	The northernmost of them	♏ 4⅓	+9¼	<4	ξ Sco
	12	The rearmost of the 3 stars between the claws	♏ 3⅓	+0½	6	λ Lib
	13	The northernmost of the other 2 in advance [of the latter]	♏ 0⅔	*+0⅓[2]	5	*41 Lib[3]

[1] χηλαί, literally 'claws' (of Scorpius). Both ζυγός ('balance', hence Libra) and χηλαί are found in the Greek texts, but Ptolemy always uses the latter except at IX 7 (H267,14), which is a quotation from an earlier observation. See Boll-Gundel cols. 963-5.
[2] The variant 3 is found in the Greek ms. B and in part of the Arabic tradition (see S 53).
[3] The identification of nos. 13 and 14 is highly uncertain. The stars I have designated are in approximately the same relative positions as Ptolemy indicates. But, if the identifications are correct, why does Ptolemy mention κ Lib? P–K identify as κ Lib and 0[h] Arg 14782 (which is *BSC* 5810, adopted by me), Manitius as 41 Lib and κ Lib. Another problem is the magnitudes: κ is 4.72, 41 is 5.38, and *BSC* 5810 only 5.94.

	[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
	23	The star in the hind thighs[219]	♌ $20\frac{1}{3}$	$+5\frac{5}{6}$	3	ι Leo
	24	The star in the hind leg-bends	♌ $21\frac{2}{3}$[220]	$+1\frac{1}{4}$	4	σ Leo
	25	The one south of this, about in the lower legs	♌ $24\frac{2}{3}$	$-0\frac{5}{6}$	4	τ Leo
	26	The star on the hind claw-clutches	♌ $27\frac{1}{2}$	*$-3\frac{1}{6}$[221]	5	υ Leo
	27	The star on the end of the tail	♌ $24\frac{1}{2}$	$+11\frac{5}{6}$	<1	β Leo
		{27 stars, 2 of the first magnitude, 2 of the second, 6 of the third, 8 of the fourth, 5 of the fifth, 4 of the sixth}				
		Stars around Leo outside the constellation:				
	28	The more advanced of the 2 over the back	♌ 6[222]	$+13\frac{1}{3}$	5	41 LMi
	29	The rearmost of them	♌ $8\frac{1}{6}$	$+15\frac{1}{2}$	5	54 Leo
	30	The northernmost of the 3 under the flank	♌ $17\frac{1}{3}$	$+1\frac{1}{6}$	<4	χ Leo
	31	The middle one of these	♌ $17\frac{1}{6}$	$-0\frac{1}{3}$	5	59(c) Leo
	32	The southernmost of them	♌ 18	$-2\frac{2}{3}$	5	58(d) Leo
	33	The northernmost part of the nebulous mass between the edges of Leo and Ursa [Major], called Coma [Berenices][223]	♌ $24\frac{5}{6}$	+30	*f.[224]	*15(c) Com
	34	The most advanced of the southern outrunners of Coma	♌ $24\frac{1}{3}$	+25	f.	*7(h) Com
	35	The rearmost of them, shaped like an ivy leaf	♌ $28\frac{1}{2}$	$+25\frac{1}{2}$	f.	*23(k) Com
		{5 stars, 1 of the fourth magnitude, 4 of the fifth, plus Coma}				
H102		[XXVII] Constellation of Virgo				
	1	The southernmost of the 2 stars in the top of the skull	*♌ $26\frac{1}{3}$[225]	$+4\frac{1}{4}$	5	ν Vir
	2	The northernmost of them	*♌ 27	$+5\frac{2}{3}$	5	ξ Vir

[219] The lion is represented with both hind legs together. Cf. nos. 24 to 26, and e.g. Thiele Fig. 26 p. 99.
[220] The variant $24\frac{2}{3}$ occurs in the Greek (A^1D) and later Arabic traditions (see S 43).
[221] Reading γ ς´ (with D, adopted by Manitius) for γ ε´ ($3\frac{1}{5}$) at H101,6. The latter fraction would be unique in the whole catalogue. The Arabic tradition (see S 44) varies between $0\frac{1}{3}$ and 3. P–K adopt the latter, which might be correct.
[222] The variant $0\frac{1}{6}$ occurs in the later Arabic tradition (see S 45).
[223] One can make out many of the stars of this cluster with the naked eye. But it is dubious whether one should identify the points named by Ptolemy with individual stars, as I have done following Manitius and P–K. For here Ptolemy uses the neuter (τὸ βορειότατον), not the masculine (which would imply ἀστήρ, 'star'). The group was named πλόκαμος ('lock') in honour of the lock of Berenice by Conon: see the poem of Callimachus, *Aetia* fr. 110.
[224] On the peculiar designation of the magnitude in most Greek mss., namely ἀμαυρός ('faint') with λαμπρός ('bright') over it, see P–K p. 103.
[225] The longitude of no. 1 should be greater than that of no. 2, but the only alternative ms. reading for the longitude of no. 1, 25 (A^1BC) is even smaller. Hence P–K interchange the longitudes of the two stars. Manitius (p. 403) would prefer to correct the longitude of no. 2 to 26.

	[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
	3	The northernmost of the 2 stars to the rear of these, in the face	♍ $0\frac{2}{3}$	+8	5	ο Vir
	4	The southernmost of them	♍ $0\frac{1}{3}$	$+5\frac{1}{2}$	5	π Vir
	5	The star on the tip of the southern, left wing	♌ 29	*$+0\frac{1}{6}$[226]	3	β Vir
	6	The most advanced of the 4 stars in the left wing	♍ $8\frac{1}{3}$	*$+1\frac{1}{6}$[227]	3	η Vir
	7	The one to the rear of this	♍ $13\frac{1}{6}$	$+2\frac{5}{6}$	3	γ Vir
	8	The one to the rear again of this	♍ $17\frac{1}{2}$	*$+2\frac{5}{6}$[228]	5	*46 Vir
	9	The last and rearmost of the 4	♍ 21	$+1\frac{2}{3}$	4	θ Vir
	10	The star in the right side under the girdle	♍ $14\frac{1}{3}$[229]	$+8\frac{1}{2}$	3	δ Vir
	11	The most advanced of the 3 stars in the right, northern wing	♍ $8\frac{1}{6}$	*$+13\frac{1}{2}$[230]	5	ρ Vir
	12	The southernmost of the other 2	♍ $10\frac{1}{6}$[231]	$+11\frac{2}{3}$	6	32(d^2) Vir
	13	The northernmost of these, called 'Vindemiatrix'[232]	♍ $12\frac{1}{6}$	*$+15\frac{1}{6}$[233]	>3	ε Vir
	14	The star on the left hand, called 'Spica'[234]	♍ $26\frac{2}{3}$	−2	1	α Vir
	15	The star under the apron,[235] just about over the right buttock	♍ $24\frac{5}{6}$	$+8\frac{2}{3}$	3	ζ Vir
	16–19	The quadrilateral in the left thigh:[236]				
	16	the northern star on the advance side	♍ $26\frac{1}{3}$	$+3\frac{1}{3}$	5	*74(l^2) Vir
H104	17	the southern star on the advance side	*♍ $27\frac{1}{4}$	$+0\frac{1}{6}$[237]	6	*76(h) Vir
	18	the northernmost of the 2 stars on the rear side	♎ 0	$+1\frac{1}{2}$	<4	*82(m) Vir
	19	the southernmost star on the rear side	♍ 28	−3[238]	5	*68(i) Vir

[226] Reading ς´ at H103,7 for γ´ ($0\frac{1}{3}$), the reading of D. $0\frac{1}{6}$ is the reading of the Greek mss. BC (confirmed by CCAG I cod. 12 f.142^b, 13) and the later Arabic tradition (see S 46). It is adopted by P–K. A and the rest of the Arabic tradition have 6.
[227] Reading α ς´ (with all mss. except D) for α ∠´ ($1\frac{1}{2}$) at H103,8. Corrected by P–K.
[228] Reading β ∠´ γ´ (with all mss. except D, '$2\frac{1}{2}$' and T, '2;10') for β ∠´ ($2\frac{1}{2}$) at H103,10. Corrected by P–K.
[229] The variant $11\frac{1}{3}$ occurs in the early Arabic tradition according to S 47.
[230] The variant $13\frac{5}{6}$, adopted by P–K, is the reading of Ar.
[231] The variant 16 is found in the Greek (A^1BC) and later Arabic traditions (see S 48).
[232] προτρυγητήρ, 'the harbinger of vintage'.
[233] This is the reading of D,Ar. The other Greek mss. have $20\frac{1}{6}$. P–K adopt 16 on no ms. authority.
[234] στάχυς, an ear of wheat or other cereal.
[235] περίζωμα (clothing worn about the loins), probably different from the ζώνη (girdle) in no. 10.
[236] P–K and Manitius agree on the identification of these four stars, but as Manitius points out (p. 403), it is hard to see them as forming any kind of a 'quadrilateral'. To remedy this P–K (no. 515 p. 104) suggest an implausible interchange in the coordinates of nos. 19 and 20.
[237] The variant 6 is found in part of the Arabic tradition (see S 49).
[238] The variant $0\frac{1}{3}$ is found in the Arabic tradition (see S 51).

[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
10	The middle one of these	♐ 17⅔	+1½	4	ο Sgr
H114 11	The rearmost of the three	♐ 19⅙	+2	4	π Sgr
12	The southernmost of the 3 stars in the northern cloak-attachment[13]	♐ 21⅓	+2⅚	5	43(d) Sgr
13	The middle one of these	♐ 22⅓	+4½	4	ρ Sgr
14	The northernmost of the three	♐ 22⅚	+6½	4[14]	υ Sgr
15	The faint star to the rear of these three	*♐ 25⅔[15]	+5½	6	55(e) Sgr
16	The northernmost of the 2 stars on the southern cloak-attachment	♐ 29½	+5⅚	5	61(g) Sgr
17	The southernmost of them	♐ 27⅔	+2	6	*57 Sgr
18	The star on the right shoulder	*♐ 22⅔[16]	−1⅚	5	*χ^1 Sgr
19	The star on the right elbow	♐ 24⅚	−2⅚	4	*51(h^1) + 52(h^2) Sgr
20–22	The three stars in the back:				
20	the one just above the place between the shoulders	♐ 20	−2½	5	ψ Sgr
21	the middle one, just above the shoulder-blade	♐ 17⅔	−4½	>4	τ Sgr
22	the other one, under the armpit	♐ 16⅓	−6¾	3	ζ Sgr
23	The star on the front left hock	♐ 17⅔	−23	2	*$\beta^1 + \beta^2$ Sgr
24	The one on the knee of the same leg	♐ 17	−18	<2[17]	α Sgr
25	The star on the front right hock	♐ 6⅔	−13	3	η Sgr
26	The star on the left thigh	♐ 27⅓	−13½	3	*$\kappa^1 + \kappa^2$ Sgr

[13] ἐφαπτίς. This word is mistreated in the dictionaries. It is a piece of cloth which was attached (hence the name) to a mounted soldier's cloak at the shoulder, and which was, in theory, used to wrap round the arm as a guard (defined by Pollux IV 116, ed. Bethe I 235, συστρεμμάτιον πορφυροῦν ἢ φοινικοῦν, ὃ περὶ τὴν χεῖρα εἶχον οἱ πολεμοῦντες ἢ οἱ θηρῶντες), but in practice was largely decorative, being often of purple or embroidered (see Athenaeus V 194f and 196f, passages from Hellenistic authors), and streaming free from the shoulder as the wearer galloped. This is how it (or they, one on each shoulder) appeared in the depictions of Sagittarius (e.g. Thiele Fig. 42 on p. 117), where they may be a Greek adaptation of the wings of a Babylonian original (see e.g. King, *Babylonian Boundary Stones* Pl. XXIX A; but it is only a plausible conjecture that this figure represents a constellation. See Seidl, *Kudurru-Reliefs* 177, with further literature). This attribute of Sagittarius is as early as Hipparchus (e.g. *Comm. in Arat.* 2.5.16, ed. Manitius 198,27). I do not know whether Hephaestion (ed. Pingree 1,3,10), in referring to 'wings or cloak-attachments', preserves a Babylonian tradition or is misinterpreting a picture of Sagittarius.
[14] S 56 records the variant 1 (!) in the earlier Arabic tradition.
[15] 25⅓ in some Greek mss. 25⅔ in AD,Ar.
[16] Ar has 22⅓, adopted by P–K.
[17] For the variants 3 and 4 in the Arabic tradition see S 58.

[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
27	The star on the right hind lower leg	*♐ 26⅚[18]	*−20⅙[19]	3	ι Sgr
28–31	The four stars [forming a quadrilateral] in the place where the tail joins [the body]:				
28	the advance star on the northern side	*♐ 27⅔[20]	−4⅚	5	ω Sgr
29	the rear star on the northern side	♐ 28⅚	−4⅚	5	60(A) Sgr
30	the advance star on the southern side	♐ 28⅚	−5⅚	5	59(b) Sgr
H116 31	the rear star on the southern side	♐ 29⅔	−6½	5	62(c) Sgr
	[31 stars, 2 of the second magnitude, 9 of the third, 9 of the fourth, 8 of the fifth, 2 of the sixth, [1] nebulous]				
	[XXXI] Constellation of Capricornus				
1	The northernmost of the 3 stars in the rear horn	♑ 7⅓	+7⅓	3	$\alpha^1 + \alpha^2$ Cap
2	The middle one of these	♑ 7⅔	+6⅔	6	ν Cap
3	The southernmost of the three	♑ 7⅓	+5	3	β Cap
4	The star on the tip of the advance horn	*♑ 5[21]	+8	6	$\xi^1 + \xi^2$ Cap
5	The southernmost of the 3 stars in the muzzle	♑ 9	+0¾	6	ο Cap
6	The more advanced of the other two	♑ 8⅔	+1¾	6	π Cap
7	The rearmost of these	♑ 8⅚	+1½	6	ρ Cap
8	The star in advance of the [above] 3, under the right eye	♑ 6⅙	+0⅔	5	σ Cap
9	The northernmost of the 2 stars in the neck	♑ 11⅔	+3⅚	6	τ Cap
10	The southernmost of them	♑ 11⅚	*+0⅚[22]	5	υ Cap
11	The star on the left, doubled-up knee[23]	♑ 11⅔	−8⅔	4	ω Cap
12	The star under the right knee	♑ 10⅚	−6½	4	ψ Cap
13	The star on the left shoulder	♑ 16⅔	−7⅔	4	24(A) Cap
14	The more advanced of the 2 stars close together under the belly	♑ 20⅙	−6⅚	4	ζ Cap

[18] Reading κς ∠′ γ′ (with Ar, adopted by P–K) for κγ ∠′ γ′ (23⅚) at H115,18.
[19] Most Greek mss. have κς (26). 20⅙ is the reading of Ar (except for T^1, which agrees with D in 4⅚).
[20] This is the reading of D,Ar. Most Greek mss. have 27⅓.
[21] This is the reading of D and most of the Arabic tradition (L,T,F). The other Greek mss. and some Arabic (Ger) have 9.
[22] This is the reading of D,Ar. Other Greek mss. have ∠′ ς′ (½ + ⅙), but that is not the way ⅔ is normally written.
[23] Compare Thiele Fig. 41 on p. 116, where, however, it is the *right* knee which is doubled up (cf. Introduction p. 15).

[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
14	The southernmost of them	♏ 1⅙	−1½	4	*BSC 5810
15	The most advanced of the 3 stars south of the southern claw	♎ 23	−7½	3	20 Lib
16	The northernmost of the other, rear 2	♏ 1⅙	*−8½[4]	4	39 Lib
17	The southernmost of them	♏ 2	−9⅔	4	40 Lib
	[9 stars, 1 of the third magnitude, 5 of the fourth, 2 of the fifth, 1 of the sixth]				
	[XXIX] Constellation of Scorpius				
1	The northernmost of the 3 bright stars in the forehead	♏ 6⅓	+1⅓	3	β Sco
H110 2	The middle one of these	♏ 5⅔	−1⅔	3	δ Sco
3	The southernmost of the three	♏ 5⅔	−5	3	π Sco
4	The star south again of this, on one of the legs	♏ 6	−7⅚	3	ρ Sco
5	The northernmost of the 2 stars adjacent to the northernmost of the [3] bright ones [no. 1]	♏ 7	+1⅔	4	ν Sco
6	The southernmost of these	♏ 6⅓	+0½	4	*ω¹+ω² Sco
7	The most advanced of the 3 bright stars in the body	♏ 10⅔	−3¾[5]	3	σ Sco
8	The middle one of these, which is reddish and called 'Antares'	♏ 12⅔	−4	2	α Sco
9	The rearmost of the 3	♏ 14½	−5½	3	τ Sco
10	The advance star of the 2 under these, approximately on the last leg	♏ 9⅓	*−6½[6]	5	13(c²) Sco
11	The rearmost of these	♏ 10⅓	−6⅔	5	d Sco = BSC 6070
12	The star in the first [tail-] joint from the body	♏ 18½	−11	3	ε Sco
13	The one after this, in the 2nd joint	♏ 18⅚	−15	3	μ¹ + μ² Sco
14	The northern star of the double-star in the 3rd joint	*♏ 20⅙	*−18	4	*ζ² Sco[7]
15	The southern star of the double-star	*♏ 20	*−18⅔	4	*ζ¹ Sco
16	The one following, in the 4th joint	♏ 23⅙	−19½	3	η Sco

[4] This is the reading of all Greek mss. except D, which has 8⅓ (so too Ar; adopted by P–K).
[5] S 54 records the variant 6¾ in the Syriac version.
[6] The variant 6⅓ is found in D,Ar.
[7] It is generally agreed that nos. 14 and 15 are to be identified with ζ¹ and ζ² Sco, but it is not clear which is which. Furthermore what Ptolemy calls the southern one has (in the mss.) a more northerly latitude (−18°) than what he calls the northern one (−18⅔°). I have therefore, dubiously, reversed the data of 14 and 15. Manitius reverses the latitudes only. P–K (no. 560 on p. 105) identify 14 as ζ¹ and 15 as ζ², emending −18 to −19. Everything is uncertain.

[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
17	The one after that, in the 5th joint	♏ 28⅙	−18⅚	3	θ Sco
18	The next one again, in the 6th joint	♐ 0½	−16⅔	3	ι¹ Sco
19	The star in the 7th joint, the joint next to the sting	♏ 29	−15⅙	3	κ Sco
20	The rearmost of the 2 stars in the sting	♏ 27½	−13⅓	3	λ Sco
H112 21	The more advanced of these	♏ 27	−13½	4	υ Sco
	[21 stars, 1 of the second magnitude, 13 of the third, 5 of the fourth, 2 of the fifth]				
	Stars around Scorpius outside the constellation:				
22	The nebulous star to the rear of the sting	♐ 1⅙	−13¼	neb.	G Sco + CGlo 6441[8]
23	The most advanced of the 2 stars to the north of the sting	♏ 25½	−6⅙	>5	45(d) Oph
24	The rearmost of them	*♏ 29½[9]	*−4⅙[10]	5	*3 Sgr
	[3 stars, 2 of the fifth magnitude, 1 nebulous]				
	[XXX] Constellation of Sagittarius[11]				
1	The star on the point of the arrow	♐ 4½	−6½	3	γ Sgr
2	The star in the [bow-]grip held by the left hand	♐ 7⅔	−6½	3	δ Sgr
3	The star in the southern portion of the bow	♐ 8	−10⅚	3	ε Sgr
4	The southernmost of the [2] stars in the northern portion of the bow	♐ 9	−1½	3	λ Sgr
5	The northernmost of these, on the tip of the bow	♐ 6⅔	+2⅚	4	μ Sgr
6	The star on the left shoulder	♐ 15⅓	−3⅒	3	σ Sgr
7	The one in advance of this, just over the arrow	♐ 13	*−3½[12]	4	φ Sgr
8	The star on the eye, which is nebulous and double	♐ 15⅙	+0¼	neb.	ν¹ + ν² Sgr
9	The most advanced of the 3 stars in the head	♐ 15⅔	+2⅙	4	ξ² Sgr

[8] Manitius identifies this as G Scorpii, P–K as γ Telescopii, an obsolete designation which is the same as G Scorpii (*BSC* 6630). But the description 'nebulous' obviously includes the globular cluster (cf. P–K no. 567 p. 105 and Burnham III 1689).
[9] Reading κθ ∠′ (with Ar) for κε ∠′ (25½) at H113,7. This correction, adopted by P–K, is confirmed by the description ('to the rear', cf. no. 23).
[10] This is the reading of D,Ar. Most Greek mss. have 1⅙. The identification depends on the coordinates one adopts With those of the translation, 3 Sgr (adopted by P–K) seems right.
[11] The archer, represented as a centaur.
[12] The variant 3⅚, found in the Arabic (L,T²,E, Ger), is adopted by P–K.

	[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
H122	24	the one next to the latter towards the south	♒ $14\frac{5}{6}$	$+0\frac{1}{6}$	4	λ Aqr
	25	the one next to this, after [the beginning of] the bend[36]	♒ $17\frac{2}{3}$	$-1\frac{1}{6}$	4	83(h) Aqr
	26	the one to the rear again of this	♒ 20	$-0\frac{1}{2}$	4	φ Aqr
	27	the one in the bend to the south of this	♒ $20\frac{1}{2}$[37]	$-1\frac{2}{3}$	4	χ Aqr
	28	the northernmost of the 2 stars to the south of this	♒ 19	$-3\frac{1}{2}$	4	ψ^1 Aqr
	29	the southernmost of the two	♒ $19\frac{5}{6}$	$-4\frac{1}{6}$	4	*ψ^3 Aqr
	30	the lone star at some distance from these [two] towards the south	♒ $20\frac{5}{6}$	$-8\frac{1}{4}$	5	*BSC 8958[38]
	31	the more advanced of the 2 stars close together after the latter	*♒ $22\frac{2}{3}$[39]	−11	5	ω^1 Aqr
	32	the rearmost of them	♒ $23\frac{1}{6}$	$-10\frac{5}{6}$	5	ω^2 Aqr
	33	the northernmost of the 3 stars in the next group	♒ $21\frac{2}{3}$	−14	5	*103(A^1) + 104(A^2) Aqr
	34	the middle one of the three	♒ $22\frac{1}{6}$	$-14\frac{3}{4}$	5	106(i^1) Aqr
	35	the rearmost of them	♒ $23\frac{1}{6}$	$-15\frac{2}{3}$	5	*108(i^3) Aqr
	36	the northernmost of the next 3 [arranged] likewise	♒ 17	$-14\frac{1}{6}$	4	98(b^1) Aqr
	37	the southernmost of the three	♒ $18\frac{1}{3}$	$-15\frac{3}{4}$	4	101(b^3) Aqr
	38	the middle one of the three	♒ $17\frac{1}{2}$	−15	4	99 (b^2) Aqr
	39	the most advanced of the 3 stars in the remaining group	♒ $11\frac{5}{6}$	*$-14\frac{3}{4}$[40]	4	86(c^1) Aqr
	40	the southernmost of the other 2	♒ $12\frac{1}{3}$	$-15\frac{1}{3}$	4	89(c^3) Aqr
	41	the northernmost of them	♒ $13\frac{1}{6}$	−14	4	88(c^2) Aqr
H124	42	the star at the end of the water and on the mouth of Piscis Austrinus	♒ 7	$-20\frac{1}{3}$[41]	1	α PsA
		[42 stars, 1 of the first magnitude, 9 of the third, 18 of the fourth, 13 of the fifth, 1 of the sixth]				

[36] This is the best sense I can make of μετὰ τὴν καμπήν. μετά here cannot be the equivalent of ἑπόμενος, as in Pisces (XXXIII) no. 29, and the situation of the star forbids us to translate 'after the bend', for the star actually in the bend comes later (no. 27).

[37] The variant $10\frac{1}{2}$ was in the Syriac version according to S 62.

[38] This identification is my proposal. P–K prefer 94 Aquarii, but this involves so great a longitudinal error that Peters had to emend the longitude to $17\frac{5}{6}$, on no authority.

[39] This is the reading of D,Ar; most Greek mss. have $22\frac{1}{3}$.

[40] The identification of nos. 39-41 is not in doubt. But the latitude of 86 Aqr is considerably to the south of 89 Aqr. Hence Manitius (p. 405) interchanges the latitudes of nos. 39 and 40. P–K, more plausibly, emend the latitude of 39, but their emendation, $16\frac{1}{4}$, is palaeographically implausible as well as without authority (the only plausible variant is $14\frac{5}{6}$ in D,Ar, which is still too small). Against making any change is Ptolemy's description. If no. 39 is indeed south of no. 40, why did he not simply describe it as the southernmost of the three? Probably he got the latitude of no. 39 wrong.

[41] The variant 23 is found in the Greek (D) and Arabic traditions (see S 63).

	[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
		Stars round Aquarius outside the constellation:				
	43	The most advanced of the 3 stars to the rear of the bend in the water	♒ $26\frac{2}{3}$	$-15\frac{1}{2}$	>4	2 Cet
	44	The northernmost of the other 2	♒ $29\frac{2}{3}$	$-14\frac{2}{3}$	>4	6 Cet
	45	The southernmost of them	♒ 29	$-18\frac{1}{4}$	>4	7 Cet
		[3 stars of magnitude greater than the fourth]				
		[XXXIII] Constellation of Pisces				
	1	The star in the mouth of the advance fish	♒ $21\frac{2}{3}$	$+9\frac{1}{4}$	*4[42]	β Psc
	2	The southernmost of the 2 stars in the top of its head	♒ $24\frac{1}{6}$	$+7\frac{1}{2}$[43]	4	γ Psc
	3	The northernmost of them	♒ 26	$+9\frac{1}{3}$	4	7(b) Psc
	4	The more advanced of the 2 stars in the back	♒ $28\frac{1}{6}$	$+9\frac{1}{2}$	4	θ Psc
	5	The rearmost of them	♓ $0\frac{2}{3}$	$+7\frac{1}{2}$	4	ι Psc
	6	The more advanced of the 2 stars in the belly	♒ 26	$+4\frac{1}{2}$	4	κ Psc
	7	The rearmost of them	♒ $29\frac{2}{3}$	$+3\frac{1}{2}$	4	λ Psc
	8	The star in the tail of the same [advance] fish	♓ 6	$+6\frac{1}{3}$	4	ω Psc
	9-18	The stars forming its fishing-line:[44]				
	9	the first after the tail	♓ 11	$+5\frac{3}{4}$	6	41(d) Psc
	10	the one to the rear[45]	♓ 13	$+3\frac{3}{4}$	6	51 Psc
H126	11	the most advanced of the 3 following bright stars	♓ $17\frac{1}{6}$	$+2\frac{1}{4}$	4	δ Psc
	12	the middle one of these	*♓ $20\frac{1}{2}$[46]	$+1\frac{1}{6}$	4	ε Psc
	13	the rearmost of the three	♓ 23	*$-0\frac{1}{6}$[47]	4	ζ Psc
	14	the northernmost of the 2 small stars under these, in the bend	*♓ $22\frac{1}{3}$[48]	−2	6	80(e) Psc

[42] A and most Arabic mss. have > 4.

[43] The variants $7\frac{3}{4}$ and $9\frac{1}{4}$ are found in the Arabic tradition (see S 64).

[44] The line joining the tails of the two fishes: see Thiele Fig. 35 on p. 110. In fact there are two lines joined in a knot (see no. 19).

[45] Perhaps one should emend αὐτῶν at H124,19 to αὐτῷ ('the one to the rear of the latter'). For no. 10 is not the rearmost of all the stars on the fishing-line (nos. 9-18), but only of the first two.

[46] $20\frac{1}{2}$ D,Ar, $20\frac{1}{6}$ the other Greek mss.

[47] The Greek mss. have 6. Heiberg adopted $0\frac{1}{6}$ from a conjecture of Bode. It was in fact the reading of the older Ma'mūn version according to S 65. The reading $1\frac{1}{3}$, found in some mss. of both the al-Ḥajjāj and Thābit versions (ibid.), and in Ger, is also possible.

[48] $22\frac{1}{2}$ BC,F,T; $22\frac{1}{3}$ A'D,E,Ger.

	[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
18	15	The rearmost of these	♑ 20⅓[24]	−6	5	36(b) Cap
	16	The rearmost of the 3 stars in the middle of the body	♑ 18½	−4¼	5	φ Cap
	17	The southernmost of the other, advance 2	♑ 16⅔	−4	5	χ Cap
	18	The northernmost of them	♑ 16⅔	−2⅚	5	η Cap
	19	The more advanced of the 2 stars in the back	♑ 16⅔	−0[25]	4	θ Cap
	20	The rearmost of them	♑ 21	−0⅚	4	ι Cap
	21	The more advanced of the 2 stars in the southern spine[26]	♑ 23⅓	−4¾	4	ε Cap
	22	The rearmost of them	♑ 25	−4½	4	κ Cap
	23	The more advanced of the 2 stars in the section [of the body] next to the tail	*♑ 24⅚[27]	−2⅙	3	γ Cap
	24	The rearmost of them	♑ 26⅓	−2	3	δ Cap
	25	The most advanced of the 4 stars on the northern portion of the tail	♑ 26⅚	+0⅓	4	42(d) Cap
	26	The southernmost of the other 3	♑ 28⅔	+0	5	μ Cap
	27	The middle one of these	♑ 27⅔	+2⅚	5	λ Cap
	28	The northernmost of them, on the end of the tail-fin[28]	♑ 28⅔	+4⅓	5	46(c¹) Cap
		{28 stars, 4 of the third magnitude, 9 of the fourth, 9 of the fifth, 6 of the sixth}				
		[XXXII] Constellation of Aquarius				
	1	The star on the head of Aquarius	♒ 0⅓	+15¾[29]	5	25(d) Aqr
	2	The brighter of the 2 stars in the right shoulder	♒ 6⅓	+11	3	α Aqr
20	3	The fainter one, under it	♒ 5⅙	+9⅔	5	ο Aqr
	4	The star in the left shoulder	♑ 26½	+8⅚[30]	3	β Aqr

[24] The variant 23 occurs in the later Arabic tradition (see S 59).
[25] The direction 'south' attached to the coordinate '0' perhaps indicates that the star is very slightly south of the ecliptic. Contrast no. 26.
[26] 'Spine' (ἀκάνθη) here means a projection from the fish-tail. Manitius (p. 404) emends νοτίῳ ('southern') to νωτιαίᾳ 'the spine on [projecting from] the back'), comparing H128,1 ἐπὶ τῆς νωτιαίας ἀκάνθης, and H166,22 ἐπὶ τῆς νωτιαίας νοτίου ἀκάνθης. Although the conjecture is superficially attractive, the location of this projection on the figure seems indeed to be south of the main tail: see Thiele Fig. 41 on p. 116.
[27] 21⅚ all Greek mss. except A. 24⅓ Ar.
[28] For this meaning of οὐραῖον cf. Cetus (XXXIV) nos. 21 and 22, and LSJ s.v. οὐραῖος 2.
[29] The variant 5¾ occurs in the earlier Arabic tradition according to S 60.
[30] The Syriac translation had the variant 18⅚ according to S 61.

[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
5	The one under that, in the back, approximately under the armpit	♑ 27⅓	+6¼	5	ξ Aqr
6	The rearmost of the three stars in the left arm, on the coat	♑ 17⅔	+5½	3	ν Aqr
7	The middle one of these	♑ 16⅙	+8	4	μ Aqr
8	The most advanced of the three	♑ 14⅔	+8⅔	3	ε Aqr
9	The star in the right forearm	♒ 9⅓	+8¾	3	γ Aqr
10	The northernmost of the 3 stars on the right hand	♒ 11⅔	+10¾	3	π Aqr
11	The more advanced of the other 2 to the south[31]	♒ 12	+9	3	ζ Aqr
12	The rearmost of them	♒ 13⅓	+8½	3	η Aqr
13	The more advanced of the 2 stars close together in the hollow of the right [hip]	♒ 6⅙	+3	4	θ Aqr
14	The rearmost of them	♒ 7	*+3⅙[32]	5	ρ Aqr
15	The star on the right buttock	♒ 8⅔	−0⅚	4	σ Aqr
16	The southernmost of the 2 stars in the left buttock	♒ 1⅔	−1⅔	4	ι Aqr
17	The northernmost of them	♒ 3⅙	*+0¼[33]	6	38(e) Aqr
18	The southernmost of the 2 stars in the right lower leg	♒ 11⅔	−7½	3	δ Aqr
19	The northernmost of them, under the knee-bend	♒ 11⅓	−5	4	τ Aqr
20	The star in the back of the left thigh	♒ 4⅔	−5⅔	5	53(f) Aqr
21	The southernmost of the 2 stars in the left lower leg	♒ 8⅓	−10	5	68(g²) Aqr
22	The northernmost of these, under the knee	♒ 7⅚	−9	5	66(g¹) Aqr
23–42	The stars on the flow of water:				
23	the most advanced [in the section] beginning at the hand[34]	♒ 15	+2	4	*κ Aqr[35]

[31] Reading νοτίων for βορείων ('to the north') at H120,10. Although all Greek mss. have βορείων, the sense requires the emendation, which is confirmed unanimously by the Arabic translations.
[32] ρ Aqr should have a latitude somewhat to the south of θ Aqr (no. 13). Hence Manitius (p.404) interchanges the latitudes of 13 and 14. Perhaps it would be preferable to adopt, for 14, the latitude 2¼, found in E, Ger.
[33] DT have 'southerly' for the latitudinal direction (adopted by Manitius). The reading 4 is found in all Arabic mss. I have examined.
[34] I take this to mean 'the most advanced in the section up to the bend'; for it cannot mean 'the most advanced in the whole flow of water', since the stars at the bottom of the flow (e.g. no. 42) are certainly 'in advance' of no. 23. But perhaps one should read πρῶτος ('first'): A has πρ´, BC πρ̅; cf. the exactly similar formulation Pisces (XXXIII) no. 20, where προηγούμενος cannot be interpreted as here (see n.51 there).
[35] If this star is correctly identified as κ Aqr, the coordinates are considerably in error. For the various solutions which have been proposed see Manitius p. 404 and P–K p. 106 nos. 651 and 652. Although no. 23 has a greater longitude than no. 24 it is 'the most advanced' of nos. 23–6. For when one converts Ptolemy's coordinates to right ascension and declination (with $\varepsilon = 23;51,20°$) one finds $\alpha(23) = 316;56°$ and $\alpha(24) = 317;20°$. Cf. p. 340 n.93.

H132

[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
7	The one in advance of this, about on the mane	*♈ $7\frac{2}{3}$[61]	$-4\frac{1}{6}$	4	ξ¹ Cet
8-11	The quadrilateral in the chest:				
8	the northernmost star on the advance side	♈ 3	$-24\frac{1}{2}$	4	ρ Cet
9	the southernmost one on the advance side	♈ $3\frac{1}{3}$	−28	4	σ Cet
11	the northernmost one on the rear side	♈ $6\frac{2}{3}$	$-25\frac{1}{6}$	4	ε Cet
11	the southernmost one on the rear side	♈ 7	$-27\frac{1}{2}$	3	π Cet
12	The midmost of the 3 stars in the body	♓ 22	$-25\frac{1}{3}$	3	τ Cet
13	The southernmost of them	♓ 23	$-30\frac{5}{6}$	4	υ Cet
14	The northernmost of the three	♓ 25	−20	3	ζ Cet
15	The rearmost of the 2 stars by the section next to the tail	♓ $19\frac{2}{3}$	*$-15\frac{2}{3}$[62]	3	θ Cet
16	The more advanced of them	♓ 15	$-15\frac{2}{3}$	3	η Cet
17-20	The quadrilateral in the section next to the tail:				
17	the northernmost star on the rear side	♓ 11	$-13\frac{2}{3}$	5	*φ² Cet[63]
18	the southernmost one on the rear side	♓ $10\frac{2}{3}$	$-14\frac{2}{3}$[64]	5	*BSC 227
19	the northernmost one on the advance side	♓ $9\frac{1}{3}$	−13	>5	*φ¹ Cet
20	the southernmost one on the advance side	♓ 9	−14	>5	*BSC 190
21-22	The 2 stars at the ends of the tail-fins:				
21	the one on the northern [tail-fin]	♓ $4\frac{1}{3}$	$-9\frac{2}{3}$	<3	ι Cet
22	the one on the end of the southern tail-fin	♓ $5\frac{2}{3}$	$-20\frac{1}{3}$	3	β Cet
	[22 stars, 10 of the third magnitude, 8 of the fourth, 4 of the fifth]				
	[XXXV] Constellation of Orion				
1	The nebulous star in the head of Orion	♉ 27	*$-13\frac{1}{2}$[65]	neb.	*λ Ori[66]
2	The bright, reddish star on the right shoulder	♊ 2	−17	<1	α Ori

[61] So D,Ar; ζ γ′ Γο (?i.e. $7\frac{1}{3}$ *or* $7\frac{2}{3}$) ABC. P–K adopt $7\frac{1}{3}$.
[62] $15\frac{1}{2}$ D,Ar, adopted by P–K.
[63] I have adopted the identifications of P–K for nos. 17 to 20, but they seem highly dubious, particularly because of the errors in the relative magnitudes. Manitius has different identifications (see his note on Wallisch 17, p. 405) which would require considerable errors in the coordinates.
[64] The variant $11\frac{2}{3}$ is attested in the earlier Arabic tradition (see S 68).
[65] So D; $16\frac{1}{2}$ the other Greek mss. See S 69 for the Arabic tradition: best attested is $13\frac{5}{6}$ (adopted by P–K), but $16\frac{5}{6}$ and $18\frac{5}{6}$ (also in Ger.) are found too.
[66] This is the identification of Manitius and P–K, but perhaps one should identify it with the diffuse nebula surrounding λ and φ¹ Ori.

H134

[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
3	The star on the left shoulder	♉ 24	$-17\frac{1}{2}$	2	γ Ori
4	The one under this to the rear	♉ 25	−18	<4	32(A) Ori
5	The star on the right elbow[67]	♊ $4\frac{1}{3}$	$-14\frac{1}{2}$	4	μ Ori
6	The star on the right forearm	♊ $6\frac{1}{3}$	$-11\frac{5}{6}$	6	74(k) Ori
7-10	The quadrilateral in the right hand:				
7	the rear, double star on the southern side	♊ $6\frac{1}{2}$	−10	4	*ξ Ori[68]
8	the advance star on the southern side	♊ 6	$-9\frac{1}{4}$	4	ν Ori
9	the rear one on the northern side	♊ $7\frac{1}{3}$	$-8\frac{1}{4}$	6	72(f²) Ori
10	the advance one on the northern side	♊ $6\frac{2}{3}$	$-8\frac{1}{4}$	6	69(f¹) Ori
11	The more advanced of the 2 stars in the staff[69]	♊ $1\frac{2}{3}$	$-3\frac{3}{4}$	5	χ¹ Ori
12	The rearmost of them	*♊ $4\frac{1}{3}$[70]	$-4\frac{1}{4}$	5	χ² Ori
13	The rearmost of the 4 stars almost on a straight line just over the back	♉ $27\frac{5}{6}$	$-19\frac{2}{3}$	4	ω Ori
14	The one in advance of this	♉ $26\frac{1}{3}$	−20	6	38(n²) Ori
15	The one in advance again of this	♉ $25\frac{1}{3}$	$-20\frac{1}{6}$	6	33(n¹) Ori
16	The last and most advanced of the 4	♉ $24\frac{1}{6}$	$-20\frac{2}{3}$	5	ψ Ori
17-25	Stars in the pelt[71] on the left arm:				
17	the northernmost	♉ $20\frac{1}{2}$	−8	4	15(y²) Ori
18	the 2nd from the northernmost	♉ $19\frac{1}{3}$	$-8\frac{1}{6}$	4	11(y¹) Ori
19	the 3rd from the northernmost	♉ 18	$-10\frac{1}{4}$	4	o² Ori
20	the 4th from the northernmost	♉ $16\frac{1}{3}$	$-12\frac{5}{6}$	4	π¹ Ori
21	the 5th from the northernmost	♉ $15\frac{1}{6}$	$-14\frac{1}{4}$	4	π² Ori
22	the 6th from the northernmost	♉ $14\frac{5}{6}$	$-15\frac{5}{6}$	3	π³ Ori
23	the 7th from the northernmost	♉ $14\frac{5}{6}$	$-17\frac{1}{6}$	3	π⁴ Ori
24	the 8th from the northernmost	♉ $15\frac{1}{3}$	$-20\frac{1}{3}$	3	π⁵ Ori
25	the last and southernmost of those in the pelt	♉ $16\frac{1}{3}$	$-21\frac{1}{2}$	3	π⁶ Ori
26	The most advanced of the 3 stars on the belt	♉ $25\frac{1}{3}$	$-24\frac{1}{6}$	2	δ Ori

[67] Nos. 5 and 6, which are north of no. 4, are described as being on the arm because the right arm is raised, holding the staff.
[68] As Manitius notes (p. 405, cf. P–K no. 740 p. 108), ξ Ori is not a double star, but there are two small stars close together just below it, which may have led to this description.
[69] κολλόροβον. cf. p. 346 n.118. Thiele Fig. 45 on p. 120 shows a curved stick, more like a λαγώβολον.
[70] $4\frac{1}{3}$ D,L,E,Ger., adopted by P–K; $4\frac{1}{2}$T¹,F.
[71] As a huntsman, Orion carries an animal pelt as a garment or an arm-guard. Cf. Thiele Fig. 45 on p. 120, and Pl. IV (lower).

[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
15	the southernmost of them	*♓ 23[49]	−5	6	89(f) Psc
16	the most advanced of the 3 stars after the bend	♓ 26½	−2⅓	4	μ Psc
17	the middle one of these	*♓ 28⅔[50]	−4⅔	4	ν Psc
18	the rearmost of the three	♈ 0⅓	−7¾	4	ξ Psc
19	The star on the knot joining the 2 fishing-lines	♈ 2½	−8½	3	α Psc
20-23	Stars in the northern fishing-line:				
20	the first[51] in the section beginning at the knot	♈ 0½	−1⅔[52]	4	ο Psc
21	the southernmost of the 3 stars following after that	♈ 0⅙	+1⅚	5	π Psc
22	the middle one of these	*♈ 0⅔[53]	+5⅓	3	η Psc
23	the northernmost of the 3, which is also on the end of the tail	♈ 0½	+9	4	ρ Psc
24	The northernmost of the 2 stars in the mouth of the rear fish	♈ 2	+21¾	5	82(g) Psc
25	The southernmost of them	♈ 1⅔	+21⅓	5	τ Psc
26	The rearmost of the 3 small stars in the head	♓ 28⅔	+20	6	68(h) Psc
27	The middle one of these	♓ 27⅔	+19⅚	6	67(k) Psc
H128 28	The most advanced of the three	♓ 27	*+20⅓[54]	6	65(i) Psc
29	The most advanced of the 3 stars on the spine in the back, following [i.e. to the rear of] the star on the elbow of Andromeda [XX no. 11]	♓ 25⅔	+14⅓	4	ψ^1 Psc
30	The middle one of the three	*♓ 26⅓[55]	*+13¼[56]	4	ψ^2 Psc
31	The rearmost of the three	♓ 27⅔	+12	4	*χ Psc[57]

[49] So D,L¹,T,F,Ger; 20⅓A, 23½ BC,L²,E.
[50] So AD,Ar; 28½ BC.
[51] προηγούμενος, which is normally 'the most advanced'. But that cannot be so here, since no. 20 is 'to the rear' of no. 21. Perhaps one should emend to πρῶτος, cf. Aquarius (XXXII) no. 23, with n.34.
[52] The variant 'northern' is found in the Greek (BC) and Arabic traditions (see S 66).
[53] So ABC. 0½ D, Ar, adopted by P–K.
[54] 23 the Greek mss. Heiberg adopts 20⅓ from the *editio princeps*. According to ibn aṣ-Ṣalāḥ (S 67) all the Arabic translations except the original Isḥāq version had 20⅓, but in the extant mss. 23 is also found (see Kunitzsch's reports ibid.).
[55] 26⅓ ABCT², adopted by P–K. 26⅙ D,L,E,F,Ger.
[56] 13¼ AD, 13 BC,Ar.
[57] The identifications of nos. 31 and 34 are very uncertain. P–K identify 31 as ψ^1 Psc and 34 as χ Psc, Manitius as χ Psc and 99 (Hevelius) respectively. The identification of 34 as χ Psc fits the description but involves a serious error in the longitude. There are in any case bad errors in the coordinates of all nos. 31 to 34.

[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
32	The northernmost of the 2 stars in the belly	♈ 2⅙	+17	4	υ Psc
33	The southernmost of them	♓ 29⅚	+15⅓	4	φ Psc
34	The star in the rear spine, near the tail	♈ 0	+11¾	4	*ψ^3 Psc
	[34 stars, 2 of the third magnitude, 22 of the fourth, 3 of the fifth, 7 of the sixth]				
	Stars round Pisces outside the constellation:				
35-38	The quadrilateral under the advance fish:				
35	the more advanced of the 2 northern stars	♓ 1⅙	−2⅔	4	27 Psc
36	the rearmost of them	♓ 2¼	−2½	4	29 Psc
37	the more advanced star on the southern side	♓ 0⅔	−5½	4	30 Psc
38	the rearmost one on the southern side	♓ 2⅓	−5½	4	33 Psc
	[4 stars of the fourth magnitude]				
	[Total for the zodiac: 346 stars, 5 of the first magnitude, 9 of the second, 64 of the third, 133 of the fourth, 105 of the fifth, 27 of the sixth, 3 nebulous, and Coma [Berenices]]				
H130	[XXXIV] Constellation of Cetus				
1	The star on the tip of the nostrils	♈ 17⅔	−7¾	4	λ Cet
2-4	The three stars in the snout:				
2	the rearmost, on the end of the jaw	♈ 17⅔	−12⅓	3	α Cet
3	the middle one, in the middle of the mouth	♈ 12⅔	−11½	3	γ Cet
4	the most advanced of the 3, on the cheek	♈ 10½	−14	3	δ Cet
5	The star on the eyebrow and the eye	*♈ 10⅙[58]	−8⅙	4	*ν Cet[59]
6	The one to the north of this, about on the hair[60]	♈ 12⅔	−6⅓	4	*ξ^2 Cet

[58] So Ar and most Greek mss.; 16⅙ BC.
[59] The identifications of nos. 5 and 6 are extremely uncertain. See P–K nos. 716, 717, p. 107.
[60] θρίξ. Manitius takes this as 'the hair on the forehead.' It is in any case distinct from the 'mane' (χαίτη) of no. 7. The representation in Thiele Fig. 49 p. 125 is little help, unless one supposes that the long ears are a distortion of brow-hair in the original. For ancient representations of serpents with manes reaching over the brow see e.g. Allinson, *Lucian*, photo opp. p. 108 (the serpent Glykon of Alexander of Abonouteichos); a remarkable life-size statue of Glykon found at Tomis is illustrated in Robert, *A travers l'Asie Mineure*, Fig. 8 on p. 398.

[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
28	The southernmost of them	♉ 5	$-51\frac{3}{4}$	4	υ^2 Eri
29	The rearmost of the next 2 stars after the bend	♈ $28\frac{1}{6}$	$-53\frac{5}{6}$	4	υ^3 Eri
30	The more advanced of them	♈ $25\frac{5}{6}$	$-53\frac{1}{6}$	4	υ^4 Eri
31	The rearmost of the 3 stars in the next interval	♈ $17\frac{5}{6}$	-53	4	*g Eri[84]
32	The middle one	♈ $14\frac{5}{6}$	$-53\frac{1}{2}$	4	*f Eri
33	The most advanced of the three	♈ $11\frac{5}{6}$	$-52\frac{1}{2}$	4	*h Eri
34	The last star of the river, the bright one	♈ $0\frac{1}{6}$	$-53\frac{1}{2}$	*1[85]	θ Eri
	{34 stars, 1 of the first magnitude, 5 of the third, 26 of the fourth, 2 of the fifth}				
	[XXXVII] Constellation of Lepus				
1-4	The quadrilateral just over the ears:				
1	the northern star on the advance side	♉ $19\frac{2}{3}$	-35	5	ι Lep
2	the southern star on the advance side	♉ $19\frac{5}{6}$	$-36\frac{1}{2}$	5	κ Lep
3	the northern star on the rear side	♉ $21\frac{1}{3}$	$-35\frac{2}{3}$	5	ν Lep
4	the southern star on the rear side	♉ $21\frac{1}{3}$	$-36\frac{2}{3}$	5	λ Lep
H142 5	The star in the cheek	♉ $19\frac{1}{6}$	$-39\frac{1}{4}$	>4	μ Lep
6	The star on the left front foot	♉ $16\frac{1}{6}$	$-45\frac{1}{4}$	>4	ε Lep
7	The star in the middle of the body	♉ $25\frac{5}{6}$	$-41\frac{1}{2}$	3	α Lep
8	The star under the belly	*♉ $24\frac{5}{6}$[86]	$-44\frac{1}{3}$	3	β Lep
9	The northernmost of the 2 stars in the hind legs	♊ 1	$-44\frac{1}{6}$	>4	δ Lep
10	The southernmost of them	♉ 29	$-45\frac{5}{6}$	>4	γ Lep
11	The star on the rump	♊ 0	$-38\frac{1}{3}$	>4	ζ Lep
12	The star on the tip of the tail	♊ $2\frac{2}{3}$	$-38\frac{1}{6}$	>4	η Lep
	{12 stars, 2 of the third magnitude, 6 of the fourth, 4 of the fifth}				

[84] On alternative identifications for nos. 31-3 see P-K. Their identifications correspond to *BSC* 1214 (Lacaille i), *BSC* 1195 (Lacaille g) and *BSC* 1143 (Lacaille h).
[85] θ Eri is not of 1st magnitude, but a double star of 3rd and 4th magnitudes (combined magnitude 2.9). Hence P-K suggest emending 1 to 4 (A to Δ). This is contradicted by the subtotal, but see p. 363 n.191.
[86] This is the reading of all Greek mss., E, F and Ger. The variant $24\frac{1}{3}$, found in T,L, is adopted by P-K.

[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
	[XXXVIII] Constellation of Canis Major[87]				
1	The star in the mouth, the brightest, which is called 'the Dog' and is reddish[88]	♊ $17\frac{2}{3}$	$-39\frac{1}{6}$	1	α CMa
2	The star on the ears	♊ $19\frac{2}{3}$	-35	4	θ CMa
3	The star on the head	♊ $21\frac{1}{3}$	$-36\frac{1}{2}$	5	μ CMa
4	The northernmost of the 2 stars in the neck	♊ $23\frac{1}{3}$	$-37\frac{3}{4}$	4	γ CMa
5	The southernmost of them	*♊ $25\frac{1}{3}$[89]	-40	4	ι CMa
6	The star on the chest	♊ $20\frac{1}{2}$	$-42\frac{2}{3}$	5	π CMa
7	The northernmost of the 2 stars on the right knee	♊ $16\frac{1}{6}$	$-41\frac{1}{4}$	5	ν^3 CMa
H144 8	The southernmost of them	♊ 16	$-42\frac{1}{2}$	5	ν^2 CMa
9	The star on the end of the front leg	♊ 11	$-41\frac{1}{3}$	3	β CMa
10	The more advanced of the 2 stars in the left knee	♊ $14\frac{2}{3}$	$-46\frac{1}{2}$	5	ξ^1 CMa
11	The rearmost of them	♊ $16\frac{1}{6}$	$-45\frac{5}{6}$	5	ξ^2 CMa
12	The rearmost of the 2 stars in the left shoulder	♊ $24\frac{2}{3}$	$-46\frac{1}{6}$	4	o^2 CMa
13	The more advanced of them	♊ $21\frac{2}{3}$	-47	5	o^1 CMa
14	The star in the place where the left thigh joins [the body]	♊ $26\frac{2}{3}$	$-48\frac{3}{4}$	<3	δ CMa
15	The star below the belly, in the middle of the thighs	♊ $23\frac{2}{3}$	$-51\frac{1}{2}$	3	ε CMa
16	The star on the joint of the right leg	*♊ 23[90]	$-55\frac{1}{6}$	4	κ CMa
17	The star on the end of the right leg	♊ $9\frac{1}{3}$	$-53\frac{3}{4}$	3	ζ CMa
18	The star on the tail	♋ $2\frac{1}{6}$	$-50\frac{2}{3}$	<3	η CMa
	{18 stars, 1 of the first magnitude, 5 of the third, 5 of the fourth, 7 of the fifth}				
	Stars round Canis Major outside the constellation:				
19	The star to the north of the top of Canis	♊ $19\frac{1}{2}$	$-25\frac{1}{4}$[91]	4	*22 Mon[92]

[87] Ptolemy calls it simply 'the dog' (κύων), since to the constellation now known as 'Canis Minor' he gives the name 'Procyon'.
[88] Ptolemy calls this star κύων ('the dog'), not Σείριος ('Sirius'), although the latter name is as old as Hesiod (*Works and Days* 587). By 'brightest' he means 'brightest of all fixed stars'. Although Sirius is not a red star today, there is considerable evidence that it was in antiquity (cf. See, 'Change in the Color of Sirius').
[89] This coordinate is greatly in error, but is found in all mss. Manitius adopts $21\frac{1}{3}$, on no authority, P-K $20\frac{1}{2}$, from aṣ-Ṣūfī. The error may be Ptolemy's.
[90] This is the reading of all mss. P-K emend to 21.
[91] The variant $65\frac{1}{4}$ is found in the Arabic tradition (see S 71).
[92] So P-K. Manitius has 19 Monocerotis.

[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
27	The middle one	♉ 27⅓	−24⅚	2	ε Ori
28	The rearmost of the three	♉ 28⅙	−25⅔	2	ζ Ori
29	The star near the handle of the dagger[72]	♉ 23⅚	−25⅚	3	η Ori
H136 30	The northernmost of the 3 stars joined together at the tip of the dagger	♉ 26½	•−28⅓[73]	4	•42 + 45 Ori[74]
31	The middle one	♉ 26⅔	−29⅙	<3	•θ¹ + θ² Ori
32	The southernmost of the three	♉ 27	−29⅚	3	•ι Ori
33	The rearmost of the 2 stars under the tip of the dagger	♉ 27⅔	−30⅔	4	49(d) Ori
34	The more advanced of them	•♉ 26½[75]	−30⅚	4	υ Ori
35	The bright star in the left foot, which is [applied in] common to the water [of Eridanus]	♉ 19⅚	−31½	1	β Ori
36	The star to the north of it[76] in the lower leg, over the ankle-joint	♉ 21	−30¼	>4	τ Ori
37	The star under the left heel, outside	♉ 23⅓	−31⅙	4	29(e) Ori
38	The star under the right, rear knee	♊ 0⅙	−33½	>3	κ Ori
	[38 stars, 2 of the first magnitude, 4 of the second, 8 of the third, 15 of the fourth, 3 of the fifth, 5 of the sixth, [1] nebulous]				
	[XXXVI] Constellation of Eridanus[77]				
1	The star after the one in the foot of Orion [XXXV no. 35], at the beginning of the river	♉ 18⅓	−31⅚[78]	>4	λ Eri
2	The one north of this, in the curve near the shin of Orion	♉ 18½	−28¼	4	β Eri
3	The rearmost of the 2 stars next in order after this	♉ 18	−29⅚	4	ψ Eri
4	The more advanced of them	♉ 14⅔	−28¼	4	ω Eri

[72] μαχαίρα, a hunting-knife or short sword.

[73] P–K adopt 28⅔, the reading of D,Ar (28;12 E).

[74] The identifications adopted for nos.30–2 are those of P–K. Although there is no doubt about the group as a whole, the precise identifications of the particular stars, which are close together, remain doubtful.

[75] 26⅙ D,Is., adopted by P–K. 26;20 L.

[76] Reading αὐτοῦ (with B,Ar) at H136,8 for αὐτῶν ('to the north of them'). Corrected by Manitius.

[77] Ptolemy has ποταμός ('river'). The identification with a particular river Eridanus is at least as early as Aratus (359 ff.) This was the mythical river into which the burning chariot of Phaethon plunged. It was later identified with the Po. See Boll-Gundel cols. 989–92

[78] The variant 30⅚ was in the Syriac version according to S 70.

[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
H138 5	The rearmost of the next 2 in order again	♉ 13⅙	−25⅚	4	μ Eri
6	The more advanced of them	•♉ 10⅙[79]	−25⅓	4	ν Eri
7	The rearmost of the 3 stars after this	♉ 6⅓	−26	5	ξ Eri
8	The middle one of these	♉ 5½	−27	4	o² Eri
9	The most advanced of the three	♉ 2⅚	−27⅚	4	o¹ Eri
10	The rearmost of the four stars in the next interval	♈ 27	−32⅚	3	γ Eri
11	The one in advance of this	♈ 24⅓	−31	4	π Eri
12	The one in advance again of this	♈ 24⅙	−28⅚	3	δ Eri
13	The most advanced of the 4	♈ 22	−28	3	ε Eri
14	The rearmost of the 4 stars in the next interval again	♈ 17⅙	−25½	3	ζ Eri
15	The one in advance of this	♈ 14½	−23⅚	4	•ρ³ + ρ² Eri[80]
16	The one in advance again of this	♈ 12⅙	−23½	3	•η Eri
17	The most advanced of the 4	♈ 10½	−23¼	4	•BSC 859
18	The first star in the bend[81] of the river, which [star] touches the chest of Cetus	♈ 5⅙	−32⅙	4	τ¹ Eri
19	The one to the rear of this	♈ 5⅚	−34⅚	4	τ² Eri
20	The most advanced of the next [group of] three	♈ 8⅚	−38½	4	τ³ Eri
21	The middle one of these	♈ 13⅚	−38⅙	4	τ⁴ Eri
22	The rearmost of the three	♈ 17½	−39	4	τ⁵ Eri
23–26	The next four stars, nearly forming a trapezium:				
23	the northern one on the advance side	♈ 21⅓	−41⅓	4	τ⁶ Eri
H140 24	the southernmost on the advance side	♈ 21½	−42½	5	τ⁷ Eri
25	the more advanced one on the rear side	♈ 22⅙	−43¼	4	τ⁸ Eri
26	the last of the 4, the rear one on that side	♈ 24⅔	−43⅓	4	τ⁹ Eri
27	The northernmost of the 2 stars close together at some distance to the east	♉ 4⅙	•−50⅓[82]	4	υ¹ Eri[83]

[79] This is the reading of A. ις (16), the reading of the other Greek mss., cannot be right, since that would not be 'more advanced'.

[80] The identifications of nos. 15 to 17 are of the utmost uncertainty. I give those dubiously proposed by P–K (see their discussion pp. 108–9). Manitius gives: 15 = ρ², 16 = ρ³ (these are certainly wrong, but one might reverse them), 17 = η. One might also consider, for 17, *BSC* 784.

[81] 'bend', ἐπιστροφή, i.e. a change of direction (see Bayer Tab. 36), in contrast to ἐπικάμπιον 'curve', in no. 2.

[82] So D,Ar (50;30 T¹, F); 53⅓ the other Greek mss.

[83] Considerable confusion arises in the identifications of nos. 27–33 from differences in the modern nomenclature of these stars (see P–K on nos. 798–804, p. 110). Thus Manitius' identifications appear to be completely different from those of P–K, but in fact are only partly so. To avoid this confusion I give the *BSC* and (where applicable) the Fl [amsteed] nos. of my identifications, which are those of P–K, though named differently: 27 = *BSC* 1453 = Fl 50; 28 = *BSC* 1464 = Fl 52; 29 = *BSC* 1393 = Fl 43; 30 = *BSC* 1347 = Fl 41; 31 = *BSC* 1195; 32 = *BSC* 1143; 33 = *BSC* 1190.

[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
18	the more advanced of the 2 faint stars under the bright one	♋ 18 1/6	−60	5	a Pup (BSC 3080)
19	the rearmost of them	♋ 21	−59 1/3	5	BSC 3162
20	the more advanced of the 2 stars over the above-mentioned bright one	*♋ 23 1/6[108]	−56 2/3	5	h^1 Pup (BSC 3225)
21	the rearmost of them	♋ 24 1/3	−57 2/3	5	h^2 Pup (BSC 3243)
22	The northernmost of the 3 stars on the little shields, about on the mast-holder	♌ 5 2/3	−51 1/2	>4	BSC 3439
H150 23	The middle one	♌ 6 1/6	−55 2/3	>4	d Vel (BSC 3477)
24	The southernmost of the three	♌ 4	−57 1/6	>4	e Vel (BSC 3426)
25	The northernmost of the 2 stars close together under these	♌ 9 1/6	−60	>4	*a Vel (BSC 3487)[109]
26	The southernmost of them	♌ 9	−61 1/4	>4	*b Vel (BSC 3445)
27	The southernmost of the 2 stars in the middle of the mast	♌ 0 1/6	*−51 5/6[110]	3	β Pyx
28	The northernmost of them	♋ 29 1/3	−49	3	α Pyx
29	The more advanced of the 2 stars by the tip of the mast	♋ 28	−43 1/3	4	γ Pyx
30	The rearmost of them	♋ 29	−43 1/2	4	δ Pyx
31	The star below the 3rd and rearmost little shield	♌ 14 1/6	−54 1/2	2	λ Vel
32	The star on the cut-off[111] of the deck	♌ 17 1/2	−51 1/4	<2	ψ Vel
33	The star between the steering-oars,[112] in the keel	♋ 11 1/6	−63	4	*σ Pup
34	The faint star to the rear of this	♋ 19	−64 1/2	6	*Ṗ Pup (BSC 3055)
35	The bright star to the rear of this, under the deck	♌ 0	−63 5/6	2	γ Vel

[108] This is the reading of A,Ger (most Arabic mss. have 23;0). The other Greek mss. have 26, adopted by P–K.
[109] The identifications of nos. 25 and 26 are those of P–K, but it is possible that they are instead g Vel (*BSC* 3520) and a Vel respectively.
[110] 51 1/2 Ar, adopted by P–K.
[111] The constellation is represented as only the stern-half of the ship. Cf. Thiele Fig. 48 on p. 123 and Pl. II, and p. 361 n.174.
[112] Two steering-oars are clearly visible in the illustration Thiele Fig. 67 on p. 157, less clearly in Fig. 48 on p. 123.

[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
36	The bright star to the south of this, on the lower [part of the] keel	♌ 8 1/2	−69 2/3	2	*χ Car
37	The most advanced of the 3 stars to the rear of this	♌ 15 1/6	−65 2/3	3	*o Vel[113]
38	The middle one	♌ 21 1/3	−65 5/6	3	*δ Vel
39	The rearmost of the three	♌ 26	−67 1/3	2	*f Car (BSC 3498)
40	The more advanced of the 2 stars to the rear of these, near the cut-off	♍ 1	−62 5/6	3	κ Vel
H152 41	The rearmost of them	♍ 8	−62 1/4	3	N Vel (BSC 3803)
42	The more advanced of the 2 stars in the northern, advance steering-oar	♊ 4	−65 5/6	>4	η Col
43	The rearmost of them	♊ 20 1/6	−65 2/3	>3	ν Pup
44	The more advanced of the 2 stars in the other steering-oar, called Canopus	♊ 17 1/6	−75	1	α Car
45	The other, rearmost star	♊ 29	−71 3/4	>3	τ Pup
	{45 stars, 1 of the first magnitude, 6 of the second, 11 of the third, 19 of the fourth, 7 of the fifth, 1 of the sixth}				
	[XLI] Constellation of Hydra[114]				
1-5	The 5 stars in the head:				
1	the southernmost of the 2 advance ones, which is on the nostrils	♋ 14	−15	4	σ Hya
2	the northernmost of these [2], which is above the eye	♋ 13 1/3	−13 1/6	4	δ Hya
3	the northernmost of the 2 to the rear of these, which is about on the skull	♋ 15 1/3	−11 1/2	4	ε Hya
4	the southernmost of them, on the gaping jaws	♋ 15 1/2	*−14 1/4[115]	4	η Hya
5	the rearmost of all, about on the cheek	♋ 17 1/2	*−12 1/4[116]	4	ζ Hya

[113] The identifications I give for nos. 37-9 are those of P–K. But the actual magnitude of f Carinae is much too small, and the positions are in poor agreement. Manitius gives δ, κ, φ Vel, which produces better agreement for the magnitudes but even worse for the positions.
[114] The water-snake. Ptolemy, like Hipparchus (e.g. *Comm. in Arat.* I.11.9, ed. Manitius 116,5) calls it ὕδρος (masculine); but it is feminine (ὕδρα) in Aratus, 444. Somewhat confusingly, there is a different modern constellation called Hydrus (far south of this).
[115] 14 3/4 Ar, adopted by P–K.
[116] 12 Ar, adopted by P–K.

H146

[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
20	The southernmost of the 4 stars almost on a straight line under the hind legs	•♊ 10[93]	-61½	4	θ Col
21	The one north of this	♊ 11⅓	-58¾	4	κ Col
22	The one north again of this	♊ 13	-57	4	δ Col
23	The last and northernmost of the 4	♊ 14⅙	-56	4	λ CMa
24	The most advanced of the 3 stars almost on a straight line to the west of the [above] four[94]	♉ 28	-55½	4	μ Col
25	The middle one	♊ 0⅓	-57⅔	4	λ Col
26	The rearmost of the three	♊ 2⅓	-59⅚	4	γ Col
27	The rearmost of the 2 bright stars under these	♉ 29	-59⅓[95]	2	β Col
28	The more advanced of them	♉ 26	-57⅓	2	α Col
29	The last star, to the south of the above	♉ 22⅙	-59½	4	ε Col
	{11 stars, 2 of the second magnitude, 9 of the fourth}				
	[XXXIX] Constellation of Canis Minor[96]				
1	The star in the neck	♊ 25	-14	4	β CMi
2	The bright star just over the hindquarters, called Procyon	•♊ 29⅙[97]	-16⅙	1	α CMi
	{2 stars, 1 of the first magnitude, 1 of the fourth}				
	[XL] Constellation of Argo[98]				
1	The more advanced of the 2 stars in the stern-ornament	♋ 10⅓[99]	-42⅔	5	11(e) Pup
2	The rearmost of them	♋ 14⅓	-43⅓	3	ρ Pup

[93] P–K adopt 7 on no authority.

[94] There is no doubt that the Greek must mean this. Accordingly Manitius (p. 405) emends τοῖς τέσσαρσιν at H146,2 to τῶν τεσσάρων, to restore the genitive normal after directions. But the use of the dative here may be explained by the desire to avoid the double genitive τῶν . . . τῶν.

[95] 59½ Ger, adopted by P–K. All Arabic mss. examined by me have 59⅓.

[96] Ptolemy calls this 'Procyon' (προκύων, 'harbinger of Sirius'), after its principal star.

[97] This is the reading of D,Ar. The other Greek mss. have 29⅓.

[98] This large constellation has in modern times been subdivided into the three constellations Puppis, Vela and Carina.

[99] The variant 13 is found in the later Arabic tradition (see S 72).

H148

[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
3	The northernmost of the 2 stars close together over the little shield in the poop	♋ 8⅚	-45	4	ξ Pup
4	The southernmost of them	♋ 8⅔	•-46[100]	4	o Pup
5	The star in advance of these	♋ 5⅓	-45½	4	m Pup (BSC 2944)
6	The bright star in the middle of the little shield	♋ 6⅓	-47¼	3	BSC 2948 + 2949
7	The most advanced of the 3 stars under the little shield	♋ 5⅓	•-49½[101]	4	p Pup (BSC 2922)
8	The rearmost of them	♋ 9⅓	•-49½[102]	4	3 Pup
9	The middle one of the three	♋ 8½	-49¼	4	1 Pup
10	The star on the goose[-neck][103]	♋ 14	-49⅚	4	BSC 3113[104]
11	The northernmost of the 2 stars in the stern-keel	♋ 4	-53	4	?[105]
12	The southernmost of them	♋ 4	-58⅔	3	π Pup
13-21	Stars in the poop-deck:				
13	the northernmost	♋ 10⅙[106]	-55½	5	f Pup (BSC 2937)
14	the most advanced of the next 3	♋ 12⅙	-58⅔	5	•BSC 2961 + 2964[107]
15	the middle one	♋ 13⅔	-57¼	4	c Pup (BSC 3017)
16	the rearmost of the three	♋ 16½	-57⅚	4	b Pup (BSC 3084)
17	the bright star on the deck to the rear of these	♋ 21⅙	-58⅔	2	ζ Pup

[100] Reading μς (with all mss., Greek and Arabic, except D) for μς ς' (46⅙), the reading of D, at H147,18. Corrected by P–K.

[101] Reading μθ ∠' (with AD,Ar, adopted by P–K) at H149,4 for μθ ∠' δ' (49¾) of the other Greek mss.

[102] This is the reading of D,Ar; the other Greek mss. have 49⅚.

[103] The top of the post on the stern, which was often given this shape. See LSJ s.v. II for other references.

[104] This seems the only likely candidate in the right region. P–K assign to the star they identify (Piazzi VII 277) the magnitude 6.5. Perhaps Peters confused the two stars (very close together) *BSC* 3113 (mag. 4.78) and *BSC* 3099 (mag. 6.36, which is too faint to be considered).

[105] This might be any of the 5th-magnitude stars (all close together) *BSC* 2819, 2823, 2834, or some combination of them. All are in the modern constellation Canis Major.

[106] The variant 16 occurs in the Greek (AD) and later Arabic traditions (see S 73).

[107] This may include more of the numerous small stars close together (P–K give $d^1+d^2+d^3$).

	[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
	3	The star in the breast	♍ 16⅔	−18⅙	5	ζ Crv
	4	The star in the advance, right wing	♍ 13½	−14⅚	3	γ Crv
H158	5	The more advanced of the 2 stars in the rear wing	♍ 16⅔	−12½	3	δ Crv
	6	The rearmost of them	♍ 17	−11¾	4	η Crv
	7	The star on the end of the leg, which is [applied in] common to Hydra	♍ 20½	−18⅙	3	β Crv
		[7 stars, 5 of the third magnitude, 1 of the fourth, 1 of the fifth]				
		[XLIV] Constellation of Centaurus				
	1	The southernmost of the 4 stars in the head	♎ 10½	−21⅔	>5	2(g) Cen
	2	The northernmost of them	♎ 10	−18⅚	>5	4(h) Cen
	3	The more advanced of the other, middle 2	♎ 9⅙	−20½	>4	1(i) Cen
	4	The rearmost of these, the last of the 4	♎ 10	−20	>5	3(k) Cen
	5	The star on the left, advance shoulder	♎ 6⅙	−25⅔	3	ι Cen
	6	The star on the right shoulder	♎ 15⅔	−22½	3	θ Cen
	7	The star on the left shoulder-blade	♎ 9⅙	−27½	4	BSC 5089 (d Cen)
	8-11	The 4 stars in the thyrsus:[129]				
	8	the northernmost of the advance 2	♎ 18⅙	−22⅓	4	ψ Cen
	9	the southernmost of these	♎ 19⅙	−23¾	4	BSC 5378 (a Cen)
	10	that one of the other two which is at the tip of the thyrsus[130]	♎ 22	−18¼	4	BSC 5485 + 5489 (c^1 + c^2 Cen)
	11	the last one, south of the latter	♎ 22½	−20⅚	4	BSC 5471 (b Cen)
	12	The most advanced of the 3 stars in the right side	♎ 13⅓	−28⅓	>4	ν Cen
	13	The middle one	♎ 14	−29⅓	>4	μ Cen
H160	14	The rearmost of the three	♎ 15⅙	−28	>4	φ Cen

[129] The thyrsus was a branch carried by followers of Dionysus, tipped with vine-leaves, pine-cone, or other Dionysiac emblems. See A. J. Reinach s.v. in Daremberg-Saglio V, 287-96, with illustrations. The attribution to a centaur is rare, but attested (ibid. 293 n.20).

[130] Manitius and P–K identify this as c^1 Cen, but c^2 and c^1 are so close together that one cannot decide between them: it is better to assume that Ptolemy refers to both.

	[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
	15	The star on the right upper arm	♎ 16⅓	−26½	>4	χ Cen
	16	The star on the right forearm	♎ 22⅚	−25¼	3	η Cen
	17	The star in the right hand	♎ 27½	−24¼	4	κ Cen
	18	The bright star in the place where the human body joins [the horse's]	♎ 18	−33½	>3	ζ Cen
	19	The rearmost of the 2 faint stars to the north of this	♎ 17⅔	−31	5	υ^2 Cen
	20	The more advanced of them	♎ 16⅚	*−30⅓[131]	5	υ^1 Cen
	21	The star on the place where the back joins [the horse's body]	♎ 12⅙	−34⅚	5	ω Cen[132]
	22	The star in advance of this, on the horse's back	♎ 9	−37⅔	5	*BSC 4940 (f Cen)
	23	The rearmost of the stars on the rump	♎ 5⅚	−40	3	γ Cen
	24	The middle one	♎ 5	*−40⅓[133]	4	τ Cen
	25	The most advanced of the three	♎ 2⅔	−41	5	σ Cen
	26	The more advanced of the 2 stars close together on the right thigh	♎ 2⅔	−46⅙	3	δ Cen
	27	The rearmost of them	♎ 3½	−46¾	4	ρ Cen
	28	The star in the chest, under the horse's armpit	♎ 18⅓	−40¾	4	BSC 5172 (M Cen)[134]
	29	The more advanced of the 2 stars under the belly	♎ 16⅓	−43	2	ε Cen
	30	The rearmost of them	♎ 17⅔	−43¼	3	Q Cen
	31	The star on the knee-bend of the right [hind] leg	♎ 10	−51⅙	2	γ Cru
H162	32	The star in the hock of the same leg	♎ 15⅓	−51⅓	2	β Cru
	33	The star under the knee-bend of the left [hind] leg	♎ 6⅓	−55⅙	4	δ Cru
	34	The star on the frog of the hoof[135] on the same leg	♎ 11⅙	−55⅓	2	α Cru
	35	The star on the end of the right front leg	♏ 8⅓	*−41⅙[136]	1	α Cen

[131] Reading λ γ′ at H161,8 (with Ar, adopted by P–K). The Greek mss. have the reading λγ (33), but, with these identifications of nos. 19 and 20, the Arabic tradition is almost certainly the correct one. Manitius identifies 19 as υ^1 and 20 as υ^2, but υ^2 is definitely 'to the rear' of υ^1.

[132] As P–K note, ω is not a single star, but a globular cluster (no. 5139).

[133] Reading μ γ′ (40⅓), which is abundantly attested in the Arabic tradition (see S 81) at H161,12 for μγ (43) of the Greek tradition. P–K also adopt 40⅓.

[134] For the identifications of nos. 28-37 see P–K. nos. 962-71 on p. 112. The identifications they suggest are probably correct, in spite of the large errors in the coordinates, which are perhaps due to the difficulty of observing stars with extreme southern declinations.

[135] βατράχιον. The *Oxford English Dictionary* defines 'frog' (s.v. 2) as 'an elastic, horny substance growing in the middle of the sole of a horse's hoof'.

[136] This is the reading of D,Ar and an alternative reading in A. Other Greek mss. have 44⅙. −41⅙ is more correct, but all other stars in this group are assigned too great a southern latitude, so −44⅙ may have been Ptolemy's measurement. It is adopted by P–K.

[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
6	The more advanced of the 2 stars in the place where the neck joins [the head]	♋ 20⅓	−11⅚[117]	5	ω Hya
7	The rearmost of them	♋ 23⅓	−13⅔[118]	4	θ Hya
8	The middle star of the following three in the bend of the neck	♋ 28⅚	−15⅓	4	τ^2 Hya
9	The rearmost of the 3	♌ 0⅔	−14⅚	4	ι Hya
10	The southernmost of them	♋ 28½	−17⅙	4	τ^1 Hya
11	The faint, northernmost star of the 2 close together to the south	♋ 29⅙	−19¾	6	*BSC 3750[119]
12	The bright one of these two close stars	♌ 0	*−20½[120]	2	α Hya
13	The most advanced of the 3 stars to the rear, after the bend [in the neck]	♌ 6	−26½	4	κ Hya
14	The middle one	♌ 8⅔	−26	4	υ^1 Hya
15	The rearmost of the three	♌ 11⅙	*−23¼[121]	4	υ^2 Hya
16	The most advanced of the next 3 stars almost on a straight line	♌ 18	−24⅔	3	μ Hya
17	The middle one	♌ 20	−23¼	4	φ Hya
18	The rearmost of the three	♌ 23[122]	−22⅙[123]	3	ν Hya
19	The northernmost of the 2 stars after [i.e. to the rear of] the base of Crater[124]	♍ 1½	−25¾	>4	β Crt
20	The southernmost of them	♍ 2⅓	−30⅙	4	χ^1 Hya
21	The most advanced of the 3 stars after these, as it were in a triangle	♍ 12⅙	−31⅓	4	ξ Hya
22	The middle and southernmost one	♍ 14½	−33⅙	4	o Hya
23	The rearmost of the three	♍ 16⅙	−31⅓	3	β Hya
24	The star after Corvus, in the section by the tail	♎ 0	−13⅔	>4	γ Hya

H154

[117] The variant 14⅚ occurs in the later Arabic tradition (see S 74).
[118] The variant 19⅓ is attested for the later Arabic tradition by S 75.
[119] *BSC* 3750 is P–K's W.9ʰ439. Another possible identification is 28 Hya. 29 Hya, adopted by Manitius, is impossible, since it is south of α Hya (no. 12).
[120] P–K's emendation, κγ (23) for κ ∠′, is very plausible.
[121] The Greek mss are unanimous for 26¼ (so too T^1). Heiberg adopts 23¼ from an emendation by Bode, which is however found in the Arabic tradition (L,T^2,E,F).
[122] The variant 20⅓ was found in the margin of Isḥāq's autograph according to S 78.
[123] The variants 29⅙ and 22½ were found in the Arabic tradition according to S 78.
[124] The figures of Crater (the mixing-bowl) and Corvus (the raven, cf. no. 24) were depicted as sitting on the back of Hydra: see Thiele Fig. 54 on p. 129 and Pl. V (lower).

[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
25	The star on the tip of the tail	♎ 13½	*−17⅔[125]	>4	π Hya
	[25 stars, 1 of the second magnitude, 3 of the third, 19 of the fourth, 1 of the fifth, 1 of the sixth]				
	Stars round Hydra outside the constellation:				
26	The star to the south of the head	♋ 12½	−23¼	3	BSC 3314[126]
27	The star some distance to the rear of those in the neck [nos. 6–15]	♌ 11	*−16⅓	3	*ε Sex[127]
	[2 stars of the third magnitude]				
	[XLII] Constellation of Crater				
1	The star in the base of bowl, which is [applied in] common to Hydra	♌ 23⅙	−23	4	α Crt
2	The southernmost of the 2 stars in the middle of the bowl	♍ 2½	−19½	4	γ Crt
3	The northernmost of them	♍ 0	−18	4	δ Crt
4	The star on the southern rim of the mouth	♍ 7	−18⅓	>4	ζ Crt
5	The star on the northern rim	♌ 29⅓	−13⅔	4	ε Crt
6	The star on the southern handle	♍ 9⅙	−16⅙	<4	η Crt
7	The star on the northern handle	*♍ 1⅔	−11½	4	θ Crt
	[7 stars of the fourth magnitude]				
	[XLIII] Constellation of Corvus				
1	The star in the beak, which is [applied in] common to Hydra[128]	♍ 15⅓	−21⅔	3	α Crv
2	The star in the neck, by the head	♍ 14⅓	−19⅔	3	ε Crv

H156

[125] Since there is no doubt about the identification, the latitude is so wrong that one should consider emendation. Manitius (p. 405) suggests 13;40, on no authority.
[126] This identification is the same as that of Manitius and P–K, who use the obsolete nomenclature 30 Monocerotis (the star is now included in the constellation Hydra).
[127] The identification is highly uncertain. My suggestion has coordinates not impossibly different from Ptolemy's, but its magnitude is less than 5. P–K suggest 24 Sex, but this involves emending the latitude to 10⅚ (adopting the variant found in D,Ar of ις (16) for ις γ′), and the magnitude is still bad. Their alternative, α Sex, is not much better. Should one emend the magnitude to 6 (ς for γ)?
[128] For the description of nos. 1 and 7 cf. p. 392 n.124 and Thiele fig. 54 on p. 129, which depicts the raven standing on and pecking the water-snake.

[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
4	The one to the rear again of this	♐ 14⅚	−20	4	ζ CrA
5	The one after this, before the knee of Sagittarius	♐ 16⅙	−18½	5	δ CrA
6	The one after this, which is north of the bright star in the knee [of Sagittarius, XXX no. 24]	♐ 17	−17⅙	4	β CrA
7	The star to the north of this	♐ 16⅚[150]	−16	4	α CrA
8	The one to the north again of this	♐ 16½	−15⅙	4	γ CrA
9	The rearmost of the 2 stars after this, in advance, in the northern rim	♐ 15⅙	−15⅓	6	ε CrA
10	The more advanced of these 2 faint stars	♐ 14⅔	−14⅚	6	BSC 7129[151]
11	The star quite some distance in advance of this	♐ 11⅚	−14⅔	5	λ CrA
12	The one in advance again of this	♐ 9⅔	−15⅚	5	*BSC 6942[152]
13	The last one, which is south of the aforementioned star	♐ 9⅙	−18½	5	θ CrA
	{13 stars, 5 of the fourth magnitude, 6 of the fifth, 2 of the sixth}				
	[XLVIII] Constellation of Piscis Austrinus				
1	The star in the mouth, which is the same as the beginning of the water [= XXXII no. 42][153]	♒ 7	−20⅓	1	α PsA
2	The most advanced of the 3 stars on the southern rim of the head	♒ 0⅔	−20⅓	4	β PsA
3	The middle one	♒ 4⅙	−22¼	4	γ PsA
4	The rearmost of the three	♒ 5⅓	−22½	4	δ PsA
5	The star by the gills	♒ 4⅓	−16¼	>4	ε PsA
6	The star on the southernmost spine on the back	♑ 25⅙	−19½	5	μ PsA
H168 7	The rearmost of the 2 stars in the belly	♒ 1⅙	−15⅙	5	ζ PsA
8	The more advanced of them	♑ 28⅚	−14⅔	4	λ PsA
9	The rearmost of the 3 stars on the northern spine	♑ 25⅙	−15	4	η PsA
10	The middle one	♑ 21⅚	−16½	4	θ PsA
11	The most advanced of the three	♑ 21	−18⅙	4	ι PsA

150 The variant 20⅚ was found in the earliest Arabic tradition according to S 85.
151 This is the star which P–K call 'v Coronae Australis'; I do not know what their authority for this appellation is.
152 This is P–K's Lac. 7748. Manitius suggests κ CrA, which is certainly possible.
153 In Aquarius (XXXII 42) this is called 'the end of the water'.

[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
12	The star on the tip of the tail	♑ 20⅙[154]	−22¼	4	γ Gru
	{11 stars,[155] 9 of the fourth magnitude, 2 of the fifth}				
	Stars round Piscis Austrinus outside the constellation:				
13	The most advanced of the 3 bright stars in advance of Piscis [Austrinus]	♑ 8	−22⅓	<3	*η Mic[156]
14	The middle one	♑ 11⅙	−22⅙	<3	*θ¹ Mic
15	The rearmost of the three	♑ 14[157]	−21⅙	<3	*ξ Gru
16	The faint star in advance of this	♑ 12	−20⅚	5	*θ² Mic
17	The southernmost of the remaining 2 stars to the north	♑ 13⅚	−17	4	*γ Mic
18	The northernmost of them	♑ 13⅚	−14⅚	4	*α Mic
	{6 stars, 3 of the third magnitude, 2 of the fourth, 1 of the fifth}				
	{Total for the southern region 316 stars, 7 of the first magnitude, 18 of the second, 63 of the third, 164 of the fourth, 54 of the fifth, 9 of the sixth, 1 nebulous}				
	{Total for all stars 1022, 15 of the first magnitude, 45 of the second, 208 of the third, 474 of the fourth, 217 of the fifth, 49 of the sixth, 9 faint, 5 nebulous, plus Coma [Berenices]}				

154 The variant 26 occurs in the Arabic tradition (see S 88).
155 Only 11, because no. 1 has already been counted as Aquarius (XXXII) no. 42. Compare the remarks of ibn aṣ-Ṣalāḥ on pp. 74–75 of Kunitzsch's edition.
156 The identifications of nos. 13–18 are mine, but are very uncertain. P–K propose (13) α Mic, (14) γ Mic, (15) ε Mic, (16) Piazzi XX 445 = *BSC* 8076, (17) Piazzi XXI 12 = *BSC* 8110, (18) 24(A) Cap. These may be right, since they are in approximately correct relative positions, but they involve huge errors in the coordinates and (for no. 18) taking a star which has already been identified as Capricorn (XXXI) no. 13, where it has completely different coordinates.
157 Reading ιδ (with Ar, found as a correction in A) at H169,12 for ια (11) or δ (4) of the other Greek mss. The correction is certain, since no. 16 is 'in advance' of no. 15. It is adopted by P–K and Manitius.

[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
36	The star on the knee of the left [front] leg	♎ 24 1/6	−45 1/3	2	β Cen
37	The star outside, under the right hind leg	*♎ 14 2/3[137]	−49 1/6	4	μ Cru
	[37 stars, 1 of the first magnitude, 5 of the second, 7 of the third, 16 of the fourth, 8 of the fifth]				
	[XLV] Constellation of Lupus[138]				
1	The star at the end of the hind leg, by the [right] hand of Centaurus	♎ 28	−24 5/6	3	β Lup
2	The star on the bend in the same leg	♎ 25 5/6	−29 1/6	3	α Lup
3	The more advanced of the 2 stars just over the shoulder-blade	♏ 1	−21 1/4	4	δ Lup
4	The rearmost of them	♏ 4 1/6	−21	4	γ Lup
5	The star in the middle of the body of Lupus	♏ 3	−25 1/6	4	ε Lup
6	The star in the belly, under the flank	♏ 0 1/6	−27	5	λ Lup
7	The star on the thigh	♏ 0 1/2	−29	5	π Lup
8	The northernmost of the 2 stars near the place where the thigh joins [the body]	♏ 4 2/3	−28 1/2	5	μ Lup
9	The southernmost of them	♏ 3 2/3	−30 1/6	5	κ Lup
10	The star on the end of the rump	♏ 5 2/3	−33 1/6	5	ζ Lup
11	The southernmost of the 3 stars in the end of the tail	*♎ 22[139]	−31 1/3	5	*ρ Lup
12	The middle one of the three	♎ 21 5/6	−30 1/2	4	* ι Lup
13	The northernmost of them	♎ 23	−29 1/3	>4	$\tau^1 + \tau^2$ Lup
H164 14	The southernmost of the 2 stars in the neck	♏ 8 5/6	−17	4	η Lup
15	The northernmost of them	♏ 9 1/3	−15 1/3	>4	θ Lup
16	The more advanced of the 2 stars in the snout	♏ 5 2/3	−13 1/3	4	*$\psi^1 + \psi^2$ Lup[140]

[137] D has 11 2/3: as P–K remark (no. 971 on p. 112), this would be more consistent than 14 2/3 with the errors of the other stars in this group.

[138] Ptolemy does not identify this as a wolf or any particular animal, but calls it the 'beast' (θηρίον). It is depicted as being held by its hind legs in the right hand of Centaurus: see Thiele Fig. 53 on p. 128, and cf. no. 1 here.

[139] The mss. are unanimous for 22 (including the Arabic, despite the statement of P–K, no. 982 on pp. 112–13, that they have 20 1/3). Peters emends to 26 without authority. The identification of this star is dubious: see P–K's discussion, l.c. Manitius' identifications, here and elsewhere in Lupus, are mostly unacceptable.

[140] For the identifications of nos. 16 and 17 P–K prefer χ and ξ Lupi, but mine (which are also proposed by Manitius) seem more in accord with the relative positions.

[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
17	The rearmost of them	♏ 6 2/3	−11 5/6	4	*χ Lup
18	The southernmost of the 2 stars in the front leg	*♎ 27 1/6[141]	*−11 5/6[142]	>4	1(i) Lup
19	The northernmost of them	♎ 26 1/2	−10	>4	2(i) Lup
	[19 stars, 2 of the third magnitude, 11 of the fourth, 6 of the fifth]				
	[XLVI] Constellation of Ara[143]				
1	The northernmost of the 2 stars in the base	♏ 27 2/3	−22 2/3	5	σ Ara
2	The southernmost of them	*♐ 3[144]	−25 1/4	4	θ Ara
3	The star in the middle of the little altar	♏ 26 1/3[145]	−26 1/2	>4	α Ara
4	The northernmost of the 3 stars in the brazier	♏ 20 2/3	−30 1/3[146]	5	ε^1 Ara
5	The southernmost of the other 2 which are close together	♏ 25 1/6	−34 1/6	>4	γ Ara
6	The northernmost of these [2]	♏ 25	−33 1/3	4	β Ara
7	The star on the end of the burning-apparatus	♏ 20 5/6	*−34 1/4[147]	4	ζ Ara
	[7 stars, 5 of the fourth magnitude, 2 of the fifth]				
	[XLVII] Constellation of Corona Australis				
1	The most advanced of the stars on the southern rim, outside [the crown]	♐ 9 1/6	−21 1/2	4	*α Tel[148]
H166 2	The star to the rear of this,[149] on the crown	♐ 11 2/3	−21	5	$\eta^1 + \eta^2$ CrA
3	The one to the rear of this	♐ 13 1/6	−23	5	BSC 7122

[141] L,T,E, Ger have 27 1/3, adopted by P–K.

[142] L,T^2,E, Ger have 11 1/2, adopted by P–K.

[143] θυμιατήριον, actually an incense-burner. It is depicted upside-down (i.e. base towards the north).

[144] BC have 3 1/6. Much of the Arabic tradition, and Ger, have 0 1/3, but 3 is also found (see S 84).

[145] 26 1/6 is found as an alternative reading in A, and in Is. It is adopted by P–K. The 'little altar' (βώμισκος) is evidently the same as the 'brazier' (ἐπίπυρον) in no. 4: see p. 400 n.160.

[146] Reading λ γ′ (with Ar) at H165,13 for α γ′ (1 1/3), the unanimous reading of the Greek mss. Heiberg (ad loc.) realized that this correction should be made, and Manitius made it.

[147] This is the reading of A; 31 1/4 BCD, 34 Ar, adopted by P–K.

[148] This is Manitius' identification. P–K prefer $\delta^1 + \delta^2$ Telescopii.

[149] Reading αὐτῷ (implied by Ar) at H166,2 for αὐτῶν 'that one of those', which has no reference.

H170 ## 2. {*On the situation of the circle of the Milky Way*}[158]

Such, then, is the way in which we may set out the order of the fixed stars. To this we shall join, as the logical order demands, our discussion of the disposition of the circle of the Milky Way, to the best of our ability, with our observations of each of its sections: we shall try to describe the form which the individual parts appear to take.

Now the Milky Way is not strictly speaking a circle, but rather a belt of a sort of milky colour overall (whence it got its name); moreover this belt is neither uniform nor regular, but varies in width, colour, density and situation, and in one section is bifurcated. [All] that is very apparent even to the casual eye, but the details, which can only be determined by a more careful examination, we find to be as follows.

The bifurcated part of the belt has one of its 'forks', so to speak, near Ara, and the other in Cygnus. But, whereas the advance [part of the] belt is in no way attached to the other part, since it forms gaps both at the fork by Ara and at the
H171 fork by Cygnus, the rearmost part is joined to the remainder of the Milky Way and forms [with it] a single belt, through which the great circle drawn approximately along the middle of it would pass. It is this belt which we shall describe first, beginning with its southernmost section.

This [section] goes through the legs of Centaurus, and is rather less dense and less bright [than the rest]. The star on the knee-bend of the right hind leg [XLIV 31] is a little farther south than the line [bounding] the milk to the north, and so are the star on the left front knee [XLIV 36] and the star under the right hind hock [XLIV 32]. But the star in the left hind lower leg [XLIV 33] lies in the middle of the milk, and the stars on the hock of the same leg [XLIV 34][159] and on the right front hock [XLIV 35] are to the north of its southern rim, by about 2° (where the great circle is 360°). It is slightly denser in the region near the hind legs.

Next in order, the northern rim of the milk is about 1$\frac{1}{2}$° from the star on the rump of Lupus [XLV 10], and the southern rim encloses the star on the burning-apparatus of Ara [XLVI 7], but just grazes the northernmost of the two stars close together in the brazier [XLVI 6] and the southernmost of the two stars in the base [XLVI 2], while the star in the northern part of the brazier and
H172 the one in the middle of the brazier [XLVI 4, 3][160] lie right in the milk. These sections are rather less dense.

Next, the northern part of the milk encloses the three joints before the sting of Scorpius [XXIX 17, 18, 19] and the nebulous mass to the rear of the sting [XXIX 22], while the southern rim touches the star in the right front hock of Sagittarius [XXX 25], and encloses the star on his left hand [XXX 2]. The star on the southern portion of the bow [XXX 3][161] is outside the milk, but the star

[158] I have appended to the stars named in this chapter references to their place in the catalogue (VII 5 and VIII 1).

[159] In the catalogue this star is described, not as 'on the hock', but as 'on the frog of the hoof'.

[160] In the catalogue this last star is called 'the star in the middle of the little altar'.

[161] Reading τόξου for Τοξότου ('Sagittarius') at H172,8, with Is. The same correction has to be made for the next star (H172,11). Corrected by Manitius. In the catalogue (H112,12-14) Heiberg rightly prints τόξου, although there too all or most Greek mss. have τοξότου in all three places.

on the point of the arrow [XXX 1] lies in the middle of it, while the stars in the northern part of the bow [XXX 4, 5] also lie in it, each of them being a little more than 1° removed from one of the rims, the southern star from the southern rim, the northern star from the opposite rim. The area [of the Milky Way] near the three joints [of Scorpius] is somewhat denser, while the area round the point [of the arrow of Sagittarius] is very dense indeed and appears smoky.

The following section is a little less dense. It extends along [the constellation] Aquila, maintaining about the same width throughout. The star on the tip of the tail of the snake [Serpens, XIV 18] held by Ophiuchus lies in the open,[162] a little more than one degree away from the advance rim of the milk, while the H173
two most advanced of the bright stars below it lie right in the milk: the southern one [XVI 15] is 1° from the rear rim, and the northern one [XVI 12], 2° [from it]. The rearmost of the [two] stars in the right shoulder of Aquila [XVI 8] touches the same rim, while the more advanced one [XVI 7] is cut off inside it, as is also the more advanced, bright star of those in the left wing [XVI 5].[163] Furthermore, the bright star on the place between the shoulders [XVI 3] and the two stars which lie on a straight line with it[164] fall a little short of touching the same rim. Next, Sagitta is enclosed entirely within the milk. The star on the arrowhead [XV 1] lies one degree from the eastern rim, while the star on the notch [XV 5] lies two degrees from the western rim. The section round Aquila is slightly denser, and the remainder slightly less dense.

Next the milk extends towards Cygnus. Its north-western rim is defined in a reentrant angle[165] by the star in the southern shoulder of Cygnus [IX 11],[166] the star under it in the same [southern] wing [IX 10], and the two stars on *the southern leg* [IX 13, 14]. *Its south-eastern rim is defined* by the star in the tip of the southern wing-feathers [IX 12], and encloses the two stars under the same wing outside the constellation [IX 18, 19], which are about 2° from it [the rim]. The section around the wing is slightly denser. The next H174
section is continuous with that belt, but is much denser and seems to have a different starting-point.[167] For it points towards the end parts of the other belt,[168] but leaves a gap between it [and itself]: on its southern side it joins the belt which we are currently describing, which is very rarefied at the junction; but after the point where it forms a gap with the other belt it gets denser,

[162] Literally 'in the open air', i.e. outside the Milky Way.

[163] In the catalogue these stars are described as being 'in the left shoulder'.

[164] This does not correspond to any description in the catalogue. Manitius identifies the two stars as XVI 2 and 4 (β and ο Aql). These are indeed approximately on a straight line with XVI 3 (α Aql), but they hardly fit the rest of the description, since β Aql lies well outside the Milky Way as viewed by Ptolemy. More appropriate would be φ Aql (XVI 6) and υ Aql. However, the latter star seems not to be mentioned in the catalogue.

[165] ἐν ἐπικαμπίῳ. Explained by what follows: this is where the other (western) branch of the Milky Way joins; since, according to Ptolemy, the part north of this is aligned with the end of that branch, it forms a reentrant angle with the present, eastern branch. This is best seen on a star globe.

[166] In the catalogue this is called 'the star in the middle of the left wing'.

[167] Translating Heiberg's emendation, ὁρμώμενα (supported by Is: 'ibtada'a') for the ὁρώμενα of the Greek mss. and L. The latter could perhaps be translated as 'and is seen, as it were, from a different starting-point', but this is very harsh.

[168] I.e. the other branch of the Milky Way which is mentioned above (p. 400) and described below (p. 403).

beginning from the bright star in the rump of Cygnus [IX 5][169] and the nebulous mass in the northern knee [IX 17]. Then it makes a slight bend as far as the star on the southern knee [IX 14], and continues, gradually diminishing in density, up to the tiara of Cepheus. The northern side is delimited by the southernmost of the three stars in the tiara [IV 9] and the star to the rear of those three [IV 13], at which it also forms two outrunners, one verging to the north and east, the other to the south and east.

Next the milk encloses the whole of Cassiopeia except for the star in the end of the leg [X 7]. The southern rim is defined by the star in the head of Cassiopeia [X 1], and the northern rim by the star in the foot of the throne [X 11] and the star in the lower leg of Cassiopeia [X 6]. The other stars [of Cassiopeia] and all
H175 those round about this [constellation] lie in the milk. The areas near the rims are of thinner consistency, but those at the middle of Cassiopeia display a dense patch running the length [of the Milky Way].

Next, the righthand parts of Perseus are enclosed in the milk. Furthermore, its northern edge, which is very rarefied, is defined by the lone star outside the right knee of Perseus [XI 28], and its southern edge, which is very dense, by the bright star on his right side [XI 7] and by the two rearmost stars of the three to the south of that [bright star, XI 9, 10]. Enclosed in it also are the nebulous mass on the hilt [XI 1],[170] the star in the head [XI 5], the star in the right shoulder [XI 3] and the star on the right elbow [XI 2]. The quadrilateral in the right knee [XI 16, 17, 18, 19] and also the star on the same [right] calf [XI 20] lie in the midst of the milk, while the star in the right heel [XI 21][171] is also inside it, a little distance from the southern border.

Next the belt goes through Auriga, displaying a slightly thinner consistency. The star on the left shoulder, called Capella [XII 3], and the two stars on the right forearm [XII 5, 6] fall just short of touching the north-eastern rim of the milk, while the small star over the left foot in the lower hem [of the garment, XII 14] defines the south-western edge. The star over the right foot [XII 12] lies half
H176 a degree within the same edge, and the two stars close together on the left forearm, called Haedi [XII 8, 9], lie in the middle of the belt.

Next the milk goes through the legs of Gemini, displaying a certain amount of density in elongated form just over the stars at the ends of the legs. Now the advance edge of the milk is defined by the rearmost of the 3 stars on a straight line under the right foot of Auriga [XXIV 19], by the rearmost star of the two in the staff of Orion [XXXV 12] and by the northernmost [two] of the four stars on his [Orion's] hand [XXXV 9, 10]; the brilliant star under the right hand of Auriga [XXIV 20] and the star in the rear foot of the rear twin [XXIV 18] are approximately 1° inside the rear edge, while the stars in the other feet [XXIV 14, 15, 16, 17] lie in the midst of the milk.

Thence the belt passes by Canis Minor [Procyon] and Canis Major: it leaves the whole of Canis Minor outside the milk no small distance to the east, and

[169] This is called 'the star in the tail' in the catalogue.

[170] In the catalogue this conglomeration is said to be 'on the right hand'. Perseus holds his weapon, the ἅρπη (cf. Hipparchus 2.5.15, ed. Manitius 198,10), the hilt of which Ptolemy refers to here, in his right hand.

[171] In the catalogue this is described as 'on the right ankle'.

leaves Canis Major too outside to the west, almost in its entirety; for the star on its ears[172] [XXXVIII 2] is caught by a sort of cloud which projects [from the Milky Way] and which then almost touches the three stars in the neck of Canis Major next to that [star] towards the rear [XXXVIII 3, 4, 5], while the lone star over the head of Canis Major, outside it and at some distance [XXXVIII 19], is about $2\frac{1}{2}°$ inside the eastern rim. The consistency in this whole region[173] is somewhat thinner.

After that the milk passes through Argo. The western rim of the belt is defined H177
by the northernmost and most advanced of the stars in the little shield in the poop [XL 5]. The star in the middle of the little shield [XL 6], the two stars close together under it [XL 8, 9], the bright star at the beginning of the deck near the steering-oar [XL 17] and the midmost of the three stars in the keel [XL 38] are just short of touching the same [western] edge. The northernmost of the three stars in the mast-holder [XL 22] defines the eastern rim, while the bright star in the stern-ornament [XL 2] is 1° within the same [eastern] edge, and the bright star under the rearmost little shield in the deck [XL 31] is the same amount, 1°, outside the same [eastern] edge. The southernmost of the two brilliant stars in the middle of the mast [XL 27] touches the same edge, and the two bright stars at the point where the keel is cut off[174] [XL 35, 36] are about 2° inside the advance rim. At that point the milk joins the belt through the legs of Centaurus.[175] The consistency in this area too, throughout Argo, is somewhat rarefied, but the sections of it around the little shield, the mast-holder and the point where the keel is cut off are more dense.

The belt we mentioned previously[176] forms a gap, as we said, between [itself and] the one we have [just] described, at Ara. Beginning at that point, it
encloses the three joints of Scorpius' [tail] nearest the body [XXIX 12, 13, 14], H178
but leaves the rearmost star of the three in the body [XXIX 9] 1° outside its western rim. The star in the fourth joint [XXIX 16] lies in the open space between the two belts, about the same distance from each, a little more than 1°.

After that the advance belt turns aside to the east, in the shape of a segment of a circle, defining the advance edge of the milk by the star on the right knee of Ophiuchus [XIII 12], and the rear edge by the star on the same [right] shin [XIII 13], while the most advanced of the stars at the end of the same [right] leg [XIII 14] touches that same [rear] edge. Subsequently the western rim is defined by the star under the right elbow of Ophiuchus [XIII 9], and the eastern rim by the more advanced of the two stars in the same [right] hand [XIII 10].

[172] Reading ἐπὶ τῶν ὤτων (with Ar; D^1 has ἐπὶ τῶν νώτων) for ἐπὶ τῷ νώτῳ ('on the back') at H176,18. The correction was made by Kunitzsch, *Der Almagest* no. 533 on p. 322. It is confirmed by the whole context, and especially by the position of the star, θ CMa. Manitius identifies the star here with XXXVIII 12, which is said in the catalogue to be 'in the left shoulder', but this star (o^2 CMa) lies well outside the Milky Way as viewed by Ptolemy.

[173] Reading τό χύμα ὅλον τοῦτο ἠρέμα ἀραιότερον (with D) at H176,24, to get a normal word order, for τὸ χύμα τοῦτο ἠρέμα ὅλον ἀραιότερον.

[174] Reading ἐν τῇ ἀποτομῇ (with D^1,Ar) at H177, 13–4 for ἐν τῇ αὐτῇ ἀποτομῇ ('in the same cut-off of the keel'), which is senseless.

[175] I.e. the point where Ptolemy began the description, p. 400.

[176] I.e. the western 'fork' mentioned on p. 400. But it is tempting to follow Is, who has 'advance' (i.e. προηγουμένη) here and 'mentioned previously' below at H178,7 (i.e. προειρημένη for προηγουμένη, 'advance', of the Greek mss.)

From that point on there is a considerable gap of open space, in which lie the two stars on the tail of Serpens [XIV 16, 17] next to the star in the tip [of the tail, XIV 18]. The whole of the section of this belt which we have [just] finished describing consists of an extremely fine and almost aery substance, except for the area enclosing the three joints [of Scorpius], which is somewhat more concentrated.

H179 After the gap the milk again makes a fresh beginning at the four stars to the rear of the right shoulder of Ophiuchus [XIII 25, 26, 27, 28]. The eastern rim of this belt is defined (being just grazed) by the lone brilliant star under[177] the tail of Aquila [XVI 9], while the opposite rim is defined by the star which is some distance to the north of the four just mentioned [XIII 29]. From there on this belt, besides being rarefied, is also contracted into a narrow space in the area which is in advance of the star in the beak of Cygnus [IX 1], so as to produce the appearance of a gap. However, the remainder of it, from the star in the beak up to the star in the breast of Cygnus [IX 4], is wider and considerably denser. The star in the neck of Cygnus [IX 3] lies in the middle of the dense section. A rarefied section branches off to the north from the star[178] in the breast as far as the star in the shoulder of the right wing [IX 6] and the two stars close together in the right foot [IX 15, 16]. From this point, as we said, occurs a clear gap to the other belt, [a gap] stretching from the above-mentioned stars in Cygnus up to the bright star in the rump [IX 5].

3. {*On the construction of a solid globe*}[179]

Such, then, is the disposition of the phenomena associated with the Milky Way. But we also wish to provide a representation [of the fixed stars] by means of a
H180 solid globe in accordance with the hypotheses which we have demonstrated concerning the sphere of the fixed stars, according to which, as we saw, this sphere too, like those of the planets, is carried around by the primary [daily] motion from east to west about the poles of the equator, but also has a proper motion in the opposite direction about the poles of the sun's, ecliptic circle. To this end we shall carry out the construction of the solid globe and the delineation of the constellations in the following fashion.

We make the colour of the globe in question somewhat deep, so as to resemble, not the daytime, but rather the nighttime sky, in which the stars actually appear. We take two points on it precisely diametrically opposite, and with these as poles draw a great circle: this will at all times be in the plane of the ecliptic. At right angles to the latter and through its poles we draw another [great] circle, and starting from one of the intersections of this with the first

[177] Reading ὑπὸ (with D,Ar) for παρὰ ('by') at H179,4. Compare the description of XVI 9 (p. 357).

[178] Reading ἀπὸ τοῦ ἐν τῷ στήθει (with Ar) at H179,14–15 for καὶ τῶν ἐν τῷ στήθει. Corrected by Manitius (ἀπὸ already suggested by Heiberg ad loc.)

[179] On this 'precession-globe' see *HAMA* II 890–92, with Figs. 79–80 on p. 1399 (for an error in Neugebauer's account see p. 405 n. 181). On the history of the star-globe in antiquity see Schlachter, *Der Globus*.

circle we divide the ecliptic into the [conventional] 360 degrees, and write by it the numbers at intervals of however many degrees seems convenient. Then we make, from a tough and unwarped[180] material, two rings with rectangular cross-section, accurately turned on the lathe in all dimensions: one should be smaller [than the other], and fit closely to the globe on the whole of its inner surface, while the other should be a little larger than this. In the middle of the H181
convex face of each ring we draw a line accurately bisecting its width. Using these lines as guides, we cut out[181] one of the latitudinal sections[182] defined by the line over half of the circumference, and divide [each of] the semi-circular recessed sections [thus created] into 180 degrees. When this is done, we take the smaller of the rings as the one which will always represent the circle through both poles, that of the equator and that of the ecliptic, and also through the solstitial points ([this circle runs] along the plane surface of the above-mentioned recessed section), and, boring holes through the middle of it at the diametrically opposite points at the ends of the recessed section, we attach it, by means of pins [through those holes], to the poles of the ecliptic which we took on the globe, in such a way that the ring can revolve freely over the whole spherical surface.

Since it is not reasonable to mark the solstitial and equinoctial points on the actual zodiac of the globe (for the stars depicted [on the globe] do not retain a constant distance with respect to these points), we need to take some fixed starting-point in the delineated fixed stars. So we mark the brightest of them, namely the star in the mouth of Canis Major [Sirius], on the circle drawn at right angles to the ecliptic at the division forming the beginning of the graduation, at the distance in latitude from the ecliptic towards its south pole H182
recorded [in the star catalogue]. Then, for each of the other fixed stars in the catalogue in order, we mark the position by rotating the ring with the graduated recessed face about the poles of the ecliptic: we turn the face of its recessed section to that point on the [globe's] ecliptic which is the same distance from the beginning of the numbered graduation (at Sirius) as the star in question is from Sirius in the catalogue;[183] then we go to that point on the

[180] εὐτόνου καὶ τεταμένης. The meaning of both adjectives is disputable. The context requires that the material (certainly wood, although ὕλη does not *mean* wood here, *pace Manitii*) be strong in the sense that it can be cut into thin strips and bored through. Cf. Heron, *Belopoeica* 94, ed. Marsden p.30.12, where the side-pieces of a catapult must be made ἐξ εὐτόνου ξύλου. εὔτονος occurs frequently in that work, and is usually applied to sinews or elements requiring *elastic* strength (e.g. 110, ibid.p.38.2; cf. Heron, *Pneumatica*, ed. Schmidt p. 200, where it is used of pieces of horn). But it seems improbable that Ptolemy means 'flexible' wood here and the meaning 'rigidly strong' is certain in one passage of Heron's *Mechanics*, preserved in Pappus, *Synagoge* VIII, 1132, 6–14. τεταμένης means literally 'stretched'. I know of no real parallel, but take it to be a synonym of ἀστραβής, 'unwarped', found frequently in Theophrastus, *Historia Plantarum*, e.g. 5.2.1.

[181] I.e., cut out along the central line so that half the width of the ring is removed for half the circumference of the ring. The purpose of this is that the graduated face may be flush with the surface of the globe, and coincide with a great circle. The result is depicted in *HAMA* Fig. 80A p. 1399, lower part. Neugebauer is wrong (p. 891) in saying that the text implies the making of a central slit in the rings: he has been misled by Manitius' translation.

[182] Reading πλευρῶν (with D) for πλευρὰς at H181,5. Corrected by Manitius.

[183] Since Sirius has in the catalogue (XXXVIII 1) the longitude ♊ 17⅔°, this means that one subtracts 77;40° from the catalogue longitudes. Wherever my translation has 'Sirius', Ptolemy has κύων ('the Dog'). Cf. p. 387 n.88.

graduated face which we have [thus] positioned which is, again, the same distance from the ecliptic as the star is in the catalogue, either towards the north or towards the south pole of the ecliptic as the particular case may be, and at that point we mark the position of the star; then we apply to it a spot of yellow colouring (or, for some stars, the colour they are noted [in the catalogue] as having), of a size appropriate to the magnitude of each star.

As for the configurations of the shapes of the individual constellations, we make them as simple as possible, connecting the stars within the same figure only by lines, which moreover should not be very different in colour from the general background of the globe. The purpose of this is, [on the one hand], not to lose the advantages of this kind of pictorial description, and [on the other] not
H183 to destroy the resemblance of the image to the original by applying a variety of colours, but rather to make it easy for us to remember and compare when we actually come to examine [the starry heaven], since we will be accustomed to the unadorned appearance of the stars in their representation on the globe too.

We also, then, mark the location of the Milky Way on [the globe], in accordance with its positions, arrangements, densities and gaps as described above. Then we attach the larger of the rings, which will always represent a meridian, to the smaller ring which fits around the globe, on poles coinciding with those of the equator. These points [the poles of the equator] are, in the case of the larger, meridian [ring], attached, again, at the diametrically opposite ends of the recessed and graduated face (which will represent the [section of the meridian] above the earth); but in the case of the smaller ring, [which passes] through both poles, they will be fixed at the ends of the diametrically opposite arcs which stretch the 23;51° of the obliquity from each of the poles of the ecliptic. We leave small solid pieces in the recessed parts of the rings, to receive the bore-holes for the attachments [of the pins representing the poles].

Now the recessed face of the smaller of the rings must, clearly, always coincide with the meridian through the solstitial points. So on any occasion
H184 [when we want to use the globe], we set it to that point of the ecliptic graduation whose distance from the starting-point defined by Sirius is equal to the distance of Sirius from the summer solstice at the time in question (e.g. at the beginning of the reign of Antoninus, $12\frac{1}{3}°$ in advance). Then we fix the meridian ring in position perpendicular to the horizon defined by the stand [of the globe],[184] in such a way that it is bisected by the visible surface of the latter, but can be moved round in its own plane: this is in order that we may, for any particular application, raise the north pole from the horizon by the appropriate arc for the latitude in question, using the graduation of the meridian [to place the ring correctly].

We shall suffer no disadvantage from our inability to mark the equator and the solstitial points on the globe itself. For since the face of the meridian is graduated, the point between the poles of the equator which is 90° of the quadrant distant from both will be equivalent to points on the equator, while the points 23;51° distant from that point will be equivalent to points on the two solstitial circles, the one to the north to those on the summer solstitial circle, and

[184] This has not been described. For a schematic representation, with a suggestion for how the motion in the plane of the meridian may be achieved, see *HAMA* p.1399 Fig. 80C.

the one to the south to those on the winter solstitial circle. Thus, when any required star is rotated with the primary, east-to-west rotation to the graduated face of the meridian, we can again, by means of that same graduation, H185 determine its distance from the equator or the solstitial circles, as measured on the great circle through the poles of the equator.

4. {*On the configurations particular to the fixed stars*}

Now that we have demonstrated the distinctive features of the pictorial representation of the fixed stars, it remains to discuss their configurations. The configurations involving the fixed stars, then, are, apart from those fixed configurations with respect to each other (e.g. such and such stars lie on a straight line, form a triangle, and the like), as follows:
[1] those considered with respect to the planets, sun and moon, or the parts of the zodiac alone;
[2] those considered with respect to the earth alone;
[3] those considered with respect both to the earth and at the same time to the planets, sun and moon, or the parts of the zodiac.

[1] Those configurations of the fixed stars with the planets and the parts of the zodiac alone which are accepted are
[a] for all stars in general, when fixed star and planet come to be on the same circle through the poles of the ecliptic, or on circles which are different, but at H186 intervals [of a regular polygon] with three, four or six angles,[185] i.e., which enclose an angle which is either a right angle or a third of a right angle greater or less than a right angle;
[b] for some stars in particular, those for which one of the planets can pass directly below it (these are the stars fixed in that narrow band[186] of the zodiac containing the latitudinal motions of the planets) - for these, [configurations] with the five planets concern their apparent contacts[187] or their occultations, and with the sun and moon, their last visibilities, conjunctions and first visibilities. We give the name 'last visibility' to the situation when a star falls within the rays of [one of] the luminaries and begins to become invisible; 'conjunction', when it is covered by the centre of [one of] them;[188] and 'first visibility',[189] when it escapes their rays and begins to be visible.

[185] These are the relationships trine, quartile and sextile, commonly applied in astrology: see Bouché-Leclercq, e.g. 165-79.

[186] πρίσμα, literally 'a sawn-out section'. This is probably the term that Ptolemy used for the 'drums' containing the planetary models in Bk. II of his *Planetary Hypotheses* (preserved only in Arabic translation); see e.g. *Op. Min.* p. 113. The word has nothing to do with the geometrical 'prism' here.

[187] κολλήσεις. This is a technical term in astrology. It includes certain kinds of close approach, besides actual occultations. For details see Bouché-Leclercq 245, quoting Porphyrius. See also Vettius Valens, index p. 380, s.v. At *Almagest* IX 2 (H213,3), it appears to mean actual contact.

[188] Reading αὐτῶν (with D) for αὐτοῦ at H186,13.

[189] Literally 'rising' (ἐπιτολή). For the planets Ptolemy uses the more appropriate word φάσις. For an explanation of the full panoply of terms associated in traditional Greek astronomy with the risings and settings of stars see below pp. 409-10, and cf. Autolycus περὶ ἐπιτολῶν I introduction (ed. Mogenet 214).

[2] The configurations of the fixed stars with the earth alone are four in number. The term applied by some people to all in common is 'cardines'.[190] Their individual titles are 'ascendant', 'culmination above the earth', 'descendant' and 'culmination below the earth'.[191] Now in the region where the equator is in the zenith all the fixed stars rise and set and once in every revolution reach culmination above the earth, and once culmination below the
H187 earth; for in that situation the poles of the equator lie on the horizon, and do not make any of the parallel circles either always visible or always invisible. And in the regions where [one of] the poles is in the zenith, none of the fixed stars either rises or sets. For in that situation the equator assumes the position of the horizon, and one of the hemispheres into which it divides [the heavens] rotates always above the earth, while the other rotates always below the earth. Hence each star repeats the same type of culmination twice in one revolution, some reaching culmination above the earth twice, the others culmination below the earth twice. But at the other, intermediate, terrestrial latitudes, some of the [parallel] circles are always visible, and some always invisible; so the stars cut off between these and the poles neither rise nor set, and perform two culminations in each revolution; those stars in the region which is always visible [culminate twice] above the earth, and those in the region which is always invisible [culminate twice] below the earth. The remaining stars, which lie on parallels greater [than the always visible and invisible parallels], both rise and set, and culminate once above the earth and once below the earth in each revolution. For these stars the time [of travel] from any one of the cardines back to the same one is the same at every place: it comprises one revolution, to the senses.[192] The
H188 time from one cardine to the one diametrically opposite is the same at every place when one considers meridian [passage], since it comprises half a revolution. When one considers horizon [passage] it is again constant where the equator is in the zenith: each of the two intervals [from rising to setting and from setting to rising] comprises half a revolution, since in that case all the parallel circles are bisected, not only by the meridian, but also by the horizon. However, at all other terrestrial latitudes, if one takes separately the time spent above the earth and the time spent below the earth [by a star], neither is the same for all stars [at a given latitude]; nor is the time spent above the earth for any particular star equal to the time it spends below the earth, except for those stars which happen to lie precisely on the equator: for the latter is the only circle which is bisected by the horizon at *sphaera obliqua* too, whereas all the other [parallels] are divided [by the horizon] into arcs which are neither similar nor equal. Furthermore, in accordance with this, the time from rising or setting to one or other of the culminations is equal to the time from the same culmination to setting or rising, since the meridian bisects those segments of the parallels which are above and below the earth; but the times from rising or setting to the two [opposite] culminations are unequal at *sphaera obliqua*, but equal at *sphaera recta*, since only

[190] κέντρα. The primary importance of these points is in astrology: see Bouché-Leclercq 257-9.

[191] The two types of culmination are usually known in modern times as 'upper' and 'lower' culmination (see Introduction p. 19). I retain the literal terminology here for obvious reasons.

[192] The qualification 'to the senses' is inserted because of precession (the effect of which is negligible over one daily revolution).

in the latter situation are the whole segments [of the parallel circles] above the earth equal to the segments below the earth.[193] Hence, for *sphaera recta*, [heavenly bodies] which culminate simultaneously always rise and set simultaneously too (in so far as their motion about the poles of the ecliptic is H189 imperceptible);[194] but, for *sphaera obliqua*, [heavenly bodies] which culminate simultaneously neither rise nor set simultaneously, but the more southerly ones always rise later and set sooner than the more northerly.

[3] The accepted configurations of the fixed stars considered with respect to the earth and at the same time to the planets or the parts of the zodiac are:
[a] in general, their risings, culminations or settings which are simultaneous with those of one of the planets or with some part of the zodiac;
[b] in particular, their configurations with respect to the sun, which are of 9 types.

The first type of configuration is that called 'dawn easterly', when the star is on the eastern horizon together with the sun. One variety of this is called 'dawn invisible later rising', when the star, which is just at last visibility, rises immediately after the sun; another is called 'dawn true simultaneous rising', when the star arrives at the eastern horizon at precisely the same time as the sun; the third is called 'dawn visible earlier rising', when the star, which is just at first H190 visibility, rises before the sun.

The second type of configuration is that called 'dawn culmination', when the sun is on the eastern horizon while the star is at the meridian, either above or below the earth. Of this too there are varieties: one is called 'dawn invisible later culmination', when the star culminates immediately after sunrise; a second is called 'dawn true simultaneous culmination', when the star culminates at the same time as the sun rises; and the third is called 'dawn earlier culmination', when the star culminates immediately before sunrise. When the latter is a culmination above the earth it is visible.

The third type of configuration is that called 'dawn westerly', when the sun is on the eastern horizon and the star on the western. This too has varieties: one is called 'dawn invisible later setting', when the star sets immediately after sunrise;[195] a second is called 'dawn true simultaneous setting', when the star sets at exactly the same time as the sun rises; and the third is called 'dawn visible earlier setting', when the sun rises immediately after the star has set.[196]

The fourth type of configuration is that called 'meridian easterly', when the H191 sun is on the meridian and the star is on the eastern horizon. This too has varieties: one during the day and invisible, when the sun is culminating above the earth as the star is rising; the other during the night and visible, when the sun is culminating below the earth as the star is rising.

The fifth type of configuration is that called 'meridian culmination', when sun and star both reach the meridian at the same time. This too has varieties:

[193] If *a* is the time from rising to upper culmination, *b* from upper culmination to setting, *c* from setting to lower culmination, and *d* from lower culmination to rising, then a = b and c = d but (at *sphaera obliqua*) a ≠ c and b ≠ d.

[194] This implies that Ptolemy is thinking of planets as well as fixed stars.

[195] Reading ἀνατείλαντος (with D) for ἀνατέλλοντος at H190,18. Corrected by Manitius.

[196] Reading καταδύναντος (with D) for καταδύνοντος at H190,22.

two are during the day and invisible, when the sun is culminating above the earth and the star is either culminating above the earth together with the sun, or else culminating below the earth opposite it; and two are during the night, when the sun is culminating below the earth; of these one is invisible, when the star too culminates below the earth together with the sun, and the other is visible, when the star culminates above the earth opposite it.

The sixth type of configuration is that called 'meridian westerly', when the sun is on the meridian and the star is on the western horizon. This too has
H192 varieties: one during the day and invisible, when the sun is culminating above the earth as the star is setting; the other during the night and visible, when the sun is culminating below the earth as the star is setting.

The seventh type of configuration is that called 'evening easterly', when the sun is on the western horizon and the star on the eastern. This again has varieties: one is called 'evening visible later rising', when the star rises immediately after the sun has set; another is called 'evening true simultaneous rising', when the star rises at the same time as the sun sets; the third is called 'evening invisible earlier rising', when the sun sets immediately after the star has risen.

The eighth type of configuration is that called 'evening culmination', when the sun is on the western horizon and the star is on the meridian either above or below the earth. This too has varieties: one is called 'evening later culmination', when the star culminates immediately after sunset (when the culmination is above the earth, this is visible);[197] another is called 'evening true simultaneous
H193 culmination', when the star culminates at the same time as the sun sets; the third is called 'evening invisible earlier culmination', when the sun sets immediately after the star has culminated.

The ninth type of configuration is that called 'evening westerly', when the star is on the western horizon together with the sun. This too has varieties: one is called 'evening visible later setting', when the star, just at last visibility, sets immediately after the sun; another is called 'evening true simultaneous setting', when the star sets at exactly the same time as the sun; and the third is called 'evening invisible earlier setting', when the star, which is just at first visibility, sets [just] before the sun.

5. {*On simultaneous risings, culminations and settings of the fixed stars*}[198]

Given the above definitions, the times of the true simultaneous risings, culminations and settings, which are taken with respect to the sun's centre, can be found by us immediately from the position of [the stars in question] in the delineation of the stars [on the solid globe], by purely geometrical methods.
H194 For the points on the ecliptic with which each fixed star simultaneously

[197] Adopting the reading of D,Ar, which omits φαινόμενον at H192,19 and adds καὶ τὸ ὑπὲρ γῆν τούτου φαινόμενον γίνεται after μεσουρανήσῃ at H192,20. The text printed by Heiberg falsely implies that both upper and lower culminations are visible.

[198] See *HAMA* 32-4, 39.

culminates, rises or sets can be derived geometrically by means of the theorems [already] established.[199]

First, to demonstrate the simultaneous culminations, let [Fig. 8.1][200] the circle through both poles, that of the equator and that of the ecliptic, be ABGD. Let AEG be a semi-circle of the equator about pole Z, and BED a semi-circle of the ecliptic about pole H. Draw through the poles of the ecliptic the great circle segment HΘKL, and take on it point Θ as the required fixed star (for it is with respect to such circles [i.e. great circles through the poles of the ecliptic] that we have observed and recorded the positions of the fixed stars). Also, draw through the poles of the equator and the star at Θ the great circle segment ZΘMN.

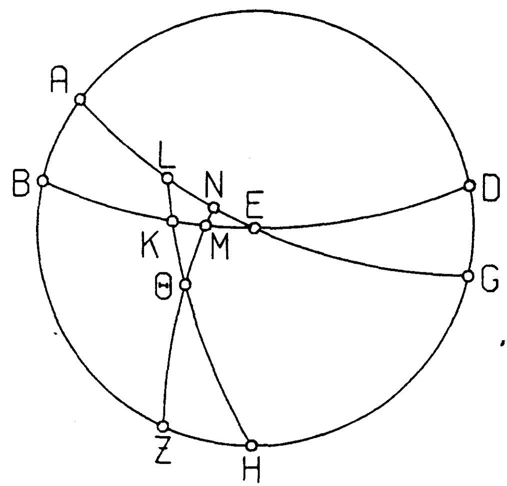

Fig. 8.1

Now it is obvious that the star at Θ culminates simultaneously with points M and N of the ecliptic and equator [respectively]. But these points, and arc ΘN, are given, as will be clear from the following considerations. From what we proved at the beginning of our treatise [I 13], since the [two] great circle arcs HL and NZ have been drawn to meet the two great circle arcs AH and AN, Crd arc 2HA:Crd arc 2AZ = H195

(Crd arc 2HL:Crd arc 2LΘ).(Crd arc 2NΘ:Crd arc 2ZN). [M.T. I]

But, immediately by hypothesis, each of the arcs AZ, ZN and HK are given as quadrants; from the catalogue, arc KΘ is given from the star's latitude and arc KB from its longitude; and arc ZH and arc KL are given from the demonstrated obliquity of the ecliptic.[201] Hence it is clear that, of the arcs in question, arc HA [= arc AZ + arc ZH], arc AZ, arc HL [= arc HK + arc KL], arc LΘ

[199] In I 13, I 16 and II 7–8.

[200] Heiberg's version of Fig. 8.1, derived from ms. A, is defective, since it contains a redundant point Ξ. I follow the correct version in D.Ar.

[201] arc ZH = ε, arc KL = δ of point K.

[= arc KL + arc KΘ] and also arc NZ are given. Hence the remaining arc, NΘ, will also be given.

Again, since

Crd arc 2ZH:Crd arc 2HA =

H196 (Crd arc 2ZΘ:Crd arc 2ΘN).(Crd arc 2NL:Crd arc 2LA), [M.T. II]

and, by the above, of the arcs in question, arc ZH, arc HA, arc ZΘ [= arc ZN − arc NΘ] and arc ΘN are given, and arc LA is given from [the given] arc KB, by means of [the arcs of] the equator which rise together with [those of] the ecliptic at *sphaera recta*, the remaining arc, NL, will also be given. Similarly [by means of the rising-times at *sphaera recta*] arc MB of the ecliptic will be given from arc NA, the sum [of arc NL + arc LA].

Moreover the points on the equator and ecliptic which rise or set simultaneously with a fixed star can readily be found by means of the simultaneous culminations, in the following manner.

Let [Fig. 8.2] ABGD be a meridian, AEG a semi-circle of the equator about pole Z, and BED a semi-circle of the horizon. Let the star rise at point H of the horizon, and draw the great circle quadrant ZHΘ through points Z, H.

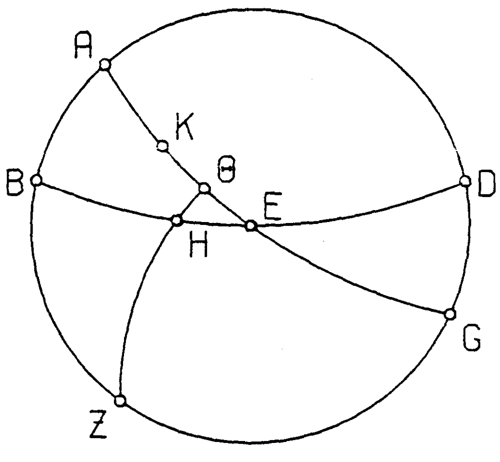

Fig. 8.2

Then again, since [two] great circle arcs ZΘ and EB have been drawn to meet

H197 two great circle arcs AZ and AE,

Crd arc 2ZB:Crd arc 2BA =

(Crd arc 2ZH:Crd arc 2HΘ).(Crd arc 2ΘE:Crd arc 2AE). [M.T. II]

But, of the arcs in question, arc ZA, arc ZΘ and arc EA each comprise a quadrant, arc ZB [and hence arc BA = arc ZA − arc ZB] is given from the elevation of the pole, and point Θ of the equator and arc ΘH [and hence arc HZ = arc ZΘ − arc ΘH] from the simultaneous culmination. Therefore the remaining [arc], ΘE, will be given.

For the simultaneous settings, too, it can easily be seen that if we cut off an arc, ΘK, in advance of Θ equal to arc ΘE, the star will set together with point K of the equator. For in that situation the setting takes place on an arc [of the horizon measured from the meridian] equal to arc BH, and cuts off an angle in advance of the meridian equal to that enclosed to the rear [of it] by arc AZ and arc ZΘ in the present situation.

Furthermore, from the arcs of the equator and ecliptic which rise and set together which we have computed for each clima [II 8], there will immediately be given that point on the ecliptic which rises together with point E of the equator and the star, and that point which sets together with point K and the H198
star. It is clear that at the moment when the sun is exactly in those points of the ecliptic, there will come to pass the risings, culminations and settings of the fixed star [in question] taken with respect to the sun's centre which are called 'true simultaneous cardinal positions'.[202]

6. {*On first and last visibilities of the fixed stars*}[203]

However, in the case of the first and last visibilities [of the fixed stars], we find that the geometrical method expounded [above], using only their position [in latitude and longitude], is no longer adequate. For it is not possible[204] to find the size of the arc by which the sun must be removed below the horizon in order for a given star to have its first or last visibility by methods similar to the geometrical procedures by which, e.g., one demonstrates the point on the ecliptic with which that star rises. For that arc [the *arcus visionis*] cannot be the same for all stars nor the same for a given star at all places [on earth], but varies according to the magnitude of the star, its distance in latitude from the sun, and the change in the inclinations of the ecliptic [with respect to the horizon].

For if we imagine [Fig. 8.3] a meridian circle ABGD, a semi-circle of the H199
ecliptic AEZG, and a semi-circle of the horizon BED about pole H, it is clear that, given a star rising simultaneously with point E of the ecliptic,[205] if a star of greater magnitude has its first visibility when the sun is at a distance of, e.g., arc EZ below the earth, a star of lesser magnitude, even one at an equal distance in latitude from the sun, will have its first visibility when the sun is at a greater distance than arc EZ, and [thus] the effect of its rays is weaker. Again, for stars of equal magnitude, if a star which is closer in latitude to point E has its first visibility at a distance [of the sun from the horizon] of arc EZ, a star which is farther than that [from point E in latitude] will have its first visibility at a lesser [solar] distance. For, given the same distance of the sun below the horizon, the rays in the vicinity of the ecliptic and of the sun itself are denser[206] than those

[202] συγκεντρώσεις, cf. p. 408 n.190, on κέντρα.

[203] See *HAMA* II 927–8.

[204] Reading δυνατὸν εἶναι with the mss. at H198.18. Heiberg deletes εἶναι, since one expects an indicative verb. But for the infinitive after words like ἐπειδή in *oratio obliqua* see Kühner-Gerth II 551, quoting Xenophon, *Mem.* 1.2.13, ἐθαύμαζε . . . ἐπεὶ καὶ τοὺς μέγιστον φρονοῦντας οὐ ταὐτὰ δοξάζειν ἀλλήλοις.

[205] Ptolemy says 'of those stars which rise simultaneously with point E'. However, he does not mean to compare a number of stars rising simultaneously with some fixed point of the ecliptic; for that would not allow the third situation envisaged, in which two different stars with the same latitude cross the horizon together with point E, and the angle at E is different in the two cases.

[206] Literally 'more numerous'.

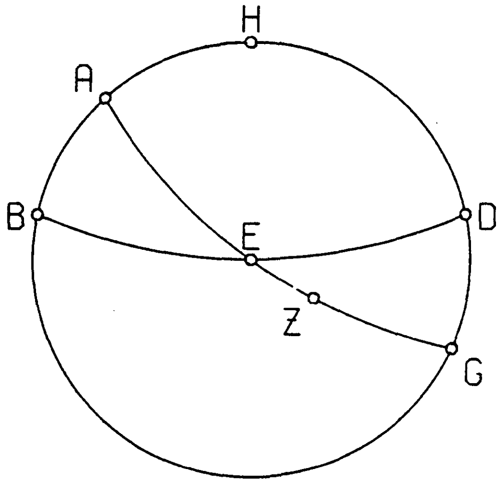

Fig. 8.3

farther away. [Finally], in the case of the stars of equal magnitude which rise at
H200 equal distances in latitude [from the sun], the more the ecliptic is inclined to the horizon, [thus] making angle DEZ smaller, the greater the [solar] distance EZ at which the star will have its first visibility.

For if, as in the following figure [Fig. 8.4], we also draw in the semi-circle HΘZK through the poles of the horizon and the sun at Z,[207] which will, obviously, be perpendicular to the horizon, the [vertical] distance of the sun below the earth will always remain equal to ZΘ for the same star, since, for an equal interval so taken, the [effect of] the rays above the earth will be similar; but if arc ΘZ is kept constant, arc EZ will, as we said, become less as the ecliptic is raised more towards a perpendicular position, and greater as it is more inclined[208] [to the horizon].

Therefore we need observations for each individual fixed star in order to determine the [required] distance of the sun below the earth as measured along the ecliptic. And if even the distance vertical to the horizon (for instance, in the present figure [8.4], ZΘ) does not remain the same for the same stars at all
H201 locations on earth, because the rays of similar density do not have the same obscuring effect[209] in the thicker air of the more northerly terrestrial latitudes, we will need observations, not merely at one terrestrial latitude, but at each of the others alike. However, if the arc corresponding to ZΘ remains constant everywhere on earth for the same stars (as seems likely, since the fixed stars too must be affected by the variation in the atmosphere in the same way as the rays are), the distances observed at a single terrestrial latitude will suffice us to determine those at the other latitudes: [we can do this] by geometrical methods,

[207] Taking the reading of D at H200, 6, τοῦ κατὰ τὸ Z (for τὸ κατὰ τὸ Z), and at H200,7, HΘZK (also in Ar) for ΘZK. Corrected by Manitius.

[208] Reading ἐγκλινομένου (with D) for κεκλιμένου at H200,13.

[209] καταλάμπειν, 'shine on so as to obscure'. See p. 470 n.8.

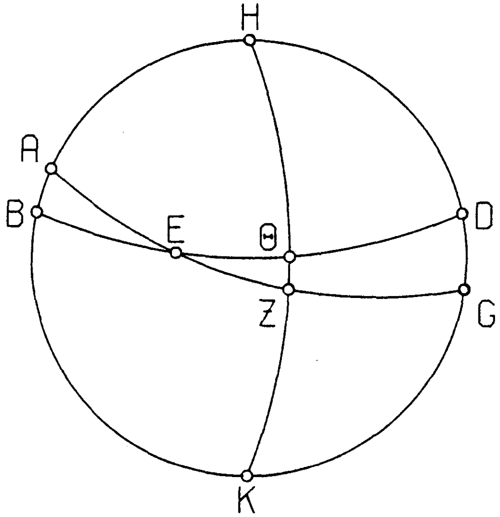

Fig. 8.4

whether the variation in the inclination of the ecliptic is due to the terrestrial location or to the demonstrated motion of the sphere of the fixed stars towards the rear with respect to it [the ecliptic].

[To show this], in the figure described [Fig. 8.4], let the distance EZ be given from an observation at any one terrestrial latitude whatever. Then since, again, the [two great circle arcs] BΘ and ZA have been drawn to meet the two great circle arcs HB and HZ,

Crd arc 2AB:Crd arc 2BH =

(Crd arc 2AE:Crd arc 2EZ).(Crd arc 2ZΘ:Crd arc 2ΘH). [M.T. II]

But, of the arcs in question, arc BH and arc ΘH are immediately [given, being] each a quadrant; and since point E, with which the star rises, is given by hypothesis, A, the culminating point, is also given, by means of the section on rising-times [II 9, p. 104]: thus arc AE too is given by this means, and arc EZ by the observation; and arc AH too [and hence arc AB = arc BH − arc AH] is given, being derived from the distance of point A from the equator (which is given from the Table of Inclination [I 15]) and from the distance of the equator from the zenith along the same meridian (which equals the elevation of the pole). Therefore the remaining [arc], ZΘ, will be given. H202

Once this [arc ZΘ] has been found, and provided that it remains the same for all locations, we can use it to derive the amounts of arc EZ at [all] other terrestrial latitudes from the same considerations. For again [in Fig. 8.4]

Crd arc 2HB:Crd 2AB =

(Crd arc 2HΘ:Crd arc 2ZΘ).(Crd arc 2ZE:Crd arc 2EA). [M.T. II]

And, of the arcs in question, arc ZΘ is now given by hypothesis; and since E, the point which rises together with the star at the terrestrial latitude in question, is given by the procedure demonstrated above [VIII 5 p. 412], and similarly arcs

EA and BA are given,[210] the remaining arc, which is arc EZ of the ecliptic, is also given.

H203 We shall take the same method of operation for granted for the last visibilities, which occur near the setting-point. Practically the only difference will be that in the same figure [Fig. 8.4] the ecliptic will be drawn on the other side [of BED], in accordance with the way it is inclined when the horizon [arc] BD is taken as the western part [see Fig. N].

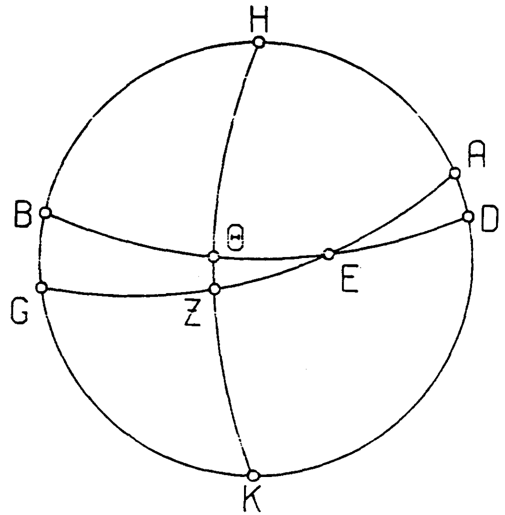

Fig. N

We think that the above suffices as an indication of the methods in this type of theoretical investigation, enough [at least] so that it cannot be said that we have neglected this topic. However, seeing that the computation of this kind of prediction is of great complexity, not only because of the great number of different terrestrial latitudes and inclinations of the ecliptic involved, but also because of the sheer multitude of the fixed stars; seeing, too, that, in respect of the actual observations of the phases[211] it is laborious and uncertain, since [differences between] the observers themselves and the atmosphere in the regions of observation can produce variation in and doubt about the time of the first suspected occurrence, as has become clear, to me at least, from my own experience and from the disagreements in this kind of observations; seeing, furthermore, that because of the motion [through the ecliptic] of the sphere of

[210] As before, (p. 415), from E, the horoscope, we find A, the culminating point, by the procedure II 9 (p. 104). Thus we have arc EA. arc AB = arc BH − arc AH, where arc BH = 90° and arc AH = $\varphi - \delta$ (A).

[211] Reading κατ' αὐτὰς τῶν φάσεων τηρήσεις, with D, at H203.14, i.e. taking it as following ἕνεκεν and understanding τοῦ before ἐργωδές τε εἶναι. Heiberg prints τὸ κατ' αὐτὰς τὰς τῶν ⟨τῶν⟩ ἀστέρων φάσεων τηρήσεις, presumably understanding παρὰ before it, but this is very harsh. By phases (φάσεις) Ptolemy means here both first and last visibilities.

the fixed stars, even for the individual terrestrial latitudes the simultaneous risings, culminations and settings cannot remain forever identical with the present ones, which would take such a vast amount of numerical and geometrical computations to calculate, we have decided to dispense with such a H204
time-consuming operation. For the time being we content ourselves with the approximate [phases] which can be derived either from[212] earlier records[213] or from actual manipulation of the [star-]globe for any particular star. Moreover, we notice that the prognostications concerning the states of the atmosphere derived from first or last visibilities (if indeed one assigns these as the cause [of changes in the weather], and not rather the positions [of the sun] in the ecliptic), are almost always approximations, and do not exhibit a perfect regularity and invariability: it seems that this causal factor has only general application, and derives its strength, not so much from the actual times of the first or last visibility, as from the configurations with respect to the sun, taken as intervals in round numbers, and, in part, the inclinations[214] of the moon at those configurations.

[212] Reading ἀπὸ, with D, for ἀπ' αὐτῶν at H204,3.

[213] In his later work, *Phaseis*, of which only Bk. II is preserved, Ptolemy lists many of these.

[214] προσνεύσεις. From the *Tetrabiblos* (II 13, ed. Boll-Boer 100,7–9) it appears that Ptolemy means the direction ('wind') towards which the moon 'points' in its motion in [argument of] latitude. But see also ibid. II 14,5 (ed. Boll-Boer 102,2–3) where it seems to be the direction towards which the sickle or gibbous moon points.

Book IX

1. {*On the order of the spheres of sun, moon and the 5 planets*}

Such, then, more or less, is the sum total of the chief topics one may mention as having to do with the fixed stars, in so far as the phenomena [observed] up to now provide the means of progress in our understanding. There remains, to [complete] our treatise, the treatment of the five planets. To avoid repetition we shall, as far as possible, explain the theory of the latter by means of an exposition common [to all five], treating each of the methods [for all planets] together.

First, then, [to discuss] the order of their spheres, which are all situated [with their poles] nearly coinciding with the poles of the inclined, ecliptic circle: we see that almost all the foremost astronomers agree that all the spheres are closer to the earth than that of the fixed stars, and farther from the earth than that of the moon, and that those of the three [outer planets] are farther from the earth than those of the other [two] and the sun, Saturn's being greatest, Jupiter's the next in order towards the earth, and Mars' below that. But concerning the H207
spheres of Venus and Mercury, we see that they are placed below the sun's by the more ancient astronomers, but by some of their successors these too are placed above [the sun's],[1] for the reason that the sun has never been obscured by them [Venus and Mercury] either. To us, however, such a criterion seems to have an element of uncertainty, since it is possible that some planets might indeed be below the sun, but nevertheless not always be in one of the planes through the sun and our viewpoint, but in another [plane], and hence might not be seen passing in front of it, just as in the case of the moon, when it passes below [the sun] at conjunction, no obscuration results in most cases.[2]

And since there is no other way, either, to make progress in our knowledge of this matter, since none of the stars[3] has a noticeable parallax (which is the only phenomenon from which the distances can be derived), the order assumed by the older [astronomers] appears the more plausible. For, by putting the sun in the middle, it is more in accordance with the nature [of the bodies] in thus

[1] There is a good deal of evidence for the identity of some of those who held the second opinion, including Plato, Eratosthenes and Archimedes. For details on this and other ancient arrangements see *HAMA* II 690-3.

[2] I.e. no transits of Venus or Mercury had been observed. Neugebauer has shown (*HAMA* 227-30) that transits are in fact predictable from Ptolemy's own theory. Ptolemy later seems to have realized this, for in the *Planetary Hypotheses* (ed. Goldstein 2,28,10-12) he says: 'if a body of such small size (as a planet) were to occult a body of such large size and with so much light (as the sun), it would necessarily be imperceptible, because of the smallness of the occulting body and the state of the parts of the sun's body which remain uncovered.' (Goldstein's translation here, p.6, is inaccurate).

[3] This includes both fixed stars and planets.

separating those which reach all possible distances from the sun and those which do not do so, but always move in its vicinity; provided only that it does not remove the latter close enough to the earth that there can result a parallax of any size.[4]

H208 2. {*On our purpose in the hypotheses of the planets*}

So much, then, for the arrangements of the spheres. Now it is our purpose to demonstrate for the five planets, just as we did for the sun and moon, that all their apparent anomalies can be represented by uniform circular motions, since these are proper to the nature of divine beings, while disorder and non-uniformity are alien [to such beings]. Then it is right that we should think success in such a purpose a great thing, and truly the proper end of the mathematical part of theoretical philosophy.[5] But, on many grounds, we must think that it is difficult, and that there is good reason why no-one before us has yet succeeded in it.[6] For, [firstly], in investigations of the periodic motions of a planet, the possible [inaccuracy] resulting from comparison of [two] observations (at each of which the observer may have committed a small observational error) will, when accumulated over a continuous period, produce a noticeable difference [from the true state] sooner when the interval [between the observations] over which the examination is made is shorter, and less soon when it is longer. But we have records of planetary observations only from a time which is recent in comparison with such a vast enterprise: this makes prediction for a time many times greater [than the interval for which observations are available] insecure. [Secondly], in investigation of the anomalies, considerable confusion stems from the fact that it is apparent that each planet exhibits two anomalies, which are moreover unequal both in their
H209 amount and in the periods of their return: one [return] is observed to be related to the sun, the other to the position in the ecliptic; but both anomalies are continuously combined, whence it is difficult to distinguish the characteristics of each individually. [It is] also [confusing] that most of the ancient [planetary] observations have been recorded in a way which is difficult to evaluate, and crude. For [1] the more continuous series of observations concern stations and phases [i.e. first and last visibilities].[7] But detection of both of these particular

[4] In his *Planetary Hypotheses* (see Goldstein's edition) Ptolemy proposes a system in which the spheres of the planets are contiguous; thus the greatest distance from the earth attained by a planet is equal to the least distance attained by the one next in order outwards. This appears to provide support for the order he adopts here, since it results in a solar distance very nearly the same as that obtained by a different method in *Almagest* V 15. Since this system also brings Mercury, at its least distance, to the moon's greatest distance (64 earth-radii), Mercury ought to exhibit a considerable parallax, contrary to what is enunciated here.

[5] Cf. I 1 p. 35.

[6] We cannot doubt that not only planetary theories but planetary tables had been constructed before Ptolemy: the proof is supplied by Indian astronomy, which is based on Greek theories which are largely, if not entirely, pre-Ptolemaic, and indeed by Ptolemy's own reference to the 'Aeon-tables' below (p. 422). What he means is that all previous efforts were, by his criteria, unsatisfactory.

[7] Ptolemy is certainly thinking of the Babylonian planetary observations, which are characteristically of this type. They have become available to us through the 'diaries' (see Sachs[2]), but to Ptolemy were probably known only through Hipparchus' compilation (see p. 421).

phenomena is fraught with uncertainty: stations cannot be fixed at an exact moment, since the local motion of the planet for several days both before and after the actual station is too small to be observable; in the case of the phases, not only do the places [in which the planets are located] immediately become invisible together with the bodies which are undergoing their first or last visibility, but the times too can be in error, both because of atmospherical differences and because of differences in the [sharpness of] vision of the observers. [2] In general, observations [of planets] with respect to one of the fixed stars, when taken over a comparatively great distance, involve difficult computations and an element of guesswork in the quantity measured, unless one carries them out in a manner which is thoroughly competent and knowledgeable. This is not only because the lines joining the observed stars do not always form right angles with the ecliptic, but may form an angle of any size (hence one may expect considerable error in determining the position in latitude and longitude, due to H210 the varying inclination of the ecliptic [to the horizon frame of reference]); but also because the same interval [between star and planet] appears to the observer as greater near the horizon, and less near mid-heaven;[8] hence, obviously, the interval in question can be measured as at one time greater, at another less than it is in reality.

Hence it was, I think, that Hipparchus, being a great lover of truth, for all the above reasons, and especially because he did not yet have in his possession such a groundwork of resources in the form of accurate observations from earlier times as he himself has provided to us,[9] although he investigated the theories of the sun and moon, and, to the best of his ability, demonstrated with every means at his command that they are represented by uniform circular motions, did not even make a beginning in establishing theories for the five planets, not at least in his writings which have come down to us.[10] All that he did was to make a compilation of the planetary observations arranged in a more useful way,[11] and to show by means of these that the phenomena were not in agreement with the hypotheses of the astronomers of that time. For, we may presume, he thought that one must not only show that each planet has a twofold anomaly, or that each planet has retrograde arcs which are not constant, and are of such and such sizes (whereas the other astronomers had constructed their geometrical proofs on the basis of a single unvarying anomaly and retrograde arc); nor [that it was sufficient to show] that these anomalies can in fact be represented either

[8] This appears to be the only reference to the effect of refraction (if that is what it is) in the Almagest, despite its obvious relevance e.g. to the observations of Mercury's greatest elongations in IX 7. Ptolemy discusses it (as a theoretical problem) in some detail in *Optics* V 23–30 (ed. Lejeune 237–42).

[9] This seems to imply that Hipparchus recorded planetary observations of his own, which Ptolemy used to establish his theories. This may be true, but it is strange that Ptolemy cites not a single such observation by Hipparchus. Could Ptolemy mean merely that Hipparchus had not 'yet' assembled the compilation of earlier planetary observations which he mentions just below?

[10] The circulation of books in antiquity was so fortuitous that, even for one, like Ptolemy, who had access to the great resources of the libraries at Alexandria, this was a necessary *caveat*.

[11] I have little doubt that all the older planetary observations cited in the Almagest are derived from this compilation (cf. p. 452 n.66), and that part of Hipparchus' 'rearrangement' was to give their dates in the Egyptian calendar. For a similar service he rendered for the listing of lunar eclipses see *HAMA* 320–21.

H211 by means of eccentric circles or by circles concentric with the ecliptic, and carrying epicycles, or even by combining both, the ecliptic anomaly being of such and such a size, and the synodic anomaly of such and such (for these representations have been employed by almost all those who tried to exhibit the uniform circular motion by means of the so-called 'Aeon-tables',[12] but their attempts were faulty and at the same time lacked proofs: some of them did not achieve their object at all, the others only to a limited extent); but, [we may presume], he reckoned that one who has reached such a pitch of accuracy and love of truth throughout the mathematical sciences will not be content to stop at the above point, like the others who did not care [about the imperfections]; rather, that anyone who was to convince himself and his future audience must demonstrate the size and the period of each of the two anomalies by means of well-attested phenomena which everyone agrees on, must then combine both anomalies, and discover the position and order of the circles by which they are brought about, and the type of their motion; and finally must make practically all the phenomena fit the particular character of the arrangement of circles in his hypothesis. And this, I suspect, appeared difficult even to him.

The point of the above remarks was not to boast [of our own achievement]. Rather, if we are at any point compelled by the nature of our subject to use a procedure not in strict accordance with theory (for instance, when we carry out proofs using without further qualification the circles[13] described in the
H212 planetary spheres by the movement [of the body, i.e.] assuming that these circles lie in the plane of the ecliptic,[14] to simplify the course of the proof); or [if we are compelled] to make some basic assumptions which we arrived at not from some readily apparent principle, but from a long period of trial and application,[15] or to assume a type of motion or inclination of the circles which is not the same and unchanged for all planets;[16] we may [be allowed to] accede [to this compulsion], since we know that this kind of inexact procedure will not affect the end desired, provided that it is not going to result in any noticeable error; and we know too that assumptions made without proof, provided only that they are found to be in agreement with the phenomena, could not have been found without some careful methodological procedure, even if it is difficult

[12] διὰ τῆς καλουμένης αἰωνίου κανονοποιίας. In my opinion, Ptolemy is referring to a type of work in which the mean motions of the planets were represented by integer numbers of revolutions in some huge period, in which they all return to the beginning of the zodiac, and the planetary equations were calculated by a combination of epicycles or of eccentre and epicycle which was not reducible to a geometrically consistent kinematic model, i.e. to a class of Greek works which were the ancestors of the Indian siddhāntas. In this I am in agreement with van der Waerden, 'Ewige Tafeln', except that I believe that the αἰών implied by the title of these tables does not mean 'eternity' (cf. the conventional translation, 'Eternal Tables', which is philologically possible, but not necessary), but refers to the immense common period in which the planets return (cf. the Greek inscription of Keskinto, *HAMA* 698–705, and the Indian Mahāyuga). The other two references to these tables in antiquity (P. Lond. 130, see Neugebauer-van Hoesen, *Greek Horoscopes* p. 21, I 12–13, and Vettius Valens VI 1, ed. Kroll 243,8) are consistent with, but do not require, this interpretation.

[13] Literally 'as if the circles were bare [circles]'.

[14] Ptolemy in fact carries out all the proofs involving the longitudinal motions of the planets (in Bks. IX–XII) as if the motions lay in the plane of the ecliptic.

[15] The paradigm case of this is the introduction of the equant.

[16] E.g. the special model for the longitudinal motions of Mercury, or the special inclinations attributed to the inner planets for their latitudinal motions.

to explain how one came to conceive them (for, in general, the cause of first principles is, by nature, either non-existent or hard to describe); we know, finally, that some variety in the type of hypotheses associated with the circles [of the planets] cannot plausibly be considered strange or contrary to reason (especially since the phenomena exhibited by the actual planets are not alike [for all]); for, when uniform circular motion is preserved for all without exception, the individual phenomena are demonstrated in accordance with a principle which is more basic and more generally applicable than that of similarity of the hypotheses [for all planets].

The observations which we use for the various demonstrations are those H213
which are most likely to be reliable, namely [1] those in which there is observed actual contact or very close approach to a star or the moon, and especially [2] those made by means of the astrolabe instruments. [In these] the observer's line of vision is directed, as it were, by means of the sighting-holes on opposite sides of the rings, thus observing equal distances as equal arcs in all directions, and can accurately determine the position of the planet in question in latitude and longitude with respect to the ecliptic, by moving the ecliptic ring on the astrolabe, and the diametrically opposite sighting-holes on the rings[17] through the poles of the ecliptic, into alignment with the object observed.

3. {*On the periodic returns of the five planets*}[18]

Now that we have completed the above discussion, we will first set out, for each of the 5 planets, the smallest period in which it makes an approximate return in both anomalies, as computed by Hipparchus.[19] These [periods] have been corrected by us, on the basis of the comparison of their positions which became possible after we had demonstrated their anomalies, as we shall explain at that point.[20] However, we anticipate and put them here, so as to have the individual mean motions in longitude and anomaly set out in a convenient form for the H214
calculations of the anomalies. But it would in fact make no noticeable difference in those calculations[21] even if one used more roughly computed mean positions.

[17] It is not clear why the plural ('rings') is used (contrast the singular at V 1, H354,13). Although the sights are attached only to ring 1 in Fig. F (p. 218). Ptolemy is presumably referring to both ring 1 and ring 2, since ring 2 has first to be moved to the correct sighting position on the ecliptic ring (no. 3).

[18] See *HAMA* 150–2. Pedersen (270) has fallen into some confusion about Ptolemy's procedure: see Toomer[3] 144–5.

[19] If Ptolemy means, as we may presume, that the periods 'computed by Hipparchus' are the relationships in integers, '57 returns in anomaly correspond to 59 years and 2 revolutions in longitude', etc., then he seems ignorant of the fact that these are well-known (to us) Babylonian period relationships (for details see *HAMA* 151).

[20] This is a reference to the chapters on the 'corrections of the mean motions', IX 10, X 4, X9, XI 3 and XI7. The 'comparison' refers to the use in these chapters of *two* positions, separated by a long time-interval, to derive the mean motions. On the problem of the actual derivation of the corrections given here, and of the mean motions, see Appendix C.

[21] Ptolemy means that where he uses the mean motions in determining the eccentricity (e.g. X 7 p. 484) over the short periods involved (a few years) one could use quite crude parameters (e.g. the mean motions given by the uncorrected Babylonian periods) without seriously affecting the final result. He is right (see p. 484 n.33). The corrected mean motions are given here merely for convenience. Cf. the procedure for the lunar mean motion table, p. 179.

As a general definition, we mean by 'motion in longitude' the motion of the centre of the epicycle around the eccentre, and by 'anomaly' the motion of the body around the epicycle.

We find, then, that

[1] for Saturn, 57 returns in anomaly correspond to 59 solar years (as defined by us, i.e. returns to the same solstice or equinox), plus about $1\frac{3}{4}$ days, and to 2 revolutions [in longitude] plus 1;43° (for in the case of the 3 planets which are always overtaken by the sun[22] the number of revolutions of the sun during the period of return is always, for each of them, the sum of the number of revolutions in longitude and the number of returns in anomaly of the planet);

[2] for Jupiter, 65 returns in anomaly correspond to 71 solar years (defined as above) less about $4\frac{9}{10}$ days, and to 6 revolutions of the planet from a solstice back
H215 to the same solstice, less $4\frac{5}{6}$°;

[3] for Mars, 37 returns in anomaly correspond to 79 solar years (as defined by us) plus about 3;13 days,[23] and to 42 revolutions of the planet from a solstice back to the same solstice, plus $3\frac{1}{6}$°;

[4] for Venus, 5 returns in anomaly correspond to 8 solar years (as defined by us) less about 2;18 days,[24] and to a number of [longitudinal] revolutions of the planet equal to that of the sun, 8, less $2\frac{1}{4}$°;

[5] for Mercury, 145 returns in anomaly correspond to 46 of the same kind of years plus about $1\frac{1}{30}$ days, and to a number of [longitudinal] revolutions which is, again, equal to that of the sun, 46, plus 1°.

But if, for each planet, we reduce the period of return to days, in accordance with the length of the year as demonstrated by us, and the number of returns in anomaly to degrees according to the system in which a circle contains 360°, we will get:

for Saturn, 21551;18^d and 20520° of anomaly
H216 for Jupiter, 25927;37^d and 23400°[25] of anomaly
for Mars, 28857;43^d[26] and 13320° of anomaly
for Venus, 2919;40^d and 1800° of anomaly
for Mercury, 16802;24^d[27] and 52200° of anomaly.

So we divide the degrees of anomaly proper to each by the appropriate number of days, and get the following for the approximate mean daily motions in anomaly:[28]

Saturn	0;57,7,43,41,43,40°
Jupiter	0;54,9,2,46,26,0°

[22] περικαταλαμβανομένων. Cf. περικατάληψις HI 24,13. This feature distinguishes the three outer planets from the two inner ones, since the latter (usually) overtake the sun.

[23] Expressed by Ptolemy as $3 + \frac{1}{6} + \frac{1}{20}$.

[24] Expressed by Ptolemy as $2 + \frac{1}{4} + \frac{1}{20}$.

[25] Reading $\overline{\gamma\upsilon}$, with D^1,Ar, for $\overline{\zeta\upsilon}$ (27400) at H216,1. Corrected by Manitius.

[26] Reading $\overline{\mu\gamma}$ for $\overline{\nu\gamma}$ (53) at H216,2. Multiplying the Ptolemaic length of the year, 365;14,48^d, by 79 and adding 3;13 produces 28857;42,12, of which 28857;43 is the rounding. The ms. tradition is solid for 53, but nothing in the previous or subsequent calculations favours it.

[27] Precise calculation (cf. n.26) gives 16802;22,48. Possibly we should change $1\frac{1}{30}$ days (above) to $1\frac{1}{20}$ days (reading κ′ for λ′ at H215,11).

[28] For the problem of precisely how Ptolemy arrives at the parameters he gives for the planetary mean motions, which is not as simple as it appears here, see Appendix C.

Mars 0;27,41,40,19,20,58°
Venus 0;36,59,25,53,11,28°
Mercury 3;6,24,6,59,35,50°.

For each of these we take $\frac{1}{24}$th to get the following mean hourly motions in anomaly: H217

Saturn 0;2,22,49,19,14,19,10°
Jupiter 0;2,15,22,36,56,5°
Mars 0;1,9,14,10,48,22,25°
Venus 0;1,32,28,34,42,58,40°
Mercury 0;7,46,0,17,28,59,35°.

Then we multiply the daily motion of each by 30 to get the following mean monthly motions in anomaly:

Saturn 28;33,51,50,51,50,0°
Jupiter 27;4,31,23,13,0,0°
Mars 13;50,50,9,40,29,0°
Venus 18.29.42.56.35.44.0°
Mercury 93;12,3,29,47,55,0°.

Similarly, we multiply the daily motions by 365, the number of days in one Egyptian year, to get the following mean yearly motions in anomaly:

Saturn 347;32,0,48,50,38,20°
Jupiter 329;25,1,52,28,10,0°
Mars 168;28,30,17,42,32,50°
Venus 225;1,32,28,34,39,15°[29]
Mercury 53;56,42,32,32,59,10° (increment [over complete circles]).

In the same way, we multiply each of the annual motions by 18 (just as we did in the construction of tables for the luminaries), to get the following increments in mean anomaly for the period of 18 Egyptian years: H218

Saturn 135;36,14,39,11,30,0°
Jupiter 169;30,33,44,27,0,0°
Mars 152;33,5,18,45,51,0°
Venus 90;27,44,34,23,46,30°
Mercury 251;0,45,45,53,45,0°.

We can also find the mean motions in longitude corresponding to the above without reducing the number of [longitudinal] revolutions to degrees and dividing them by [the number of days in] the period set out above for each planet. For Venus and in Mercury, it is obvious that we can do this by taking the same mean motions as we set out previously for the sun; for the other three planets, by taking the difference between the [mean motion in] anomaly and the corresponding solar [mean] motion for each individual entry.[30] By this method we get the following mean motions in longitude:

[29] This corresponds to a mean daily motion of 0;36,59,25,53,11,27°, i.e. one less in the last place than that given above. Thus the mean motion table of Venus is based on different parameters in different parts: on 28 in the last place for hours, days and months, and on 27 in the last place for years and 18-year periods. On the possible significance of this see Appendix C p. 671 n.11.

[30] Venus and Mercury have the same mean motion in longitude as the sun. For the other planets, for any length of time, the sum of anomaly and mean motion equals the sun's mean motion, because of the relationship stated at p. 424.

	Daily:	
	Saturn	0;2,0,33,31,28,51°
	Jupiter	0;4,59,14,26,46,31°
	Mars	0;31,26,36,53,51,33°.
	Hourly:	
H219	Saturn	0;0,5,1,23,48,42,7,30°
	Jupiter	0;0,12,28,6,6,56,17,30°
	Mars	0;1,18,36,32,14,38,52,30°.[31]
	Monthly:	
	Saturn	1;0,16,45,44,25,30°
	Jupiter	2;29,37,13,23,15,30°
	Mars	15;43,18,26,55,46,30°.
	Yearly:	
	Saturn	12;13,23,56,30,30,15°
	Jupiter	30;20,22,52,52,58,35°[32]
	Mars	191;16,54,27,38,35,45°.
	For 18 years:	
	Saturn	220;1,10,57,9,4,30° in mean motion
	Jupiter	186;6,51,51,53,34,30° } increment [over
	Mars	203;4,20,17,34,43,30° } complete circles].

So once again, for the user's convenience, we shall set out, for each of the planets in order, tables of the above mean motions derived by successive summation [of the motions for the appropriate time-interval]. Like the other [mean motion tables], these will be in 45 lines and 3 sections: the first section will contain the entries (obtained by successive summation) for the 18-year periods; the second will contain those for the years and hours, and the third those for the months and days.

The tables are as follows.

H220–49 4. {*Tables of the mean motions in longitude and anomaly of the five planets*}[33]

[See pp. 427–41.]

H250 5. {*Preliminary notions [necessary] for the hypotheses of the 5 planets*}[34]

Now that these [mean motions] have been tabulated, our next task is to discuss the anomalies which occur in connection with the longitudinal positions of the five planets. The way we have approached it, to give the general outlines, is as follows.

[31] Reading $\overline{\lambda\eta}$ $\overline{\nu\beta}$ $\overline{\lambda}$ (38,52,30) for $\overline{\lambda\theta}$ (39) at H219,2, with D,Ar. Although the figure is rounded to 39 in the table, there is no reason why it should be (for Mars alone) here.

[32] Reading $\overline{\nu\beta}$ $\overline{\nu\eta}$ $\overline{\lambda\epsilon}$ for $\overline{\nu\beta}$ $\overline{\lambda\eta}$ $\overline{\lambda\epsilon}$ (52,38,35) at H219,7, with D,Ar. Corrected by Manitius.

[33] Corrections to Heiberg:
H235,24 (Mars, longitude, 3ʰ, last place) read νς for ς (6). Misprint.
H238,3 (Venus, epoch in longitude) read ο $\overline{\mu\epsilon}$ for $\overline{\mu\epsilon}$ (45°), with D^2.
Corrected by Manitius, but this is not (*pace Manitii*) a misprint in Heiberg.

[34] On chs. 5 and 6 see *HAMA* 149–50.

Saturn 18-Year Periods	[Epoch] Position in [Mean] Longitude : ♑26;43° Longitude							[Epoch] Position in Anomaly : 34,2° [Epoch] Position of Apogee : ♏14;10° Anomaly						
18	220°	1	10	57	9	4	30	135°	36	14	39	11	30	0
36	80	2	21	54	18	9	0	271	12	29	18	23	0	0
54	300	3	32	51	27	13	30	46	48	43	57	34	30	0
72	160	4	43	48	36	18	0	182	24	58	36	46	0	0
90	20	5	54	45	45	22	30	318	1	13	15	57	30	0
108	240	7	5	42	54	27	0	93	37	27	55	9	0	0
126	100	8	16	40	3	31	30	229	13	42	34	20	30	0
144	320	9	27	37	12	36	0	4	49	57	13	32	0	0
162	180	10	38	34	21	40	30	140	26	11	52	43	30	0
180	40	11	49	31	30	45	0	276	2	26	31	55	0	0
198	260	13	0	28	39	49	30	51	38	41	11	6	30	0
216	120	14	11	25	48	54	0	187	14	55	50	18	0	0
234	340	15	22	22	57	58	30	322	51	10	29	29	30	0
252	200	16	33	20	7	3	0	98	27	25	8	41	0	0
270	60	17	44	17	16	7	30	234	3	39	47	52	30	0
288	280	18	55	14	25	12	0	9	39	54	27	4	0	0
306	140	20	6	11	34	16	30	145	16	9	6	15	30	0
324	0	21	17	8	43	21	0	280	52	23	45	27	0	0
342	220	22	28	5	52	25	30	56	28	38	24	38	30	0
360	80	23	39	3	1	30	0	192	4	53	3	50	0	0
378	300	24	50	0	10	34	30	327	41	7	43	1	30	0
396	160	26	0	57	19	39	0	103	17	22	22	13	0	0
414	20	27	11	54	28	43	30	238	53	37	1	24	30	0
432	240	28	22	51	37	48	0	14	29	51	40	36	0	0
450	100	29	33	48	46	52	30	150	6	6	19	47	30	0
468	320	30	44	45	55	57	0	285	42	20	58	59	0	0
486	180	31	55	43	5	1	30	61	18	35	38	10	30	0
504	40	33	6	40	14	6	0	196	54	50	17	22	0	0
522	260	34	17	37	23	10	30	332	31	4	56	33	30	0
540	120	35	28	34	32	15	0	108	7	19	35	45	0	0
558	340	36	39	31	41	19	30	243	43	34	14	56	30	0
576	200	37	50	28	50	24	0	19	19	48	54	8	0	0
594	60	39	1	25	59	28	30	154	56	3	33	19	30	0
612	280	40	12	23	8	33	0	290	32	18	12	31	0	0
630	140	41	23	20	17	37	30	66	8	32	51	42	30	0
648	0	42	34	17	26	42	0	201	44	47	30	54	0	0
666	220	43	45	14	35	46	30	337	21	2	10	5	30	0
684	80	44	56	11	44	51	0	112	57	16	49	17	0	0
702	300	46	7	8	53	55	30	248	33	31	28	28	30	0
720	160	47	18	6	3	0	0	24	9	46	7	40	0	0
738	20	48	29	3	12	4	30	159	46	0	46	51	30	0
756	240	49	40	0	21	9	0	295	22	15	26	3	0	0
774	100	50	50	57	30	13	30	70	58	30	5	14	30	0
792	320	52	1	54	39	18	0	206	34	44	44	26	0	0
810	180	53	12	51	48	22	30	342	10	59	23	37	30	0

Single Years	Saturn Longitude							Saturn Anomaly						
1	12°	13	23	56	30	30	15	347°	32	0	48	50	38	20
2	24	26	47	53	1	0	30	335	4	1	37	41	16	40
3	36	40	11	49	31	30	45	322	36	2	26	31	55	0
4	48	53	35	46	2	1	0	310	8	3	15	22	33	20
5	61	6	59	42	32	31	15	297	40	4	4	13	11	40
6	73	20	23	39	3	1	30	285	12	4	53	3	50	0
7	85	33	47	35	33	31	45	272	44	5	41	54	28	20
8	97	47	11	32	4	2	0	260	16	6	30	45	6	40
9	110	0	35	28	34	32	15	247	48	7	19	35	45	0
10	122	13	59	25	5	2	30	235	20	8	8	26	23	20
11	134	27	23	21	35	32	45	222	52	8	57	17	1	40
12	146	40	47	18	6	3	0	210	24	9	46	7	40	0
13	158	54	11	14	36	33	15	197	56	10	34	58	18	20
14	171	7	35	11	7	3	30	185	28	11	23	48	56	40
15	183	20	59	7	37	33	45	173	0	12	12	39	35	0
16	195	34	23	4	8	4	0	160	32	13	1	30	13	20
17	207	47	47	0	38	34	15	148	4	13	50	20	51	40
18	220	1	10	57	9	4	30	135	36	14	39	11	30	0

Hours	Longitude							Anomaly						
1	0°	0	5	1	23	48	42	0°	2	22	49	19	14	19
2	0	0	10	2	47	37	24	0	4	45	38	38	28	38
3	0	0	15	4	11	26	6	0	7	8	27	57	42	57
4	0	0	20	5	35	14	48	0	9	31	17	16	57	17
5	0	0	25	6	59	3	31	0	11	54	6	36	11	36
6	0	0	30	8	22	52	13	0	14	16	55	55	25	55
7	0	0	35	9	46	40	55	0	16	39	45	14	40	14
8	0	0	40	11	10	29	37	0	19	2	34	33	54	33
9	0	0	45	12	34	18	19	0	21	25	23	53	8	52
10	0	0	50	13	58	7	1	0	23	48	13	12	23	12
11	0	0	55	15	21	55	43	0	26	11	2	31	37	31
12	0	1	0	16	45	44	25	0	28	33	51	50	51	50
13	0	1	5	18	9	33	8	0	30	56	41	10	6	9
14	0	1	10	19	33	21	50	0	33	19	30	29	20	28
15	0	1	15	20	57	10	32	0	35	42	19	48	34	47
16	0	1	20	22	20	59	14	0	38	5	9	7	49	7
17	0	1	25	23	44	47	56	0	40	27	58	27	3	26
18	0	1	30	25	8	36	38	0	42	50	47	46	17	45
19	0	1	35	26	32	25	20	0	45	13	37	5	32	4
20	0	1	40	27	56	14	2	0	47	36	26	24	46	23
21	0	1	45	29	20	2	45	0	49	59	15	44	0	42
22	0	1	50	30	43	51	27	0	52	22	5	3	15	2
23	0	1	55	32	7	40	9	0	54	44	54	22	29	21
24	0	2	0	33	31	28	51	0	57	7	43	41	43	40

Months	Saturn Longitude							Saturn Anomaly						
30	1°	0	16	45	44	25	30	28°	33	51	50	51	50	0
60	2	0	33	31	28	51	0	57	7	43	41	43	40	0
90	3	0	50	17	13	16	30	85	41	35	32	35	30	0
120	4	1	7	2	57	42	0	114	15	27	23	27	20	0
150	5	1	23	48	42	7	30	142	49	19	14	19	10	0
180	6	1	40	34	26	33	0	171	23	11	5	11	0	0
210	7	1	57	20	10	58	30	199	57	2	56	2	50	0
240	8	2	14	5	55	24	0	228	30	54	46	54	40	0
270	9	2	30	51	39	49	30	257	4	46	37	46	30	0
300	10	2	47	37	24	15	0	285	38	38	28	38	20	0
330	11	3	4	23	8	40	30	314	12	30	19	30	10	0
360	12	3	21	8	53	6	0	342	46	22	10	22	0	0

Days	Longitude							Anomaly						
1	0°	2	0	33	31	28	51	0°	57	7	43	41	43	40
2	0	4	1	7	2	57	42	1	54	15	27	23	27	20
3	0	6	1	40	34	26	33	2	51	23	11	5	11	0
4	0	8	2	14	5	55	24	3	48	30	54	46	54	40
5	0	10	2	47	37	24	15	4	45	38	38	28	38	20
6	0	12	3	21	8	53	6	5	42	46	22	10	22	0
7	0	14	3	54	40	21	57	6	39	54	5	52	5	40
8	0	16	4	28	11	50	48	7	37	1	49	33	49	20
9	0	18	5	1	43	19	39	8	34	9	33	15	33	0
10	0	20	5	35	14	48	30	9	31	17	16	57	16	40
11	0	22	6	8	46	17	21	10	28	25	0	39	0	20
12	0	24	6	42	17	46	12	11	25	32	44	20	44	0
13	0	26	7	15	49	15	3	12	22	40	28	2	27	40
14	0	28	7	49	20	43	54	13	19	48	11	44	11	20
15	0	30	8	22	52	12	45	14	16	55	55	25	55	0
16	0	32	8	56	23	41	36	15	14	3	39	7	38	40
17	0	34	9	29	55	10	27	16	11	11	22	49	22	20
18	0	36	10	3	26	39	18	17	8	19	6	31	6	0
19	0	38	10	36	58	8	9	18	5	26	50	12	49	40
20	0	40	11	10	29	37	0	19	2	34	33	54	33	20
21	0	42	11	44	1	5	51	19	59	42	17	36	17	0
22	0	44	12	17	32	34	42	20	56	50	1	18	0	40
23	0	46	12	51	4	3	33	21	53	57	44	59	44	20
24	0	48	13	24	35	32	24	22	51	5	28	41	28	0
25	0	50	13	58	7	1	15	23	48	13	12	23	11	40
26	0	52	14	31	38	30	6	24	45	20	56	4	55	20
27	0	54	15	5	9	58	57	25	42	28	39	46	39	0
28	0	56	15	38	41	27	48	26	39	36	23	28	22	40
29	0	58	16	12	12	56	39	27	36	44	7	10	6	20
30	1	0	16	45	44	25	30	28	33	51	50	51	50	0

Jupiter 18-Year Periods	[Epoch] Position in [Mean] Longitude : ♎4;41° Longitude							[Epoch] Position in Anomaly : 146;4° [Epoch] Position of Apogee : ♍2;9° Anomaly						
18	186°	6	51	51	53	34	30	169°	30	33	44	27	0	0
36	12	13	43	43	47	9	0	339	1	7	28	54	0	0
54	198	20	35	35	40	43	30	148	31	41	13	21	0	0
72	24	27	27	27	34	18	0	318	2	14	57	48	0	0
90	210	34	19	19	27	52	30	127	32	48	42	15	0	0
108	36	41	11	11	21	27	0	297	3	22	26	42	0	0
126	222	48	3	3	15	1	30	106	33	56	11	9	0	0
144	48	54	54	55	8	36	0	276	4	29	55	36	0	0
162	235	1	46	47	2	10	30	85	35	3	40	3	0	0
180	61	8	38	38	55	45	0	255	5	37	24	30	0	0
198	247	15	30	30	49	19	30	64	36	11	8	57	0	0
216	73	22	22	22	42	54	0	234	6	44	53	24	0	0
234	259	29	14	14	36	28	30	43	37	18	37	51	0	0
252	85	36	6	6	30	3	0	213	7	52	22	18	0	0
270	271	42	57	58	23	37	30	22	38	26	6	45	0	0
288	97	49	49	50	17	12	0	192	8	59	51	12	0	0
306	283	56	41	42	10	46	30	1	39	33	35	39	0	0
324	110	3	33	34	4	21	0	171	10	7	20	6	0	0
342	296	10	25	25	57	55	30	340	40	41	4	33	0	0
360	122	17	17	17	51	30	0	150	11	14	49	0	0	0
378	308	24	9	9	45	4	30	319	41	48	33	27	0	0
396	134	31	1	1	38	39	0	129	12	22	17	54	0	0
414	320	37	52	53	32	13	30	298	42	56	2	21	0	0
432	146	44	44	45	25	48	0	108	13	29	46	48	0	0
450	332	51	36	37	19	22	30	277	44	3	31	15	0	0
468	158	58	28	29	12	57	0	87	14	37	15	42	0	0
486	345	5	20	21	6	31	30	256	45	11	0	9	0	0
504	171	12	12	13	0	6	0	66	15	44	44	36	0	0
522	357	19	4	4	53	40	30	235	46	18	29	3	0	0
540	183	25	55	56	47	15	0	45	16	52	13	30	0	0
558	9	32	47	48	40	49	30	214	47	25	57	57	0	0
576	195	39	39	40	34	24	0	24	17	59	42	24	0	0
594	21	46	31	32	27	58	30	193	48	33	26	51	0	0
612	207	53	23	24	21	33	0	3	19	7	11	18	0	0
630	34	0	15	16	15	7	30	172	49	40	55	45	0	0
648	220	7	7	8	8	42	0	342	20	14	40	12	0	0
666	46	13	59	0	2	16	30	151	50	48	24	39	0	0
684	232	20	50	51	55	51	0	321	21	22	9	6	0	0
702	58	27	42	43	49	25	30	130	51	55	53	33	0	0
720	244	34	34	35	43	0	0	300	22	29	38	0	0	0
738	70	41	26	27	36	34	30	109	53	3	22	27	0	0
756	256	48	18	19	30	9	0	279	23	37	6	54	0	0
774	82	55	10	11	23	43	30	88	54	10	51	21	0	0
792	269	2	2	3	17	18	0	258	24	44	35	48	0	0
810	95	8	53	55	10	52	30	67	55	18	20	15	0	0

Single Years	Jupiter Longitude							Jupiter Anomaly						
1	30°	20	22	52	52	58	35	329°	25	1	52	28	10	0
2	60	40	45	45	45	57	10	298	50	3	44	56	20	0
3	91	1	8	38	38	55	45	268	15	5	37	24	30	0
4	121	21	31	31	31	54	20	237	40	7	29	52	40	0
5	151	41	54	24	24	52	55	207	5	9	22	20	50	0
6	182	2	17	17	17	51	30	176	30	11	14	49	0	0
7	212	22	40	10	10	50	5	145	55	13	7	17	10	0
8	242	43	3	3	3	48	40	115	20	14	59	45	20	0
9	273	3	25	55	56	47	15	84	45	16	52	13	30	0
10	303	23	48	48	49	45	50	54	10	18	44	41	40	0
11	333	44	11	41	42	44	25	23	35	20	37	9	50	0
12	4	4	34	34	35	43	0	353	0	22	29	38	0	0
13	34	24	57	27	28	41	35	322	25	24	22	6	10	0
14	64	45	20	20	21	40	10	291	50	26	14	34	20	0
15	95	5	43	13	14	38	45	261	15	28	7	2	30	0
16	125	26	6	6	7	37	20	230	40	29	59	30	40	0
17	155	46	28	59	0	35	55	200	5	31	51	58	50	0
18	186	6	51	51	53	34	30	169	30	33	44	27	0	0
Hours	Longitude							Anomaly						
1	0°	0	12	28	6	6	56	0°	2	15	22	36	56	5
2	0	0	24	56	12	13	52	0	4	30	45′	13	52	10
3	0	0	37	24	18	20	48	0	6	46	7	50	48	15
4	0	0	49	52	24	27	45	0	9	1	30	27	44	20
5	0	1	2	20	30	34	41	0	11	16	53	4	40	25
6	0	1	14	48	36	41	37	0	13	32	15	41	36	30
7	0	1	27	16	42	48	34	0	15	47	38	18	32	35
8	0	1	39	44	48	55	30	0	18	3	0	55	28	40
9	0	1	52	12	55	2	26	0	20	18	23	32	24	45
10	0	2	4	41	1	9	22	0	22	33	46	9	20	50
11	0	2	17	9	7	16	19	0	24	49	8	46	16	55
12	0	2	29	37	13	23	15	0	27	4	31	23	13	0
13	0	2	42	5	19	30	11	0	29	19	54	0	9	5
14	0	2	54	33	25	37	8	0	31	35	16	37	5	10
15	0	3	7	1	31	44	4	0	33	50	39	14	1	15
16	0	3	19	29	37	51	0	0	36	6	1	50	57	20
17	0	3	31	57	43	57	56	0	38	21	24	27	53	25
18	0	3	44	25	50	4	53	0	40	36	47	4	49	30
19	0	3	56	53	56	11	49	0	42	52	9	41	45	35
20	0	4	9	22	2	18	45	0	45	7	32	18	41	40
21	0	4	21	50	8	25	42	0	47	22	54	55	37	45
22	0	4	34	18	14	32	38	0	49	38	17	32	33	50
23	0	4	46	46	20	39	34	0	51	53	40	9	29	55
24	0	4	59	14	26	46	31	0	54	9	2	46	26	0

Months	Jupiter Longitude							Jupiter Anomaly						
30	2°	29	37	13	23	15	30	27°	4	31	23	13	0	0
60	4	59	14	26	46	31	0	54	9	2	46	26	0	0
90	7	28	51	40	9	46	30	81	13	34	9	39	0	0
120	9	58	28	53	33	2	0	108	18	5	32	52	0	0
150	12	28	6	6	56	17	30	135	22	36	56	5	0	0
180	14	57	43	20	19	33	0	162	27	8	19	18	0	0
210	17	27	20	33	42	48	30	189	31	39	42	31	0	0
240	19	56	57	47	6	4	0	216	36	11	5	44	0	0
270	22	26	35	0	29	19	30	243	40	42	28	57	0	0
300	24	56	12	13	52	35	0	270	45	13	52	10	0	0
330	27	25	49	27	15	50	30	297	49	45	15	23	0	0
360	29	55	26	40	39	6	0	324	54	16	38	36	0	0
Days	Longitude							Anomaly						
1	0°	4	59	14	26	46	31	0°	54	9	2	46	26	0
2	0	9	58	28	53	33	2	1	48	18	5	32	52	0
3	0	14	57	43	20	19	33	2	42	27	8	19	18	0
4	0	19	56	57	47	6	4	3	36	36	11	5	44	0
5	0	24	56	12	13	52	35	4	30	45	13	52	10	0
6	0	29	55	26	40	39	6	5	24	54	16	38	36	0
7	0	34	54	41	7	25	37	6	19	3	19	25	2	0
8	0	39	53	55	34	12	8	7	13	12	22	11	28	0
9	0	44	53	10	0	58	39	8	7	21	24	57	54	0
10	0	49	52	24	27	45	10	9	1	30	27	44	20	0
11	0	54	51	38	54	31	41	9	55	39	30	30	46	0
12	0	59	50	53	21	18	12	10	49	48	33	17	12	0
13	1	4	50	7	48	4	43	11	43	57	36	3	38	0
14	1	9	49	22	14	51	14	12	38	6	38	50	4	0
15	1	14	48	36	41	37	45	13	32	15	41	36	30	0
16	1	19	47	51	8	24	16	14	26	24	44	22	56	0
17	1	24	47	5	35	10	47	15	20	33	47	9	22	0
18	1	29	46	20	1	57	18	16	14	42	49	55	48	0
19	1	34	45	34	28	43	49	17	8	51	52	42	14	0
20	1	39	44	48	55	30	20	18	3	0	55	28	40	0
21	1	44	44	3	22	16	51	18	57	9	58	15	6	0
22	1	49	43	17	49	3	22	19	51	19	1	1	32	0
23	1	54	42	32	15	49	53	20	45	28	3	47	58	0
24	1	59	41	46	42	36	24	21	39	37	6	34	24	0
25	2	4	41	1	9	22	55	22	33	46	9	20	50	0
26	2	9	40	15	36	9	26	23	27	55	12	7	16	0
27	2	14	39	30	2	55	57	24	22	4	14	53	42	0
28	2	19	38	44	29	42	28	25	16	13	17	40	8	0
29	2	24	37	58	56	28	59	26	10	22	20	26	34	0
30	2	29	37	13	23	15	30	27	4	31	23	13	0	0

Mars 18-Year Periods	[Epoch] Position in [Mean] Longitude : ♈ 3;32° Longitude							[Epoch] Position in Anomaly : 327;13° [Epoch] Position of Apogee : ♋ 16;40° Anomaly						
18	203°	4	20	17	34	43	30	152°	33	5	18	45	51	0
36	46	8	40	35	9	27	0	305	6	10	37	31	42	0
54	249	13	0	52	44	10	30	97	39	15	56	17	33	0
72	92	17	21	10	18	54	0	250	12	21	15	3	24	0
90	295	21	41	27	53	37	30	42	45	26	33	49	15	0
108	138	26	1	45	28	21	0	195	18	31	52	35	6	0
126	341	30	22	3	3	4	30	347	51	37	11	20	57	0
144	184	34	42	20	37	48	0	140	24	42	30	6	48	0
162	27	39	2	38	12	31	30	292	57	47	48	52	39	0
180	230	43	22	55	47	15	0	85	30	53	7	38	30	0
198	73	47	43	13	21	58	30	238	3	58	26	24	21	0
216	276	52	3	30	56	42	0	30	37	3	45	10	12	0
234	119	56	23	48	31	25	30	183	10	9	3	56	3	0
252	323	0	44	6	6	9	0	335	43	14	22	41	54	0
270	166	5	4	23	40	52	30	128	16	19	41	27	45	0
288	9	9	24	41	15	36	0	280	49	25	0	13	36	0
306	212	13	44	58	50	19	30	73	22	30	18	59	27	0
324	55	18	5	16	25	3	0	225	55	35	37	45	18	0
342	258	22	25	33	59	46	30	18	28	40	56	31	9	0
360	101	26	45	51	34	30	0	171	1	46	15	17	0	0
378	304	31	6	9	9	13	30	323	34	51	34	2	51	0
396	147	35	26	26	43	57	0	116	7	56	52	48	42	0
414	350	39	46	44	18	40	30	268	41	2	11	34	33	0
432	193	44	7	1	53	24	0	61	14	7	30	20	24	0
450	36	48	27	19	28	7	30	213	47	12	49	6	15	0
468	239	52	47	37	2	51	0	6	20	18	7	52	6	0
486	82	57	7	54	37	34	30	158	53	23	26	37	57	0
504	286	1	28	12	12	18	0	311	26	28	45	23	48	0
522	129	5	48	29	47	1	30	103	59	34	4	9	39	0
540	332	10	8	47	21	45	0	256	32	39	22	55	30	0
558	175	14	29	4	56	28	30	49	5	44	41	41	21	0
576	18	18	49	22	31	12	0	201	38	50	0	27	12	0
594	221	23	9	40	5	55	30	354	11	55	19	13	3	0
612	64	27	29	57	40	39	0	146	45	0	37	58	54	0
630	267	31	50	15	15	22	30	299	18	5	56	44	45	0
648	110	36	10	32	50	6	0	91	51	11	15	30	36	0
666	313	40	30	50	24	49	30	244	24	16	34	16	27	0
684	156	44	51	7	59	33	0	36	57	21	53	2	18	0
702	359	49	11	25	34	16	30	189	30	27	11	48	9	0
720	202	53	31	43	9	0	0	342	3	32	30	34	0	0
738	45	57	52	0	43	43	30	134	36	37	49	19	51	0
756	249	2	12	18	18	27	0	287	9	43	8	5	42	0
774	92	6	32	35	53	10	30	79	42	48	26	51	33	0
792	295	10	52	53	27	54	0	232	15	53	45	37	24	0
810	138	15	13	11	2	37	30	24	48	59	4	23	15	0

Single Years	Mars Longitude							Mars Anomaly						
1	191°	16	54	27	38	35	45	168°	28	30	17	42	32	50
2	22	33	48	55	17	11	30	336	57	0	35	25	5	40
3	213	50	43	22	55	47	15	145	25	30	53	7	38	30
4	45	7	37	50	34	23	0	313	54	1	10	50	11	20
5	236	24	32	18	12	58	45	122	22	31	28	32	44	10
6	67	41	26	45	51	34	30	290	51	1	46	15	17	0
7	258	58	21	13	30	10	15	99	19	32	3	57	49	50
8	90	15	15	41	8	46	0	267	48	2	21	40	22	40
9	281	32	10	8	47	21	45	76	16	32	39	22	55	30
10	112	49	4	36	25	57	30	244	45	2	57	5	28	20
11	304	5	59	4	4	33	15	53	13	33	14	48	1	10
12	135	22	53	31	43	9	0	221	42	3	32	30	34	0
13	326	39	47	59	21	44	45	30	10	33	50	13	6	50
14	157	56	42	27	0	20	30	198	39	4	7	55	39	40
15	349	13	36	54	38	56	15	7	7	34	25	38	12	30
16	180	30	31	22	17	32	0	175	36	4	43	20	45	20
17	11	47	25	49	56	7	45	344	4	35	1	3	18	10
18	203	4	20	17	34	43	30	152	33	5	18	45	51	0
Hours	Longitude							Anomaly						
1	0°	1	18	36	32	14	39	0°	1	9	14	10	48	22
2	0	2	37	13	4	29	18	0	2	18	28	21	36	44
3	0	3	55	49	36	43	56	0	3	27	42	32	25	7
4	0	5	14	26	8	58	35	0	4	36	56	43	13	29
5	0	6	33	2	41	13	14	0	5	46	10	54	1	52
6	0	7	51	39	13	27	53	0	6	55	25	4	50	14
7	0	9	10	15	45	42	32	0	8	4	39	15	38	36
8	0	10	28	52	17	57	11	0	9	13	53	26	26	59
9	0	11	47	28	50	11	49	0	10	23	7	37	15	21
10	0	13	6	5	22	26	28	0	11	32	21	48	3	44
11	0	14	24	41	54	41	7	0	12	41	35	58	52	6
12	0	15	43	18	26	55	46	0	13	50	50	9	40	29
13	0	17	1	54	59	10	25	0	15	0	4	20	28	51
14	0	18	20	31	31	25	4	0	16	9	18	31	17	13
15	0	19	39	8	3	39	43	0	17	18	32	42	5	36
16	0	20	57	44	35	54	22	0	18	27	46	52	53	58
17	0	22	16	21	8	9	0	0	19	37	1	3	42	21
18	0	23	34	57	40	23	39	0	20	46	15	14	30	43
19	0	24	53	34	12	38	18	0	21	55	29	25	19	5
20	0	26	12	10	44	52	57	0	23	4	43	36	7	28
21	0	27	30	47	17	7	36	0	24	13	57	46	55	50
22	0	28	49	23	49	22	15	0	25	23	11	57	44	13
23	0	30	8	0	21	36	54	0	26	32	26	8	32	35
24	0	31	26	36	53	51	33	0	27	41	40	19	20	58

Months	Mars Longitude							Mars Anomaly						
30	15°	43	18	26	55	46	30	13°	50	50	9	40	29	0
60	31	26	36	53	51	33	0	27	41	40	19	20	58	0
90	47	9	55	20	47	19	30	41	32	30	29	1	27	0
120	62	53	13	47	43	6	0	55	23	20	38	41	56	0
150	78	36	32	14	38	52	30	69	14	10	48	22	25	0
180	94	19	50	41	34	39	0	83	5	0	58	2	54	0
210	110	3	9	8	30	25	30	96	55	51	7	43	23	0
240	125	46	27	35	26	12	0	110	46	41	17	23	52	0
270	141	29	46	2	21	58	30	124	37	31	27	4	21	0
300	157	13	4	29	17	45	0	138	28	21	36	44	50	0
330	172	56	22	56	13	31	30	152	19	11	46	25	19	0
360	188	39	41	23	9	18	0	166	10	1	56	5	48	0
Days	Longitude							Anomaly						
1	0°	31	26	36	53	51	33	0°	27	41	40	19	20	58
2	1	2	53	13	47	43	6	0	55	23	20	38	41	56
3	1	34	19	50	41	34	39	1	23	5	0	58	2	54
4	2	5	46	27	35	26	12	1	50	46	41	17	23	52
5	2	37	13	4	29	17	45	2	18	28	21	36	44	50
6	3	8	39	41	23	9	18	2	46	10	1	56	5	48
7	3	40	6	18	17	0	51	3	13	51	42	15	26	46
8	4	11	32	55	10	52	24	3	41	33	22	34	47	44
9	4	42	59	32	4	43	57	4	9	15	2	54	8	42
10	5	14	26	8	58	35	30	4	36	56	43	13	29	40
11	5	45	52	45	52	27	3	5	4	38	23	32	50	38
12	6	17	19	22	46	18	36	5	32	20	3	52	11	36
13	6	48	45	59	40	10	9	6	0	1	44	11	32	34
14	7	20	12	36	34	1	42	6	27	43	24	30	53	32
15	7	51	39	13	27	53	15	6	55	25	4	50	14	30
16	8	23	5	50	21	44	48	7	23	6	45	9	35	28
17	8	54	32	27	15	36	21	7	50	48	25	28	56	26
18	9	25	59	4	9	27	54	8	18	30	5	48	17	24
19	9	57	25	41	3	19	27	8	46	11	46	7	38	22
20	10	28	52	17	57	11	0	9	13	53	26	26	59	20
21	11	0	18	54	51	2	33	9	41	35	6	46	20	18
22	11	31	45	31	44	54	6	10	9	16	47	5	41	16
23	12	3	12	8	38	45	39	10	36	58	27	25	2	14
24	12	34	38	45	32	37	12	11	4	40	7	44	23	12
25	13	6	5	22	26	28	45	11	32	21	48	3	44	10
26	13	37	31	59	20	20	18	12	0	3	28	23	5	8
27	14	8	58	36	14	11	51	12	27	45	8	42	26	6
28	14	40	25	13	8	3	24	12	55	26	49	1	47	4
29	15	11	51	50	1	54	57	13	23	8	29	21	8	2
30	15	43	18	26	55	46	30	13	50	50	9	40	29	0

Venus 18-Year Periods	[Epoch] Position in [Mean] Longitude : ♓ 0;45° Longitude							[Epoch] Position in Anomaly : 71;7° [Epoch] Position of Apogee : ♉ 16;10° Anomaly						
18	355°	37	25	36	20	34	30	90°	27	44	34	23	46	30
36	351	14	51	12	41	9	0	180	55	29	8	47	33	0
54	346	52	16	49	1	43	30	271	23	13	43	11	19	30
72	342	29	42	25	22	18	0	1	50	58	17	35	6	0
90	338	7	8	1	42	52	30	92	18	42	51	58	52	30
108	333	44	33	38	3	27	0	182	46	27	26	22	39	0
126	329	21	59	14	24	1	30	273	14	12	0	46	25	30
144	324	59	24	50	44	36	0	3	41	56	35	10	12	0
162	320	36	50	27	5	10	30	94	9	41	9	33	58	30
180	316	14	16	3	25	45	0	184	37	25	43	57	45	0
198	311	51	41	39	46	19	30	275	5	10	18	21	31	30
216	307	29	7	16	6	54	0	5	32	54	52	45	18	0
234	303	6	32	52	27	28	30	96	0	39	27	9	4	30
252	298	43	58	28	48	3	0	186	28	24	1	32	51	0
270	294	21	24	5	8	37	30	276	56	8	35	56	37	30
288	289	58	49	41	29	12	0	7	23	53	10	20	24	0
306	285	36	15	17	49	46	30	97	51	37	44	44	10	30
324	281	13	40	54	10	21	0	188	19	22	19	7	57	0
342	276	51	6	30	30	55	30	278	47	6	53	31	43	30
360	272	28	32	6	51	30	0	9	14	51	27	55	30	0
378	268	5	57	43	12	4	30	99	42	36	2	19	16	30
396	263	43	23	19	32	39	0	190	10	20	36	43	3	0
414	259	20	48	55	53	13	30	280	38	5	11	6	49	30
432	254	58	14	32	13	48	0	11	5	49	45	30	36	0
450	250	35	40	8	34	22	30	101	33	34	19	54	22	30
468	246	13	5	44	54	57	0	192	1	18	54	18	9	0
486	241	50	31	21	15	31	30	282	29	3	28	41	55	30
504	237	27	56	57	36	6	0	12	56	48	3	5	42	0
522	233	5	22	33	56	40	30	103	24	32	37	29	28	30
540	228	42	48	10	17	15	0	193	52	17	11	53	15	0
558	224	20	13	46	37	49	30	284	20	1	46	17	1	30
576	219	57	39	22	58	24	0	14	47	46	20	40	48	0
594	215	35	4	59	18	58	30	105	15	30	55	4	34	30
612	211	12	30	35	39	33	0	195	43	15	29	28	21	0
630	206	49	56	12	0	7	30	286	11	0	3	52	7	30
648	202	27	21	48	20	42	0	16	38	44	38	15	54	0
666	198	4	47	24	41	16	30	107	6	29	12	39	40	30
684	193	42	13	1	1	51	0	197	34	13	47	3	27	0
702	189	19	38	37	22	25	30	288	1	58	21	27	13	30
720	184	57	4	13	43	0	0	18	29	42	55	51	0	0
738	180	34	29	50	3	34	30	108	57	27	30	14	46	30
756	176	11	55	26	24	9	0	199	25	12	4	38	33	0
774	171	49	21	2	44	43	30	289	52	56	39	2	19	30
792	167	26	46	39	5	18	0	20	20	41	13	26	6	0
810	163	4	12	15	25	52	30	110	48	25	47	49	52	30

Single Years	Venus Longitude							Venus Anomaly						
1	359°	45	24	45	21	8	35	225°	1	32	28	34	39	15
2	359	30	49	30	42	17	10	90	3	4	57	9	18	30
3	359	16	14	16	3	25	45	315	4	37	25	43	57	45
4	359	1	39	1	24	34	20	180	6	9	54	18	37	0
5	358	47	3	46	45	42	55	45	7	42	22	53	16	15
6	358	32	28	32	6	51	30	270	9	14	51	27	55	30
7	358	17	53	17	28	0	5	135	10	47	20	2	34	45
8	358	3	18	2	49	8	40	0	12	19	48	37	14	0
9	357	48	42	48	10	17	15	225	13	52	17	11	53	15
10	357	34	7	33	31	25	50	90	15	24	45	46	32	30
11	357	19	32	18	52	34	25	315	16	57	14	21	11	45
12	357	4	57	4	13	43	0	180	18	29	42	55	51	0
13	356	50	21	49	34	51	35	45	20	2	11	30	30	15
14	356	35	46	34	56	0	10	270	21	34	40	5	9	30
15	356	21	11	20	17	8	45	135	23	7	8	39	48	45
16	356	6	36	5	38	17	20	0	24	39	37	14	28	0
17	355	52	0	50	59	25	55	225	26	12	5	49	7	15
18	355	37	25	36	20	34	30	90	27	44	34	23	46	30

Hours	Longitude							Anomaly						
1	0°	2	27	50	43	3	1	0°	1	32	28	34	42	58
2	0	4	55	41	26	6	2	0	3	4	57	9	25	57
3	0	7	23	32	9	9	3	0	4	37	25	44	8	56
4	0	9	51	22	52	12	5	0	6	9	54	18	51	54
5	0	12	19	13	35	15	6	0	7	42	22	53	34	53
6	0	14	47	4	18	18	7	0	9	14	51	28	17	52
7	0	17	14	55	1	21	9	0	10	47	20	3	0	50
8	0	19	42	45	44	24	10	0	12	19	48	37	43	49
9	0	22	10	36	27	27	11	0	13	52	17	12	26	48
10	0	24	38	27	10	30	12	0	15	24	45	47	9	46
11	0	27	6	17	53	33	14	0	16	57	14	21	52	45
12	0	29	34	8	36	36	15	0	18	29	42	56	35	44
13	0	32	1	59	19	39	16	0	20	2	11	31	18	42
14	0	34	29	50	2	42	18	0	21	34	40	6	1	41
15	0	36	57	40	45	45	19	0	23	7	8	40	44	40
16	0	39	25	31	28	48	20	0	24	39	37	15	27	38
17	0	41	53	22	11	51	21	0	26	12	5	50	10	37
18	0	44	21	12	54	54	23	0	27	44	34	24	53	36
19	0	46	49	3	37	57	24	0	29	17	2	59	36	34
20	0	49	16	54	21	0	25	0	30	49	31	34	19	33
21	0	51	44	45	4	3	27	0	32	22	0	9	2	32
22	0	54	12	35	47	6	28	0	33	54	28	43	45	30
23	0	56	40	26	30	9	29	0	35	26	57	18	28	29
24	0	59	8	17	13	12	31	0	36	59	25	53	11	28

Months	Venus Longitude							Venus Anomaly						
30	29°	34	8	36	36	15	30	18°	29	42	56	35	44	0
60	59	8	17	13	12	31	0	36	59	25	53	11	28	0
90	88	42	25	49	48	46	30	55	29	8	49	47	12	0
120	118	16	34	26	25	2	0	73	58	51	46	22	56	0
150	147	50	43	3	1	17	30	92	28	34	42	58	40	0
180	177	24	51	39	37	33	0	110	58	17	39	34	24	0
210	206	59	0	16	13	48	30	129	28	0	36	10	8	0
240	236	33	8	52	50	4	0	147	57	43	32	45	52	0
270	266	7	17	29	26	19	30	166	27	26	29	21	36	0
300	295	41	26	6	2	35	0	184	57	9	25	57	20	0
330	325	15	34	42	38	50	30	203	26	52	22	33	4	0
360	354	49	43	19	15	6	0	221	56	35	19	8	48	0
Days	Longitude							Anomaly						
1	0°	59	8	17	13	12	31	0°	36	59	25	53	11	28
2	1	58	16	34	26	25	2	1	13	58	51	46	22	56
3	2	57	24	51	39	37	33	1	50	58	17	39	34	24
4	3	56	33	8	52	50	4	2	27	57	43	32	45	52
5	4	55	41	26	6	2	35	3	4	57	9	25	57	20
6	5	54	49	43	19	15	6	3	41	56	35	19	8	48
7	6	53	58	0	32	27	37	4	18	56	1	12	20	16
8	7	53	6	17	45	40	8	4	55	55	27	5	31	44
9	8	52	14	34	58	52	39	5	32	54	52	58	43	12
10	9	51	22	52	12	5	10	6	9	54	18	51	54	40
11	10	50	31	9	25	17	41	6	46	53	44	45	6	8
12	11	49	39	26	38	30	12	7	23	53	10	38	17	36
13	12	48	47	43	51	42	43	8	0	52	36	31	29	4
14	13	47	56	1	4	55	14	8	37	52	2	24	40	32
15	14	47	4	18	18	7	45	9	14	51	28	17	52	0
16	15	46	12	35	31	20	16	9	51	50	54	11	3	28
17	16	45	20	52	44	32	47	10	28	50	20	4	14	56
18	17	44	29	9	57	45	18	11	5	49	45	57	26	24
19	18	43	37	27	10	57	49	11	42	49	11	50	37	52
20	19	42	45	44	24	10	20	12	19	48	37	43	49	20
21	20	41	54	1	37	22	51	12	56	48	3	37	0	48
22	21	41	2	18	50	35	22	13	33	47	29	30	12	16
23	22	40	10	36	3	47	53	14	10	46	55	23	23	44
24	23	39	18	53	17	0	24	14	47	46	21	16	35	12
25	24	38	27	10	30	12	55	15	24	45	47	9	46	40
26	25	37	35	27	43	25	26	16	1	45	13	2	58	8
27	26	36	43	44	56	37	57	16	38	44	38	56	9	36
28	27	35	52	2	9	50	28	17	15	44	4	49	21	4
29	28	35	0	19	23	2	59	17	52	43	30	42	32	32
30	29	34	8	36	36	15	30	18	29	42	56	35	44	0

Mercury 18-Year Periods	[Epoch] Position in [Mean] Longitude : ♓ 0;45° Longitude							[Epoch] Position in Anomaly: 21;55° [Epoch] Position of Apogee : ♎ 1;10° Anomaly						
18	355°	37	25	36	20	34	30	251°	0	45	45	53	45	0
36	351	14	51	12	41	9	0	142	1	31	31	47	30	0
54	346	52	16	49	1	43	30	33	2	17	17	41	15	0
72	342	29	42	25	22	18	0	284	3	3	3	35	0	0
90	338	7	8	1	42	52	30	175	3	48	49	28	45	0
108	333	44	33	38	3	27	0	66	4	34	35	22	30	0
126	329	21	59	14	24	1	30	317	5	20	21	16	15	0
144	324	59	24	50	44	36	0	208	6	6	7	10	0	0
162	320	36	50	27	5	10	30	99	6	51	53	3	45	0
180	316	14	16	3	25	45	0	350	7	37	38	57	30	0
198	311	51	41	39	46	19	30	241	8	23	24	51	15	0
216	307	29	7	16	6	54	0	132	9	9	10	45	0	0
234	303	6	32	52	27	28	30	23	9	54	56	38	45	0
252	298	43	58	28	48	3	0	274	10	40	42	32	30	0
270	294	21	24	5	8	37	30	165	11	26	28	26	15	0
288	289	58	49	41	29	12	0	56	12	12	14	20	0	0
306	285	36	15	17	49	46	30	307	12	58	0	13	45	0
324	281	13	40	54	10	21	0	198	13	43	46	7	30	0
342	276	51	6	30	30	55	30	89	14	29	32	1	15	0
360	272	28	32	6	51	30	0	340	15	15	17	55	0	0
378	268	5	57	43	12	4	30	231	16	1	3	48	45	0
396	263	43	23	19	32	39	0	122	16	46	49	42	30	0
414	259	20	48	55	53	13	30	13	17	32	35	36	15	0
432	254	58	14	32	13	48	0	264	18	18	21	30	0	0
450	250	35	40	8	34	22	30	155	19	4	7	23	45	0
468	246	13	5	44	54	57	0	46	19	49	53	17	30	0
486	241	50	31	21	15	31	30	297	20	35	39	11	15	0
504	237	27	56	57	36	6	0	188	21	21	25	5	0	0
522	233	5	22	33	56	40	30	79	22	7	10	58	45	0
540	228	42	48	10	17	15	0	330	22	52	56	52	30	0
558	224	20	13	46	37	49	30	221	23	38	42	46	15	0
576	219	57	39	22	58	24	0	112	24	24	28	40	0	0
594	215	35	4	59	18	58	30	3	25	10	14	33	45	0
612	211	12	30	35	39	33	0	254	25	56	0	27	30	0
630	206	49	56	12	0	7	30	145	26	41	46	21	15	0
648	202	27	21	48	20	42	0	36	27	27	32	15	0	0
666	198	4	47	24	41	16	30	287	28	13	18	8	45	0
684	193	42	13	1	1	51	0	178	28	59	4	2	30	0
702	189	19	38	37	22	25	30	69	29	44	49	56	15	0
720	184	57	4	13	43	0	0	320	30	30	35	50	0	0
738	180	34	29	50	3	34	30	211	31	16	21	43	45	0
756	176	11	55	26	24	9	0	102	32	2	7	37	30	0
774	171	49	21	2	44	43	30	353	32	47	53	31	15	0
792	167	26	46	39	5	18	0	244	33	33	39	25	0	0
810	163	4	12	15	25	52	30	135	34	19	25	18	45	0

Single Years	Mercury Longitude							Mercury Anomaly						
1	359°	45	24	45	21	8	35	53°	56	42	32	32	59	10
2	359	30	49	30	42	17	10	107	53	25	5	5	58	20
3	359	16	14	16	3	25	45	161	50	7	37	38	57	30
4	359	1	39	1	24	34	20	215	46	50	10	11	56	40
5	358	47	3	46	45	42	55	269	43	32	42	44	55	50
6	358	32	28	32	6	51	30	323	40	15	15	17	55	0
7	358	17	53	17	28	0	5	17	36	57	47	50	54	10
8	358	3	18	2	49	8	40	71	33	40	20	23	53	20
9	357	48	42	48	10	17	15	125	30	22	52	56	52	30
10	357	34	7	33	31	25	50	179	27	5	25	29	51	40
11	357	19	32	18	52	34	25	233	23	47	58	2	50	50
12	357	4	57	4	13	43	0	287	20	30	30	35	50	0
13	356	50	21	49	34	51	35	341	17	13	3	8	49	10
14	356	35	46	34	56	0	10	35	13	55	35	41	48	20
15	356	21	11	20	17	8	45	89	10	38	8	14	47	30
16	356	6	36	5	38	17	20	143	7	20	40	47	46	40
17	355	52	0	50	59	25	55	197	4	3	13	20	45	50
18	355	37	25	36	20	34	30	251	0	45	45	53	45	0
Hours	Longitude							Anomaly						
1	0°	2	27	50	43	3	1	0°	7	46	0	17	28	59
2	0	4	55	41	26	6	2	0	15	32	0	34	57	59
3	0	7	23	32	9	9	3	0	23	18	0	52	26	58
4	0	9	51	22	52	12	5	0	31	4	1	9	55	58
5	0	12	19	13	35	15	6	0	38	50	1	27	24	57
6	0	14	47	4	18	18	7	0	46	36	1	44	53	57
7	0	17	14	55	1	21	9	0	54	22	2	2	22	57
8	0	19	42	45	44	24	10	1	2	8	2	19	51	56
9	0	22	10	36	27	27	11	1	9	54	2	37	20	56
10	0	24	38	27	10	30	12	1	17	40	2	54	49	55
11	0	27	6	17	53	33	14	1	25	26	3	12	18	55
12	0	29	34	8	36	36	15	1	33	12	3	29	47	55
13	0	32	1	59	19	39	16	1	40	58	3	47	16	54
14	0	34	29	50	2	42	18	1	48	44	4	4	45	54
15	0	36	57	40	45	45	19	1	56	30	4	22	14	53
16	0	39	25	31	28	48	20	2	4	16	4	39	43	53
17	0	41	53	22	11	51	21	2	12	2	4	57	12	52
18	0	44	21	12	54	54	23	2	19	48	5	14	41	52
19	0	46	49	3	37	57	24	2	27	34	5	32	10	52
20	0	49	16	54	21	0	25	2	35	20	5	49	39	51
21	0	51	44	45	4	3	27	2	43	6	6	7	8	51
22	0	54	12	35	47	6	28	2	50	52	6	24	37	50
23	0	56	40	26	30	9	29	2	58	38	6	42	6	50
24	0	59	8	17	13	12	31	3	6	24	6	59	35	50

Months	Mercury Longitude							Mercury Anomaly						
30	29°	34	8	36	36	15	30	93°	12	3	29	47	55	0
60	59	8	17	13	12	31	0	186	24	6	59	35	50	0
90	88	42	25	49	48	46	30	279	36	10	29	23	45	0
120	118	16	34	26	25	2	0	12	48	13	59	11	40	0
150	147	50	43	3	1	17	30	106	0	17	28	59	35	0
180	177	24	51	39	37	33	0	199	12	20	58	47	30	0
210	206	59	0	16	13	48	30	292	24	24	28	35	25	0
240	236	33	8	52	50	4	0	25	36	27	58	23	20	0
270	266	7	17	29	26	19	30	118	48	31	28	11	15	0
300	295	41	26	6	2	35	0	212	0	34	57	59	10	0
330	325	15	34	42	38	50	30	305	12	38	27	47	5	0
360	354	49	43	19	15	6	0	38	24	41	57	35	0	0

Days	Longitude							Anomaly						
1	0°	59	8	17	13	12	31	3°	6	24	6	59	35	50
2	1	58	16	34	26	25	2	6	12	48	13	59	11	40
3	2	57	24	51	39	37	33	9	19	12	20	58	47	30
4	3	56	33	8	52	50	4	12	25	36	27	58	23	20
5	4	55	41	26	6	2	35	15	32	0	34	57	59	10
6	5	54	49	43	19	15	6	18	38	24	41	57	35	0
7	6	53	58	0	32	27	37	21	44	48	48	57	10	50
8	7	53	6	17	45	40	8	24	51	12	55	56	46	40
9	8	52	14	34	58	52	39	27	57	37	2	56	22	30
10	9	51	22	52	12	5	10	31	4	1	9	55	58	20
11	10	50	31	9	25	17	41	34	10	25	16	55	34	10
12	11	49	39	26	38	30	12	37	16	49	23	55	10	0
13	12	48	47	43	51	42	43	40	23	13	30	54	45	50
14	13	47	56	1	4	55	14	43	29	37	37	54	21	40
15	14	47	4	18	18	7	45	46	36	1	44	53	57	30
16	15	46	12	35	31	20	16	49	42	25	51	53	33	20
17	16	45	20	52	44	32	47	52	48	49	58	53	9	10
18	17	44	29	9	57	45	18	55	55	14	5	52	45	0
19	18	43	37	27	10	57	49	59	1	38	12	52	20	50
20	19	42	45	44	24	10	20	62	8	2	19	51	56	40
21	20	41	54	1	37	22	51	65	14	26	26	51	32	30
22	21	41	2	18	50	35	22	68	20	50	33	51	8	20
23	22	40	10	36	3	47	53	71	27	14	40	50	44	10
24	23	39	18	53	17	0	24	74	33	38	47	50	20	0
25	24	38	27	10	30	12	55	77	40	2	54	49	55	50
26	25	37	35	27	43	25	26	80	46	27	1	49	31	40
27	26	36	43	44	56	37	57	83	52	51	8	49	7	30
28	27	35	52	2	9	50	28	86	59	15	15	48	43	20
29	28	35	0	19	23	2	59	90	5	39	22	48	19	10
30	29	34	8	36	36	15	30	93	12	3	29	47	55	0

There are, as we said,[35] two types of motion which are simplest and at the same time sufficient for our purpose, [namely] that produced by circles eccentric to [the centre of] the ecliptic, and that produced by circles concentric with the ecliptic but carrying epicycles around. There are likewise two apparent anomalies for each planet: [1] that anomaly which varies according to its position in the ecliptic, and [2] that which varies according to its position relative to the sun.

For [2] we find, from a series of different [sun-planet] configurations observed round about the same part of the ecliptic,[36] that in the case of the five planets[37] the time from greatest speed to mean is always greater than the time from mean speed to least. Now this feature cannot be a consequence of the eccentric hypothesis, in which exactly the opposite occurs, since the greatest speed takes
H251 place at the perigee in the eccentric hypothesis, while the arc from the perigee to the point of mean speed is less than the arc from the latter to the apogee in both [eccentric and epicyclic] hypotheses. But it can occur as a consequence of the epicyclic hypothesis, however only when the greatest speed occurs, not at the perigee, as in the case of the moon, but at the apogee; that is to say, when the planet, starting from the apogee, moves, not as the moon does, in advance [with respect to the motion] of the universe, but instead towards the rear. Hence we use the epicyclic hypothesis to represent this kind of anomaly.[38]

But for [1], the anomaly which varies according to the position in the ecliptic, we find from [observations of] the arcs of the ecliptic between [successive] phases or [sun-planet] configurations of the same kind[39] that the opposite is true: the time from least speed to mean is always greater than the time from mean speed to greatest. This feature can indeed be a consequence of either of the two hypotheses (in the way we described in our discussion of the equivalence of the hypotheses at the beginning of our treatise on the sun [III 3]). But it is more appropriate to the eccentric hypothesis,[40] and that is the hypothesis which we actually use to represent this kind of anomaly, since, moreover, the other anomaly was found to be peculiar, so to speak, to the epicyclic hypothesis.

Now from prolonged application and comparison of observations of individual [planetary] positions with the results computed from the combin-
H252 ation of both [the above] hypotheses, we find that it will not work simply to assume[41] [as one has hitherto] that the plane in which we draw the eccentric

[35] III 3 p. 141.

[36] This eliminates the effect of the ecliptic anomaly. Examples would be observations of Mars at opposition, station and (by interpolation) conjunction all near the same point in the ecliptic.

[37] Excising καὶ before ἐπὶ τῶν πέντε πλανωμένων at H250,17. (καὶ was apparently omitted in the text translated by al-Ḥajjāj). One would have to translate Heiberg's text 'in the case of the five planets too' (as well as the sun and moon). But the situation is precisely the opposite for the sun and moon (see e.g. III 4 p. 153). Perhaps the whole phrase καὶ . . . πλανωμένων is an ancient interpolation.

[38] See Ptolemy's discussion of this point at III 3 p. 144–5. However, as Neugebauer points out (*HAMA* 149–50) it is perfectly possible for an eccentric model to represent the planetary motions, provided the apsidal line is allowed to move, and precisely this kind of eccentric model is described at XII 1, though even there Ptolemy restricts its applicability to the outer planets.

[39] This eliminates the effect of the synodic anomaly. Examples would be observations of oppositions of Mars in different parts of the ecliptic (as in X 7).

[40] Cf. III 4 p. 153, where Ptolemy prefers it on the ground that it is 'simpler'.

[41] Literally 'that the assumption that . . . cannot progress so simply'.

circles is stationary, and that the straight line through both centres (the centre of the [planet's] eccentre and the centre of the ecliptic), which defines apogee and perigee, remains at a constant distance from the solstitial and equinoctial points; nor [to assume] that the eccentre on which the epicycle centre is carried is identical with the eccentre with respect to the centre of which the epicycle makes its uniform revolution towards the rear, cutting off equal angles in equal times at [that centre]. Rather, we find that the apogee of the eccentre performs a slow motion towards the rear with respect to the solstices, which is uniform about the centre of the ecliptic, and comes to about the same for each planet as the amount determined for the sphere of the fixed stars, i.e. 1° in 100 years (at least, as far as can be estimated on the basis of available evidence). We find, too, that the epicycle centre is carried on an eccentre which, though equal in size to the eccentre which produces the anomaly, is not described about the same centre as the latter. For all planets except Mercury the centre [of the actual deferent] is the point bisecting the line joining the centre of the eccentre producing the anomaly to the centre of the ecliptic. For Mercury alone, [the centre of the deferent] is a point whose distance from the centre of the circle about which it rotates is equal to the distance of the latter point towards the apogee from the centre of the eccentre producing the anomaly, which in turn is the same distance towards the apogee from the point representing the observer; for also, in the case of this planet alone, we find that, just as for the moon, the H253 eccentre is rotated by [the movement of] the above-mentioned centre in the opposite sense to the epicycle, [i.e.] in the advance direction, one rotation per year. [This must be so] because the planet itself appears twice in the perigee in the course of one revolution, just as the moon appears twice in the perigee in one [synodic] month.

6. {*On the type of and difference between the hypotheses*}

One may more easily grasp the type of the hypotheses which we infer on the basis of the preceding [phenomena] from the following description.

First for that of the [four planets] other [than Mercury], imagine [Fig. 9.1] the eccentre ABG about centre D, with ADG as the diameter through D and the centre of the ecliptic; on this let E be taken as the centre of the ecliptic, i.e. the viewpoint of the observer, making A the apogee and G the perigee. Let DE be bisected at Z, and with centre Z and radius DA draw a circle HΘK, which must, clearly, be equal to ABG. Then on centre Θ draw the epicycle LM, and H254 join LΘMD.

First, then, although we assume that the plane of the eccentric circles is inclined to the plane of the ecliptic, and also that the plane of the epicycle is inclined to the plane of the eccentres, to account for the latitudinal motion of the planets, in accordance with what we shall demonstrate concerning that topic, nevertheless, for the motions in longitude, for the sake of convenience, let us imagine that all [those planes] lie in a single [plane], that of the ecliptic, since there will be no noticeable longitudinal difference, not at least when the inclinations are as small as those which will be brought to light for each of the

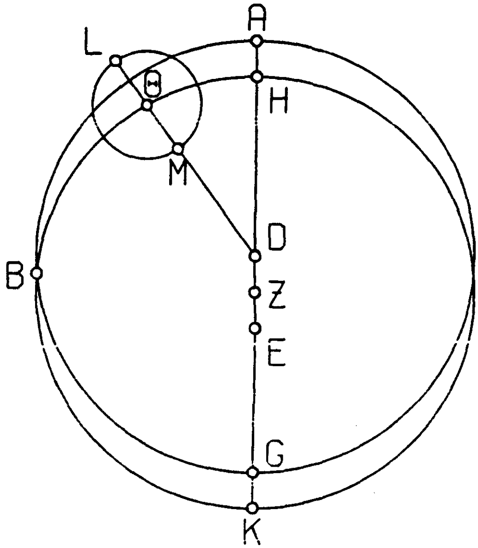

Fig. 9.1

planets. Next, we say that the whole plane [of the eccentre] moves uniformly about centre E towards the rear [i.e. in the order] of the signs, shifting the position of apogee and perigee 1° in 100 years, and that diameter LΘM of the epicycle rotates uniformly about centre D, again towards the rear [i.e. in the
H255 order] of the signs, with a speed corresponding to the planet's return in longitude, and that it carries with it points L and M of the epicycle, and centre Θ of the epicycle (which always moves on the eccentre HΘK), and also carries with it the planet; the planet, for its part, moves with uniform motion on the epicycle LM and performs its return always with respect to that diameter [of the epicycle] which points towards centre D, with a speed corresponding to the mean period of the synodic anomaly, and [a sense of rotation] such that its motion at the apogee L takes place towards the rear.

We can visualise the peculiar features of the hypothesis for Mercury as follows. Let [Fig. 9.2] the eccentre producing the anomaly be ABG about centre D, and let the diameter through D and centre E of the ecliptic be ADEG, [passing] through the apogee at A. On AG take DZ towards the apogee A, equal to DE. Then everything else remains the same, namely the whole plane, [revolving] about centre E, shifts the apogee towards the rear by the same amount as for the other planets, the epicycle is revolved uniformly about centre D towards the rear, as [here] by the line DB, and furthermore the planet moves on the epicycle in the same way as the others. But in this case the centre of the other eccentre, which is, again, equal in size to the first eccentre, and on which
H256 the epicycle centre is always located, is carried around point Z in the opposite sense to the motion of the epicycle, namely in advance [i.e. in the reverse order] of the signs, but uniformly and with the same speed as the epicycle, as [here] by the line ZHΘ. Thus in one year each of the lines DB and ZHΘ performs one

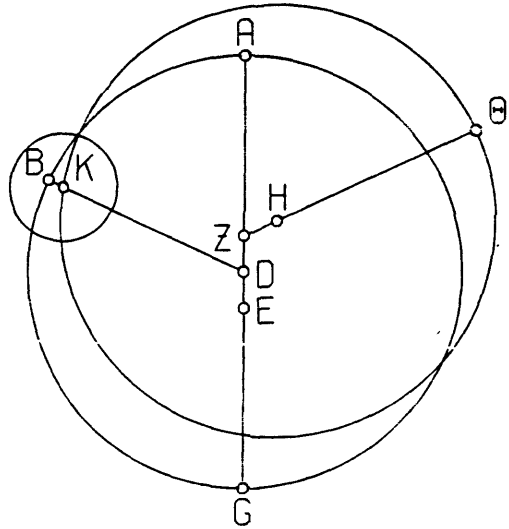

Fig. 9.2

return with respect to a [given] point of the ecliptic, but, with respect to each other, obviously, two returns. And [the centre of that eccentre] will always be at a constant distance from point Z, and that distance too will be equal to both ED and DZ (as [here] ZH). Thus the small circle described by its motion in advance, with centre Z and radius ZH, always has on its boundary the point D (the centre of the first, fixed eccentre) too; and the moving eccentre, at any given moment, can be described with centre H and radius HΘ equal to DA (as here ΘK), the epicycle always having its centre on it (as here at point K).

We shall get an even clearer grasp of these hypotheses from the demonstrations we shall make [in determining] the parameters for each planet individually. In those demonstrations will also frequently become clear, [at least] in outline, the motives which somehow led us to adopt these hypotheses. H257

However, one must make the preliminary point that the longitudinal periods do not bring the planet back to the same position both with respect to a point on the ecliptic and [simultaneously] with respect to the apogee or perigee of the eccentre; this is due to the shift in position which we assign to the latter. Hence the mean motions in longitude which we tabulated above represent, not the returns [of the planets] defined with respect to the apogees of the eccentres, but the returns defined with respect to the solstitial and equinoctial points, agreeing with the length of the year as we have determined it.[42]

Now we must prove first that from these hypotheses too it follows that, for equal distances of the planet in mean longitudinal motion on opposite sides of apogee or perigee, the equation of ecliptic anomaly on one side [of apogee or

[42] In other words, the mean motions tabulated by Ptolemy are tropical, not sidereal mean motions, and since the apogees are, by his definition, sidereally fixed, a return in longitude (to the same point in the ecliptic) must differ slightly from a return to the apogee.

perigee] is equal to that on the other side, and that the greatest elongation on the epicycle from the mean position [on one side is equal to that] in the same direction [on the other side].[43]

Let [Fig. 9.3] the eccentric circle on which the epicycle centre moves be ABGD on centre E, with diameter AEG, on which Z is taken as the centre of the ecliptic, and H as the centre of the eccentre producing the anomaly, i.e. the point about which we say the uniform motion of the epicycle takes place. Draw
H258 BHΘ and DHK at equal distances from apogee A (so that ∠ AHB = ∠ AHD), draw on points B and D epicycles of equal size, and join BZ and DZ. From Z, the observer, draw ZL and ZM as tangents to the [two] epicycles in the same direction [i.e. both towards the perigee].

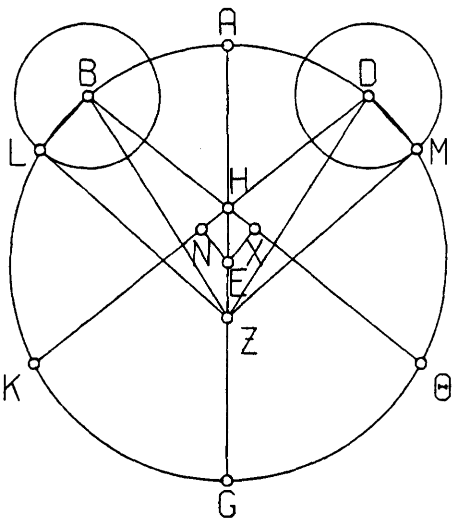

Fig. 9.3

I say [1] that the angles of the equation of ecliptic anomaly

∠ ZBH = ∠ HDZ

[2] similarly, that the greatest elongations on the epicycle

∠ BZL = ∠ DZM.

(For, [if these statements are true], the amounts of the greatest elongations from the mean [position] resulting from the combination [of the hypotheses] will also be equal [on opposite sides of the apsides]).[44]

[Proof:] Drop perpendiculars BL and DM from B and D on to ZL and ZM, and perpendiculars EN and EX from E on to DK and BΘ.

[43] By 'in the same direction' is meant 'both towards apogee' or 'both towards perigee'. This is explained by Fig. 9.3. Ptolemy is carrying out the proof of symmetry analogous to that performed for the models of the sun and moon (III 3 pp. 151-3).

[44] ∠ BZL etc. are the true maximum elongations (as seen from the earth). In what follows Ptolemy is going to compare the mean maximum elongations, and it is essential to his proof that these too be symmetrical about the line of the apsides. Since the latter differ from the angles BZL etc. by an angle equal to the equation of centre, or ∠ ZBH etc., the symmetry is guaranteed by the equations [1] and [2].

Then, since $\angle$ XHE = $\angle$ NHE[45]
and the angles at N and X are right
and EH is common to the equiangular triangles [NHE, XHE],
NH = XH
and perpendicular EN = perpendicular EX.
Therefore lines BΘ and DK are equidistant from centre E. H259
Therefore they are equal to one another,[46]
and their halves are equal to one another [i.e. BX = DN].
Therefore, by subtraction [of XH from BX and NH from DN],
BH = DH.
But HZ is common [to triangles BHZ, DHZ]
and $\angle$ BHZ = $\angle$ DHZ[47].
Therefore base BZ = base DZ
and $\angle$ HBZ = $\angle$ HDZ.
But also BL = DM (radii of the epicycle),
and the angles at L and M are right.
$\therefore$ $\angle$ BZL = $\angle$ DZM.

Q.E.D.

Again, to represent the hypothesis for Mercury, let [Fig. 9.4] ABG be the diameter through the centres and apogee of the [eccentric] circles, and let A be taken as the centre of the ecliptic, B as the centre of the eccentre producing the anomaly, and G as the point about which rotates the centre of the eccentre carrying the epicycle. Draw, again on both sides [of the apogee], lines BD and BE, representing the uniform motion of the epicycle towards the rear, and lines

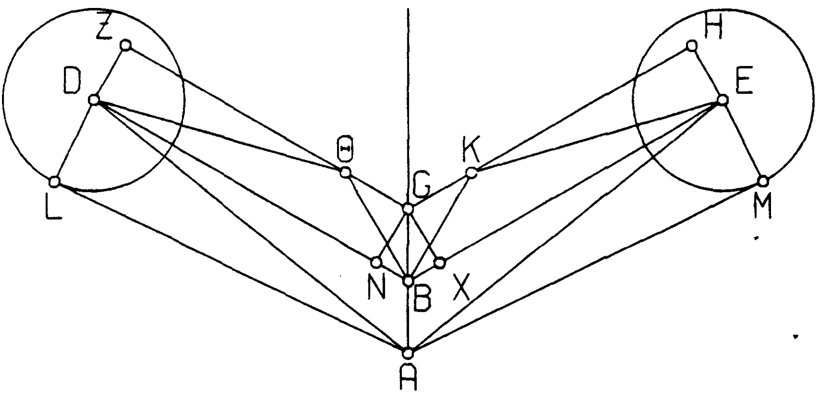

Fig. 9.4

[45] Because they are vertically opposite the equal angles AHB and AHD.

[46] Euclid III 14.

[47] Excising ἡ ὑπὸ τὰς ἴσας πλευράς at H259,4–5. Heiberg emended to ἡ ὑπὸ τῶν ἴσων πλευρῶν (the normal expression). It would mean 'the angles enclosed by the equal sides', and was presumably interpolated to make explicit the condition of Euclid I 4, 'If two triangles have two sides equal to two sides, and have the angles enclosed by the equal straight lines equal, they will also have the base equal to the base'. The reason for the equality of the angles is that they are the supplements of the equal angles AHB and AHD.

GZ and GH representing the revolution of the eccentre in advance with a speed equal [to the epicycle's]. (So it is clear that the angles at G and B must be equal,
H260 and BD must be parallel to GZ, and BE to GH). On GZ and GH take the centres of the [moving] eccentres - let them be Θ and K - and let the eccentres drawn on those centres (on which the epicycles are located), pass through points D and E. On points D and E draw epicycles (again equal), join AD and AE, and draw AL and AM tangent to the epicycles on the same side [of the epicycles].

Then we must prove that in this situation too the angles of the equation[48] of ecliptic anomaly

$$\angle ADB = \angle AEB,$$

and that the angles of greatest elongation on the epicycle

$$\angle DAL = \angle EAM.$$

[Proof:] Join BΘ, BK, ΘD and KE,
and drop perpendiculars GN and GX from G on to BD and BE,
perpendiculars DZ and EH from D and E on to GZ and GH,
and perpendiculars DL and EM from D and E on to AL and AM.
Then, since ∠ GBN = ∠ GBX [by hypothesis]
H261 and the angles at N and X are right
and line GB is common [to triangles GBN, GBX],
GN = GX
i.e. DZ = EH.[49]
And also ΘD = KE[50]
and the angles at Z and H are right.
So ∠ DΘZ = ∠ EKH.
And because [in triangles GΘB, GKB]
ΘG = GK (by hypothesis)
and GB is common
and ∠ ΘGB = ∠ KGB,
hence ∠ GΘB = ∠ GKB.
Therefore, by subtraction, ∠ BΘD = ∠ BKE,[51]
and base BD = base BE.[52]
But again [in triangles BAD, BAE]
BA is common
and ∠ DBA = ∠ EBA [by hypothesis].
So base AD = base AE
and ∠ ADB = ∠ AEB.
By the same reasoning [as before]
since DL = EM [epicycle radii]
and the angles at L and M are right,
∠ DAL = ∠ EAM.

Q.E.D.

[48] Reading τοῦ παρὰ τὴν ζῳδιακὴν ἀνωμαλίαν διαφόρου at H260.8. Heiberg, following the Greek mss., omits the last word, which was restored by Halma (followed by Manitius), apparently without authority. It was in fact read by Is.

[49] GZDN and GHEX are parallelograms.

[50] Although one can see that this must be so by symmetry, the proof is quite intricate. For the radii of the deferent in its two positions are not ΘD and KE, but KD and ΘE. Cf. Manitius p. 435.

[51] ∠ BΘD = 180° − (∠ DΘZ + ∠ GΘB). ∠ BKE = 180° − (∠ EKH + ∠ GKB).

[52] In the congruent triangles BΘD, BKE.

7. {*Demonstration of* [*the position of*] *the apogee of Mercury and of its displacement*}[53]

After establishing the above theory, we determined, first, in what part of the ecliptic Mercury's apogee lies, by the following method. H262

We sought out observations of greatest elongations in which the distance [of Mercury] as morning-star from the mean longitude of the sun (i.e. from the mean longitude of the planet) is equal to its distance as evening-star. For once we had found such a situation, it necessarily follows from our [above] demonstrations that the point on the ecliptic halfway between the two positions [of Mercury as morning-star and evening-star] occupies the apogee of the eccentre.

The observations which we used for this purpose are few in number, because precisely such combinations [of planet and sun positions] rarely occur; nevertheless they are sufficient to exhibit the desired result. The more recent of them are the following.

[1] In the sixteenth year of Hadrian, Phamenoth [VII] 16/17 in the Egyptian calendar [132 Feb. 2/3], in the evening, we observed Mercury, by means of the astrolabe instrument, at its greatest distance from the mean longitude of the sun. Also, from a sighting with respect to the bright star in the Hyades, it was seen then to occupy a longitude of ♓ 1°. At the time in question the sun's mean longitude was ♒ $9\frac{3}{4}$°. So the greatest elongation from the mean as evening-star comes out as $21\frac{1}{4}$°.[54]

[2] And, in the eighteenth year of Hadrian, Epiphi [XI] 18/19 in the Egyptian calendar [134 June 3/4], at dawn, Mercury [was observed] at greatest elongation, appearing very small and dim; from a sighting with respect to the bright star in the Hyades it was seen to occupy ♉ $18\frac{3}{4}$°.[55] Now at that time the mean sun was in ♊ 10°. Here too, then, the greatest elongation from the mean as morning-star was $21\frac{1}{4}$°, equal [to the elongation in [1]]. H263

So, since the mean position of the planet was ♒ $9\frac{3}{4}$° at one of the observations, and ♊ 10° at the other, and the point of the ecliptic halfway between these occupies ♈ $9\frac{7}{8}$°, the diameter through the apogee must lie in that position at that time.

[3] Again, in the first year of Antoninus, Epiphi [XI] 20/21 in the Egyptian calendar [138 June 4/5], in the evening, we observed Mercury by means of the astrolabe at its greatest distance from the sun's mean longitude. From a sighting

[53] See *HAMA* 159–61, Pedersen 309–312. An acute critique of the method employed by Ptolemy for determining the apsidal line of the inner planets was made by Sawyer, 'Ptolemy's Determination of the Apsidal Line for Venus'. He shows that mere equality of mean maximum morning and evening elongations is an insufficient criterion for positing symmetry to the apsidal line, although the observations Ptolemy actually chose are in fact (*grosso modo*) symmetric. For other criticisms see Wilson, 'Inner Planets', 225 ff.

[54] The star in question is α Tau, which has in the catalogue (XXIII 14) a longitude of ♉ $12\frac{2}{3}$°. In order to find the result he does, Ptolemy should have observed on the instrument a longitudinal difference of $71\frac{2}{3}$°, which is so large as to cast doubt on the validity of the observation. But, by using the same star as reference-point in both observations [1] and [2], Ptolemy may have thought that he was minimizing any error resulting from faulty determination of the star's ecliptic position.

[55] I.e. on this occasion the observed longitudinal difference was only $6\frac{1}{12}$°. (see n.54).

at that moment with respect to the star on the heart of Leo it was seen to occupy ♋ 7°.[56] But at the time in question the mean sun was in ♊ 10½°. Therefore the greatest elongation [of Mercury] as evening-star comes out as 26½°.

[4] Similarly, in the fourth year of Antoninus, Phamenoth [VII] 18/19 in the Egyptian calendar [141 Feb. 1/2], at dawn, [Mercury was observed], again, at greatest elongation: from a sighting with respect to the star called Antares it was seen to occupy ♑ 13½°,[57] while the mean sun was in ♒ 10°. Here too, then, the greatest elongation from the mean as morning-star was 26½°, equal [to the elongation in [3]].

So, since the mean position of the planet was ♊ 10½° at one of the observations and ♒ 10° at the other, and the point of the ecliptic halfway between them occupies ♎ 10¼°, the diameter through the apogee must lie in that position at that time.

From these observations, then, we find that the apogee falls at about 10° of Aries or Libra, whereas from the ancient observations made near the greatest elongations we find it at about 6° of the same signs, as can be calculated from the following kind [of data].

[5] In the 23rd year in Dionysius' calendar, Hydron 21,[58] at dawn, Stilbon[59] was 3 moons to the north of the brightest star in the tail of Capricorn. At that time the star in question had a position, according to [the coordinate system defined by] our origin, namely that beginning with the solstitial or equinoctial points, of ♑ 22⅓°.[60] Mercury, obviously, had the same longitude,
H265 and[61] the mean sun was in ♒ 18⅙°: for that moment was in the 486th year from Nabonassar, Choiak [IV] 17/18 in the Egyptian calendar [−261 Feb. 11/12], dawn. Therefore the greatest elongation from the mean [of Mercury] as morning-star was 25⅚°.

Now we did not find a greatest elongation from the mean as evening-star which was precisely equal to that, at least in the observations which have reached us: but we calculated the [position with] equal [elongation] by means of two observations which were very close [to the required situation], in the following manner.

[6] [Firstly], in the same 23rd year in Dionysius' calendar, Tauron 4,

[56] The star (Regulus, α Leo) has in the catalogue (XXVI 8) the longitude ♌ 2½°. Thus the observed difference should have been 34½°.

[57] The star (α Sco) has in the catalogue (XXIX 8) the longitude ♏ 12⅔°. Thus the observed longitudinal difference should have been the large one of 60⅚°.

[58] Reading κα´ (with D²G,Ar) at H264,18 for κθ´ (29). The correction was made by Böckh, following Lepsius, in his discussion of the calendar of Dionysius (*Sonnenkreise* 294–95), on which see Introduction pp. 13–14.

[59] Mercury. The names Φαίνων, Φαέθων, Πυρόεις, Φωσφόρος and Στίλβων for Saturn, Jupiter, Mars, Venus and Mercury are found in Hellenistic texts (and occasionally later, as an archaism). An excellent discussion of the evidence for their use and the reason for their introduction (the nomenclature used by Ptolemy, 'star of Kronos [Saturn]', etc. is undoubtedly earlier) is given by Cumont, 'Les noms des planètes'. The occurrences in the *Almagest* (here and at H288,11, both connected with Dionysius, i.e. earlier third century B.C.) are the earliest dated examples of the nomenclature.

[60] The star in question is identified by Ptolemy with no. XXXI 24 in his catalogue (δ Cap). The longitude there is ♑ 26⅓°, from which he subtracts 4° to account for precession in the intervening 398 or so years. A 'moon', as measurement, is about half a degree.

[61] Reading δὲ (with D,Ar) at H264,24 for δηλονότι ('and, obviously'). The position of the mean sun is not obvious, but has to be computed.

in the evening, [Mercury] was 3 moons behind [i.e. to the rear of] the straight line through the horns of Taurus, and it seemed as if it was going to be more than 3 moons to the south of that one common [to Auriga and Taurus] when it passed by it.[62] Thus its position according to our coordinates was ♉ 23⅔°. That moment was again in the 486th year from Nabonassar, [Mechir [VI]] 30/ Phamenoth [VII] 1[63] in the Egyptian calendar [−261 Apr. 25/26], evening, at which time the longitude of the mean sun was ♈ 29½°. So the greatest elongation from the mean as evening-star was 24⅙°.

[7] [Secondly], in the 28th year in Dionysius' calendar, Didymon 7, in the evening, [Mercury] was practically on a straight line with [the stars in] the heads of Gemini, and lay to the south of the southern one by ⅓ of a moon less than twice the distance between [the stars in] the heads.[64] Thus at that time, according to our coordinates, Mercury was in ♊ 29⅓°. This moment is in the H266
491st year from Nabonassar, Pharmouthi [VIII] 5/6 in the Egyptian calendar

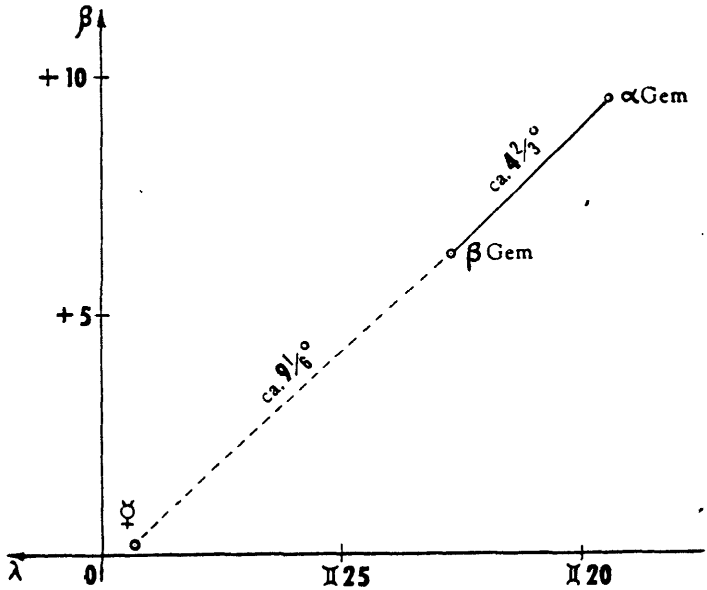

Fig. O

[62] The stars in question are, in the catalogue, XXIII 19 and 21 (ζ and β Tau). The latter is also counted as Auriga [XII] no. 11. Subtracting 4° from the catalogue longitudes for precession, we get the coordinates at the observation as: southern horn, λ ♉ 23⅔°, β −2½°; northern horn, λ ♉ 21⅔°, β +5°. Ptolemy concludes that the longitude of Mercury was the same as that of the southern horn.

[63] There is no doubt that this is what is intended. The Greek mss. have, at H265,16, Φαμενὼθ λ΄ εἰς τὴν α΄, which seems hardly possible. Petavius, followed by Ideler and Böckh, emended to Μεχὶρ λ′ εἰς τὴν α′ Φαμενὼθ; Halma, followed by Manitius, to λ′ εἰς τὴν α′ Φαμενὼθ. The Arabic translations suggest that one must read Φαμενὼθ εἰς τὴν ά, i.e. simply excise. λ΄. For the expression cf. p. 456 n.84.

[64] These are, in the catalogue, XXIV 1 and 2 (α and β Gem), with coordinates (corrected for precession): northern head, λ ♊ 19⅓°, β +9½°; southern head, λ ♊ 22⅔°, β +6¼°. See Fig. O, which shows that Mercury's 'distance to the south' is measured along the line between the stars.

[−256 May 28/29], evening, at which time the longitude of the mean sun was ♊ $2\frac{5}{6}$°. Thus this [greatest] elongation was $26\frac{1}{2}$°.

Now, when the mean position was in ♈ $29\frac{1}{2}$°, the greatest elongation was $24\frac{1}{6}$°, and when the mean position was in ♊ $2\frac{5}{6}$°, the [greatest] elongation was $26\frac{1}{2}$°; and the [greatest elongation] as morning-star, to which we were seeking the corresponding [greatest elongation as evening-star], was $25\frac{5}{6}$°. So we derived the location of the mean position for a [greatest] evening elongation of $25\frac{5}{6}$° from the difference between the above two observations: the difference between the mean positions at the two observations is $33\frac{1}{3}$°, and the difference between the greatest elongations $2\frac{1}{3}$°. Thus to $1\frac{2}{3}$° (which is the amount by which $25\frac{5}{6}$° exceeds $24\frac{1}{6}$°) correspond approximately 24°.[65] If we add this amount to ♈ $29\frac{1}{2}$°, we shall get the mean position at which the greatest evening elongation is calculated to be equal to the greatest morning elongation of $25\frac{5}{6}$°:
H267 this point is ♉ $23\frac{1}{2}$°. And the point halfway between ♒ $18\frac{1}{6}$° and ♉ $23\frac{1}{2}$° is at ♈ $5\frac{5}{6}$°.

[8] Again, in the 24th year in Dionysius' calendar, Leonton 28, in the evening, [Mercury] was a little more than 3° in advance of Spica, according to Hipparchus' reckoning.[66] Thus at that moment its longitude according to our coordinates was ♍ $19\frac{1}{2}$°. That moment is in the 486th year from Nabonassar, Payni [X] 30 in the Egyptian calendar [−261 Aug. 23], evening, at which time the longitude of the mean sun was ♌ $27\frac{5}{6}$°. Therefore the greatest elongation from the mean as evening-star was $21\frac{2}{3}$°. We again calculated [the position of] the morning elongation precisely corresponding to that from two of the available [observations].

[9] In the 75th year in the Chaldaean calendar,[67] Dios 14, at dawn, [Mercury] was half a cubit [ca. 1°] above [the star on] the southern scale [of Libra]. Thus at that time it was in ♎ $14\frac{1}{6}$°, according to our coordinates.[68] This moment is in the 512th year from Nabonassar, Thoth [I] 9/10 in the Egyptian calendar [−236 Oct. 29/30], dawn, at which time the longitude of the mean sun was ♏ $5\frac{1}{6}$°. Therefore the greatest morning elongation was 21°.

H268 [10] In the 67th year in the Chaldaean calendar, Apellaios 5, at dawn, [Mercury] was a half a cubit [ca. 1°] above the northern [star in the] forehead of Scorpius. Thus at that time it was in ♏ $2\frac{1}{3}$°, according to our coordinates.[69] This moment is in the 504th year from Nabonassar, Thoth [I] 27/28 in the Egyptian calendar [−244 Nov. 18/19], dawn, at which time the

[65] This is a crudely rounded result. In fact $33\frac{1}{3} \times 1\frac{2}{3} / 2\frac{1}{3} \approx 23;49°$, so a reasonable approximation would have been $23\frac{5}{6}$. However, linear interpolation is itself a crude procedure here.

[66] This is proof that this observation (?by Dionysius) was one of those which Hipparchus 'arranged in a more useful way' (see IX 2 p. 421, with n.11), and it is a plausible surmise that all of these Mercury observations were derived by Ptolemy from that compilation. The longitude of Spica (catalogue XXVII 14) was, according to Ptolemy, ♍ $22\frac{2}{3}$° in Dionysius' time; thus he takes Mercury as being $3\frac{1}{6}$° in advance of Spica.

[67] The Seleucid era. See Introduction p. 13.

[68] The star is catalogue XXVIII 1 (α Lib, there said to be on the 'southern claw') to which Ptolemy assigns the longitude ♎ 18° in his own time. Here, then, he has subtracted $3\frac{5}{6}$° to account for the precession in 373 years (one would have expected $3\frac{3}{4}$°).

[69] The star is catalogue XXIX 1. Its longitude there is ♏ $6\frac{1}{3}$°, so Ptolemy has subtracted 4° for the precession in 381 years, again more than one would have expected.

longitude of the mean sun was ♏ $24\frac{5}{6}°$. Therefore this [greatest morning] elongation was $22\frac{1}{2}°$.[70]

In these two observations again, then, since the difference between the two mean positions is $19\frac{2}{3}°$, and the difference between the greatest elongations is $1\frac{1}{2}°$, it follows that to $\frac{2}{3}°$ (which is the amount by which the $21\frac{2}{3}°$ of the required elongation exceeds the 21° of the lesser [of these two]) corresponds about 9°.[71] If we add the latter to ♏ $5\frac{1}{6}°$, we get the mean position at which the greatest morning elongation becomes equal to the greatest evening elongation of $21\frac{2}{3}°$: this point is ♏ $14\frac{1}{6}°$. And the point halfway between ♌ $27\frac{5}{6}°$ and ♏ $14\frac{1}{6}°$ is, H269
again, about ♎ 6°.[72]

From the above, and also because the phenomena associated with the other planets individually fit [the assumption], we find it consistent [with the facts to assume] that the diameters through the apogees and perigees of the five planets shift about the centre of the ecliptic towards the rear through the signs, and that this shift has the same speed as that of the sphere of the fixed stars. For the latter moves about 1° in 100 years, as we demonstrated [p. 328]; and here too the interval from the ancient observations, in which[73] the apogee of Mercury was in about the 6th degree [of the signs in question],[74] to the time of our observations, during which it has moved about 4° (since it [now] occupies the 10th degree), is found to comprise approximately 400 years.

8. {*That the planet Mercury, too, comes closest to the earth twice in one revolution*}[75]

In accordance with the above we investigated the size of the greatest elongations which occur when the mean longitude of the sun is exactly in the apogee, and again, when it is diametrically opposite that point. We cannot H270
derive this from the ancient observations, but we can do so from our own observations made with the astrolabe. For it is in this situation that one can best appreciate the usefulness of this way of making observations, since, even if those stars with previously determined positions which are visible are not near the planet being observed (which is generally the case with Mercury, since, for the majority of the fixed stars, it is rare that they are visible when they are [only] as

[70] Observations [9] and [10] are proven to be Babylonian by several marks: use of the Seleucid era (called by Ptolemy 'according to the Chaldaeans'); the use of the 'cubit' as an astronomical measurement; and also the fact that both the stars used as markers belong to the small group used in Babylonian texts for precisely this purpose and known as 'normal stars' (see *HAMA* 545; Sachs [1] 46).

[71] This linear interpolation, like the earlier one (see p. 452 n.65) is inaccurate. $8\frac{1}{4}°$ would be much more reasonable.

[72] On this occasion the half-way point is at precisely 6°.

[73] One would expect, at H269,12, καθ' ἅς, referring to τηρήσεων, rather than καθ' ὅν, referring to χρόνον, since the latter means 'interval'. But apparently, since χρόνος can also mean 'epoch', Ptolemy has somewhat illogically assimilated the relative pronoun to it (cf. τὸν [sc. χρόνον] in the next line, where it certainly means 'epoch').

[74] It has not yet been decided whether the apogee lies in Aries or Libra.

[75] See *HAMA* 161, Pedersen 314–15. 'too' refers to the moon (picking up Ptolemy's remark IX 5 p. 443). On the term περιγειότατος as applied to Mercury see p. 461 n.94.

far from the sun as Mercury is),[76] one can still determine positions of the planet in question accurately in latitude and longitude, by sighting stars which are at a considerable distance.

[Firstly] then, in the nineteenth year of Hadrian, Athyr [III] 14/15 in the Egyptian calendar [134 Oct. 2/3], at dawn, Mercury, which was around its greatest elongation, was sighted with respect to the star on the heart of Leo, and was seen to have a longitude of ♍ $20\frac{1}{5}$°.[77] The mean sun was at about ♎ $9\frac{1}{4}$°, so the greatest elongation was $19\frac{1}{20}$°.

[Secondly], in the same year, Pachon [IX] 19 [135 Apr. 5], in the evening, [Mercury], which was again around its greatest elongation, was sighted with respect to the bright star in the Hyades, and was seen to have a longitude of ♉
H271 $4\frac{1}{3}$°.[78] The mean sun had a longitude of ♈ $11\frac{1}{12}$°. Hence in this case one calculates the greatest elongation as $23\frac{1}{4}$°, and it is immediately obvious that the apogee of the eccentre is in Libra and not in Aries.

With these data, let [Fig. 9.5][79] the diameter through the apogee be ABG. Let B be taken as the centre of the ecliptic, at which the observer is, A as the point at ♎ 10°, and G as the point at ♈ 10°. Describe equal epicycles with points D and

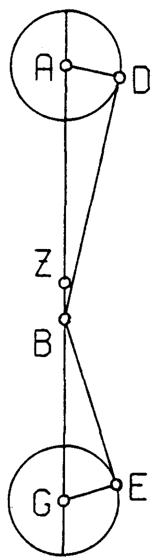

Fig. 9.5

[76] Since Mercury's maximum elongation from the sun is never much more than 20°, it is only visible for a short time after sunset or before dawn, when the sky in its region is too illuminated for any but very bright stars to be visible. The 'ancient observations' (i.e. those by Babylonians or earlier Greeks) were made by giving the position with respect to nearby stars; but in some regions of the ecliptic there is a scarcity of bright stars.

[77] The star had a longitude of ♌ $2\frac{1}{2}$° according to Ptolemy's catalogue (XXVI 8), so the observed interval was 47;42°.

[78] The star had a longitude of ♉ $12\frac{2}{3}$°, according to the catalogue (XXIII 14), so the observed interval was only $8\frac{1}{3}$°.

[79] Heiberg has made an error in the figure on p. 271: Z is on the wrong side of B. Corrected by Manitius.

E [on their circumferences] about A and G [respectively], and draw from B tangents to them, BD and BE. Drop perpendiculars AD and GE from the centres to the points of tangency.

Now since the greatest elongation from the mean as morning-star in Libra was observed as $19\frac{1}{20}°$,

$$\angle \text{ABD} = \begin{cases} 19;3° & \text{where 4 right angles} = 360° \\ 38;6^{\circ\circ} & \text{where 2 right angles} = 360^{\circ\circ}. \end{cases}$$

Therefore in the circle about right-angled triangle ABD

arc AD = 38;6°

and its chord, AD ≈ 39;9^p where hypotenuse AB = 120^p. H272

Again, since the greatest elongation from the mean as evening-star in Aries was observed as $23\frac{1}{4}°$,

$$\angle \text{GBE} = \begin{cases} 23;15° & \text{where 4 right angles} = 360° \\ 46;30^{\circ\circ} & \text{where 2 right angles} = 360^{\circ\circ}. \end{cases}$$

Therefore in the circle about right-angled triangle GBE

arc GE = 46;30°

and its chord, GE = 47;22^p where hypotenuse BG = 120^p.

Therefore where GE = 39;9^p and AB = 120^p

(for AD = GE, radii of the epicycle),

BG = 99;9^p

and, by addition [of AB to BG], ABG = 219;9^p.

So if it is bisected at point Z,

its half, AZ = 109;34^p } in the same units.

and the distance between points B and Z = 10;25^p }

Now it is clear that either point Z is the centre of the eccentre on which the centre of the epicycle is always located, or else the centre of that [eccentre] moves about point Z. For those are the only conditions under which the centre of the epicycle could be equidistant from Z at both the above diametrically opposite situations, as demonstrated. But if Z were the actual centre of the eccentre on which the epicycle centre is always located, that eccentre would be stationary, and the situation in Aries would be the closest to the earth of all H273 situations [i.e. the perigee], since BG is the shortest of [all] lines drawn from B to the circle described on centre Z.[80] However, we find that the situation in Aries is not the closest to the earth of all, but the situations in Gemini and Aquarius are even closer to the earth than that, and approximately equal to each other. Hence it is clear that the centre of the eccentre in question rotates about point Z, in the opposite sense to the revolution of the epicycle (i.e. in advance with respect to the signs), it too making one rotation in one revolution [of the epicycle]. For if this is so the epicycle centre will be closest to the earth twice [in one revolution] on the eccentre.

As for the fact that the epicycle is closer to the earth in Gemini and Aquarius than in the [above] situation in Aries, this is easily seen to be an immediate consequence of the observations already detailed. For in the observation of the 16th year of Hadrian, Phamenoth 16 [p. 449 no. 1], the greatest elongation from the mean as evening-star was $21\frac{1}{4}°$, and in the observation of the 4th year of

[80] Euclid III 7.

Antoninus, Phamenoth 19[81] [p. 450 no. 4], the greatest elongation from the mean as morning-star was $26\frac{1}{2}°$, while in both observations the mean sun was near ♒ 10°. Again, in the observation of the 18th year of Hadrian, Epiphi 19 [p. 449 no.2], the greatest elongation from the mean as morning-star was $21\frac{1}{4}°$, and
H274 in the observation of the 1st year of Antoninus, Epiphi 20 [p. 449 no. 3], the greatest elongation from the mean as evening-star was $26\frac{1}{2}°$, the mean sun in both these observations being near ♊ 10°. Thus both in Aquarius and in Gemini the sum of the opposite greatest elongations comes to $47\frac{3}{4}°$, while the sum of the two [greatest] elongations in Aries is [only] $46\frac{1}{2}°$, since the evening elongation (which is equal to the morning elongation) was observed as $23\frac{1}{4}°$.

9. {*On the ratio and the amount of the anomalies of Mercury*}[82]

Having completed the above preliminary investigation, we have still to demonstrate the position of the point on line AB about which takes place the annual revolution of the epicycle in uniform motion towards the rear with respect to the signs, and the distance from Z of the centre of that eccentre which performs its revolution in advance in the same period [as the epicycle]. For this investigation we used two observations of greatest elongations, one as morning-star and one as evening-star, but in both of which the mean position was a quadrant from the apogee on the same side: that is the situation in which,
H275 approximately, the greatest equation of ecliptic anomaly occurs.

[1] In the fourteenth year of Hadrian, Mesore [XII] 18 in the Egyptian calendar [130 July 4], in the evening, as we found in the observations we got from Theon,[83] he says that [Mercury] was at its greatest distance from the sun, $3\frac{5}{6}°$ behind [i.e. to the rear of] the star on the heart of Leo. Thus, according to our coordinates, its longitude was about ♌ $6\frac{1}{3}°$, while the longitude of the mean sun at that moment was about ♋ $10\frac{1}{12}°$. Thus the greatest evening elongation was $26\frac{1}{4}°$.

[2] In the second year of Antoninus, Mesore [XII] [20]/21[84] in the Egyptian calendar [139 July 4/5], at dawn, we observed its greatest distance by means of the astrolabe: sighting it with respect to the bright star in the Hyades, we found its longitude as ♊ $20\frac{1}{12}°$. The mean sun was, again, near ♋ $10\frac{1}{3}°$. Thus the greatest morning elongation was $20\frac{1}{4}°$.

With the above as data, let [Fig. 9.6] the diameter through ♎ 10° and ♈ 10° again be AZBG, and, as in the previous figure [9.5], let A be taken as the point

[81] Reading ιθ′ (with D, Ar) for ιη′ (18) at H273,19. Ptolemy gives a double date (18/19) in the passage in question. Since the observation was taken at dawn, the second date is preferable, and agrees with the practice just below (Epiphi 19, for the earlier 18/19 at dawn).

[82] *HAMA* 161–2, Pedersen, 318–19.

[83] Other observations by this man are used by Ptolemy in X 1 and X 2. There (p. 469) he says that they were 'given to us by the mathematician Theon', implying personal contact. He has often been identified with Theon of Smyrna. This is chronologically possible, but given the frequency of the name, especially in Roman Egypt, the identification is highly uncertain.

[84] Reading Μεσορὴ εἰς τὴν κα′ (with D,Ar) for Μεσορὴ εἰς τὴν κδ′ (24th) at H275,13. The date is determined by the longitude of the mean sun (computed for Nabonassar 886 XII 20/21, 6 a.m., as 100;19°). Neugebauer (*HAMA* 162 n.3) suggests reading Μεσορὴ ⟨κ′⟩ εἰς τὴν κα′, but for the above form cf. p. 451 n.63.

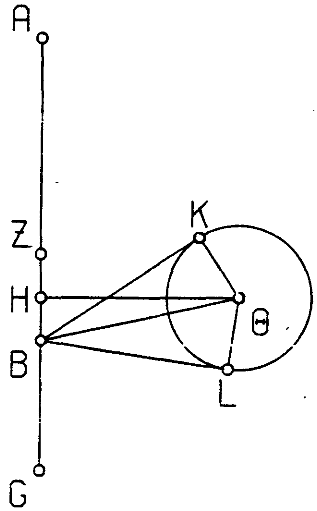

Fig. 9.6

at which the epicycle centre is found when its longitude is ♎ 10°, G as the point at which it is found when its longitude is ♈ 10°, B as the centre of the ecliptic, and Z as the point about which the centre of the eccentre rotates in advance. H276

Let the first problem be to find the distance from point B of the centre about which we say the uniform motion of the epicycle towards the rear takes place.

Let that centre be H, and draw a straight line through H at right angles to AG, so that its [angular] distance from the apogee is a quadrant. On this line take Θ, the centre of the epicycle at the above observations (for at those observations the mean longitude of the sun was a quadrant from the apogee, since it was near ♋ 10°). Draw the epicycle KL on centre Θ, and draw the tangents to it from B, BK and BL. Join ΘK, ΘL and BΘ.

Then, since at the mean position in question the greatest morning elongation from the mean is given as $20\frac{1}{4}°$, and the greatest evening elongation as $26\frac{1}{4}°$,

$\angle$ KBL = [$20\frac{1}{4}° + 26\frac{1}{4}°$ =] 46;30° where 4 right angles = 360°. H277

Therefore its half, $\angle$ KBΘ = 46;30°° where 2 right angles = 360°°.[85]

Therefore in the circle about right-angled triangle BΘK

arc ΘK = 46;30°

and its chord, ΘK = 47;22^p where hypotenuse BΘ = 120^p.

Therefore where ΘK, the radius of the epicycle, is 39;9^p

and, as was shown, BZ = 10;25^p,

BΘ = 99;9^p.

Again, the difference between the above greatest elongations, 6°, comprises twice the equation of the ecliptic anomaly; and the latter is represented by $\angle$ BΘH, as we proved previously.[86]

[85] Note that this is exactly equal to $\angle$ GBE in IX 8 (p. 455), which implies that the distance of the epicycle from the observer is the same at quadrature (here) and at 180° from apogee (there).

[86] IX 6 p. 448. But it is assumed rather than 'proven' there.

$$\text{Therefore } \angle\, B\Theta H = \begin{cases} 3^\circ & \text{where 4 right angles} = 360^\circ \\ 6^{\circ\circ} & \text{where 2 right angles} = 360^{\circ\circ}. \end{cases}$$

Therefore in the circle about right-angled triangle BHΘ

$$\text{arc } BH = 6^\circ$$

$$\text{and } BH = 6;17^p \text{ where hypotenuse } B\Theta = 120^p.$$

Therefore where $B\Theta = 99;9^p$, and likewise $BZ = 10;25^p$,

$$BH = 5;12^p.$$

Therefore BH is approximately half BZ,

H278 and $BH \approx HZ \approx 5;12^p$, where the radius of the epicycle is $39;9^p$.

Again, in the same figure [Fig. 9.7], draw line ZMN through Z at right angles to AG, but on the opposite side to HΘ. Because lines HΘ and ZN perform their returns to the same point in the same period, but in opposite senses, the centre of

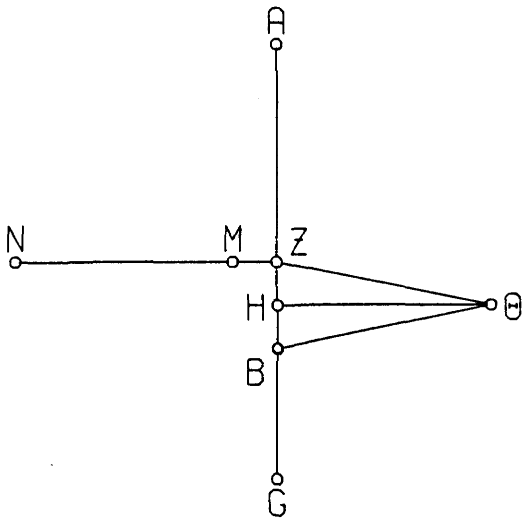

Fig. 9.7

that eccentre on which the epicycle centre Θ is located will, obviously, lie on ZMN at that moment. Let ZN be equal to ZA: thus ZN, like AZ, is the sum of the radius of the eccentre and the distance between the centres ([i.e.] between the centre of the eccentre and point Z). Take on ZN the centre of the eccentre, M,
H279 and join ZΘ.

Now ∠ MZH is right, and ∠ ΘZH is practically a right angle (hence NZΘ, too, is practically a straight line);[87]

and it has been demonstrated that where the epicycle radius is $39;9^p$

$$NZ = AZ = 109;34^p$$

$$\text{and } Z\Theta = B\Theta = 99{:}9^p.^{88}$$

[87] This simplification is necessary in order to solve the problem at all: for one does not know *a priori* where on ZM the point M lies, only that it lies on a circle with center Z.

[88] See p. 455.

Therefore, by addition, NZΘ = 208;43^p
and its half, NM, the radius of the eccentre, is about 104;22^p,
and by subtraction [of NM from NZ],
ZM, the distance between the centres, is 5;12^p.
But we showed that both BH and HZ were the same amount, 5;12^p.
Thus we have computed that
where the radius of the eccentre is 104;22^p
each of the distances between the centres [BH, HZ, ZM] is 5;12^p
and the radius of the epicycle is 39;9^p.
Therefore where the radius of the eccentre is 60^p,
each of the distances between the centres is 3;0^p
and the radius of the epicycle is 22;30^p.

Q.E.D.

With the above [elements] given, the [computed] greatest elongations at the points closest to the earth are in agreement with those observed (i.e. when the mean position is at ♒ 10° or ♊ 10°, and [thus] its distance from the apogee is the side of the [inscribed] triangle [i.e. 120°], the angle subtended by the epicycle at the eye is about 47¾°), as we can deduce by the following. H280

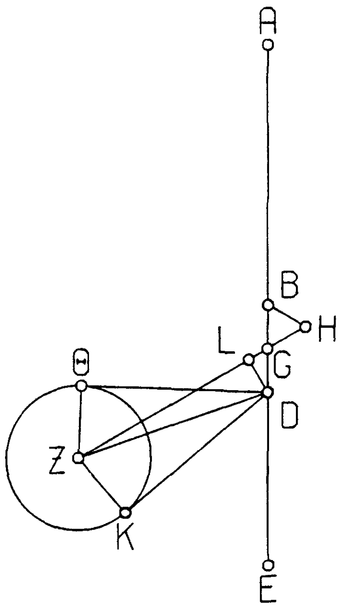

Fig. 9.8

Let [Fig. 9.8] the diameter through the apogee be ABGDE, on which point A is taken as the apogee, B as the point about which the centre of the eccentre performs its motion in advance, G as the point about which the epicycle centre performs its [uniform] motion towards the rear, and D as the centre of the ecliptic. Let each of the [above] motions have gone through the side of the [inscribed] triangle [i.e. 120°] (performed uniformly and with equal speed

about its own centre) from the apogee A on opposite sides of it. Let the straight
H281 line rotating the epicycle be GZ, and that rotating the centre of the eccentre be BH, and let the centre of the eccentre be H and the centre of the epicycle, Z. With the latter as centre describe the epicycle, draw tangents to the epicycle, DΘ and DK, join GH, DZ, ZΘ and ZK, and drop perpendicular DL from D on to GZ.

We have to show that

$$\angle \Theta DK = 47\tfrac{3}{4}^\circ \text{ where 4 right angles} = 360^\circ.$$

Now both ∠ ABH and ∠ AGL subtend the side of the [inscribed] triangle and are equal to 120° where 2 right angles = 180°;

$$\text{so } \angle GBH = \angle DGL = 60^\circ;$$

$$\text{and } \angle BHG = \angle BGH \text{ (BG = BH, by hypothesis).}$$

$$\text{But } \angle BHG + \angle BGH = 120^\circ \text{ (supplement [to } \angle GBH = 60^\circ\text{]).}$$

$$\therefore \angle BHG = \angle BGH = 60^\circ.$$

So triangle BGH is equiangular and equilateral.

$$\text{And } \angle DGL = \angle BGH.$$

So points H, G and Z lie on a straight line.

Hence HZ, the radius of the eccentre $= 60^p$ where GH (which equals GD) $= 3^p$, the distance between the centres. Therefore, by subtraction [of GH from HZ], GZ $= 57^p$ in the same units.

Again, since

H282

$$\angle DGL = \begin{cases} 60^\circ & \text{where 4 right angles} = 360^\circ \\ 120^{\circ\circ} & \text{where 2 right angles} = 360^{\circ\circ}, \end{cases}$$

in the circle about right-angle triangle GDL

$$\text{arc } DL = 120^\circ$$

$$\text{and arc } GL = 60^\circ \text{ (supplement).}$$

Therefore the corresponding chords

$$\left.\begin{aligned} DL &= 103;55^p \\ \text{and } GL &= 60^p \end{aligned}\right\} \text{where hypotenuse } GD = 120^p.$$

$$\text{Therefore where } DG = 3^p \text{ and } GZ = 57^p$$

$$DL = 2;36^p$$

$$\text{and } GL = 1;30^p;$$

and, by subtraction [of GL from GZ], LZ $= 55;30^p$.

$$\text{And since } LZ^2 + DL^2 = DZ^2,$$

$$DZ = 55;34^{p}\text{[89]}$$

where the radius of the epicycle (i.e. ZΘ and ZK) $= 22;30^p$, by hypothesis.

$$\text{Therefore where hypotenuse } DZ = 120^p$$

$$\Theta Z = ZK = 48;35^p;$$

$$\text{and } \angle ZD\Theta = \angle ZDK = 47;46^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ}.$$

Therefore, by addition [of ∠ ZDΘ to ∠ ZDK], ∠ ΘDK $= 47;46^\circ$ where 4 right angles $= 360^\circ$.

Q.E.D.

[89] This is, according to Ptolemy, the least distance of the center of Mercury's epicycle (cf. XI 10 p. 546). It was shown by Hartner, 'Mercury Horoscope' 109–17 (cf. Pedersen 321–4) that, with the parameters of Ptolemy's model, the least distance actually occurs at about $120\tfrac{1}{2}^\circ$ from apogee, and is less than 55;34 (about 55;33,38). These differences are utterly negligible for practical purposes.

10. {*On the correction of the periodic motions of Mercury*}[90] H283

The sequel to the above is the establishment of the periodic motions of Mercury and their epochs.[91] Now the [motion and epoch] in longitude, that is, of the epicycle in its uniform motion about point G, are given immediately from those of the sun. As for the [motion and epoch] in anomaly, that is, of the planet in its [uniform] motion on the epicycle about the epicycle centre, we have derived it from two reliable observations, one from among those recorded in our time, and the other from the ancient observations.

[Firstly], we observed the planet Mercury in the second year of Antoninus (which was the 886th year from Nabonassar), Epiphi [XI] 2/3 in the Egyptian calendar [139 May 17/18], by means of the astrolabe instrument. It had not yet reached its greatest elongation as evening-star. When sighted with respect to the star on the heart of Leo it was observed at a longitude of ♊ $17\frac{1}{2}$°; and at that moment it was also $1\frac{1}{6}$° to the rear of the moon's centre. The time at Alexandria was $4\frac{1}{2}$ equinoctial hours before midnight of [Epiphi 2/]3,[92] since, according to the astrolabe, the 12th degree of Virgo [i.e. ♍ 11°–12°] was culminating, while the sun was in about ♉ 23°. Now at that moment, the positions according to the hypotheses we have demonstrated were as follows:[93] H284

mean longitude of the sun	♉ 22;34°
mean longitude of the moon	♊ 12;14°
anomaly of the moon from the apogee of the epicycle	281;20°
hence, by computation, true position of the moon's centre	♊ 17;10°
apparent position of the moon's centre	♊ 16;20°.

Thus from this [computation] too we find that Mercury's longitude was ♊ $17\frac{1}{2}$° (since it was $1\frac{1}{6}$° to the rear of the moon's centre).

With this as datum, let [Fig. 9.9] the diameter through the apogee and perigee[94] be ABGDE, on which point A is taken as the apogee, B as the point about which the centre of the eccentre performs its motion in advance, G the point about which the centre of the epicycle performs its [uniform] motion towards the rear, and D the centre of the ecliptic. Let the epicycle centre, Z, have been carried by the line GZ about point G through the angle AGZ, and let H285
the centre of the eccentre, H, have been carried by line BH about point B through the angle ABH, which will, obviously, be equal to ∠ AGZ because of the equal speed of the motions. Draw the epicycle, ΘKL, on centre Z, and let the planet be situated at L. Join GH, HZ, DZ, ZL and DL, extend GZΘ and drop perpendiculars HM and DN on to it from H and D, and drop perpendicular ZX from Z on to DL.

[90] See *HAMA* 165–8.

[91] Reading αὐτῶν (with D,L) for αὐτοῦ ('its epochs') at H283,4.

[92] Literally 'of the midnight towards the 3rd'.

[93] These positions are computed for 7;7 p.m. Alexandria, i.e. Ptolemy has applied the equation of time (I find −25 mins. with respect to era Nabonassar). For this moment the computations are accurate (I find a longitudinal parallax of −53′ where Ptolemy applies −50′).

[94] 'perigee' (τὸ περίγειον) here and at H285,12 and 14 is taken, somewhat loosely, as the point 180° from the apogee, and *not* the point where Mercury's center is closest to the earth. For the latter Ptolemy always uses the superlative form τὸ περιγειότατον (H273,11, *al.*)

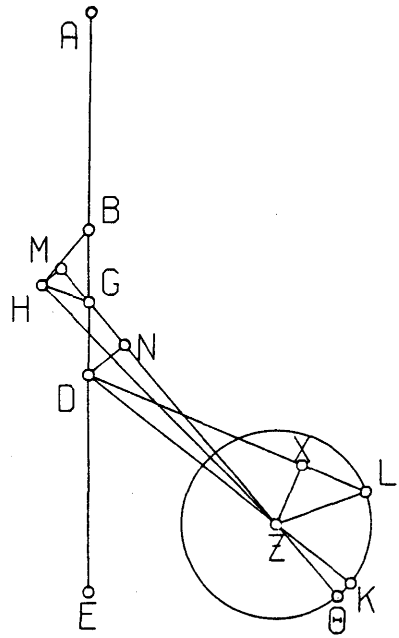

Fig. 9.9

Let us consider the problem, to find the arc of the epicycle between Θ, the apogee [of the epicycle], and the planet at L.

Now at that moment the longitude of the mean sun was ♉ 22;34°, and the perigee of the planet was at about ♈ 10°.[95]

Thus its distance from the perigee in mean longitude was 42;34°.

$$\therefore \angle\, \text{GBH} = \begin{cases} 42;34^\circ & \text{where 4 right angles} = 360^\circ \\ 85;8^{\circ\circ} & \text{where 2 right angles} = 360^{\circ\circ}. \end{cases}$$

And since BG always equals BH

$$\angle\, \text{BHG} = \angle\, \text{BGH} = 137;26^{\circ\circ} \text{ in the same units.}$$

So, in the circle about triangle BGH[96]

$$\text{arc HG} = 85;8^\circ$$
$$\text{and arc BG} = 137;26^\circ.$$

Therefore the corresponding chords

H286 $$\left.\begin{aligned} \text{GH} &= \ \ 81;10^p \\ \text{and BG} &= 111;49^p \end{aligned}\right\} \text{where the diameter of the circle is } 120^p.$$

Therefore where BG $= 3^p$, GH $= 2;11^p$.

$$\left.\begin{aligned} \text{Again, since } \angle\, \text{BGH} &= 137;26^{\circ\circ} \\ \text{and } \angle\, \text{BGM} &= \ \ 85;8^{\circ\circ} \end{aligned}\right\} \text{where 2 right angles} = 360^{\circ\circ},$$

by subtraction, $\angle$ HGM $= 52;18^{\circ\circ}$ in the same units.

Therefore in the circle about right-angled triangle GHM

[95] Cf. IX 7 p. 450 and IX 8 p. 454.

[96] This is one of the rare cases where Ptolemy applies the equivalent of the sine theorem in a triangle which is not right-angled. See Introduction p. 7 n.10.

arc $HM = 52;18^{\circ}$
and arc $GM = 127;42^{\circ}$ (supplement).
Therefore the corresponding chords

$$\left.\begin{array}{r} HM = \ 52;53^p \\ \text{and } GM = 107;43^p \end{array}\right\}\text{ where hypotenuse } GH = 120^p.$$

Therefore where $GH = 2;11^p$,
and HZ, the radius of the eccentre carrying the epicycle, is 60^p,
$HM = 0;58^p$
and $GM = 1;58^p$.
Hence MZ, being a negligible amount less than HZ, the hypotenuse [of triangle HMZ], is the same, 60^p,
and, by subtraction [of GM from MZ], $GZ = 58;2^p$.
Similarly, since $\angle DGN = 85;8^{\circ\circ}$ where 2 right angles $= 360^{\circ\circ}$,
in the circle about right-angled triangle GDN
arc $DN = 85;8^{\circ}$
and arc $GN = 94;52^{\circ}$ (supplement).
Therefore the corresponding chords

$$\left.\begin{array}{r} DN = 81;10^p \\ \text{and } GN = 88;23^p \end{array}\right\}\text{ where hypotenuse } GD = 120^p.$$

Therefore where $GD = 3^p$ and, as was demonstrated, $GZ = 58;2^p$,
$DN = 2;2^p$
and $GN = 2;13^p$; H287
and, by subtraction [of GN from GZ], $NZ = 55;49^p$.
Hence hypotenuse DZ $[= \sqrt{DN^2 + NZ^2}] \approx 55;51^p$
where the radius of the epicycle $= 22;30^p$.
Therefore in the circle about right-angled triangle DZN,
where hypotenuse $DZ = 120^p$,
$DN = 4;22^p$
and arc $DN = 4;11^{\circ}$.
$\therefore \angle DZN = 4;11^{\circ\circ}$ where 2 right angles $= 360^{\circ\circ}$,
and, by addition [of $\angle$ DZN and $\angle$ DGN], $\angle EDZ = 89;19^{\circ\circ}$.
And the whole angle $EDL = 135^{\circ\circ}$ in the same units, since the planet was observed at $67;30^{\circ}$ from the perigee.
Therefore by subtraction [of $\angle$ EDZ from $\angle$ EDL], $\angle ZDL = 45;41^{\circ\circ}$.
Therefore in the circle about right-angled triangle DZX,
arc $ZX = 45;41^{\circ}$
and $ZX = 46;35^p$ where hypotenuse $DZ = 120^p$.
Therefore where hypotenuse $DZ = 55;51^p$ and the radius of the epicycle, $ZL = 22;30^p$,
$ZX = 21;41^p$.
And, in the circle about right-angled triangle ZLX,
where hypotenuse $ZL = 120^p$,
$ZX = 115;39^p$.
$\therefore$ arc $ZX = 149;2^{\circ}$[97]

[97] The arc corresponding to $115;39^p$ is in fact $149;3^{\circ}$. But if one takes the chord as 115,38,40 (which is an accurate transformation of $46;35 \times 55;51/120$), one finds as arc $149;1,56^{\circ}$. As often, Ptolemy computes with more accuracy than he displays.

H288 and ∠ ZLX = 149;2°° where 2 right angles = 360°°.
But we showed that ∠ ZDL = 45;41°° in the same units.

[∴ ∠ LZK = ∠ ZLX + ∠ ZDL = 194;43°°.]

And ∠ ΘZK [= ∠ DZN] = 4;11°° likewise.
Therefore, by addition [of ∠ ΘZK + ∠ LZK],

$$\angle\,\Theta ZL = \begin{cases} 198;54^{\circ\circ} & \text{where 2 right angles} = 360^{\circ\circ} \\ 99;27^{\circ} & \text{where 4 right angles} = 360^{\circ}. \end{cases}$$

Therefore arc ΘKL of the epicycle, which was the distance of the planet Mercury from the apogee Θ at the observation, is 99;27°.

Q.E.D.

Secondly, in the 21st year of Dionysius' calendar (which was in the 484th year from Nabonassar), Scorpion 22, [which is] Thoth [I] 18/19 in the Egyptian calendar [−264 Nov. 14/15], at dawn, Stilbon [i.e. Mercury] was 1 moon to the rear of the straight line through the northern [star in the] forehead of Scorpius and the middle [star in the forehead], and was 2 moons to the north of the northern [star in the] forehead. Now according to our coordinates at that time the midmost of the stars in the forehead of Scorpius had a longitude of ♏ $1\frac{2}{3}$°, and is the same amount [$1\frac{2}{3}$°] south of the ecliptic, while the northernmost star had a longitude of ♏ $2\frac{1}{3}$° and is $1\frac{1}{3}$° north of the ecliptic.[98] So the planet
H289 Mercury had a longitude of about ♏ $3\frac{1}{3}$°.[99] Furthermore it is clear that it had not yet reached its greatest elongation as morning-star, since 4 days later, on Scorpion 26, it is recorded that its distance from the same straight line towards the rear was $1\frac{1}{2}$ moons: for [by that time] the elongation had become greater, the sun having moved about 4 degrees, but the planet [only] half a moon. And on Thoth 19 at dawn the longitude of the mean sun, according to our tables, was ♏ $20\frac{5}{6}$°, while the longitude of the apogee of the planet was about ♎ 6°, since the 400 or so years between the observations produce a displacement of the apogee of about 4°.

With the above as data, then, let us draw a figure [Fig. 9.10] similar to the one preceding [Fig. 9.9], but in which, because of the difference in the positions, the angles towards the apogee A [i.e. ∠ AGZ, ∠ ABH] are to be drawn as acute, the straight lines joining [points] to the planet [i.e. ZL, DL], as in advance of the epicycle [centre], and perpendicular ZX as beyond ZL, the radius of the epicycle.[100]

Then, since the mean position of the planet was [♏ $20\frac{5}{6}$° − ♎ 6° =] 44;50° from the apogee,

$$\angle\,ABH = \begin{cases} 44;50^{\circ} & \text{where 4 right angles} = 360^{\circ} \\ 89;40^{\circ\circ} & \text{where 2 right angles} = 360^{\circ\circ}. \end{cases}$$

Therefore its supplement, ∠ GBH = 270;20°°
and ∠ BGH = ∠ BHG = 44;50°° in the same units.

[98] See catalogue nos. XXIX 2 and 1. Ptolemy has subtracted 4° from the longitudes there to account for precession.

[99] It is difficult to see how Ptolemy arrives at this position from his data: see the discussion *HAMA* 166, with Fig. 151. This was an observation of a station. Cf. Ptolemy's remark about ancient observations IX 2 pp. 420–1.

[100] There is the additional difference (as noted by Manitius) that the significations of points Θ and K has been interchanged: in Fig. 9.9 Θ was the mean apogee and K the true, while in Fig. 9.10 K is the mean perigee and Θ the true.

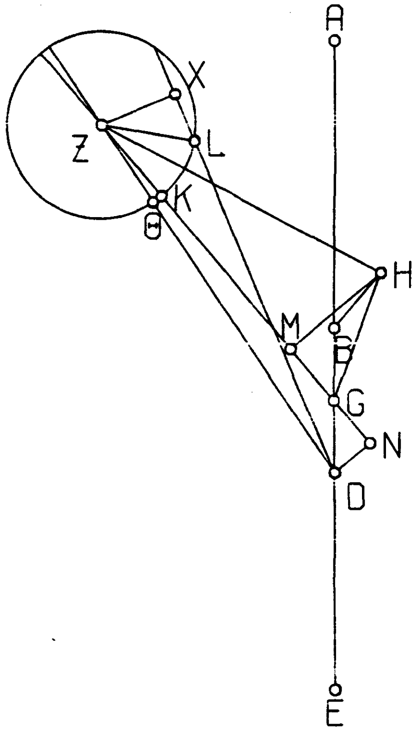

Fig. 9.10

And, by the same reasoning [as before] H290
in the circle about triangle BGH the corresponding chords

$$\left.\begin{array}{r} GH = 84;36^p \\ \text{and } BG = BH = 45;46^p \end{array}\right\} \text{where the diameter is } 120^p.$$

Therefore where BG = BH = 3^p,
GH = $5;33^p$.

Again, by hypothesis,
$\angle$ AGZ = $89;40^{\circ\circ}$ where 2 right angles = $360^{\circ\circ}$
and $\angle$ BGH = $44;50^{\circ\circ}$ in the same units,
so, by addition, $\angle$ ZGH = $134;30^{\circ\circ}$,
and, in the circle about right-angled triangle GHM
arc HM = $134;30^{\circ}$
and arc GM = $45;30^{\circ}$ (supplement).
Therefore the corresponding chords

$$\left.\begin{array}{r} MH = 110;40^p \\ \text{and } GM = 46;24^p \end{array}\right\} \text{where hypotenuse } GH = 120^p.$$ H291

Therefore where GH = $5;33^p$ (i.e. where ZH, the radius of the eccentre, is 60^p),
HM = $5;7^p$
and GM = $2;10^p$.[101]

[101] $2;9^p$ would be more accurate by any method of computation

Hence we compute ZM [$=\sqrt{ZH^2 - HM^2}$] as $59;47^p$,
and, by addition [of MG to ZM], ZMG as $61;57^p$ in the same units.
Similarly, since ∠ DGN [= ∠ AGZ] = $89;40^{\circ\circ}$ where 2 right angles = $360^{\circ\circ}$,
in the circle about right-angled triangle GDN,

arc DN = 89;40°
and arc GN = 90;20° (supplement).

So the corresponding chords

$$\left.\begin{array}{r}DN = 84;36^p\\ \text{and } GN = 85;6^p\end{array}\right\}\text{ where hypotenuse } GD = 120^p.$$

Therefore where GD = 3^p,
DN = $2;7^p$
and GN = $2;8^p$,

and, by addition [of ZG to GN], ZGN = $64;5^p$.
Hence hypotenuse ZD [$=\sqrt{ZN^2 + DN^2}$] = $64;7^p$ in the same units.
Therefore, in the circle about right-angled triangle ZDN,

where ZD = 120^p,
DN = $3;58^p$
and arc DN = 3;48°.[102]
∴ ∠ DZN = $3;48^{\circ\circ}$ where 2 right angles = $360^{\circ\circ}$,
and, by subtraction [of ∠ DZN from ∠ AGZ],
∠ ADZ = $85;52^{\circ\circ}$ in the same units.

But ∠ ADL is given as $54;40^{\circ\circ}$ in the same units
(for the planet was [♏ $3\frac{1}{3}$ − ♎ 6° =] 27;20° from the apogee at the observation).
H292 Hence, by subtraction, ∠ ZDL = $31;12^{\circ\circ}$ where 2 right angles = $360^{\circ\circ}$.
Therefore in the circle about right-angled triangle ZDX,

arc ZX = 31;12°
and ZX = $32;16^p$ where hypotenuse DZ = 120^p.

Therefore where DZ = $64;7^p$ (i.e. where ZL, the radius of the epicycle, is $22;30^p$),

XZ = $17;15^p$.

And, in the circle about right-angled triangle ZLX,

where hypotenuse ZL = 120^p,
ZX ≈ 92^p.
∴ arc ZX = 100;8°,[103]
and ∠ ZLX = $100;8^{\circ\circ}$ where 2 right angles = $360^{\circ\circ}$.

And we showed that, in the same units, ∠ ZDL = $31;12^{\circ\circ}$,

[hence ∠ ΘZL = ∠ ZLX − ∠ ZDL = $68;56^{\circ\circ}$],
and that ∠ ΘZK = $3;48^{\circ\circ}$.

Therefore, by subtraction [of ∠ ΘZK from ∠ ΘZL],

$$\angle KZL = \begin{cases} 65;8^{\circ\circ} \text{ where 2 right angles } = 360^{\circ\circ} \\ 32;34^{\circ} \text{ where 4 right angles } = 360^{\circ}. \end{cases}$$

At this observation, then, the planet was 32;34° from the epicycle perigee K, and, obviously, 212;34° from the apogee. But we showed that at the moment of

[102] 3;47° would be more accurate by any method of computation.

[103] The nearest one can get to this by any method of computation is 100;7°. More accurate calculation would give 100;4°.

our observation it was 99;27° from the apogee of the epicycle. Now the interval between the two observations is approximately H293

402 Egyptian years 283 days $13\frac{1}{2}$ hours.

This interval contains 1268 complete returns of the planet in anomaly (for 20 Egyptian years produce very nearly 63 returns, so 400 years produce 1260, and the remaining 2 years plus the additional days another 8 complete returns). Thus we have shown that in 402 Egyptian years 283 days $13\frac{1}{2}$ hours the planet Mercury moved in anomaly, beyond 1268 complete revolutions, 246;53°, which is the amount by which the position at our observation is beyond the previous one. And just about the same increment [in anomaly] results from the tables we set out before: for it was on the basis of these very same calculations that we made our correction to the periodic motions of Mercury, by reducing the above interval to days, and the above revolutions in anomaly plus the increment to degrees. For when the total of degrees is divided by the total of days, there results the mean daily motion in anomaly which we set out for Mercury in our previous discussion [IX 3].[104]

11. {*On the epoch of its* [*Mercury's*] *periodic motions*}

Then in order to establish the epochs of the five planets, as we did for the sun and moon, for the first year of Nabonassar, Thoth 1 in the Egyptian calendar, noon, we took the interval between that moment and the more ancient of the observations, which is closer to it: this is very nearly H294

483 Egyptian years 17 days 18 hours.[105]

The increment in mean anomaly corresponding to that interval is 190;39°. If we subtract the latter from the 212;34° (counted from the apogee) derived from the observation, we get the following epoch positions for Nabonassar 1, Thoth 1 in the Egyptian calendar, noon:

anomaly counted from the apogee of the epicycle	21;55°
[mean] longitude the same as the sun's, i.e.	♓ 0;45°
apogee of the eccentre in about	♎ $1\frac{1}{6}$°

(for $\frac{1}{100}$th [of a degree for each] of the above years comes to about $4\frac{5}{6}$°, which, subtracted from the [longitude] ♎ 6 ° at the observation, gives [♎] $1\frac{1}{6}$°).

[104] For the actual derivation of the mean motion in anomaly see Appendix C. In the derivation of the two positions in anomaly on which the mean motion is allegedly based Ptolemy has committed a number of small computational and rounding errors. These result in a compounded error which is not negligible, as accurate computation from his initial values reveals:

	Ptolemy	Computed
Obs. I	212;34°	212;29,18°
Obs. II	99;27°	99;33,31°
Increment	246;53°	247; 4,13°.

The difference of +11′, distributed over about 400 years, leads to +0;0,0,0,16 °/d in the mean motion.

[105] Reading ιη (with Ar) for ιη γ′ ($18\frac{1}{3}$) at H294,5. 18^h is shown to be correct both by the increment in mean motion below ($18\frac{1}{3}^h$ would give 190;42° instead) and by the interval between the two observations given above. Corrected by Manitius.

Book X

1. {*Demonstration of* [*the position of*] *the apogee of the planet Venus*}[1] H296

Such, then, was the method by which we found the hypotheses for the planet Mercury, the sizes of its anomalies, and also the precise amounts of its periodic motions, and their epochs. For the planet Venus, again, we first investigated the position in the ecliptic of the apogee and perigee of the eccentre by [finding] greatest elongations which are equal and in the same direction.[2] The available ancient observations did not supply us with exact pairs of positions [suitable] for this purpose, but we used contemporary observations for our approach, as follows.

[1] Among the observations given to us by the mathematician Theon, we found one recorded in the sixteenth year of Hadrian, on Pharmouthi [VIII] 21/22 in the Egyptian calendar [132 Mar. 8/9], at which, he says, the planet Venus was at its greatest elongation as evening-star from the sun, and was the length of the Pleiades in advance of the middle of the Pleiades; and it seemed to be passing it a little to the south. Now, according to our coordinates, the longitude of the middle of the Pleiades at that time was ♉ 3°, and its length is about 1½°:[3] so clearly Venus' longitude at that moment was ♉ 1½°. So, since the longitude H297 of the mean sun at that moment was ♓ 14¼°, the greatest distance from the mean as evening-star was 47¼°.

[2] In the fourth[4] year of Antoninus, Thoth [I] 11/12 in the Egyptian calendar [140 July 29/30], we observed Venus at its greatest elongation from the sun as morning-star. It was [the breadth of] half a full moon to the north-east of [the star in] the middle knee of Gemini. At that moment the longitude of the fixed star, according to us, was ♊ 18¼°,[5] so Venus was in about ♊ 18½°. And the

[1] On chapters 1–3 see *HAMA* 152–6, Pedersen 298–306 and (for a criticism of Ptolemy's procedure) Sawyer, 'Ptolemy's determination of the apsidal line for Venus' (cf. p. 449 n.53).

[2] See p. 446 n.43. Many of the dates of greatest elongations of Venus given here by Ptolemy are in error, some by as much as three weeks (see *HAMA* 153 n.1). We cannot doubt that he was aware of this, but he was forced by the lack of suitable observations during the limited period available to take those positions of Venus close to greatest elongation which gave the required positions of the mean sun with respect to Venus' apsidal line. The point is discussed in detail by Swerdlow and Neugebauer, Ch.5.

[3] In the catalogue (XXIII 30–32) the group of the Pleiades has longitudes between ♉ 2⅙° and ♉ 3⅔°. The length of this is indeed 1½°, but its midpoint is ♉ 2;55°, which Ptolemy has rounded to 3° (a correction for precession would make it even less than 2;55°).

[4] Reading δ´ (with D,Ar) for ιδ´ (14th) at H297,5. The date is confirmed by the computations below. Corrected by Manitius.

[5] Catalogue XXIV 11, where the description is somewhat different. Of the three knees mentioned (nos. 10, 11 and 13) this is the middle one.

mean sun was in ♌ $5\frac{3}{4}$°. So the greatest distance as morning-star was the same amount as before, $47\frac{1}{4}$°.

Therefore, since the mean position was ♓ $14\frac{1}{4}$° at the first observation, and ♌ $5\frac{3}{4}$° at the second, and the point on the ecliptic halfway between these falls in [either] ♉ 25° [or] ♏ 25°, the diameter through apogee and perigee must go through the latter [points].

[3] Similarly, in the [observations we got] from Theon, we found that in the twelfth year of Hadrian, Athyr [III] 21/22 in the Egyptian calendar [127 Oct. 11/12], Venus as morning-star had its greatest elongation from the sun when it
H298 was to the rear of the star on the tip of the southern wing of Virgo by the length of the Pleiades, or less than that amount by its own diameter; and it seemed to be passing the star one moon to the north. Now the longitude of the fixed star at that time, according to us, was ♌ $28\frac{11}{12}$°: hence the longitude of Venus was about ♍ $0\frac{1}{3}$°.[6] And the mean sun was in ♎ $17\frac{26}{30}$°. So the greatest elongation from the mean as morning-star was $47\frac{16}{30}$°.

[4] In the twenty-first year of Hadrian, Mechir [VI] 9/10 in the Egyptian calendar [136 Dec. 25/26], in the evening, we observed Venus at its greatest elongation from the sun. It was in advance of the northernmost star of the four which almost form a quadrilateral (behind the star to the rear of and on a straight line with the [two] in the groin of Aquarius):[7] [its distance from the star was] about two-thirds of a full moon, and it seemed about to obscure the star with its light.[8] Now the longitude of the fixed star at that time, according to us, was ♒ 20°; hence Venus was in about ♒ $19\frac{3}{5}$°,[9] and the mean sun's longitude was ♑ $2\frac{1}{15}$°.

Here too, then, the greatest elongation as evening-star was the same [as in [3] as morning-star], $47\frac{16}{30}$°. And the points on the ecliptic halfway between the ♎
H299 $17\frac{26}{30}$° of the first observation and the ♑ $2\frac{1}{15}$° of the second are again about ♏ 25° and ♉ 25°.

2. {*On the size of* [*Venus'*] *epicycle*}

By these means, then, we determined that in our time the apogee and perigee of [Venus'] eccentre lie in ♉ 25° and ♏ 25°. Accordingly, we again looked for greatest elongations from the mean which occur when the sun is near ♉ 25° and ♏ 25°.

[6] Literally 'a third of the first degree of Virgo'. The longitude in the catalogue (XXVII 5) is ♌ 29°. Ptolemy subtracts 5′ for 11 years' precession, adds $1\frac{1}{2}$° for the length of the Pleiades, and then subtracts 5′ for the diameter of Venus. (In the *Planetary Hypotheses*, ed. Goldstein p. 8 § 5, he estimates the apparent diameter of Venus as $\frac{1}{10}$th of the sun's, i.e. 3′).

[7] The stars in question are (according to Manitius' identification): the quadrilateral, catalogue nos. XXXII 26–9; the two in the groin, nos. 15 and 16. The differences in the description here from the catalogue are so great that we must assume that this was originally written before the catalogue existed (as the date of the observation suggests).

[8] Reading καταλάμψειν (with GD) for καταλάμπειν ('seemed to be obscuring') at H298,14–15. The word is a technical term for one bright body (the sun, as at VIII 6, H201,1, cf. καταλάμψεις at XIII 7, H591,11, or the moon, as here) coming so close to another that it 'outshines' it and makes it no longer visible.

[9] 'two-thirds of a moon' is only 20′, whereas Ptolemy subtracts 24′. Is the difference to account for the diameter of Venus?

[1] In the [observations] given to us by Theon we find that in the thirteenth year of Hadrian, Epiphi [XI] 2/3 in the Egyptian calendar [129 May 19/20], Venus was at its greatest elongation from the sun as morning-star, and was $1\frac{2}{5}°$ in advance of the straight line through the foremost of the 3 stars in the head of Aries and the star on the hind leg, while its distance from the foremost star of those in the head was approximately double its distance from the star on the leg. Now at that time, according to us, the foremost star of the 3 in the head of Aries had a longitude of [♈] $6\frac{3}{5}°$ and is $7\frac{1}{3}°$ north of the ecliptic, while the star in the hind leg of Aries had a longitude of $14\frac{3}{4}°$, and is $5\frac{1}{4}°$ south of the ecliptic.[10] Therefore the longitude of Venus was ♈ $10\frac{3}{5}°$ and it was $1\frac{1}{2}°$ south of the ecliptic. Hence, since the longitude of the mean sun at that time was ♉ $25\frac{2}{5}°$, the greatest elongation from the mean was $44\frac{4}{5}°$. H300

[2] In the twenty-first year of Hadrian, Tybi [V] 2/3 in the Egyptian calendar [136 Nov. 18/19], in the evening, we observed Venus at its greatest distance from the sun: when sighted with respect to the stars in the horns of Capricorn it was seen to occupy ♑ $12\frac{5}{6}°$, while the longitude of the mean sun was ♏ $25\frac{1}{2}°$. Hence in this position the greatest elongation from the mean comes out as $47\frac{1}{3}°$.

Hence it is clear that the apogee lies in ♉ 25°, and the perigee in ♏ 25°. Furthermore, it has also become plain to us that the eccentre of Venus carrying the epicycle is fixed, since nowhere on the ecliptic do we find the sum of the greatest elongations from the mean on both sides to be less than the sum of both in Taurus, or greater than the sum of both in Scorpius. H301

With the above as data, let [Fig. 10.1] the eccentric circle, on which Venus' epicycle is always carried, be ABG on diameter AG, on which D is taken as the centre of the eccentre, E as the centre of the ecliptic, and A as the point at ♉ 25°. About points A and G let there be drawn equal epicycles, on which lie points Z and H [respectively]. Draw the tangents EZ and EH, and join AZ, GH.

Then, since ∠ AEZ, which is at the centre of the ecliptic, subtends the greatest elongation of the planet at the apogee, which is, by hypothesis, $44\frac{4}{5}°$,

$$\angle\, AEZ = \begin{cases} 44;48° & \text{where 4 right angles} = 360° \\ 89;36°° & \text{where 2 right angles} = 360°°. \end{cases}$$

Therefore in the circle about right-angled triangle AEZ

arc AZ = 89;36°

and its chord AZ ≈ 84;33^p where hypotenuse AE = 120^p. H302

Similarly, since ∠ GEH subtends the greatest elongation at the perigee, which is, by hypothesis, $47\frac{1}{3}°$,

$$\angle\, GEH = \begin{cases} 47;20° & \text{where 4 right angles} = 360° \\ 94;40°° & \text{where 2 right angles} = 360°°. \end{cases}$$

Therefore in the circle about right-angled triangle GEH

arc GH = 94;40°

and its chord GH ≈ 88;13^p where hypotenuse EG = 120^p.

[10] The stars in question are catalogue XXII 1 and 13 (note the different descriptions there), with longitudes of $6\frac{2}{3}°$ and 15°. The difference in the longitudes given here is −4′ and −15′ respectively. One would expect about −5′ for the precession in 8 years. Hence Manitius emended $14\frac{3}{4}$ to $14\frac{11}{12}$; but it is implausible to change, as he does, ∠′ δ′ to ⌈′ δ′ ($\frac{2}{3}+\frac{1}{4}$); for $\frac{11}{12}$ is written ∠′ γ′ ιβ′ ($\frac{1}{2}+\frac{1}{3}+\frac{1}{12}$), e.g. H303,7. The stars in the alignment are too far apart to allow us to use it to check the text, so in the absence of any ms. variation I merely note the possibility of some corruption.

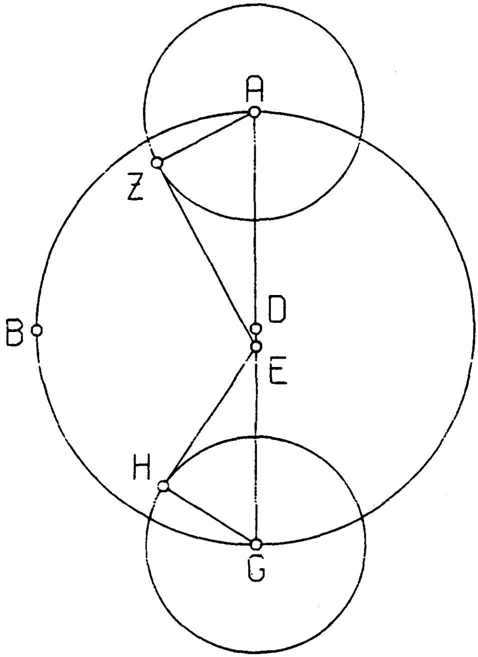

Fig. 10.1

Therefore where GH (= AZ), the radius of the epicycle, is $84;33^p$, and $AE = 120^p$,

$$EG = 115;1^p,$$

and obviously, by addition, $AG = 235;1^p$

and its half, $AD \approx 117;30^p$,

and, by subtraction, the distance between the centres, $DE = 2;29^p$.

Therefore where the radius of the eccentre, $AD = 60^p$,
the distance between the centres, $DE \approx 1\frac{1}{4}^p$,
and the radius of the epicycle, $AZ = 43\frac{1}{6}^p$.

3. {*On the ratios of the eccentricities of the planet* [*Venus*]}

But since it is not clear whether the uniform motion of the epicycle takes place
H303 about point D, here too we took two greatest elongations, in opposite directions [i.e. one as evening-star and the other as morning-star], in each of which[11] the mean motion of the sun was a quadrant from the apogee.

[1] We observed the first in the eighteenth year of Hadrian, Pharmouthi [VIII] 2/3 in the Egyptian calendar [134 Feb. 17/18]. In this Venus was at

[11] Reading ἐφ' ἑκατέρας (with CDG,Is) at H303,2 for ἐφ' ἑκάτερα ('in both directions'). Corrected by Manitius.

greatest elongation from the sun as morning-star, and when it was sighted with respect to the star called Antares [catalogue XXIX 8], its longitude was ♑ $11\frac{11}{12}$°, at which time the longitude of the mean sun was ♒ $25\frac{1}{2}$°. So the greatest elongation from the mean as morning-star was $43\frac{7}{12}$°.

[2] We observed the second in the third year of Antoninus, Pharmouthi [VIII] 4/5 in the Egyptian calendar [140 Feb. 18/19], in the evening. In this Venus was at its greatest elongation from the sun, and when it was sighted with respect to the bright star in the Hyades [catalogue XXIII 14], its longitude was ♈ $13\frac{5}{6}$°, while the longitude of the mean sun was again ♒ $25\frac{1}{2}$°. Hence in this case the greatest elongation from the mean as evening-star was $48\frac{1}{3}$°.

With the above as data, let [Fig. 10.2] the diameter through the apogee and perigee of the eccentre be ABG; let A represent the point at ♉ 25°, and let B represent the centre of the ecliptic. Let our task be to find the centre about which we say that the uniform motion of the epicycle takes place. Let that

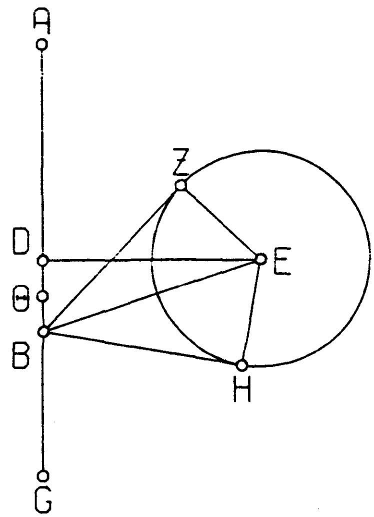

Fig. 10.2

centre be point D, and draw DE through D perpendicular to AG, in order for the mean position of the epicycle to be a quadrant from the apogee, as in the observations. On DE take E to represent the centre of the epicycle at the observations in question, draw the epicycle ZH on it as centre, draw the tangents to it from B, BZ and BH, and join BE, EZ and EH. H304

Then since, at the mean position in question, the greatest elongation from the mean as morning-star is, by hypothesis, $43\frac{7}{12}$°, and the greatest as evening-star $48\frac{1}{3}$°,

by addition, ∠ ZBH = 91;55° where 4 right angles = 360°.

Therefore its half, ∠ ZBE = 91;55°° where 2 right angles = 360°°.

Therefore in the circle about right-angled triangle BEZ

arc EZ = 91;55° H305

and EZ = 86;16^p where hypotenuse BE = 120^p.

Therefore where the radius of the epicycle, EZ = 43;10^p

$$BE = 60;3^p.$$

Again, since the difference between the above greatest elongations (which is 4;45°) comprises twice the equation of the ecliptic anomaly at that point, which is represented by ∠ BED,

$$\angle BED = \begin{cases} 2;22,30^\circ \text{ where 4 right angles} = 360^\circ \\ 4;45^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ}. \end{cases}$$

Therefore in the circle about right-angled triangle BDE

$$\text{arc } BD = 4;45^\circ$$

$$\text{and } BD \approx 4;59^p \text{ where hypotenuse } BE = 120^p.$$

Therefore where BE = 60;3^p and the radius of the epicycle is 43;10^p,

$$BD \approx 2\tfrac{1}{2}^p.$$

But we showed [p. 472] that the distance between B, the centre of the ecliptic, and the centre of the eccentre on which the epicycle centre is always carried, is $1\frac{1}{4}^p$ in the same units; thus it is half of BD.

Therefore, if we bisect BD at Θ, we have demonstrated[12] that

H306 where ΘA, the radius of the eccentre carrying the epicycle, is 60^p,
each of the distances between the centres, BΘ and ΘD = $1\frac{1}{4}^p$,
and EZ, the radius of the epicycle, is 43;10^p.

Q.E.D.

4. {*On the correction of the periodic motions of the planet* [*Venus*]}[13]

Such, then, is the method by which we determined the type of [Venus'] hypothesis and the ratios of its anomalies. For the periodic motions and epochs of the planet, once again [as for Mercury], we took two reliable observations, [one] from among ours, and [one] of the ancient ones.

[1] In the second year of Antoninus, Tybi [V] 29/30 in the Egyptian calendar [138 Dec. 15/16], we observed the planet Venus, after its greatest elongation as morning-star, using the astrolabe and sighting it with respect to Spica: its apparent longitude was ♏ 6½°. At that moment it was also between and on a straight line with the northernmost of the stars in the forehead of Scorpius and the apparent centre of the moon, and was in advance of the moon's centre 1½ times the amount it was to the rear of the northernmost of the stars in the forehead. Now the [latter] fixed star had at that time, according to our coordinates, a longitude of ♏ 6;20°, and is 1;20° north of the ecliptic.[14] The
H307 time was 4¾ equinoctial hours after midnight, since the sun was in about ♐ 23,

[12] This is the only 'demonstration' of the 'bisection of the eccentricity' in the *Almagest*, although it is also assumed for the outer planets. However, this does not prove (*contra HAMA* 155) that observations of Venus were the historical origin of Ptolemy's introduction of the equant. It seems far more likely that it arose from the considerations Ptolemy himself outlines at X 6 (see p. 480, with n.24), for which Mars must have provided the most opportune observations.

[13] On chs. 4 and 5 see *HAMA* 156–8.

[14] See catalogue XXIX 1.

and the second degree of Virgo [i.e. ♍ 1°–2°] was culminating according to the astrolabe. At that moment the positions were as follows:[15]

mean longitude of the sun	♐ 22;9°
mean longitude of the moon	♏ 11;24°
anomaly of the moon, counted from apogee	87;30°
[argument of] latitude of the moon, from the northern limit	12;22°
hence, true position of the moon's centre	♏ 5;45°
[moon's latitude]	5° north of the ecliptic
apparent position [of the moon] at Alexandria in longitude	♏ 6;45°
[apparent position of the moon in latitude]	4;40° north of the ecliptic.

From these considerations too, then, Venus' longitude was ♏ 6;30°, and it was 2;40° north of the ecliptic.

With the above as data, let [Fig. 10.3] the diameter through the apogee be ABGDE. Let A represent the point at ♉ 25°, B the point about which the epicycle moves uniformly, G the centre of the eccentre carrying the epicycle centre, and D the centre of the ecliptic. Since the mean sun had a longitude of ♐ 22;9° at the observation, the mean position of the epicycle is [♐ 22;9°–♏ 25° =] 27;9° towards the rear from the perigee at E. So let the epicycle centre be located at Z, and draw the epicycle HΘK on Z as centre. Join DZH, GZ and H308
BZΘ, and drop perpendiculars GL and DM from G and D on to BZ. Let the

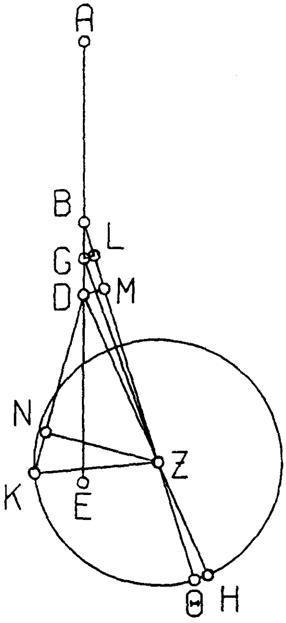

Fig. 10.3

[15] The following data are calculated, accurately, not for 4;45 a.m., but for 4;30 a.m. Since the equation of time for a solar longitude of ♐ 23° is about − 17 mins., Ptolemy's (silent) correction is justified. For 4;45 a.m. local time I find the culminating point as a little past ♍ 1°, in agreement with the text.

planet be located at point K, join DK and ZK, and drop perpendicular ZN [on to DK]. Let the problem be, to find the arc ΘK, which is the distance of the planet from the epicycle apogee Θ [at the observation].

Now since

$$\angle \text{EBZ} = \begin{cases} 27;9^\circ \text{ where 4 right angles} = 360^\circ \\ 54;18^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ}, \end{cases}$$

in the circle about right-angled triangle BGL

$$\text{arc GL} = 54;18^\circ$$

$$\text{and arc BL} = 125;42^\circ \text{ (supplement).}$$

Therefore the corresponding chords

$$\left.\begin{aligned} \text{GL} &= 54;46^p \\ \text{and BL} &= 106;47^p \end{aligned}\right\} \text{ where hypotenuse BG} = 120^p.$$

Therefore where BG = $1;15^p$ and GZ, the radius of the eccentre, is 60^p,

H309 $$\text{GL} = 0;34^p$$

$$\text{and BL} = 1;7^p.$$

And since $ZG^2 - GL^2 = ZL^2$,

$$\text{ZL} \approx 60^p \text{ in the same units.}$$

And since BG = GD

$$\text{ML} = \text{LB } [= 1;7^p],$$

$$\text{and DM} = 2\text{GL}.$$

Therefore, by subtraction [of ML from ZL], ZM = $58;53^p$

$$\text{and DM} = 1;8^p \text{ in the same units.}$$

Hence hypotenuse ZD[= $\sqrt{ZM^2 + DM^2}$] ≈ $58;54^p$.

Therefore, where ZD = 120^p, DM = $2;18^p$,

and, in the circle about right-angled triangle DZM,

$$\text{arc DM} = 2;12^\circ.$$

$$\therefore \angle \text{BZD} = 2;12^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ},$$

and, by addition [of ∠ EBZ and ∠ BZD], ∠ EDZ = $56;30^{\circ\circ}$ in the same units. And, since the planet was 18;30° in advance of the perigee at E (i.e. ♏ 25°) at the observation,

$$\angle \text{EDK} = \begin{cases} 18;30^\circ \text{ where 4 right angles} = 360^\circ \\ 37^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ}. \end{cases}$$

Therefore, by addition [of ∠ EDK to ∠ EDZ],

$$\angle \text{KDZ} = 93;30^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ},$$

and, in the circle about right-angled triangle DZN,

$$\text{arc ZN} = 93;30^\circ.$$

H310 Therefore its chord, ZN = $87;25^p$ where ZD = 120^p.

So where ZD = $58;54^p$, i.e. where the epicycle radius ZK is $43;10^p$.

$$\text{ZN} = 42;54^p.$$

$$\therefore \text{ZN} = 119;18^p \text{ where hypotenuse ZK} = 120^p,$$

and, in the circle about right angled triangle ZKN,

$$\text{arc ZN} = 167;38^\circ.$$ [16]

$$\therefore \angle \text{ZKD} = 167;38^{\circ\circ} \text{ where } \angle \text{ ZDK has already been found as } 93;30^{\circ\circ}.$$

[16] The accumulated rounding error here is considerable. ZN should be about $119;16^p$ rather than $119;18^p$. Since this chord is so close to the maximum of 120^p, the resulting error in the arc is great: accurate computation would give ZN = 167;22°, resulting in a not negligible change of 8′ in the final result (230;40°).

So, by addition, $\angle$ KZH = 261;8°°.

And we showed that $\angle$ BZD (= $\angle$ HZΘ) = 2;12°° in the same units.

Therefore, by subtraction, $\angle$ ΘZK = $\begin{cases} 258;56^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \\ 129;28^{\circ} \text{ where 4 right angles} = 360^{\circ}. \end{cases}$

So the planet Venus, at the time in question, was the above distance, 129;28°, in advance of the epicycle apogee Θ, and, [therefore], in the motion [on the epicycle] assigned to it in the hypothesis, [namely] towards the rear, it was the difference of the above from one revolution, 230;32°, which was what we had to determine.

[2] From the ancient observations we selected one which is recorded by Timocharis as follows. In the thirteenth year of Philadelphos, Mesore [XII] 17/18 in the Egyptian calendar [−271 Oct. 11/12], at the twelfth hour, Venus was seen to have exactly overtaken[17] the star opposite Vindemiatrix. That is the star which, in our descriptions [catalogue XXVII 6], is the one following the H311
star on the tip of the southern wing of Virgo, and which had a longitude of ♍ $8\frac{1}{4}$° in the first year of Antoninus. Now the year of the observation is the 476th from Nabonassar, while the first year of Antoninus is 884 [years] from Nabonassar;[18] to the 408 years of the interval corresponds a motion of the fixed stars and the apogees of about $4\frac{1}{12}$°. Hence it is clear that the longitude of Venus was ♍ $4\frac{1}{6}$°, and the longitude of the perigee of its eccentre ♏ $20\frac{11}{12}$°. And here too Venus was past its greatest elongation as morning-star; for 4 days after the above observation, on Mesore 21/22, as one can deduce from what Timocharis says, its longitude was ♍ $8\frac{5}{6}$° according to our coordinates; and the mean position of the sun was ♎ 17;3° at the first observation and ♎ 20;59° at the next: thus its elongation at the first observation comes to 42;53° and at the next 42;9°.

With the above as data, let there be drawn [Fig. 10.4] a figure similar [to the preceding], but which has the epicycle in advance of the perigee, since the mean longitude of the epicycle is ♎ 17;3°, while the longitude of the perigee is ♏ H312
20;55°. Now for that reason

$\angle$ EBZ [=♏ 20;55° − ♎ 17;3°] = $\begin{cases} 33;52^{\circ} \text{ where 4 right angles} = 360^{\circ} \\ 67;44^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ}. \end{cases}$

Therefore, in the circle about right-angled triangle BGL,

arc GL = 67;44°

and arc BL = 112;16° (supplement).

Therefore the corresponding chords

$\left.\begin{array}{r} \text{GL} = 66;52^{p} \\ \text{and BL} = 99;38^{p} \end{array}\right\}$ where hypotenuse BG = 120^{p}.

Therefore where BG = $1;15^{p}$ and the radius of the eccentre, GZ = 60^{p},

GL = $0;42^{p}$

and BL = $1;2^{p}$.

[17] Most translations interpret this word (κατειληφώς) as 'occulted'. Modern calculations show that no occultation occurred, since Venus passed about 12′ to the south of η Vir. Nevertheless, since another observation where no occultation could have occurred is unambiguously described as an occultation (see p. 522 n.16), and καταλαμβάνειν denotes occultations by the moon at H28,15, H31,5, H32,7 and H33,9, the same is probably intended here.

[18] Reading τὸ δὲ α′ ἔτος τῆς 'Αντωνίνου βασιλείας $\overline{\text{ωπδ}}$ ἔστιν ἀπὸ Ναβονασσάρου with DG,Ar) at H311, 4–5, for τὸ δὲ μέχρι τῆς 'Αντωνίνου βασιλείας ωπδ′ of the other mss. The first year of Antoninus is the 885th in the era Nabonassar, but since this observation is towards the end of the Egyptian year, Ptolemy correctly counts to the end of Nabonassar 884.

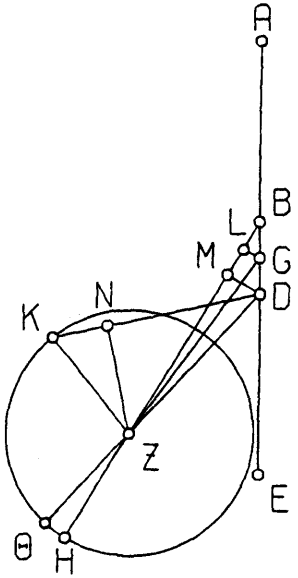

Fig. 10.4

And since $ZG^2 - GL^2 = ZL^2$,
$ZL \approx 60^p$.
And by the same reasoning [as before]
H313 $BL = LM$
and $DM = 2GL$.
Therefore, by subtraction [of LM from ZL], $ZM = 58;58^p$
and $DM = 1;24^p$ in the same units.
Hence hypotenuse $ZD[= \sqrt{ZM^2 + DM^2}] \approx 58;59^p$.
Therefore, where $ZD = 120^p$, $DM = 2;51^p$,
and, in the circle about right-angled triangle ZDM,
arc $DM = 2;44°$
$\therefore \angle BZD = 2;44^{\circ\circ}$ where 2 right angles $= 360^{\circ\circ}$.
And, by addition [of $\angle$ BZD and $\angle$ EBZ], $\angle EDZ = 70;28^{\circ\circ}$ in the same units.
And the distance of the planet in advance from the perigee,
$\angle EDK [= ♏ 20;55° - ♍ 4;10°] = \begin{cases} 76;45° \text{ where 4 right angles} = 360° \\ 153;30^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ}. \end{cases}$
Therefore, by subtraction, $\angle ZDK = 83;2^{\circ\circ}$ in the same units,
and, in the circle about right-angled triangle DZN,
arc $ZN = 83;2°$.
So its chord $ZN = 79;33^p$ where hypotenuse $DZ = 120^p$,
and where $DZ = 58;59^p$, i.e. where the epicycle radius $ZK = 43;10^p$,
$ZN = 39;7^p$.
Therefore, in the circle about right-angled triangle ZKN,
where hypotenuse $ZK = 120^p$
$ZN = 108;45^p$

and arc ZN ≈ 130°.

∴ ∠ DKZ = 130°° where ∠ ZDK has already been found as 83;2°°. H314

And, by addition, ∠ ΘZK = 213;2°° in the same units.

But we showed that ∠ BZD (= ∠ HZΘ) = 2;44°° in the same units.

Therefore, by addition, ∠ HZK = $\begin{cases} 215;46^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \\ 107;53^{\circ} \text{ where 4 right angles} = 360^{\circ}.\end{cases}$ [19]

At that moment, then, the distance of the planet Venus, [in the sense of rotation] towards the rear, from the epicycle apogee H was the difference from one revolution, 252;7°, which was what we had to determine.

Now its distance from the apogee of the epicycle, in the same sense, at the moment of our observation was 230;32°. And the interval between the two observations comprises 409 Egyptian years and about 167 days, and 255 complete revolutions in anomaly (for 8 Egyptian years produce approximately 5 revolutions, so the 408 years produce 255 revolutions, while the remaining year plus the additional days do not complete the period of one revolution). So we have demonstrated that in 409 Egyptian years 167 days the planet Venus travels on the epicycle, beyond 255 complete revolutions in anomaly,[20] 338;25°, which is the amount by which the position at our observation exceeded the H315 earlier one. And approximately the same increment results from the mean motion tables which we set out above. For our correction of the mean motions was derived from the increment over complete revolutions we have found [above]: the time-interval was reduced to days, and the revolutions plus the increment to degrees. For then, when the total in degrees is divided by the total in days, there results the mean daily motion of Venus in anomaly which we set out previously.[21]

5. {*On the epoch of* [*Venus'*] *periodic motions*}

Here, too, the task remains to establish the epochs of the periodic motions for the first year of the reign of Nabonassar, Thoth 1 in the Egyptian calendar, noon. We again took the interval between the latter moment and the moment of the more ancient observation. This comes to

475 Egyptian years 346¾ days approximately.[22]

The increment in mean motion corresponding to that interval in the columns

[19] The accumulated rounding error here amounts to 4′ (one finds 107;49°).

[20] Reading ἀνωμαλίας (with DG) for ἀνωμαλιῶν at H314,22. Corrected by Manitius.

[21] On the actual derivation of the mean motion for Venus see Appendix C. Ptolemy's increment in mean motion, 338;25°, is the motion from 252;7° (above) to 230;32° (p. 477). The accumulated rounding errors in those figures (see p. 476 n.16 and above n.19) lead to a difference in the increment of +4′, which would have an effect on the resulting mean motion. Furthermore it is unclear what interval in days Ptolemy is actually using. He gives the round number 409^y 167^d. But the time of Ptolemy's observation is given as 4;45 a.m., and of Timocharis' as 'at the 12th hour' (interpreted as 6 a.m. in X 5, see below n.22). So the interval should be 1¼ hours less than the above, or, if one corrects for the equation of time at Ptolemy's observation, (cf. p. 475 n.15) 1½ hours less.

[22] If one assumes that the observation of Timocharis (p. 477) was made just at dawn, and applies the equation of time with respect to the epoch of era Nabonassar (about −½ hour), the interval given is approximately correct. But see n.23.

H316 for anomaly is approximately 181°.[23] Subtracting the latter from the 252;7° [of the position] at the observation, we get for the first year of Nabonassar, Thoth 1 in the Egyptian calendar, noon:

epoch in anomaly: 71;7° from the apogee of the epicycle.

The mean position in longitude is again, by hypothesis, the same as the sun's namely

longitude: ♓ 0;45°.

And it is obvious that, since the apogee [of the eccentre] was at about ♉ 20;55° at the observation, and to the intervening 476 years correspond approximately $4\frac{3}{4}$° [of motion of the apogee], at the moment of epoch the apogee will be in about ♉ 16;10°.

6. {*Preliminaries for the demonstrations concerning the other* [*3 outer*] *planets*}

Such, then, were the methods which we successfully used for these two planets, Mercury and Venus, to establish the hypotheses and demonstrate [the sizes of] the anomalies. For the other three, Mars, Jupiter and Saturn, the hypothesis which we find for their motion is the same [for all] and like that established for the planet Venus, namely one in which the eccentre on which the epicycle centre is always carried is described on a centre which is the point bisecting the
H317 line joining the centre of the ecliptic and the point about which the epicycle has its uniform motion; for in the case of each of these planets too, using rough estimation, the eccentricity one finds from the greatest equation of ecliptic anomaly turns out to be about twice that derived from the size of the retrograde arcs at greatest and least distances of the epicycle. However, the demonstrations by which we calculate the amounts of both anomalies and [the positions of] the apogees cannot proceed along the same lines for these planets as for the previous two, since these reach every possible elongation from the sun, and it is not obvious from observation, as it was from the greatest elongations for Mercury and Venus, when the planet is at the point where the line of our sight is tangent to the epicycle. So, since that approach is not available, we have used observations of their oppositions to the mean position of the sun to demonstrate, first of all, the ratios of their eccentricities and [the positions of] their apogees. For only in such positions [of the planet],[24] considered from a theoretical point of view, do we find the ecliptic anomaly isolated, with no effect from the anomaly related to the sun.

H318 For let [Fig. 10.5] the planet's eccentre, on which the epicycle centre is carried, be ABG on centre D, and let the diameter through the apogee be AG, on which point E is the centre of the ecliptic, and Z the centre of that eccentre with respect to which the epicycle's mean motion in longitude is taken. Draw the epicycle HΘKL on centre B, and join ZLBΘ and HBKEM.

I say, first, that when the planet is seen along line EH through the epicycle

[23] Computing from the table (IX 4) one finds for the stated interval 180;58,34°. Ptolemy has either rounded unjustifiably, or computed for a slightly longer interval. A motion of half an hour more (i.e. neglecting the equation of time, cf. n.22) produces 180;59,20°.

[24] See *HAMA* 172. An ingenious analysis of the way in which Ptolemy arrived at the notion of the equant for the outer planets was made by Swerdlow, 'The Origin of Ptolemaic Planetary Theory'.

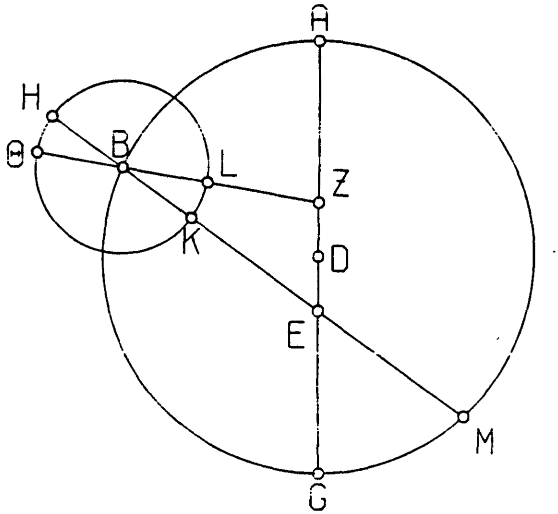

Fig. 10.5

centre B, then the mean position of the sun, too, will always be on the same line, and that when the planet is at H it will be in conjunction[25] with the mean sun (which will also, in theory, be seen towards H), and when the planet is at K it will be in opposition to the mean sun (which will be seen, in theory, towards M). [Proof:] For each of these [outer] planets, the sum of the mean motions in longitude and anomaly, counted from the apogee [of eccentre and epicycle respectively], equals the mean motion of the sun counted from the same starting-point. And the difference between the angle at centre Z (which comprises the mean motion of the planet in longitude), and the angle at E H319
(which comprises the apparent motion in longitude),[26] is always the angle at B (which comprises the mean motion on the epicycle). Hence it is clear that when the planet is at H, it will fall short of a return to the apogee Θ by ∠ HBΘ; but ∠ HBΘ added to ∠ AZB produces the angle comprising the sun's mean motion, namely ∠ AEH, which is the same as the apparent motion of the planet.[27] And when the planet is at K, its motion on the epicycle, again, will be ∠ ΘBK, and ∠ ΘBK + ∠ AZB equal the mean motion of the sun counted from the apogee A.

[25] Reading συνοδεύσει (with G. and possibly Ar, but the translations are ambiguous) for συνοδεύει ('is in conjunction') at H318,18.

[26] By this expression (ἡ φαινομένη κατὰ μῆκος κίνησις) Ptolemy means, not the true position of the planet, but the position of the epicycle center as seen from the earth. Compare the expression ἡ φαινομένη ἐπὶ τοῦ ἐπικύκλου πάροδος at XII 2 (H470,11) to denote the 'true anomaly' (i.e. as counted from true and not mean perigee of the epicycle).

[27] In fact ∠ AZB − ∠ HBΘ = ∠ AEH. But what Ptolemy means is illustrated by Figs. P1 and P2: in Fig. P1 planet and mean sun are in conjunction. In Fig. P2 (= Fig. 10.5) they are again in conjunction. The epicycle has travelled through the angle $\bar{\kappa}$ (∠ AZB), the planet on the epicycle has travelled through $\bar{\alpha}$, and the mean sun through $\kappa + 360°$. Then (from the figure) $\kappa = \bar{\kappa} - (360° - \bar{\alpha}) = \bar{\kappa} + \bar{\alpha} - 360°$. Hence the mean sun's motion $\kappa + 360° = \bar{\kappa} + \bar{\alpha}$. Failing to understand this, an interpolator has inserted τουτέστιν λειφθεισα ὑπ' αὐτῆς at H319,8, producing the strange result '∠ HBΘ added to ∠ AZB, i.e. subtracted from it.'

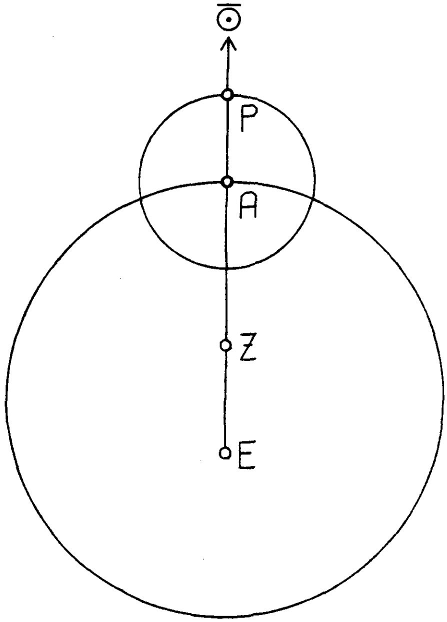

Fig. P1

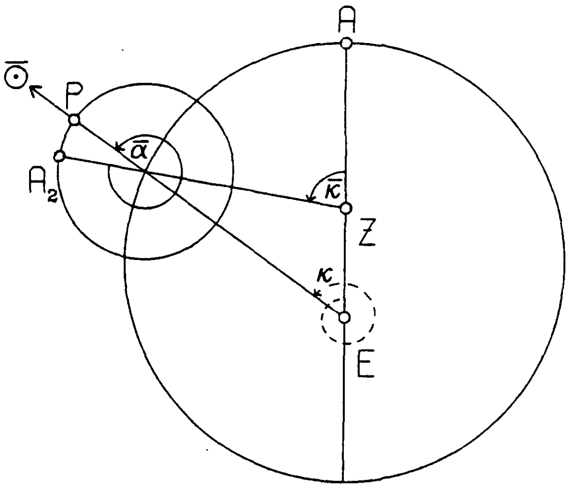

Fig. P2

Thus the latter comprises 180° + (∠ AZB − ∠ LBK) = 180° + ∠ GEM, i.e. the mean position of the sun will be opposite the apparent position of the planet.

Hence, furthermore, in such configurations [i.e. mean conjunctions and oppositions], the line joining the epicycle centre B to the planet, and the line from E, our point of view, to the mean sun, will coincide in one straight line, but H320
at all other [sun-planet] elongations [those vectors] will always be parallel to each other, although the direction in which they point will vary.

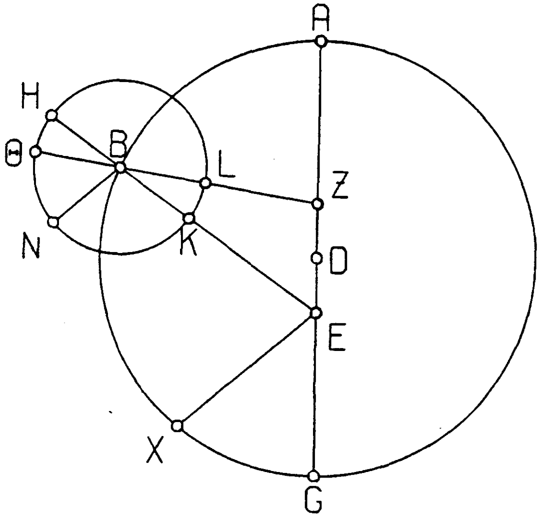

Fig. 10.6

[Proof:] In the above figure [see Fig. 10.6], if we draw the line BN from B to the planet in any situation, and the line EX from E to the mean sun, for the reasons stated above

∠ AEX = ∠ AZΘ + ∠ NBΘ,[28]
and ∠ AZΘ = ∠ AEH + ∠ HBΘ.
[∴ ∠ AEX = ∠ AEH + ∠ NBΘ + ∠ HBΘ.]

If we subtract ∠ AEH from both sides,

∠ HEX = ∠ HBN.

Therefore line EX is parallel to line BN.

Thus we find that in the above configurations of conjunction and opposition H321
with respect to the mean sun, the planet is viewed, in theory, [along the line] through the centre of the epicycle, just as if its motion on the epicycle did not exist, but instead it were itself situated on circle ABG and were carried in uniform motion by the line ZB, in the same way as the epicycle centre is. Hence it is clear that it is possible to isolate and demonstrate the ratio of the ecliptic eccentricity by [both] such types of [planetary] positions, but since the

[28] I.e. the mean motion of the sun equals the mean longitudinal motion of the planet plus the mean anomaly of the planet.

conjunctions are not visible, we are left with the oppositions[29] on which to build our demonstrations.

7. {*Demonstration of the eccentricity and apogee* [*position*] *of Mars*}[30]

In the case of the moon we took the positions and times of three lunar eclipses, and demonstrated the ratio of the anomaly and the position of the apogee geometrically. So too, here, in the same way, for each of these [outer] planets, we observed the positions of three oppositions to the mean sun, as accurately as
H322 possible, using the astrolabe instruments, computed, too, the time and position for the precise 180° elongation[31] from the position of the mean sun at [each of] the observations, and thence demonstrate the ratio of the eccentricity and [the position of] the apogee.

First, then, for Mars, we took three oppositions, which we observed as follows.[32]

[1] The first in the fifteenth year of Hadrian, Tybi [V] 26/27 in the Egyptian calendar [130 Dec. 14/15], 1 equinoctial hour after midnight, at about ♊ 21°.

[2] The second in the nineteenth year of Hadrian, Pharmouthi [VIII] 6/7 in the Egyptian calendar [135 Feb. 21/22], 3 hours before midnight, at about ♌ 28;50°.

[3] The third in the second year of Antoninus, Epiphi [XI] 12/13 in the Egyptian calendar [139 May 27/28], 2 equinoctial hours before midnight, at about ♐ 2;34°.

The intervals between the above are as follows:
From oppositions [1] to [2] 4 Egyptian years 69 days 20 equinoctial hours.
From [2] to [3] 4 years 96 days 1 equinoctial hour.

For the first interval we compute a [mean] motion in longitude, beyond complete revolutions, of 81;44°
and for the second interval, 95;28°.

H323 Even if we used the crude periods of return, which we listed above, to compute the mean motions, it would make no significant difference over such a short interval.[33]

[29] ἀκρώνυκτοι σχηματισμοί, literally 'configurations [at which the planet rises and sets] at the beginning and end of night'.

[30] On the method used to find the eccentricities of the outer planets see *HAMA* 172–7, Pedersen 273–83.

[31] Reading διαμέτρου στάσεως (with DG,Ar) for διαστάσεως 'elongation' at H322,1.

[32] The times are arrived at by computing the position of the mean sun. Therefore the computed position of the mean sun at the time stated ought to be exactly 180° different from the longitudes given. I find, from the solar mean motion tables, 260;58,55° (instead of 261°), 328;50,22° (for 328;50°) and 62;31,45° (for 62;34°). The latter discrepancy represents about half an hour in solar motion. Could Ptolemy have applied the equation of time (which is about $-25\frac{1}{2}$ mins. compared with epoch) here? If so, he was mistaken, since all the computations are in terms of mean solar days.

[33] Ptolemy is referring to the crude periods of IX 3. Thus for Mars (cf. p. 424) in 79 solar years occur 37 returns in anomaly and 42 returns in longitude. Assuming Ptolemy's year-length of $365;14,48^d$, one finds from this, for $4^y\ 69^d\ 20^h$, a longitudinal increment of 81;39°, and, for $4^y\ 96^d\ 1^h$, 95;23°. Using Ptolemy's procedure, and carrying out three iterations, I find from the above data $2e \approx 11;57^p$, distance of 3rd opposition from perigee ≈ 44°. Comparison with Ptolemy's results from the more accurate data, 12^p and 44;21°, shows that the differences are indeed negligible.

It is obvious that the apparent motion of the planet, beyond complete revolutions, is

for the first interval 67;50°
and for the second interval 93;44°.

Then [see Fig. 10.7] let there be drawn in the plane of the ecliptic three equal circles: let the circle carrying the epicycle centre of Mars be ABG on centre D, the eccentre of uniform motion EZH on centre Θ, and the circle concentric with the ecliptic KLM on centre N, and let the diameter through all [three] centres be XOPR. Let A be the point at which the epicycle centre was at the first opposition, B the point where it was at the second opposition, and G the point where it was at the third opposition. Join ΘAE, ΘBZ, ΘHG, NKA, NLB and NGM. Then arc EZ of the eccentric [equant] is 81;44°, the amount of the first interval of mean motion, and arc ZH is 95;28°, the amount of the second H324

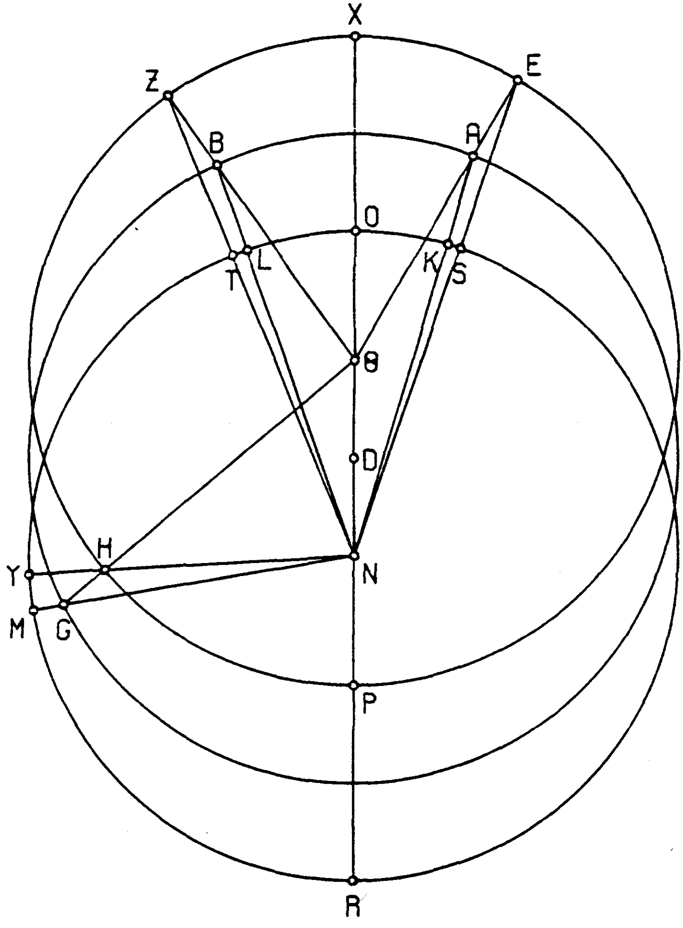

Fig. 10.7

interval. Furthermore arc KL of the ecliptic is 67;50°, the amount of the first interval of apparent motion, while arc LM is 93;44°, the amount of the second interval.

Now if arcs EZ and ZH of the eccentric [equant] were subtended by arcs KL and LM of the ecliptic, that would be all we would need in order to demonstrate the eccentricity.[34] However, as it is, they[35] [arc KL and arc LM] subtend arcs AB and BG of the middle eccentre, which are not given; and if we join NSE, NTZ, NHY, we again find that arcs EZ and ZH of the eccentric [equant] are subtended by arcs ST and TY of the ecliptic, which are, obviously, not given either. Hence the difference arcs,[36] KS, LT and MY, must first be given, in order to carry out a rigorous demonstration of the ratio of the eccentricity starting from the corresponding arcs, EZ, ZH, and ST, TY. But the latter [arcs ST and TY] cannot be precisely determined until we have found the ratio of the eccentricity and [the position of] the apogee; however, even without the previous precise determination of eccentricity and apogee, the arcs are given approximately, since the difference arcs are not large. Therefore we shall first carry out the calculation as if the[37] arcs ST, TY did not differ significantly from the arcs KL, LM.

For [see Fig. 10.8] let the eccentre of mean motion of Mars be ABG, on which
H325 A is taken as the point of the first opposition, B of the second, and G of the third. Inside the eccentre take D as the centre of the ecliptic, which is our point of view, draw in every case [where one has to carry out this kind of calculation] the lines joining the points of the three oppositions to the observer (as here AD, BD

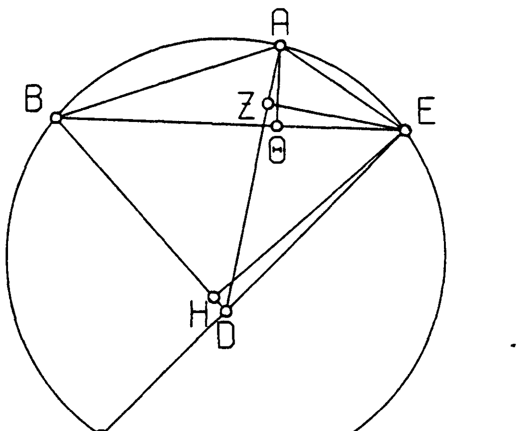

Fig. 10.8

[34] For the situation would be identical with that of the lunar hypothesis (IV 6).

[35] Reading αὗται (with A,B [not reported by Heiberg]. Ar) for αὐταὶ at H324,8.

[36] The arcs forming the differences between arc KL and arc TS, and between arc LM and arc TY.

[37] Reading παρὰ τὰς ΚΛΜ τῶν ΣΤΥ περιφερειῶν, at H324,22, for παρὰ τὰς ΚΛΜ, ΣΤΥ περιφερειῶν ('as if arcs did not differ significantly from [arcs] KLM and STY', which is senseless). My text is the reading of all mss., Greek and Arabic. Heiberg omitted τῶν through a slip or a misprint. Because Manitius did not realize this, his translation here is badly flawed.

and GD), and, as a universal rule, produce one of the three lines so drawn to meet the circumference of the eccentre on the other side (as here GDE), and draw the line joining the other two opposition points (as in this case AB). Then, from the point where the straight line produced intersects the eccentre (as E), draw the lines joining it to the other two opposition points (as here EA and EB), and drop perpendiculars [from the point corresponding to E] on to the lines joining the above-mentioned two points to the centre of the ecliptic (in this case, drop EZ on to AD, and EH on to BD). Also, drop a perpendicular from one of H326 those two points on to the line joining the other with the extra point generated on the eccentre (as here, perpendicular AΘ on to line BE). If we always observe the above rules when drawing this type of figure, we will find that the same numerical ratios result however we decide to draw it.[38] The remainder of the demonstration will become clear as follows, on the basis of the above arcs for Mars.

Since arc BG of the eccentre is given as subtending 93;44° of the ecliptic, the angle at the centre of the ecliptic,

$$\angle \text{BDG} = \begin{cases} 93;44^\circ & \text{where 4 right angles} = 360^\circ \\ 187;28^{\circ\circ} & \text{where 2 right angles} = 360^{\circ\circ}, \end{cases}$$

and its supplement, ∠ EDH = 172;32°° in the same units.

Therefore, in the circle about right-angled triangle DEH,

arc EH = 172;32°

and EH = 119;45ᵖ where hypotenuse DE = 120ᵖ.

Similarly, since arc BG = 95;28°

the angle at the circumference, ∠ BEG = 95;28°° where 2 right angles = 360°°.

But we found that ∠ BDE = 172;32°° in the same units.

Therefore the remaining angle [in triangle BDE],

∠ EBH = 92°° in the same units.

Therefore, in the circle about right-angled triangle BEH, H327

arc EH = 92°

and EH = 86;19ᵖ where hypotenuse BE = 120ᵖ.

Therefore where EH, as we showed, is 119;45ᵖ, and ED = 120ᵖ,

BE = 166;29ᵖ.

Again, since the whole arc ABG of the eccentre is given as subtending [93;44° + 67;50° =] 161;34° of the ecliptic (the sum of both intervals),

∠ ADG = 161;34° where 4 right angles = 360°,

and, by subtraction [from 180°],

$$\angle \text{ADE} = \begin{cases} 18;26^\circ & \text{where 4 right angles} = 360^\circ \\ 36;52^{\circ\circ} & \text{where 2 right angles} = 360^{\circ\circ}. \end{cases}$$

Therefore, in the circle about right-angled triangle DEZ,

arc EZ = 36;52°

and EZ = 37;57ᵖ where hypotenuse DE = 120ᵖ.

Similarly, since arc ABG of the eccentre is, by addition [of 81;44° to 95;28°], 177;12°,

∠ AEG = 177;12°° where 2 right angles = 360°°.

But we found that ∠ ADE = 36;52°° in the same units.

[38] I.e. whichever of the lines AD, BD, GD we decide to produce.

Therefore the remaining angle [in triangle ADE],
$\angle$ DAE = 145;56°° in the same units.
Therefore, in the circle about right-angled triangle AEZ,
arc EZ = 145;56°
H328 and EZ = 114;44^p where hypotenuse AE = 120^p.
Therefore, where EZ, as was shown = 37;57^p, and ED = 120^p,
AE = 39;42^p.
Again, since arc AB of the eccentre = 81;44°,
$\angle$ AEB = 81;44°° where 2 right angles = 360°°.
Therefore, in the circle about right-angled triangle AEΘ,
arc AΘ = 81;44°
and arc EΘ = 98;16° (supplement).
Therefore the corresponding chords
AΘ = 78;31^p and EΘ = 90;45^p } where hypotenuse AE = 120^p.
Therefore where AE, as was shown, is 39;42^p, and DE is given as 120^p,
ΘA = 25;58^p
and EΘ = 30;2^p.
But the whole line EB was shown to be 166;29^p in the same units.
Therefore, by subtraction, ΘB = 136;27^p where ΘA = 25;58^p.
And ΘB^2 = 18615;16,[39]
ΘA^2 = 674;16,
so $AB^2 = \Theta B^2 + \Theta A^2$ = 19289;32.
∴ AB = 138;53^p where ED = 120^p and AE = 39;42^p.
H329 But, where the diameter of the eccentre is 120^p, AB = 78;31^p,
since it subtends an arc of 81;44°.
Therefore where AB = 78;31^p, and the diameter of the eccentre is 120^p,
ED = 67;50^p
and AE = 22;44^p.
Therefore arc AE of the eccentre is 21;41°.[40]
And, by addition, arc EABG = [177;12° + 21;41° =] 198;53°.
Therefore the remaining arc GE = 161;7°
and the corresponding chord GE = 118;22^p where the diameter of the eccentre is 120^p.

Now if GE had been found equal to the diameter of the eccentre, it is obvious that the centre would lie on GE, and the ratio of the eccentricity would immediately be apparent. But, since it is not equal [to the diameter], but makes segment EABG greater than a semi-circle, it is clear that the centre of the eccentre will fall within[41] the latter. Let it be at K [Fig. 10.9], and draw through

[39] The square of 136;27 is 18618;36 to the nearest minute. The error has no significant effect on the size of AB below.

[40] There are some serious errors here. For the chord AE one should find, from Ptolemy's figures, 22;27^p, and this is indeed the reading of Ger (but not the rest of the Arabic tradition) at H329,6. The arc of the latter, however, is not 21;41°, but 21;34°. Ptolemy's result (guaranteed by his further calculations), 21;41°, is the arc of 22;*34*^{p}. It looks as if the errors are Ptolemy's own (hence the reading of Ger is a misguided emendation). Did Ptolemy compute 22;27^p → 21;34°, and then, misreading his own notes, 22;34^p → 21;41°?

[41] Reading ἐντὸς τούτου (with DG) at H329,17 for πρὸς τούτῳ ('at the latter'). Corrected by Manitius.

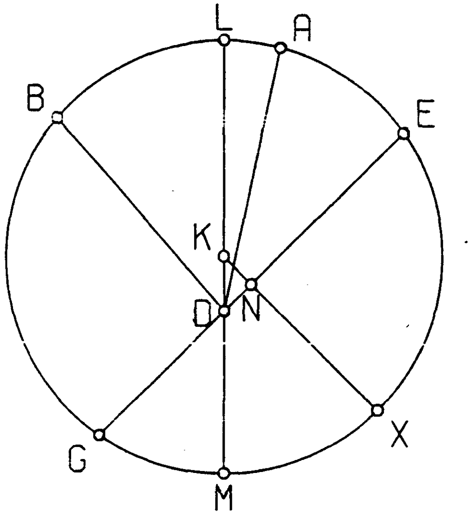

Fig. 10.9

D and K the diameter through both centres, LKDM, and drop perpendicular KNX from K on to GE.

Then, since, as we showed, EG = 118;22^p where diameter LM = 120^p, H330

and DE = 67;50^p in the same units,

by subtraction, GD = 50;32^p in the same units.

Then, since ED.DG = LD.DM,[42]

LD.DM = [67;50 × 50;32 =] 3427;51.

But (LD.DM) + DK^2 equals the square on half the whole line [LD + DM],[43]

i.e. (LD.DM) + DK^2 = LK^2.

Now the square on the half is 3600, and (LD.DM) = 3427;51,

so DK^2 = 3600 − 3427;51 = 172;9,

and the distance between the centres,

DK ≈ 13;7^p where the radius of the eccentre, KL = 60^p[44].

Furthermore, since

GN = ½GE = 59;11^p where diameter LM = 120^p,

and, as we showed, GD = 50;32^p in the same units, H331

by subtraction, DN = 8;39^p where DK was computed as 13;7^p.

Therefore in the circle about right-angled triangle DKN,

DN = 79;8^p where hypotenuse DK = 120^p,

and arc DN = 82;30°.

$$\therefore \angle \text{DKN} = \begin{cases} 82;30^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \\ 41;15^{\circ} \text{ where 4 right angles} = 360^{\circ}. \end{cases}$$

And since ∠ DKN is an angle at the centre of the eccentre,

arc MX = 41;15° also.

[42] Euclid III 35.
[43] Euclid II 5.
[44] Accurate computation from Ptolemy's original data gives about 13;2½^p.

But the whole arc GMX = ½ arc GXE [= ½. 161;7°] = 80;34°.
Therefore, by subtraction, the arc from the third opposition to the perigee,
arc GM = 39;19°.[45]

And it is obvious that, since arc BG is given as 95;28°,
by subtraction, the arc from the apogee to the second opposition,
arc LB [= 180° − (95;28° + 39;19°)] = 45;13°,
and that, since arc AB is given as 81;44°,
by subtraction, the arc from the first opposition to the apogee,
arc AL [= arc AB − arc LB] = 36;31°.

Taking the above quantities as given, let us investigate the differences which
H332 can be derived from them in the ecliptic arcs which we seek to determine at each of the oppositions [in turn]. Our investigation proceeds as follows.

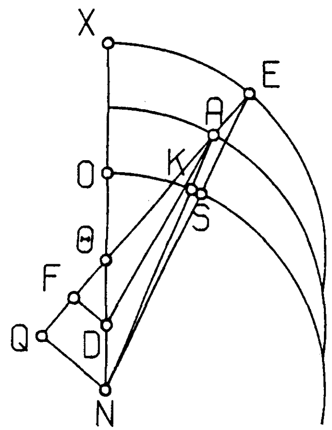

Fig. 10.10

[See Fig. 10.10.] From the previous figure [10.7] for the three oppositions let us draw separately the part representing the first opposition, draw the additional line AD, and drop perpendiculars DF and NQ from points D and N on to AΘ produced.

Then, since arc XE = 36;31°,

$$\angle E\Theta X = \begin{cases} 36;31^\circ & \text{where 4 right angles} = 360^\circ \\ 73;2^{\circ\circ} & \text{where 2 right angles} = 360^{\circ\circ}. \end{cases}$$

And the vertically opposite angle DΘF = 73;2°° in the same units also.
Therefore, in the circle about right-angled triangle DΘF,
arc DF = 73;2°
and arc ΘF = 106;58° (supplement).
Therefore the corresponding chords

$$\left.\begin{aligned} DF &= 71;25^p \\ \text{and } F\Theta &= 96;27^p \end{aligned}\right\} \text{ where hypotenuse } D\Theta = 120^p.$$

H333 Therefore where DΘ = 6;33½ᵖ and the radius of the eccentre, DA = 60ᵖ,
DF = 3;54ᵖ
and FΘ = 5;16ᵖ.

[45] Accurate computation from Ptolemy's data gives 39;10°.

And since $DA^2 - DF^2 = FA^2$,
$AF = 59;52^p$,
and, since $QF = F\Theta$,
by addition [of QF to FA], $QA = 65;8^p$
where $NQ = 2DF = 7;48^p$.
Hence hypotenuse [of right-angled triangle NAQ]
$NA = 65;36^p$ in the same units.
Therefore, where $NA = 120^p$, $NQ = 14;16^p$,
and, in the circle about right-angled triangle ANQ,
arc $NQ = 13;40°$
$\therefore \angle NAQ = 13;40^{\circ\circ}$ where 2 right angles $= 360^{\circ\circ}$.
Again, since QN was shown to be $7;48^p$ and $Q\Theta$ [$= 2F\Theta$] to be $10;32^p$,
where the radius of the eccentre, $\Theta E = 60^p$,
by addition, $Q\Theta E = 70;32^p$ in the same units,
and hence the hypotenuse [of right-angled triangle QNE]
$NE \approx 71^p$ in the same units.
Therefore, where $NE = 120^p$, $QN = 13;10^p$,[46]
and, in the circle about right-angled triangle ENQ,
arc $QN = 12;36°$.
$\therefore \angle NEQ = 12;36^{\circ\circ}$ where 2 right angles $= 360^{\circ\circ}$.
But we found that $\angle NAQ = 13;40^{\circ\circ}$ in the same units. H334
Therefore, by subtraction [of $\angle$ NEQ from $\angle$ NAQ],

$$\angle ANE = \begin{cases} 1;4^{\circ\circ} & \text{where 2 right angles} = 360^{\circ\circ} \\ 0;32^{\circ} & \text{where 4 right angles} = 360^{\circ}. \end{cases}$$

That [0;32°], then, is the amount of arc KS of the ecliptic.

Next, draw a similar figure containing [the part of] the diagram for the second opposition [Fig. 10.11].

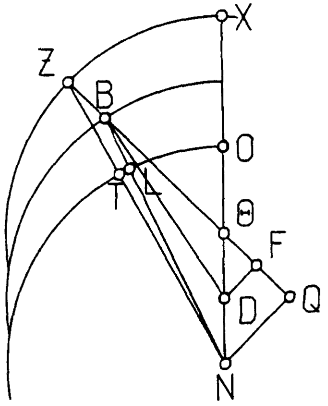

Fig. 10.11

[46] The roundings here are particularly crude: from the immediately preceding numbers one finds $NE = 70;57,48^p$, whence $QN = 13;11,24^p$. Even $NE = 71^p$ leads to $QN = 13;10,59^p$.

Then, since arc XZ is given as 45;13°,[47]

$$\angle X\Theta Z = \begin{cases} 45;13^{\circ} & \text{where 4 right angles} = 360^{\circ} \\ 90;26^{\circ\circ} & \text{where 2 right angles} = 360^{\circ\circ}, \end{cases}$$

and the vertically opposite angle DΘF = 90;26°° in the same units, also.

Therefore, in the circle about right-angled triangle DΘF,

arc DF = 90;26°

and arc FΘ = 89;34° (supplement).

Therefore the corresponding chords

$$\left.\begin{aligned} DF &= 85;10^{p} \\ \text{and } F\Theta &= 84;32^{p} \end{aligned}\right\} \text{ where hypotenuse } D\Theta = 120^{p}.$$

H335 Therefore where DΘ = $6;33\frac{1}{2}^{p}$ and the radius of the eccentre, DB = 60^p,

DF = $4;39^p$

and FΘ = $4;38^p$.

And since $DB^2 - DF^2 = BF^2$,

FB = $59;49^p$,

and, since FQ = FΘ,

by addition, QB = $64;27^p$ where NQ (= 2DF) is computed as $9;18^p$.

Therefore hypotenuse [of right-angled triangle NQB]

NB = $65;6^p$[48] in the same units.

Therefore, where NB = 120^p, NQ = $17;9^p$,

and, in the circle about right-angled triangle BNQ,

arc NQ = 16;26°

∴ ∠ NBQ = 16;26°° where 2 right angles = 360°°.

Again, since NQ was shown to be $9;18^p$, and QΘ [= 2FΘ] = $9;16^p$,

where the radius of the eccentre, ZΘ = 60^p,

by addition, QΘZ = $69;16^p$ in the same units.

Hence hypotenuse NZ [of right-angled triangle NQZ] = $69;52^p$.

Therefore, where hypotenuse NZ = 120^p, NQ ≈ 16^p,

and, in the circle about right-angled triangle ZNQ,

arc NQ = 15;20°.

H336 ∴ ∠ NZQ = 15;20°° where 2 right angles = 360°°.

But we found that ∠ NBQ = 16;26°° in the same units.

$$\text{Therefore, by subtraction, } \angle BNZ = \begin{cases} 1;6^{\circ\circ} & \text{in the same units} \\ 0;33^{\circ} & \text{where 4 right angles} = 360^{\circ}. \end{cases}$$

That [0;33°], then, is the amount of arc LT of the ecliptic.

Now, since we found arc KS as 0;32° for the first opposition, it is clear that the first interval, taken with respect to the eccentre,[49] will be greater than the interval of apparent motion by the sum of both arcs, [namely] 1;5°, and [hence] will contain 68;55°.

Then let [the part of] the diagram for the third opposition be drawn [Fig. 10.12]. Now, since arc PH is given as 39;19°,

$$\angle P\Theta H = \begin{cases} 39;19^{\circ} & \text{where 4 right angles} = 360^{\circ} \\ 78;38^{\circ\circ} & \text{where 2 right angles} = 360^{\circ\circ}. \end{cases}$$

[47] Cf. arc LB on p. 490.

[48] Reading $\overline{\xi\varepsilon}$ (with D,Ar) for $\overline{\xi\theta}$ (69;6) at H335,9. The correction is assured by the preceding and subsequent computations.

[49] I.e. the equant: this is made explicit in XI 1 p. 515. See n.7 there.

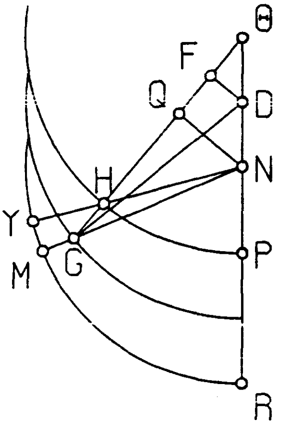

Fig. 10.12

Therefore, in the circle about right-angled triangle DΘF,

arc DF = 78;38°

and arc ΘF = 101;22° (supplement).

Therefore the corresponding chords

$$\left.\begin{array}{r} \text{DF} = 76;2^p \\ \text{and } \Theta\text{F} = 92;50^p \end{array}\right\} \text{ where hypotenuse D}\Theta = 120^p.$$

Therefore where the distance between the centres, DΘ = 6;33$\frac{1}{2}^p$, and the radius of the eccentre, DG = 60^p, H337

DF = $4;9^p$

and ΘF = $5;4^p$.

And since $GD^2 - DF^2 = GF^2$,

GF = $59;51^p$,

and, since ΘF = FQ,

by subtraction, GQ = $54;47^p$ where NQ (= 2DF) is computed as $8;18^p$.

Hence hypotenuse [of right-angled triangle NGQ]

NG = $55;25^p$ in the same units.

Therefore, where NG = 120^p, NQ = $17;59^p$,

and, in the circle about right-angled triangle GNQ,

arc NQ = 17;14°

∴ ∠ NGQ = $17;14^{\circ\circ}$ where 2 right angles = $360^{\circ\circ}$.

Again, since NQ was shown to be $8;18^p$, and ΘQ[= 2FΘ] = $10;8^p$,

where the radius of the eccentre, ΘH = 60^p,

by subtraction, QH = $49;52^p$ in the same units,

and therefore hypotenuse NH [of right-angled triangle NHQ] = $50;33^p$. H338

Therefore, where NH = 120^p, NQ = $19;42^p$,

and, in the circle about right-angled triangle HNQ,

arc NQ = 18;54°.

∴ ∠ NHQ = $18;54^{\circ\circ}$ where 2 right angles = $360^{\circ\circ}$.

But we showed that ∠ NGQ = $17;14^{\circ\circ}$ in the same units.

Therefore by subtraction, $\angle$ GNH $= \begin{cases} 1;40^{\circ\circ} & \text{in the same units.} \\ 0;50^{\circ} & \text{where 4 right angles} = 360^{\circ}. \end{cases}$

That [0;50°], then, is the amount of arc MY of the ecliptic.

Now since we found arc LT as 0;33° for the second opposition, it is clear that the second interval, taken with respect to the eccentre, will be less than the interval of apparent motion by the sum of both arcs, [namely] 1;23°, and will [thus] contain 92;21°.

Using the ecliptic arcs thus computed for the two intervals, and, once more, the original arcs assumed for the eccentric [equant], and following the theorem demonstrated above [pp. 486–9] for such elements, by means of which we determine [the position of] the apogee and the ratio of the eccentricity, we find (not to lengthen our account by going through the same [computations in detail again]),

H339 the distance between the centres, DK = 11;50^p where the radius of the eccentre is 60^p;

the arc of the eccentre from the third opposition to the perigee, GM = 45;33°.[50]

Hence arc LB = [180° − (95;28° + 45;33°)] = 38;59°

and arc AL = [81;44° − 38;59°] = 42;45°.

Next, starting from these [arcs] as data, we found from our demonstration for each of the oppositions [separately] the following amounts for the true size of each of the arcs in question:

arc KS	0;28°
arc LT, about the same,	0;28°
and arc MY	0;40.[51]

We combined the [corrections] for the first and second oppositions, added the resulting 0;56° to the ecliptic arc of the first interval, 67;50°, and got the accurate interval with respect to the eccentre as 68;46°. Again, combining the [corrections] for the second and third oppositions, and subtracting the resulting 1;8° from the apparent motion on the ecliptic over the second interval, 93;44°, we got the accurate interval with respect to the eccentre as 92;36°.

Next, using the same procedure [as before], we determined a more accurate

H340 value for the ratio of the eccentricity and [the position of] the apogee; we found the distance between the centres, DK ≈ 12^p where the radius of the eccentre, KL = 60^p,

arc GM of the eccentre = 44;21°,[52]

whence, again, arc LB = 40;11°

and arc AL = 41;33°.

Next, we shall show by means of the same [configurations] that the observed apparent intervals between the three oppositions are found to be in agreement with the above quantities.

[50] From Ptolemy's elements, $\Delta\bar{\lambda}_1 = 81;44°$, $\Delta\bar{\lambda}_2 = 95;28°$, $\Delta\lambda_1 = 68;55°$, $\Delta\lambda_2 = 92;21°$, I compute $2e$ = 11;50^p, GM = 45;28°.

[51] From a double eccentricity of 11;50^p and Ptolemy's values for arcs GM, LB and AL, I find: arc KS = 0;27,49°, arc LT = 0;26,51°, arc MY = 0;39,31°.

[52] From Ptolemy's elements I find: DK = 11;59,50^p ≈ 12^p, arc GM = 44;18,45° ≈ 44;19°. Ptolemy is quite right to terminate his calculation here, since a further iteration produces a change in the eccentricity of less than 0;0,30^p and in the line of the apsides of less than 5′.

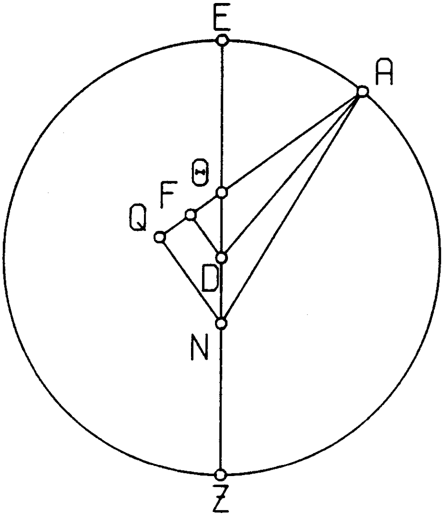

Fig. 10.13

Let there be drawn [Fig. 10.13] the diagram for the first opposition, but with only eccentre EZ, on which the epicycle centre is always carried, drawn in. Then

$\angle A\Theta E = 41;33^\circ$ where 4 right angles $= 360^\circ$,

so where 2 right angles $= 360^{\circ\circ}$,

$\angle A\Theta E = 83;6^{\circ\circ} = \angle D\Theta F$ (vertically opposite).

Therefore, in the circle about right-angled triangle $D\Theta F$,

arc $DF = 83;6^\circ$

and arc $F\Theta = 96;54^\circ$ (supplement).

Therefore the corresponding chords

$\left.\begin{array}{r} DF = 79;35^p \\ \text{and } F\Theta = 89;50^p \end{array}\right\}$ where hypotenuse $D\Theta = 120^p$.

Therefore where $D\Theta = 6^p$ and hypotenuse [of right-angled triangle DAF] $DA = 60^p$,

$DF = 3;58\frac{1}{2}^p$ H341

and $F\Theta = 4;30^p$.

And since $DA^2 - DF^2 = FA^2$,

$FA = 59;50^p$ in the same units.

Furthermore, since $F\Theta = FQ$ and $NQ = 2DF$,

by addition, $AQ = 64;20^p$ where $NQ = 7;57^p$.

Hence hypotenuse [of right-angled triangle NAQ] $NA = 64;52^p$ in the same units.

Therefore where $NA = 120^p$, $NQ = 14;44^p$,

and, in the circle about right-angled triangle ANQ,

arc $NQ = 14;6^\circ$.

$\therefore \angle NAQ = \left\{\begin{array}{l} 14;6^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \\ 7;3^\circ \text{ where 4 right angles} = 360^\circ. \end{array}\right.$

But $\angle A\Theta E = 41;33^\circ$ in the same units.

Therefore, by subtraction, the angle of the apparent position, $\angle$ ANE = 34;30°. This is the amount by which the planet was in advance of the apogee at the first opposition.

H342 Let a similar diagram [Fig. 10.14] be drawn again for the second opposition. Then the angle of the mean position of the epicycle,

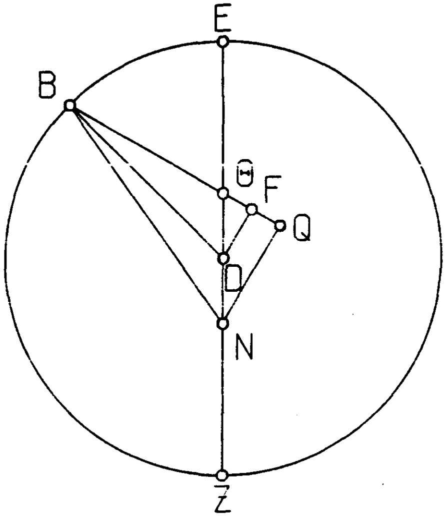

Fig. 10.14

$\angle$ BΘE = 40;11° where 4 right angles = 360°,
so where 2 right angles = 360°°,
$\angle$ BΘE = 80;22°° = $\angle$ QΘN (vertically opposite).

Therefore, in the circle about right-angled triangle DΘF,
arc DF = 80;22°
and arc FΘ = 99;38° (supplement).

Therefore the corresponding chords
DF = $77;26^p$
and FΘ = $91;41^p$ } where hypotenuse DΘ = 120^p.

Therefore where DΘ = 6^p and hypotenuse [of right-angled triangle DBF] DB = 60^p,
DF = $3;52^p$
and FΘ = $4;35^p$.

And since $DB^2 - DF^2 = BF^2$,
BF = $59;53^p$ in the same units.

And, by the same argument [as before],[53]

H343 since FΘ = FQ, and NQ = 2 DF,
by addition, BQ = $64;28^p$ where NQ = $7;44^p$.

Hence hypotenuse [of right-angled triangle BNQ] BN = $64;56^p$ in the same units.

[53] Reading κατὰ ταὐτὰ (as D, κατὰ τὰ αὐτὰ, Ar) for κατὰ ταῦτα ('according to this') at H342.23.

Therefore, where hypotenuse BN = 120^p, NQ = $14;19^p$,[54]
and, in the circle about right-angled triangle BNQ,

arc NQ = 13;42°.

$$\therefore \angle \text{NBQ} = \begin{cases} 13;42^{\circ\circ} & \text{where 2 right angles} = 360^{\circ\circ} \\ 6;51^{\circ} & \text{where 4 right angles} = 360^{\circ}. \end{cases}$$

But ∠ BΘE = 40;11° in the same units.

Therefore, by subtraction, the angle of apparent position,

∠ ENB = 33;20° in the same units.

That [33;20°], then, is the amount by which the planet, in its apparent motion, was to the rear of the apogee at the second opposition. And we showed that at the first opposition it was 34;30° in advance of the apogee. Therefore the total distance [in apparent motion] from first to second opposition comes to 67;50°, in agreement with what we derived from the observations [p. 485].

Let the diagram for the third opposition be drawn in the same way [Fig. 10.15]. In this case the angle of the mean position of the epicycle, H344

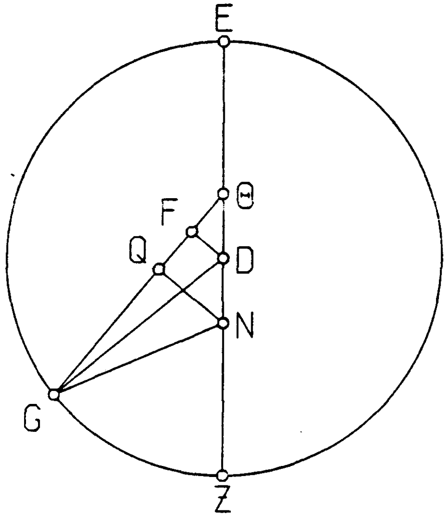

Fig. 10.15

$$\angle \text{G}\Theta\text{Z} = \begin{cases} 44;21^{\circ} & \text{where 4 right angles} = 360^{\circ} \\ 88;42^{\circ\circ} & \text{where 2 right angles} = 360^{\circ\circ}. \end{cases}$$

Therefore, in the circle about right-angled triangle DΘF,

arc DF = 88;42°

and arc FΘ = 91;18° (supplement).

Therefore the corresponding chords

$$\left.\begin{array}{r} \text{DF} = 83;53^p \\ \text{and F}\Theta = 85;49^p \end{array}\right\} \text{where hypotenuse D}\Theta = 120^p.$$

[54] 7;44 × 120/64;56 = 14;17,30, but if one carries out the above computations to 2 fractional sexagesimal places, one finds NQ = 14;18,41^p. As often, Ptolemy computed with greater accuracy than the text implies.

Therefore where DΘ = 6^p and the radius of the eccentre, DG = 60^p,
DF = $4;11\frac{1}{2}^p$
and FΘ = $4;17^p$.
And since $DG^2 - DF^2 = GF^2$,
we find that GF = $59;51^p$ in the same units.
Furthermore, since FΘ = FQ, and NQ = 2DF,
we find by subtraction that QG = $55;34^p$ where NQ = $8;23^p$.
Hence we find that hypotenuse [of right-angled triangle GNQ]
GN = $56;12^p$ in the same units.
Therefore, where hypotenuse GN = 120^p, NQ = $17;55^p$,
and, in the circle about right-angled triangle GNQ,
arc NQ = 17;10°.

H345 $\therefore \angle \Theta GN = \begin{cases} 17;10^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \\ 8;35^{\circ} \text{ where 4 right angles} = 360^{\circ}. \end{cases}$

But ∠ GΘZ = 44;21° in the same units.
Therefore, by addition, ∠ GNZ = 52;56° in the same units.
That [52;56°], then, is the amount by which the planet was in advance of the perigee at the third opposition. But we also showed that at the second opposition it was 33;20° to the rear of the apogee. So we have found 93;44° between the second and third oppositions, computed by subtraction [of the sum of 52;56° and 33;20° from 180°], in agreement with the amount observed for the second interval [p. 485].

Furthermore, since the planet, when viewed at the third opposition along line GN, had a longitude of ♐ 2;34° according to our observation [p. 484], and angle GNZ at the centre of the ecliptic was shown to be 52;56°, it is clear that the perigee of the eccentre, at point Z, had a longitude of [♐ 2;34° + 52;56° =] ♑ 25;30°, while the apogee was diametrically opposite in ♋ 25;30°.

And if [see Fig. 10.16] we draw Mars' epicycle KLM on centre G and produce
H346 line ΘGM,[55] we will have, for the moment of the third opposition:
mean motion of the epicycle counted from apogee of the eccentre: 135;39°
(for its supplement, ∠ GΘZ, was shown to be 44;21°);
mean motion of the planet from the epicycle apogee M (i.e. arc MK): 171;25°
(for ∠ ΘGN was shown to be 8;35° [above], and since it is an angle at the centre of the epicycle, the arc KL from the planet at K to the perigee at L is also 8;35°, hence the supplementary arc from the apogee M to the planet at K is, as already stated, 171;25°).

Thus we have demonstrated, among other things, that at the moment of the third opposition, i.e. in the second year of Antoninus, Epiphi 12/13 in the Egyptian calendar, 2 equinoctial hours before midnight, the mean positions of the planet Mars were:

H347 in longitude (so-called) from the apogee of the eccentre: 135;39°
in anomaly from the apogee of the epicycle: 171;25°.

Q.E.D.

[55] Reading ΘΓΜ (with al-Ḥajjāj) for ΘΓ (ΘG) at H345,22.

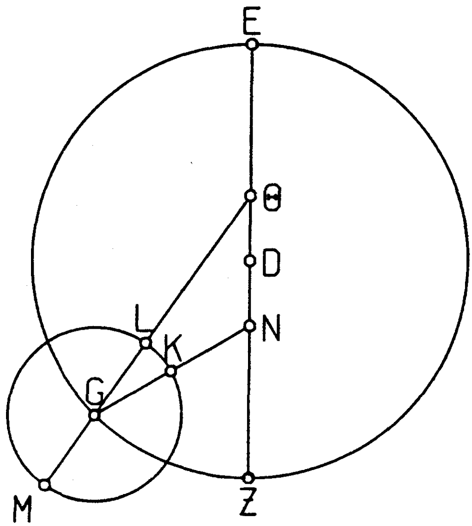

Fig. 10.16

8. {*Demonstration of the size of the epicycle of Mars*}[56]

Our next task is to demonstrate the ratio of the size of the epicycle. For this purpose we took an observation which we obtained by sighting [with the astrolabe] about three days after the third opposition, that is, in the second year of Antoninus, Epiphi [XI] 15/16 in the Egyptian calendar [139 May 30/31], 3 equinoctial hours before midnight. [That was the time,] for the twentieth degree of Libra [i.e. ♎ 19°–20°] was culminating according to the astrolabe, while the mean sun was in ♊ 5;27° at that moment. Now when the star on the ear of wheat [Spica] was sighted in its proper position [on the instrument], Mars was seen to have a longitude of ♐ 1⅗°. At the same time it was observed to be the same distance (1⅗°) to the rear of the moon's centre. Now at that moment the moon's position was as follows:[57]

mean longitude	♐ 4;20°
true longitude	♏ 29;20°
(for its distance in anomaly from the epicycle apogee was 92°)	
apparent longitude	♐ 0°,[58]

H348

So from these considerations too the longitude of Mars was ♐ 1;36°, in agreement with the [astrolabe] sighting.

Hence, clearly, it was 53;54° in advance of the perigee.[59]

[56] On the method employed here see *HAMA* 179–80, Pedersen 283–6.

[57] These positions are computed (accurately), not for 9 p.m., but for 8;37 p.m., i.e. Ptolemy has applied the equation of time with respect to epoch as −23 minutes (it should be about −25½ mins.)

[58] Literally 'at the beginning of Sagittarius'.

[59] Which was in ♑ 25;30° (X 7 p. 498).

And the interval between the third opposition and this observation comprises
in longitude about 1;32°
in anomaly about 1;21°.[60]

If we add the latter to the [mean] positions at the opposition in question[61] as demonstrated above, we get, for the moment of this observation;
distance of Mars in longitude from the apogee of the eccentre: 137;11°
distance in anomaly from the apogee of the epicycle: 172;46°.

With these elements as data, let [Fig. 10.17] the eccentric circle carrying the centre of the epicycle be ABG on centre D and diameter ADG, on which the centre of the ecliptic is taken at E, and the point of greater eccentricity [i.e. the

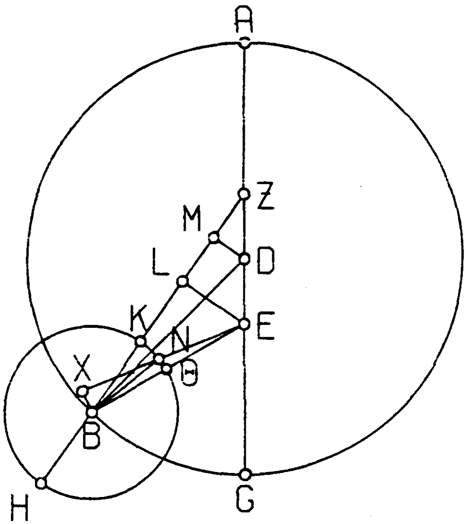

Fig. 10.17

equant] at Z. Draw the epicycle HΘK on centre B, draw ZKBH, EΘB and DB,
H349 and drop perpendiculars EL and DM from points D and E on to ZB. Let the planet be situated at point N on the epicycle, join EN, BN, and drop perpendicular BX from B on to EN produced.

Then, since the planet's distance from the apogee of the eccentre is 137;11°,

$$\angle \text{ BZG} = [180° - 137;11° =] \begin{cases} 42;49° & \text{where 4 right angles} = 360° \\ 85;38°° & \text{where 2 right angles} = 360°°. \end{cases}$$

Therefore, in the circle about right-angled triangle DZM,

arc DM = 85;38°
and arc ZM = 94;22° (supplement).

Therefore the corresponding chords

[60] These mean motions also agree better with an interval of $2^d\ 22^h\ 37^m$ than with one of $2^d\ 23^h$ (see n.57).

[61] Reading κατὰ τὴν ὑποκειμένην ἀκρώνυκτον (with D) for κατὰ τὴν ὑποκειμένην γ´ ἀκρώνυκτον ('at the third opposition, which is the one in question') at H348.9–10.

$$\left.\begin{aligned} DM &= 81;34^p \\ \text{and } ZM &= 88;1^p \end{aligned}\right\} \text{ where hypotenuse } DZ = 120^p.$$

Therefore where the distance between the centres, $DZ = 6^p$, H350
and the radius of the eccentre, $DB = 60^p$,

$$DM = 4;5^p$$
$$\text{and } ZM = 4;24^p.$$

And since $DB^2 - DM^2 = BM^2$,

$$BM = 59;52^p \text{ in the same units.}$$

Similarly, since $ZM = ML$, and $EL = 2DM$,
by subtraction, $BL = 55;28^p$ and $EL = 8;10^p$ in the same units.
Hence hypotenuse [of right-angled triangle EBL] $EB = 56;4^p$.
Therefore, where $EB = 120^p$, $EL = 17;28^p$,
and, in the circle about right-angled triangle BEL,

$$\text{arc } EL = 16;44^\circ$$
$$\therefore \angle ZBE = 16;44^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ}.$$

Furthermore, the apparent distance of the planet Mars in advance of the perigee G,

$$\angle GEX \text{ is given as} \begin{cases} 53;54^\circ & \text{where 4 right angles} = 360^\circ \\ 107;48^{\circ\circ} & \text{where 2 right angles} = 360^{\circ\circ}. \end{cases}$$

And, in the same units, $\angle ZBE = 16;44^{\circ\circ}$ (shown above),
and $\angle GZB = 85;38^{\circ\circ}$ (given),
so $\angle GEB = \angle ZBE + \angle GZB = 102;22^{\circ\circ}$.
Therefore, by subtraction [of $\angle GEB$ from $\angle GEX$],
$\angle BEX = 5;26^{\circ\circ}$ in the same units,
and, in the circle about right-angled triangle BEX

$$\text{arc } BX = 5;26^\circ.$$ H351

So $BX = 5;41^p$ where hypotenuse $EB = 120^p$.
Therefore where EB, as was shown, $= 56;4^p$,
and the radius of the eccentre is 60^p,

$$BX = 2;39^p.$$

Similarly, since the distance of point N from the epicycle apogee H was $172;46^\circ$, and [hence], from the perigee K, $7;14^\circ$,

$$\angle KBN = \begin{cases} 7;14^\circ & \text{where 4 right angles} = 360^\circ \\ 14;28^{\circ\circ} & \text{where 2 right angles} = 360^{\circ\circ}. \end{cases}$$

But $\angle K B\Theta$ was found as $16;44^{\circ\circ}$ in the same units.
Therefore, by subtraction, $\angle NB\Theta = 2;16^{\circ\circ}$,
and, by addition, [of $\angle NB\Theta$ to $\angle BEX$], $\angle XNB = 7;42^{\circ\circ}$.
Therefore, in the circle about right-angled triangle BNX,

$$\text{arc } XB = 7;42^\circ$$
$$\text{and } BX = 8;3^p \text{ where hypotenuse } BN = 120^p.$$

Therefore where $BX = 2;39^p$ and the radius of the eccentre $= 60^p$,
the epicycle radius $BN \approx 39;30^p$.
Therefore the ratio of the radius of the eccentre to the radius of the epicycle is 60 : 39;30.

Q.E.D.

H352 9. {*On the correction of the periodic motions of Mars*}[62]

In order to correct the periodic mean motions we took one of the ancient observations, in which it is declared that in the 13th year of the calendar of Dionysius, Aigon 25,[63] at dawn, Mars seemed to have occulted the northern [star in the] forehead of Scorpius. The moment of this observation is in the 52nd year from the death of Alexander, i.e. in the 476th year from Nabonassar, Athyr [III] 20/21 in the Egyptian calendar [−271 Jan. 17/18], dawn. At this time we find the longitude of the mean sun as ♑ 23;54°; and the longitude of the star on the northern part of the forehead of Scorpius was observed in our time[64] as ♏ $6\frac{1}{3}$°. So, since the 409 years from the observation to [the beginning of] the reign of Antoninus produce about 4;5° of shift in the position of the fixed stars, at the time of the observation in question the longitude of the star must have been ♏ $2\frac{1}{4}$°, and, obviously, the longitude of the planet Mars was the same. In the same way, since the longitude of the apogee of Mars in our time, that is at the
H353 beginning of the reign of Antoninus, was ♋ 25;30°, it must have been ♋ 21;25° at the observation. Thus it is clear at that moment the apparent distance of the planet from its apogee was 100;50°, while the distance of the mean sun from the same apogee was 182;29°, and, obviously, 2;29° from [Mars'] perigee.

With the above elements as data, let [Fig. 10.18] the eccentric circle carrying the epicycle centre be ABG on centre D and diameter ADG, on which the centre of the ecliptic is taken at E, and the point of the greater eccentricity [i.e. the equant] at Z. Draw the epicycle HΘ on centre B, draw ZBH and DB, and drop perpendicular ZK from Z on to DB. Let the planet be situated at point Θ
H354 of the epicycle; join BΘ and draw EL parallel to it from E; then it is clear from our earlier demonstration [X 6, pp. 480–3] that the mean position of the sun will be seen along EL. Join EΘ, and on to it drop perpendiculars DM and BN from points D and B. Also, drop perpendicular DX from D on to BN, so that the figure DMNX is a rectangular parallelogram.

Then, since the angle representing the apparent distance of the planet from the apogee,

$$\angle\ AE\Theta = 100;50° \text{ where 4 right angles} = 360°,$$

and the angle representing the mean motion of the sun [counted from the perigee],

$$\angle\ GEL = 2;29° \text{ in the same units},$$

$$\angle\ \Theta EL = \angle\ B\Theta E = [180° - 100;50° + 2;29° =] \begin{cases} 81;39° & \text{where 4 right angles} = 360° \\ 163;18^{\circ\circ} & \text{where 2 right angles} = 360^{\circ\circ}. \end{cases}$$

[62] On the method employed here see *HAMA* 180–2.

[63] Böckh (*Sonnenkreise* 294), in agreement with Lepsius, changed this to 'Aigon 26' on the basis of his reconstruction of Dionysius' calendar. He was followed by Manitius. The uncertainties are too many to justify emendation by a single day. It may be pertinent that the occultation (if there was one) must, according to modern calculations, have occurred two days earlier than the date Ptolemy gives.

[64] Catalogue no. XXIX 1.

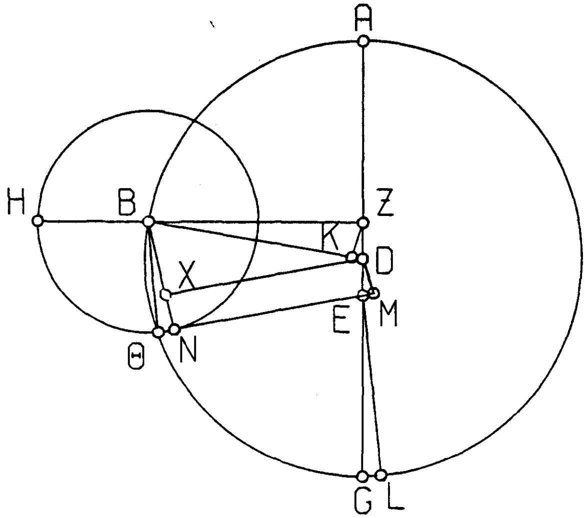

Fig. 10.18

Therefore, in the circle about right-angled triangle BΘN,

arc BN = 163;18°

and BN = 118;43ᵖ where hypotenuse BΘ = 120ᵖ.

Therefore where the radius of the epicycle, BΘ = 39;30ᵖ,

and the distance between the centres, ED = 6ᵖ,

BN = 39;3ᵖ.

Furthermore, since

$$\angle \text{AE}\Theta = \begin{cases} 100;50^\circ & \text{where 4 right angles} = 360^\circ \\ 201;40^{\circ\circ} & \text{where 2 right angles} = 360^{\circ\circ}, \end{cases}$$

and therefore its supplement, ∠ DEM = 158;20°° in the same units, H355

in the circle about right-angled triangle DEM,

arc DM = 158;20°

and DM = 117;52ᵖ where hypotenuse DE = 120ᵖ.

Therefore where DE = 6ᵖ and BN, as was shown, is 39;3ᵖ,

DM = NX = 5;54ᵖ.

So, by subtraction, BX = 33;9ᵖ where the radius of the eccentre, BD = 60ᵖ.

Therefore where hypotenuse [of right-angled triangle BDX] BD = 120ᵖ,

BX = 66;18ᵖ,

and, in the circle about right-angled triangle BDX,

arc BX ≈ 67;4°.

∴ ∠ BDX = 67;4°° where 2 right angles = 360°°,

and, by addition [of right angle XDM], ∠ BDM = 247;4°°.

But, since ∠ DEM was shown to be 158;20°°,

∠ EDM [= a right angle minus ∠ DEM] = 21;40°° in the same units.

Therefore, by subtraction, ∠ BDE is computed as 225;24°°,

and its supplement, ∠ BDA = 134;36°° in the same units.

Therefore, in the circle about right-angled triangle DZK,

arc ZK = 134;36°

and arc DK = 45;24° (supplement).

Therefore the corresponding chords

$$\left.\begin{array}{r} ZK = 110;42^p \\ \text{and } DK = 46;18^p \end{array}\right\} \text{ where hypotenuse } DZ = 120^p.$$

H356

Therefore where DZ = 6^p and the radius of the eccentre, DB = 60^p,

ZK = $5;32^p$

and DK = $2;19^p$.

And, by subtraction, KB = $57;41^p$.

Hence hypotenuse [of right-angled triangle BZK] BZ ≈ $57;57^p$ in the same units.

Therefore, where BZ = 120^p, ZK = $11;28^p$,

and, in the circle about right-angled triangle BKZ,

arc ZK = 10;58°

∴ ∠ ZBD = 10;58°° where 2 right angles = 360°°.

But ∠ BDA = 134;36°° in the same units.

Therefore, by addition, BZA = $\left\{\begin{array}{ll} 145;34^{\circ\circ} & \text{in the same units} \\ 72;47^{\circ} & \text{where 4 right angles} = 360^{\circ}. \end{array}\right.$

Therefore the mean position in longitude of the planet (i.e. of B, the centre of the epicycle) at the moment of the observation in question was 72;47° from the apogee.[65] Hence its [mean] longitude was [♋ 21;25° + 72;47° =] ♎ 4;12°.

And ∠ GEL is given as 2;29°,

and ∠ GEL plus the two right angles of semi-circle ABG equals the sum of the mean longitude, ∠ AZB, and the [mean] anomaly (i.e. the [mean] motion of the planet on the epicycle), ∠ HBΘ.

So, by subtraction [of ∠ AZB from ∠ GEL + 180°], we get

H357 ∠ HBΘ = 109;42°.

Therefore the distance of the planet in anomaly from the apogee of the epicycle at that same moment of the observation was the above 109;42°, which was what we had to determine.

Now we had [already] shown [X 7, p. 498] that at the moment of the third opposition the distance [of Mars] in anomaly from the apogee of the epicycle was 171;25°. Therefore, in the interval between the observations, which comprises 410 Egyptian years and 231⅞ days (approximately), the planet moved 61;43° beyond 192 complete revolutions. That is practically the same increment [in anomaly] which we find from the tables for Mars' mean motion we constructed. For our [mean] daily motion was derived from these very data, by dividing the number of degrees obtained from the complete revolutions plus the increment by the number of days computed from the interval between the two observations.[66]

[65] Through accumulated small computational and rounding errors Ptolemy's result is 3′ too great (accurate is 72;43.50°). This would have some effect on the resulting mean motion in anomaly.

[66] On the actual derivation of the mean motion in anomaly, which remains mysterious in the case of Mars, see Appendix C.

10. {*On the epoch of* [*Mars'*] *periodic motions*}

Furthermore, the interval from the first year of Nabonassar, Thoth 1 in the Egyptian calendar, noon, until the above observation [p. 502], is H358

475 Egyptian years and approximately 79¾ days,

and that interval comprises increments of

180;40° in longitude

and 142;29° in anomaly.[67]

If, then, we subtract the latter from the respective positions for both [longitude and anomaly] at the observation, as given above [p. 504], namely,

♎ 4;12° in longitude

and 109;42° in anomaly,

we get the following epoch positions for the periodic motion of Mars at noon Thoth 1 in the Egyptian calendar, first year of Nabonassar:

longitude	♈ 3;32°
anomaly	327;13° from the epicycle apogee.

Similarly, since, for the shift of the apogee in 475 years one gets by computation 4¾°, and the apogee of Mars was in ♋ 21;25° at the observation, it is obvious that, at the above moment of epoch,

longitude of the apogee was ♋ 16;40°.

[67] The increments over 475ʸ 79¾ᵈ are (to the nearest minute) 180;39° in longitude and 142;28° in anomaly. To get Ptolemy's figures one needs about ½ hour more of motion. Perhaps he took 'dawn' as 6:30 a.m. at Dionysius' observation. But in that case the interval between Dionysius' observation and his own (p. 504) should have been less.

Book XI

1. {*Demonstration of the eccentricity of Jupiter*}[1] H360

Now that we have established the periodic motions, anomalies and epochs of the planet Mars, we shall next deal with those of Jupiter in the same way. Once again, we first take, to demonstrate [the position of] the apogee and [the ratio of] the eccentricity, three oppositions [in which Jupiter is] directly opposite the mean sun.

[1] We observed the first of these by means of the astrolabe instrument in the seventeenth year of Hadrian, Epiphi [XI] 1/2 in the Egyptian calendar [133 May 17/18], 1 hour before midnight, in ♏ 23;11°;

[2] the second in the twenty-first year [of Hadrian], Phaophi [II] 13/14 [136 Aug. 31/Sept. 1], 2 hours before midnight, in ♓ 7;54°;

[3] and the third in the first year of Antoninus, Athyr [III] 20/21 [137 Oct. 7/8], 5 hours after midnight, in ♈ 14;23°.

For the two intervals, that from the first to the second opposition comprises:

[in time]	3 Egyptian years 106 days 23 hours
and in apparent motion of the planet	104;43°;

while that from the second to the third opposition comprises:

[in time]	1 Egyptian year 37 days 7 hours
and [in true longitude]	36;29°.

By computation we find the mean motion in longitude H361

for the first interval:	99;55°
for the second interval:	33;26°.

From these intervals, following the methods expounded for Mars, we carried out the demonstration of what we proposed to determine; first of all as if there were, again, only one eccentre. The demonstration is as follows.

Let [Fig. 11.1] the eccentre be ABG, on which point A is taken as the position of the epicycle centre at the first opposition, B that of the second opposition, and G that of the third. Within the eccentre ABG take D as the centre of the ecliptic, join AD, BD and GD, produce GD to E and draw AE, EB and AB, and drop perpendiculars EZ and EH from E on to AD and BD, and perpendicular AΘ from A on to EB.

Then, since arc BG of the eccentre is given as subtending 36;29° of the ecliptic, the angle at the centre of the ecliptic, H362

$$\angle \text{BDG } (= \angle \text{EDH}) = \begin{cases} 36;29^\circ & \text{where 4 right angles} = 360^\circ \\ 72;58^{\circ\circ} & \text{where 2 right angles} = 360^{\circ\circ}. \end{cases}$$

[1] The procedure for Jupiter and Saturn is identical to that for Mars (except that fewer iterations are required). The reader is referred to the notes on X 7–9 for elucidations of points of detail.

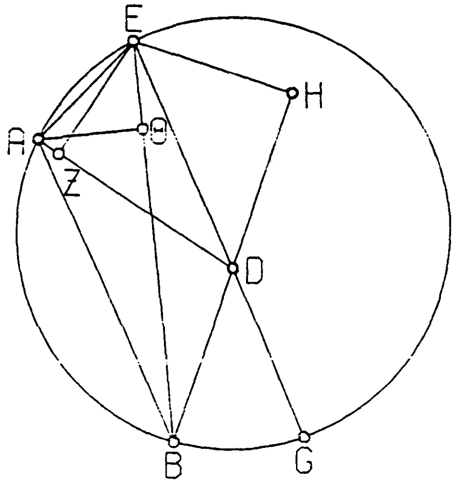

Fig. 11.1

Therefore, in the circle about right-angled triangle EDH,

arc EH = 72;58°

and EH = 71;21^p where hypotenuse DE = 120^p.

Similarly, since arc BG = 33;26°,

the angle [subtended by it] at the circumference,

∠ BEG = 33;26°° where 2 right angles = 360°°;

and, by subtraction [of ∠ BEG from ∠ EDH],

∠ EBH = 39;32°° in the same units.

Therefore, in the circle about right-angled triangle BEH,

arc EH = 39;32°

and EH = 40;35^p where hypotenuse BE = 120^p.

Therefore where EH, as we showed, is 71;21^p, and ED = 120^p,

BE = 210;58^p.

Furthermore, since the whole arc ABG of the eccentre is given as subtending 141;12° of the ecliptic (the sum of both intervals [104;43° and 36;29°]), the angle at the centre of the ecliptic,

H363 $$\angle \text{ADG} = \begin{cases} 141;12^\circ & \text{where 4 right angles} = 360^\circ \\ 282;24^{\circ\circ} & \text{where 2 right angles} = 360^{\circ\circ}, \end{cases}$$

and its complement, ∠ ADE = 77;36°° in the same units.

Therefore, in the circle about right-angled triangle DEZ,

arc EZ = 77;36°

and EZ = 75;12^p where hypotenuse DE = 120^p.

Similarly, since arc ABG of the eccentre is, by addition [of 99;55° + 33;26°], 133;21°, the angle [subtended by it] at the circumference,

∠ AEG = 133;21°° where 2 right angles = 360°°.

But ∠ ADE was found to be 77;36°° in the same units.

Therefore the remaining angle [in triangle EAD],

∠ EAZ = 149;3°° in the same units.

Therefore, in the circle about right-angled triangle AEZ,

arc EZ = 149;3°

and EZ = 115;39^p where hypotenuse EA is 120^p.

Therefore where EZ, as was shown, is 75;12^p, and ED is given as 120^p,

EA = 78;2^p.

Furthermore, since arc AB of the eccentre is 99;55°, the angle [subtended by it] at the circumference,

∠ AEB = 99;55°° where 2 right angles = 360°°.

Therefore, in the circle about right-angled triangle AEΘ,

arc AΘ = 99;55°

and arc EΘ = 80;5° (supplement).

Therefore the corresponding chords

$$\left.\begin{aligned} A\Theta &= 91;52^p \\ \text{and } E\Theta &= 77;12^p \end{aligned}\right\} \text{ where hypotenuse EA} = 120^p.$$ H364

Therefore where AE, as was shown, is 78;2^p, and DE = 120^p,

AΘ = 59;44^p

and EΘ = 50;12^p.

But the whole line EB was shown to be 210;58^p in the same units.

So, by subtraction, ΘB = 160;46^p where AΘ = 59;44^p.

And ΘB^2 = 25845;55

ΘA^2 = 3568;4,

so $\Theta B^2 + \Theta A^2 = AB^2$ = 29413;59.

∴ AB = 171;30^p where ED is 120^p and EA is 78;2^p.

Moreover, where the diameter of the eccentre is 120^p,

AB = 91;52^p (for it subtends an arc of 99;55°).

Therefore where AB = 91;52^p and the diameter of the eccentre is 120^p,

ED = 64;17^p

and EA = 41;47^p.

Therefore arc EA of the eccentre equals 40;45°,

and the whole arc EABG [= 40;45° + 133;21°] = 174;6°. H365

Hence EDG ≈ 119;50^p where the diameter of the eccentre is 120^p.

Now segment EABG is less than a semi-circle, so the centre of the eccentre will fall outside it. Let it, then, be at K [see Fig. 11.2], and draw through K and D the diameter through both centres, LKDM, and let the perpendicular from K to GE be produced as KNX.

Then, where diameter LM = 120^p,

the whole line EG was shown to be 119;50^p, and ED to be 64;17^p;

so, by subtraction, GD = 55;33^p in the same units.

So, since ED.DG = LD.DM,

LD.DM = 3570;56^p where diameter LM = 120^p.

But LD.DM + DK^2 = LK^2 (i.e. the square on half the diameter). H366

Therefore, if we subtract (LD.DM), i.e. 3570;56, from the square on half the diameter, i.e. 3600, the remainder will be the square on DK,

i.e. DK^2 = 29;4.

Therefore the distance between the centres, DK ≈ 5;23^p[2]

where the radius of the eccentre, KL = 60^p.

[2] Because of an accumulation of rounding errors this should be 5;20^p.

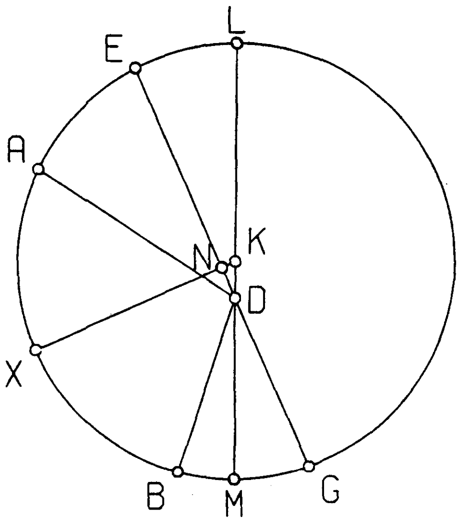

Fig. 11.2

Furthermore, since

$$GN = \tfrac{1}{2}GE = 59;55^p \text{ where diameter } LM = 120^p,$$

and GD was shown to be $55;33^p$ in the same units,

by subtraction, $DN = 4;22^p$ where $DK = 5;23^p$.

Therefore where hypotenuse [of right-angled triangle DKN] $DK = 120^p$,

$$DN = 97;20^p,$$

and, in the circle about right-angled triangle DKN,

$$\text{arc } DN = 108;24°.$$

$$\therefore \angle DKN = \begin{cases} 108;24^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \\ 54;12° \text{ where 4 right angles} = 360°. \end{cases}$$

And since DKN is an angle at the centre of the eccentre,

H367 arc MX = 54;12° also.

But the whole arc GMX, which is ½ arc GXE, equals 87;3°.

Therefore, by subtraction, the arc from the perigee to the third opposition,

arc MG = 32;51°.[3]

And clearly, since the interval BG is given as 33;26°,

by subtraction, we find the arc from the second opposition to the perigee,

arc BM = 0;35°;[4]

and since the interval AB is given as 99;55°,

by subtraction [of (arc AB + arc BM) from 180°], we find the arc from the apogee to the first opposition,

arc LA = 79;30°.

[3] Ptolemy's accumulation of rounding errors has led to the considerable discrepancy of ½° from the accurate result, 32;21°.

[4] The smallness of the corrections for this and the next opposition shows that these oppositions have been badly chosen. To display the greatest difference between the simple eccentric and equant models, all three oppositions should be near the octants (as they are for Mars).

Now if it were this eccentre on which the epicycle centre is carried, the above quantities would be sufficiently accurate to use. However, since, according to our hypothesis, [the epicycle centre] moves on a different circle, namely the circle described with centre at the point bisecting DK and with radius KL, we must once again, as we did for Mars, first calculate the differences which result in the apparent intervals [i.e. the arcs of the ecliptic between the oppositions]: we must show what the sizes of these differences would be (taking the above ratio for the eccentricity as approximately correct), if the epicycle centre were carried, not on the second eccentre, but on the first eccentre [i.e. the equant], which produces the ecliptic anomaly, i.e. the one drawn on centre K. H368

Then [see Fig. 11.3] let the eccentre carrying the epicycle centre be LM on centre D, and the eccentre of the planet's mean motion be NX on centre Z,

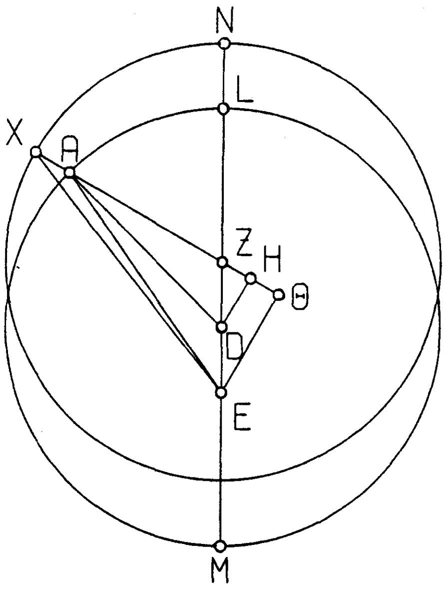

Fig. 11.3

equal to LM. Draw the diameter through the centres, NLM, and take on it the centre of the ecliptic E. Let the epicycle centre be situated, first, at A, for the first opposition. Draw DA, EA, ZAX and EX, and drop perpendiculars DH and EΘ from D and E on to AZ produced.

Then, since the angle of mean motion in longitude, ∠ NZX, was shown to be 79;30° where 4 right angles = 360°, the angle vertically opposite to it,

$$\angle\ \mathrm{DZH} = \begin{cases} 79;30^\circ \text{ where 2 right angles} = 360^\circ \\ 159^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ}. \end{cases}$$

Therefore, in the circle about right-angled triangle DZH, H369

arc DH = 159°

and arc ZH = 21° (supplement).

Therefore the corresponding chords

$$\left.\begin{array}{r} DH = 117;59^p \\ \text{and } ZH = 21;52^p \end{array}\right\} \text{ where hypotenuse } DZ = 120^p.$$

Therefore where DZ ($= \frac{1}{2}$EZ) $\approx 2;42^p$ and the radius of the eccentre, DA $= 60^p$,

DH $= 2;39^p$

and ZH $= 0;30^p$.

And since $DA^2 - DH^2 = AH^2$,

AH $= 59;56^p$ in the same units.

Similarly, since ZH = HΘ, and EΘ = 2DH,

by addition, AΘ $= 60;26^p$ where EΘ $= 5;18^p$,

and hence hypotenuse [of right-angled triangle AEΘ]

AE $= 60;40^p$ in the same units.

Therefore, where AE $= 120^p$, EΘ $= 10;29^p$,

and, in the circle about right-angled triangle AEΘ,

arc EΘ $\approx 10;1°$.

∴ ∠ EAΘ $= 10;1^{\circ\circ}$ where 2 right angles $= 360^{\circ\circ}$.

H370 Furthermore, where EΘ $= 5;18^p$,

the radius of the eccentre, ZX $= 60^p$ and ZΘ [= 2ZH] $= 1^p$,

(hence, obviously, by addition, XΘ $= 61^p$).

So we find hypotenuse [of right-angled triangle EΘX] EX as $61;14^p$ in the same units.

Therefore, where EX $= 120^p$, EΘ $= 10;23^p$,

and, in the circle about right-angled triangle EΘX,

arc EΘ $= 9;55°$.

∴ ∠ EXΘ $= 9;55^{\circ\circ}$ where 2 right angles $= 360^{\circ\circ}$.

But we showed that ∠ EAΘ $= 10;1^{\circ\circ}$ in the same units.

Therefore, by subtraction, the angle of the difference in question,

$$\angle\ AEX = \left\{\begin{array}{l} 0;6^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \\ 0;3^{\circ} \text{ where 4 right angles} = 360^{\circ}. \end{array}\right.$$

But at the first opposition the planet, viewed along the line EA, had an apparent longitude of ♏ 23;11°. Thus it is clear that, if the epicycle centre were carried, not on eccentre LM, but on [eccentre] NX, it would have been at point X on that eccentre, and the planet would have appeared along line EX, differing by 0;3° [from the actual position], and thus would have had a longitude of ♏ 23;14°.

H371 Let the diagram for the second opposition be drawn, again with a similar figure [Fig. 11.4],[5] [with the epicycle centre] depicted as a little in advance of the perigee.

Then, since arc XN of the eccentre was shown [p. 510, arc BM] to be 0;35°,

$$\angle\ XZN = \left\{\begin{array}{l} 0;35^{\circ} \text{ where 4 right angles} = 360^{\circ} \\ 1;10^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ}. \end{array}\right.$$

Therefore, in the circle about right-angled triangle DZH,

arc DH $= 1;10°$

and arc ZH $= 178;50°$ (supplement).

[5] Heiberg's figure (p. 371) is wrong: ΔΞ has been connected instead of ΔB, and Λ is misprinted as A. Corrected by Manitius.

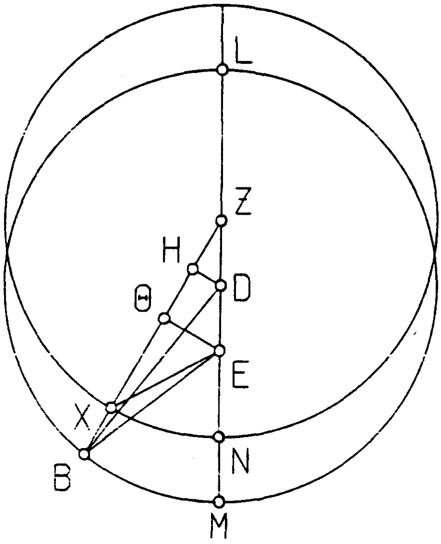

Fig. 11.4

Therefore the corresponding chords

$$\left.\begin{array}{r} DH = 1;13^p \\ \text{and } ZH \approx 120^p \end{array}\right\} \text{where hypotenuse } DZ = 120^p.$$

Therefore where $DZ = 2;42^p$ and the radius of the eccentre, $DB = 60^p$,

$$DH = 0;2^p$$
$$\text{and } ZH = 2;42^p.$$

And $HB = 60^p$ in the same units (for it is negligibly smaller than hypotenuse [of right-angled triangle HBD] BD).

Furthermore, since $\Theta H = HZ$, and $E\Theta = 2DH$, H372

by subtraction, $\Theta B = 57;18^p$ where $E\Theta = 0;4^p$.

Hence hypotenuse [of right-angled triangle $E\Theta B$] $EB = 57;18^p$ in the same units.

Therefore, where $EB = 120^p$, $E\Theta \approx 0;8^p$,

and, in the circle about right-angled triangle $BE\Theta$,

arc $E\Theta = 0;8°$ also.

$\therefore \angle EB\Theta = 0;8^{\circ\circ}$ where 2 right angles $= 360^{\circ\circ}$.

In the same way, since we showed that the whole line $Z\Theta$ [$= 2ZH$] $= 5;24^p$ where the radius of the eccentre, $ZX = 60^p$,

by subtraction, $\Theta X = 54;36^p$ where $E\Theta = 0;4^p$.

Hence hypotenuse [of right-angled triangle $E\Theta X$] $EX = 54;36^p$ in the same units.

Therefore, where $EX = 120^p$, $E\Theta \approx 0;10^p$,

and, in the circle about right-angled triangle $E\Theta X$,

arc $E\Theta = 0;10°$.

$\therefore \angle EX\Theta = 0;10^{\circ\circ}$ where 2 right angles $= 360^{\circ\circ}$,

and, by subtraction [of $\angle EB\Theta$], $\angle BEX = \left\{\begin{array}{l} 0;2^{\circ\circ} \text{ in the same units} \\ 0;1° \text{ where 4 right angles} = 360°. \end{array}\right.$

Here, then, it is clear that the planet, since its apparent longitude at the

H373 second opposition, when it was viewed along line EB, was ♓ 7;54°, would, if it had been viewed along line EX, have had a longitude of only ♓ 7;53°.

So let the diagram for the third opposition be drawn, to the rear of the perigee [Fig. 11.5].[6]

Then, since arc NX of the eccentre is given as 32;51°,

$$\angle \text{NZX} = \begin{cases} 32;51^\circ & \text{where 4 right angles} = 360^\circ \\ 65;42^{\circ\circ} & \text{where 2 right angles} = 360^{\circ\circ}. \end{cases}$$

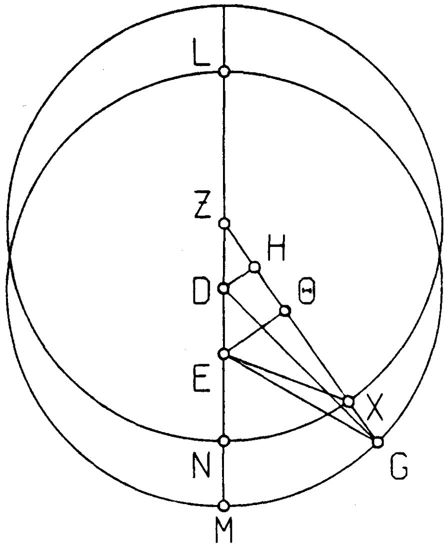

Fig. 11.5

Therefore, in the circle about right-angled triangle DZH,

arc DH = 65;42°

and arc ZH = 114;18° (supplement).

Therefore the corresponding chords

$$\left.\begin{array}{r} \text{DH} = 65;6^p \\ \text{and ZH} = 100;49^p \end{array}\right\} \text{where hypotenuse DZ} = 120^p.$$

Therefore where DZ = 2;42ᵖ and the radius of the eccentre, DG = 60ᵖ,

H374 DH = 1;28ᵖ

and ZH = 2;16ᵖ.

And since $GD^2 - DH^2 = GH^2$,

GH ≈ 59;59ᵖ.

Similarly, since ΘH = HZ, and EΘ = 2DH,

by subtraction, GΘ = 57;43ᵖ where EΘ = 2;56ᵖ.

Hence hypotenuse [of right-angled triangle EΘG] EG = 57;47ᵖ in the same units.

Therefore, where EG = 120ᵖ, EΘ = 6;5ᵖ,

[6] Heiberg's figure (p. 373) is wrong: ΔΞ has been connected instead of ΔΓ, and Λ is in the wrong place and misprinted as A. Corrected by Manitius.

and, in the circle about right-angled triangle GEΘ,

arc EΘ ≈ 5;48°.

∴ ∠ EGΘ = 5;48°° where 2 right angles = 360°°.

In the same way, since the whole line ZΘ[= 2ZH] comes to 4;32^p

where the radius of the eccentre, ZX = 60^p,

by subtraction, XΘ = 55;28^p where EΘ was found to be 2;56^p.

Hence hypotenuse [of right-angled triangle EΘX] EX = 55;33^p in the same units.

Therefore, where EX = 120^p, EΘ = 6;20^p,

and, in the circle about right-angled triangle EΘX,

arc EΘ = 6;2°.

∴ ∠ EXΘ = 6;2°° where 2 right angles = 360°°,

and, by subtraction [of ∠ EGΘ], ∠ GEX = $\begin{cases} 0;14^{\circ\circ} \text{ in the same units} \\ 0;7^{\circ} \text{ where 4 right angles} = 360^{\circ}. \end{cases}$

Hence, since the planet at the 3rd opposition, when viewed along line EG, had a longitude of ♈ 14;23°, it is clear that, if it had been on line EX, it would have had a longitude of ♈ 14;30°. And we showed that its [corrected] longitudes [would have been] H375

at the first opposition ♏ 23;14°

at the second opposition ♓ 7;53°.

Hence we calculate the apparent intervals [in longitude] of the planet, taken, not with respect to the eccentre carrying the epicycle centre, but with respect to the eccentre producing the mean motion [i.e. the equant],[7] as

from first to second oppositions	104;39°
from second to third oppositions	36;37°.

Starting from these data, by means of the previously demonstrated theorem we find the distance between the centres of the ecliptic and the eccentre producing the mean motion of the epicycle as about

5;30^p where the diameter of the eccentre is 120^p;

and, for the arcs of the eccentre,

from the apogee to the first opposition:	77;15°
from the second opposition to the perigee	2;50°
from the perigee to the third opposition	30;36°.

The above quantities have been accurately determined by this method, for the differences in the intervals [as measured along deferent and equant], when calculated from these data, are very nearly the same as the previous set.[8] That is [also] clear from the fact that the apparent intervals [in longitude] of the planet derived from the ratios we have thus found turn out to be the same as those observed; we can show this as follows. H376

Once again, let the diagram for the first opposition be drawn [Fig. 11.6], but containing only the eccentre carrying the epicycle centre. Then, since

∠ LZA was shown to be 77;15° where 4 right angles = 360°,

∠ LZA = ∠ DZH (vertically opposite) = 154;30°° where 2 right angles = 360°°.

[7] I.e. the apparent intervals which would result if the epicycle were carried, not on the actual deferent, but on the equant. Cf. XI 5 p. 529, where this is stated explicitly. Cf. also p. 492.

[8] Indeed, a further iteration produces a change of much less than 0;1^p in the eccentricity, and about 0;10° in the line of the apsides.

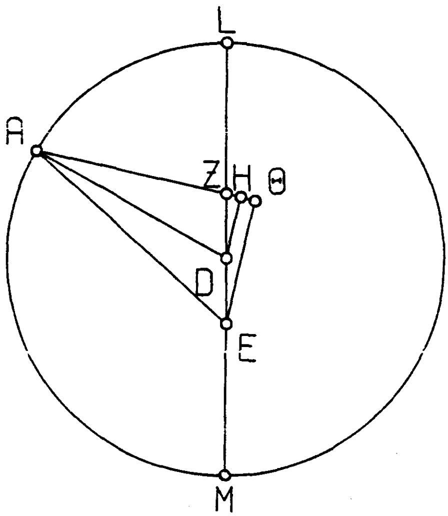

Fig. 11.6

Therefore, in the circle about right-angled triangle DZH,

arc DH = 154;30°

and arc ZH = 25;30° (supplement).

Therefore the corresponding chords

$$\left.\begin{array}{r} \text{DH} = 117;2^{p} \\ \text{and ZH} = 26;29^{p} \end{array}\right\} \text{where hypotenuse DZ} = 120^{p}.$$

H377 Therefore where ZD = 2;45^p and the radius of the eccentre DA = 60^p,

DH = 2;41^p

and ZH = 0;36^p.

Then, by the same argument as in the previous proof,

AH [= $\sqrt{AD^2 - DH^2}$] = 59;56^p in the same units,

and, by addition [of HΘ = ZH], AΘ = 60;32^p where EΘ (= 2DH) = 5;22^p. Therefore hypotenuse [of right-angled triangle AEΘ] AE comes to 60;46^p in the same units.

Therefore, where AE = 120^p, EΘ = 10;36^p,

and, in the circle about right-angled triangle AEΘ,

arc EΘ = 10;8°.

∴ ∠ EAΘ = 10;8°° where 2 right angles = 360°°,

and, by subtraction [of ∠ EAΘ from ∠ LZA],

$$\angle \text{LEA} = \begin{cases} 144;22^{\circ\circ} \text{ in the same units} \\ 72;11^{\circ} \text{ where 4 right angles} = 360^{\circ}. \end{cases}$$

That [72;11°], then, was the distance in the ecliptic[9] of the planet from its apogee at the first opposition.

[9] So we must translate τοῦ ζῳδιακοῦ (i.e. take it closely with μοίρας) at H377,16, to make any sense at all. But its position in the sentence, and redundance, make me suspect it as an interpolation, although it is in all branches of the ms. tradition.

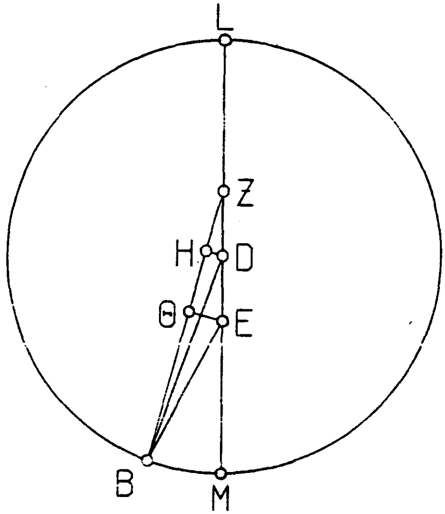

Fig. 11.7

Again, let the [corresponding] diagram for the second opposition be drawn [Fig. 11.7]. [Then,] since

$$\angle \text{ BZM is given as} \begin{cases} 2;50^\circ & \text{where 4 right angles} = 360^\circ \\ 5;40^{\circ\circ} & \text{where 2 right angles} = 360^{\circ\circ}, \end{cases}$$

in the circle about right-angled triangle DZH,

arc DH = 5;40°

and arc ZH = 174;20° (supplement). H378

Therefore the corresponding chords

$$\left.\begin{array}{r} \text{DH} = 5;55^p \\ \text{and ZH} = 119;51^p \end{array}\right\} \text{where hypotenuse DZ} = 120^p.$$

Therefore where DZ = 2;45^p and the radius of the eccentre, DB = 60^p,

DH = 0;8^p

and ZH ≈ 2;45^p.

And, by the same [argument as previously],

BH ≈ 60^p in the same units,

and, by subtraction [of HΘ = ZH], BΘ = 57;15^p where EΘ = 0;16^p.
Hence hypotenuse [of right-angled triangle EBΘ] EB comes to 57;15^p in the same units.

Therefore, where EB = 120^p, EΘ = 0;33^p,

and, in the circle about right-angled triangle BEΘ,

arc EΘ = 0;32°.

∴ ∠ EBΘ = 0;32$^{\circ\circ}$ where 2 right angles = 360$^{\circ\circ}$.

$$\text{And, by addition [of } \angle \text{ BZM], } \angle \text{ BEM} = \begin{cases} 6;12^{\circ\circ} & \text{in the same units} \\ 3;6^\circ & \text{where 4 right angles} = 360^{\circ\circ}. \end{cases}$$

Therefore the distance of the planet in advance of the perigee at the second opposition was 3;6°. And we showed [p. 516] that at the first opposition it was H379

72;11° to the rear of the apogee.[10] Thus the computed apparent interval from first to second oppositions is the supplement [of 3;6° + 72;11°], 104;43°, in agreement with the interval derived from the observations [p. 507].

So let the [corresponding] diagram for the third opposition be drawn [Fig. 11.8]. [Then,] since

$$\angle \text{MZG was shown to be} \begin{cases} 30;36^\circ \text{ where 4 right angles} = 360^\circ \\ 61;12^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ}, \end{cases}$$

in the circle about right-angled triangle DZH,

arc DH = 61;12°

and arc ZH = 118;48° (supplement).

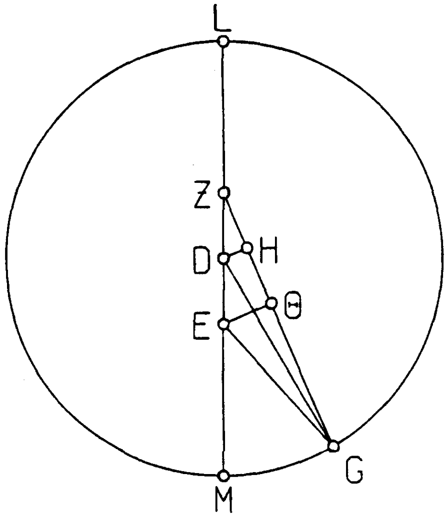

Fig. 11.8

Therefore the corresponding chords

$$\left.\begin{array}{r} \text{DH} = 61;6^p \\ \text{and ZH} = 103;17^p \end{array}\right\} \text{where hypotenuse DZ} = 120^p.$$

Therefore where DZ = $2;45^p$ and the radius of the eccentre, GD = 60^p,

DH = $1;24^p$

and ZH = $2;22^p$.

And, by the same [argument as previously],

GH = $59;59^p$,

H380 and, by subtraction [of HΘ = ZH], GΘ = $57;37^p$ where EΘ = $2;48^p$.

Therefore hypotenuse [of right-angled triangle EGΘ] EG = $57;41^p$ in the same units;

and hence, where EG = 120^p, EΘ = $5;50^p$,

and, in the circle about right-angled triangle GEΘ,

arc EΘ = 5;34°

∴ ∠ EGΘ = 5;34°° where 2 right angles = 360°°.

[10] Reading εἰς τὰ ἑπόμενα τοῦ ἀπογείου (with D,Ar) at H379,3 for εἰς τὰ ἑπόμενα ('to the rear'). Corrected by Manitius.

And, by addition [of ∠ MZG],

$$\angle \text{MEG} = \begin{cases} 66;46^{\circ\circ} \text{ in the same units} \\ 33;23^{\circ} \text{ where 4 right angles} = 360^{\circ}. \end{cases}$$

That [33;23°], then, was the distance of the planet to the rear of the perigee at the third opposition. And we showed that at the second opposition its distance in advance of the same perigee was 3;6°. Therefore the apparent interval [in longitude] from the second to the third oppositions is computed as the sum [of the above], 36;29°, once again in agreement with the observed interval [p. 507].

It is immediately clear, since the planet at the third opposition had an observed longitude of ♈ 14;23° and, as we showed, was 33;23° to the rear of the perigee, that at that moment the perigee of its eccentre had a longitude of ♓ 11°, while its apogee was diametrically opposite at ♍ 11°. H381

And if [see Fig. 11.9][11] we draw the epicycle HΘK about centre G, we will immediately have:

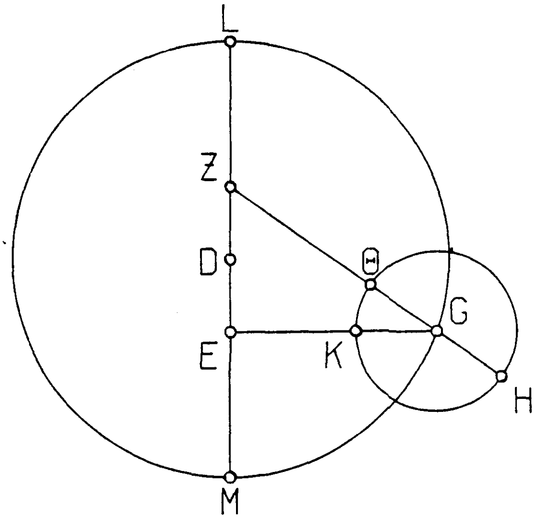

Fig. 11.9

the mean position in longitude [counted] from the apogee of the eccentre, L, as 210;36° (for we have shown that ∠ MZG = 30;36°);
and the arc ΘK of the epicycle from the perigee Θ to the planet K as 2;47° (for we showed that

$$\angle \text{EGZ} = \begin{cases} 5;34^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \\ 2;47^{\circ} \text{ where 4 right angles} = 360^{\circ}). \end{cases}$$

Therefore at the moment of the third opposition, namely *in the first year of* Antoninus, Athyr [III] 20/21 in the Egyptian calendar, 5 hours after midnight, the planet Jupiter had the following mean positions:

in longitude	210;36° from the apogee of the eccentre (i.e. its mean longitude was ♈ 11;36°)	H382
in anomaly	182;47° from the apogee of the epicycle, H.	

[11] Heiberg's figure on p. 381 is wrong: he has connected ΔΓ instead of EΓ. Corrected by Manitius.

2. {*Demonstration of the size of Jupiter's epicycle*}

Next, to demonstrate the size of the epicycle, we again took an observation, which we obtained by sighting [with the astrolabe], in the second year of Antoninus, Mesore [XII] 26/27 in the Egyptian calendar [139 July 10/11], before sunrise, i.e. about 5 equinoctial hours after midnight (for the mean longitude of the sun was ♋ 16;11°, and the second degree of Aries [i.e. ♈ 1°–2°] was culminating according to the astrolabe). At that moment Jupiter, when sighted with respect to the bright star in the Hyades, was seen to have a longitude of ♊ $15\frac{3}{4}$°, and also had the same apparent longitude as the centre of the moon, which lay to the south of it. For that moment[12] we find, by means of the [kind of] calculations [previously] explained:

moon's mean longitude ♊ 9;0°
H383 moon's [mean] anomaly counted from the epicycle apogee 272;5°
hence its true position ♊ 14;50°
and its apparent position at Alexandria ♊ 15;45°.

Thus from these considerations too Jupiter's longitude was ♊ $15\frac{3}{4}$°.

Furthermore, the time interval from the third opposition to the above observation comprises

1 Egyptian year and 276 days,

and this interval produces

in longitude: 53;17°
and in anomaly: 218;31°

(for it will make no sensible difference even if this kind of calculation is carried out rather crudely);[13] so, if we add the latter to the [mean] positions derived for the third opposition, we will get, for the moment of the present observation, [the mean positions]:

in longitude 263;53° from the apogee (which is in approximately the same position [as at the third opposition])[14]
in anomaly 41;18° from the apogee of the epicycle.

With the above as data, let the diagram for the similar demonstration in the case of Mars [Fig. 10.17] be repeated [Fig. 11.10], [but] with the epicycle in a position to the rear of the perigee of the eccentre, and with the planet past the apogee of the epicycle, in accordance with the mean positions in longitude and anomaly set out here.

Then, since the mean position in longitude from the apogee of the eccentre is
H384 263;53°,

$$\angle \mathrm{BZG} = \begin{cases} 83;53^\circ & \text{where 4 right angles} = 360^\circ \\ 167;46^{\circ\circ} & \text{where 2 right angles} = 360^{\circ\circ}. \end{cases}$$

[12] These positions were (correctly) computed, not for 5 a.m., but for 4;42 a.m., i.e. the correct equation of time with respect to epoch of era Nabonassar has been applied. Cf. p. 499 n.57.

[13] These intervals are correct to the nearest minute if one computes for exactly $1^y\,276^d$. However, for 18 mins. less (cf. n.12) one finds 218;30° for the motion in anomaly. Is it this neglect of the equation of time to which Ptolemy refers by 'rather crudely'?

[14] I.e. in less than 2 years the precessional motion of the apogee is negligible.

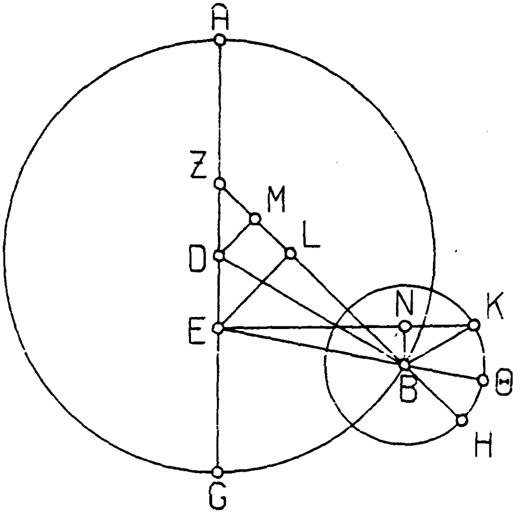

Fig. 11.10

Therefore, in the circle about right-angled triangle DZM,

arc DM = 167;46°

and arc ZM = 12;14° (supplement).

Therefore the corresponding chords

$$\left.\begin{aligned} DM &= 119;19^p \\ \text{and } ZM &= 12;47^p \end{aligned}\right\} \text{ where hypotenuse } DZ = 120^p.$$

Therefore where DZ = $2;45^p$ and the radius of the eccentre, DB = 60^p,

DM ≈ $2;44^p$

and ZM = $0;18^p$.

And since $DB^2 - DM^2 = MB^2$,

MB = $59;56^p$ in the same units.

Similarly, since ZM = ML and EL = 2DM,

by subtraction, LB = $59;38^p$ where EL is computed as $5;28^p$. H385

Hence hypotenuse [of right-angled triangle LBE] EB = $59;52^p$ in the same units.

Therefore, where EB = 120^p, EL ≈ $10:58^p$,

and, in the circle about right-angled triangle BEL,

arc EL = 10;30°.

∴ ∠ EBZ = 10;30°° where 2 right angles = 360°°.

But ∠ BZG = 167;46°° in the same units.

Therefore, by addition, ∠ BEG = 178;16°° in the same units.

Furthermore, since the approximate longitude of the perigee G is ♓ 11°, and the apparent longitude of the planet, as viewed along line EK, was ♊ 15;45°,

$$\angle\ KEG = \begin{cases} 94;45^\circ \text{ where 4 right angles} = 360^\circ \\ 189;30^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ}. \end{cases}$$

And, by subtraction [of ∠ BEG], ∠ BEK = 11;14°° in the same units.

Therefore, in the circle about right-angled triangle BEN,

arc BN = 11;14°

and BN = $11;44^p$ where hypotenuse EB = 120^p.

Therefore, where EB = 59;52^p, and the radius of the eccentre is 60^p,
BN = 5;50^p.
Similarly, since arc HK = 41;18°,

H386 $\angle$ HBK = $\begin{cases} 41;18^\circ \text{ where 4 right angles} = 360^\circ \\ 82;36^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ}. \end{cases}$

But $\angle$ EBZ (= $\angle$ HBΘ) = 10;30°° in the same units.
Therefore, by subtraction, $\angle$ ΘBK ≐ 72;6°°.
And we showed that $\angle$ KEΘ = 11;14°° in the same units.
Therefore, by subtraction, $\angle$ BKN = 60;52°° in the same units.
Therefore, in the circle about right-angled triangle BKN,
arc BN = 60;52°
and BN = 60;47^p where hypotenuse BK = 120^p.
Therefore where BN = 5;50^p and the radius of the eccentre is 60^p, the radius of the epicycle, BK ≈ 11;30^p.[15]

Q.E.D.

3. {*On the correction of the periodic motions of Jupiter*}

Next, to [determine] the periodic motions, we again took one of the precisely recorded ancient observations. In this it is declared that in the 45th year of the calendar of Dionysius, on Parthenon 10, the planet Jupiter occulted[16] the southernmost [of the 2] Aselli at dawn. Now the moment [of the observation] is in the 83rd year from the death of Alexander, Epiphi [XI] 17/18 in the Egyptian calendar [-240 Sept. 3/4], dawn. For that time we find the longitude
H387 of the mean sun as ♍ 9;56°. But the star called 'the southern Asellus' among those surrounding the nebula in Cancer had a longitude, at the time of our observation [of it], of ♋ 11⅓° [catalogue XXV 5]. Hence, obviously, its longitude at the observation in question was [♋] 7;33°, since to the 378 years between the observations[17] corresponds [a precessional motion of] 3;47°. Therefore the longitude of Jupiter at that moment (since it had occulted the star) was also ♋ 7;33°. Similarly, since the apogee was in ♍ 11° in our times, it must have had a longitude of ♍ 7;13° at the observation. Hence it is clear that the distance of the apparent planet from the then apogee of the eccentre was 300;20°, while the distance of the mean sun from that same apogee was 2;43°.

With the above elements as data, let there again be drawn [Fig. 11.11] a diagram similar to that for the [corresponding] demonstration for Mars [Fig. 10.18], but in this case in accordance with the positions given for the observation: [i.e.], have the epicycle, on centre B, positioned before the apogee A, and the point L, representing the mean position of the sun, a little after that

[15] There are a series of small miscalculations and rounding errors, which result in a not negligible final error (one finds 11;38^p to the nearest minute). No doubt Ptolemy was aiming at a convenient round number.

[16] Literally 'covered' (ἐπεκάλυψεν). Modern calculations show that Jupiter in fact passed ca. ¼° to the north of δ Cnc (cf. p. 658), but Ptolemy's wording is unambiguous here (cf. p. 477 n.17).

[17] The epoch of the star catalogue is Antoninus 1 = Nabonassar 885. And 885–507 = 378. But since the observation took place in the 11th month of the Egyptian year, 377 would have been more accurate.

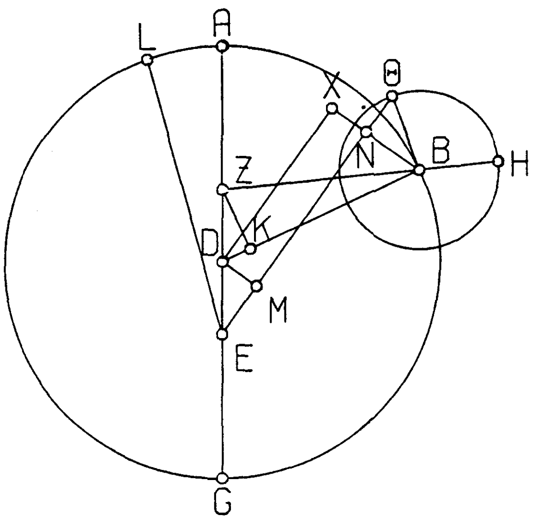

Fig. 11.11

same apogee, and hence the point Θ, representing the planet, after H, the apogee of the epicycle. And, as we always do in similar situations, we join ZBH, DB, BΘ and EΘ, and drop perpendiculars ZK on to DB, DM and BN on to EΘ, and DX on to NB (produced in this case), which forms the rectangular parallelogram DMNX. H388

Then ∠ AEΘ contains one revolution in the ecliptic less 300;20°, or 59;40°.

And ∠ AEL = 2;43°.

Therefore, by addition,

$$\angle \text{ LE}\Theta \ (= \angle \text{ B}\Theta\text{E}) = \begin{cases} 62;23^\circ \text{ where 4 right angles} = 360^\circ \\ 124;46^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ}. \end{cases}$$

Therefore, in the circle about right-angled triangle BΘN,

arc BN = 124;46°

and BN = 106;20^p where hypotenuse BΘ = 120^p; H389

Therefore where the radius of the epicycle, BΘ[18] = 11;30^p,

BN = 10;12^p.

$$\text{Again, since } \angle \text{ DEM is given as} \begin{cases} 59;40^\circ \text{ where 4 right angles} = 360^\circ \\ 119;20^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ}, \end{cases}$$

and ∠ MDE = 60;40°° in the same units (complement),

in the circle about right-angled triangle DEM

arc DM = 119;20°

and DM = 103;34^p where hypotenuse ED = 120^p.

Therefore where ED = 2;45^p and the radius of the eccentre, DB = 60^p,

DM = 2;23^p,

and, by addition, BNX = 12;35^p.

[18] Reading ἡ BΘ ἐκ τοῦ κέντρου (with D,Ar) for ἡ ἐκ τοῦ κέντρου ('the radius of the epicycle') at H389,2-3.

Therefore where hypotenuse [of right-angled triangle BDX] BD = 120^p,

$$BX = 25;10^p,$$

and, in the circle about right-angled triangle BDX,

$$\text{arc } BX = 24;14^\circ$$

$$\therefore \angle BDX = 24;14^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ},$$

and, by subtraction [from a right angle], $\angle$ BDM = $155;46^{\circ\circ}$ in the same units;
and, by addition [of $\angle$ MDE], $\angle$ BDE = $216;26^{\circ\circ}$ in the same units:
and, again by subtraction [from 2 right angles], $\angle$ BDZ = $143;34^{\circ\circ}$ in the same units.

Therefore, in the circle about right-angled triangle ZDK,

$$\text{arc } ZK = 143;34^\circ$$

$$\text{and arc } DK = 36;26^\circ \text{ (supplement).}$$

H390 Therefore the corresponding chords

$$\left.\begin{aligned} ZK &= 113;59^p \\ \text{and } DK &= 37;31^p \end{aligned}\right\} \text{ where hypotenuse } DZ = 120^p.$$

Therefore where DZ = $2;45^p$ and the radius of the eccentre, DB = 60^p,

$$KZ = 2;37^p$$

$$\text{and } DK = 0;52^p,$$

and, by subtraction [from DB], KB = $59;8^p$ in the same units.
Hence hypotenuse [of right-angled triangle ZBK] ZB = $59;12^p$ in the same units.

Therefore, where ZB = 120^p, ZK = $5;18^p$,

and, in the circle about right-angled triangle BZK,

$$\text{arc } ZK = 5;4^\circ.$$

$$\therefore \angle ZBD = 5;4^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ},$$

and, by addition [of $\angle$ BDZ],

$$\angle AZB \text{ (which comprises the mean motion in longitude)} = \begin{cases} 148;38^{\circ\circ} \text{ in the same units} \\ 74;19^\circ \text{ where 4 right angles} = 360^\circ. \end{cases}$$

And since $\angle$ HBΘ + $\angle$ BZG + 180° (i.e. here $\angle$ HBΘ − $\angle$ AZB) = $\angle$ AEL = 2;43°, we find that $\angle$ HBΘ (which comprises the planet's position [in anomaly] from the apogee of the epicycle) is 77;2°.[19]

Therefore we have shown that at the moment of the observation in question the planet Jupiter had the following mean positions:

in longitude, from the apogee of the eccentre, 285;41°
(i.e. its mean longitude was ♊ 22;54°)
in anomaly, from the apogee of the epicycle, 77;2°.

H391 And we had [already] shown that at the moment of the third opposition its distance from the apogee of the epicycle was 182;47°. Thus in the interval between the two observations, which comprises

377 Egyptian years and 128 days less approximately 1 hour,

its motion in anomaly was

105;45° beyond 345 complete revolutions.

That is, again, very nearly the same increment in anomaly as one derives from the [tables for] mean motions which we constructed. For it was from these very same elements that we derived the daily [mean motion in anomaly], by dividing

[19] There are numerous small inaccuracies and rounding errors in the preceding calculations, which to some extent cancel each other. Accurate computation gives 77;0° to the nearest minute.

the number of degrees contained in the complete revolutions plus the increment by the number of days contained in the time-interval.[20]

4. {On the epoch of Jupiter's periodic motions}

Here too again, then, since the interval from the first year of Nabonassar, Thoth 1 in the Egyptian calendar, noon, to the above-mentioned ancient observation is

506 Egyptian years and approximately 316¾ days,

and this interval comprises increments of

258;13° in longitude

and 290;58° in anomaly,[21]

if we subtract the latter from the respective [mean] positions listed above for the observation, we get, for the same moment of epoch as for the other [heavenly bodies], for Jupiter: H392

mean longitude ♎ 4;41°

mean anomaly 146;4° from the epicyclic apogee.

And, by the same [kind of computation as before],

the apogee of its eccentre will be in ♍ 2;9°.[22]

5. {Demonstration of Saturn's eccentricity and [the position of] its apogee}

To complete this topic, it remains to demonstrate the anomalies and epochs for the theory of the planet Saturn. Once again, as for the other planets, we took, first, for our investigation of [the position of] the apogee and the eccentricity, three opposition situations of the planet, in which it was diametrically opposite the sun's mean position.

[1] The first of these was observed by us, using the astrolabe instruments, in the eleventh year of Hadrian, Pachon [IX] 7/8 in the Egyptian calendar [127 Mar. 26/27], in the evening, in ♎ 1;13°;

[2] the second, in the seventeenth year of Hadrian, Epiphi [XI] 18 in the Egyptian calendar [133 June 3]. We computed the time and place of exact opposition from nearby observations as 4 hours after noon on the 18th, in ♐ 9;40°; H393

[3] we observed the third opposition in the twentieth year of Hadrian, Mesore [XII] 24 in the Egyptian calendar [136 July 8]. As before, we computed the time of exact opposition as having occurred precisely at noon on the 24th, and computed the place as ♑ 14;14°.

Of these two intervals, then, that from the first to the second opposition comprises

[20] On the actual derivation of the mean motion in anomaly for Jupiter, which remains obscure, see Appendix C.

[21] These intervals are precise (to the nearest minute) for an increment of exactly ¾ day.

[22] The apogee was in ♍ 7;13° at the observation (p. 522). In 507^y (at the rate of 1° in 100 years) it moves about 5;4°. Hence at epoch it was in ♍ 2;9°.

[in time] 6 Egyptian years 70 days 22 hours
in apparent motion of the planet 68;27°;
while that from the second to the third opposition comprises
[in time] 3 Egyptian years 35 days 20 hours
[in apparent motion] 34;34°.
And we compute for the mean motion in longitude, using rough figures,[23]
for the first interval: 75;43°
and for the second interval: 37;52°.

These intervals [in mean and true longitude] being given, we again demonstrate the required [parameters] by means of the same theorem [as before] (as if there were only one eccentre), as follows.

To avoid repetition, let there be drawn a diagram [Fig. 11.12] like those used for the same proof [previously, Figs. 10.8, 11.1]. Then since arc BG of the
H394 eccentre is given as subtending 34;34° of the ecliptic, the [corresponding] angle at the centre of the ecliptic,

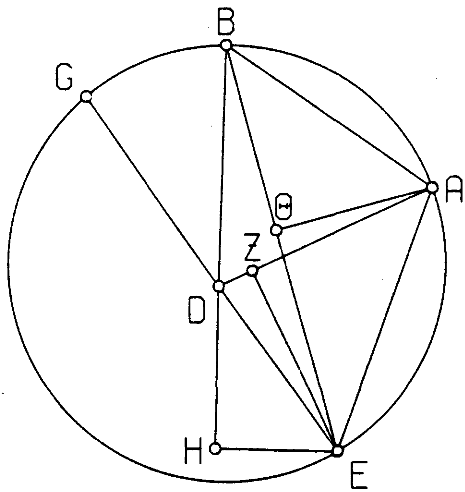

Fig. 11.12

$$\angle \text{ BDG } (= \angle \text{ EDH}) = \begin{cases} 34;34^\circ \text{ where 4 right angles} = 360^\circ \\ 69;8^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ}. \end{cases}$$

Therefore, in the circle about right-angled triangle DEH,
arc EH = 69;8°
and EH = 68;5^p where hypotenuse DE = 120^p.

Similarly, since arc BG = 37;52°, the angle at the circumference,
∠ BEG = 37;52°° where 2 right angles = 360°°,
and, by subtraction [from ∠ BDG], ∠ EBH = 31;16°° in the same units.

Therefore, in the circle about right-angled triangle EBH,
arc EH = 31;16°
and EH = 32;20^p where hypotenuse BE = 120^p.

[23] Despite Ptolemy's phrase here, the intervals in mean longitude are accurate to the nearest minute according to his own tables. Nor would the equation of time make any difference.

Therefore where EH, as we showed, is $68;5^p$, and ED = 120^p, H395

BE = $252;41^p$.

Furthermore, since the whole arc ABG subtends 103;1° of the ecliptic (the sum of both intervals [in true longitude]), the [corresponding] angle at the centre of the ecliptic,

$\angle$ ADG = 103;1° where 4 right angles = 360°.

Hence the supplementary angle, $\angle$ ADE $= \begin{cases} 76;59° & \text{in the same units} \\ 153;58^{\circ\circ} & \text{where 2 right angles} = 360^{\circ\circ}. \end{cases}$

Therefore, in the circle about right-angled triangle DEZ,

arc EZ = 153;58°

and EZ = $116;55^p$ where hypotenuse DE = 120^p.

Similarly, since arc ABG of the eccentre is found by addition [of 75;43° and 37;52°] as 113;35°, the angle at the circumference,

$\angle$ AEG = $113;35^{\circ\circ}$ where 2 right angles = $360^{\circ\circ}$.

But we found that $\angle$ ADE = $153;58^{\circ\circ}$ in the same units.

Therefore the remaining angle [in triangle ADE],

$\angle$ ZAE = $92;27^{\circ\circ}$ in the same units.

Therefore, in the circle about right-angled triangle AEZ,

arc EZ = 92;27°

and EZ = $86;39^p$ where hypotenuse AE = 120^p.

Therefore where EZ, as we showed, is $116;55^p$, and ED = 120^p

EA = $161;55^p$.

Furthermore, since arc AB of the eccentre is 75;43°, the angle at the circumference

$\angle$ AEB = $75;43^{\circ\circ}$ where 2 right angles = $360^{\circ\circ}$. H396

Therefore, in the circle about right-angled triangle AEΘ,

arc AΘ = 75;43°,

and arc EΘ = 104;17° (supplement).

Therefore the corresponding chords

$\left.\begin{array}{r} A\Theta = 73;39^p \\ \text{and } E\Theta = 94;45^p \end{array}\right\}$ where hypotenuse EA = 120^p.

Therefore where AE, as we showed, is $161;55^p$, and DE = 120^p,

AΘ = $99;23^p$[24]

and EΘ = $127;51^p$.

But we showed that the whole line EB = $252;41^p$ in the same units.

Therefore, by subtraction, ΘB = $124;50^p$ where AΘ = $99;23^p$.

And $\Theta B^2 = 15583;22$

and $A\Theta^2 = 9877;3$

and $\Theta B^2 + A\Theta^2 = AB^2 = 25460;25$.

$\therefore$ AB = $159;34^p$ where ED = 120^p and EA = $161;55^p$.

And, where the diameter of the eccentre is 120^p, AB = $73;39^p$

(for it subtends an arc of 75;43°).

Therefore where AB = $73;39^p$ and the diameter of the eccentre is 120^p, H397

ED = $55;23^p$

and EA = $74;43^p$.

[24] Reading $\overline{\kappa\gamma}$ for $\overline{\mu\gamma}$ ($99;43^p$) at H396,10 and 13. '23', which is guaranteed by the rest of Ptolemy's working, is found in Ger.

Therefore arc EA of the eccentre = 77;1°
and, by addition [of arc ABG], arc EABG = 190;36°,
and hence, by subtraction [from the circle], arc GE = 169;24°.
Therefore GDE ≈ 119;28^p where the diameter of the eccentre is 120^p.

So [see Fig. 11.13] let the centre of the eccentre be taken inside segment EAG (since it is greater than a semi-circle) as point K. Draw through K and D the diameter of the eccentre through both centres, LKDM, and let the perpendicular from K on to GE be produced [to meet the circumference] as KNX.

Then, where the diameter, LM = 120^p,
the whole line EG was shown to be 119;28^p and ED to be 55;23^p;
so, by subtraction, DG = 64;5^p in the same units.
H398 So, since ED. DG = LD.DM,
LD.DM = 3549;9^p where diameter LM is 120^p.
But LD.DM + DK^2 = LK^2 (the square on half the diameter).

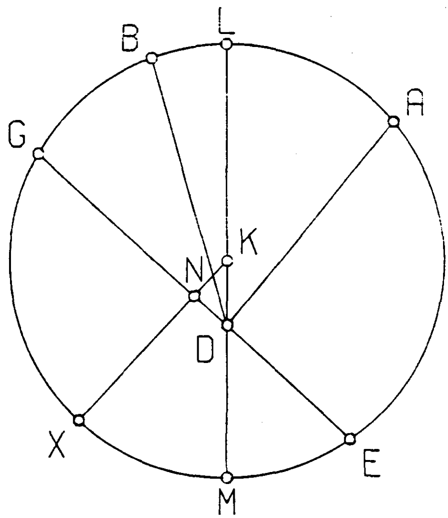

Fig. 11.13

Therefore, if from the square on half the diameter, 3600, we subtract 3549;9, we are left with DK^2 as 50;51^p in the same units.

Therefore the distance between the centres, DK ≈ 7;8^p
where the diameter of the eccentre is 120^p.[25]
Furthermore, since EN (= ½GE) = 59;44^p where diameter LM = 120^p,
and we showed that ED = 55;23^p in the same units,
by subtraction, DN = 4;21^p where DK, as we showed, = 7;8^p.
Therefore where hypotenuse [of right-angled triangle DKN] DK = 120^p,
DN = 73;11^p,

[25] DG and ED have been computed with only small inaccuracies (I find 64;5,21 and 55;23,39 for Ptolemy's 64;5 and 55;23), but the resulting value for the eccentricity, 7;3,33^p, differs significantly from Ptolemy's 7;8^p.

and, in the circle about right-angled triangle DKN

$$\text{arc DN} = 75;10^\circ.$$

$$\therefore \angle \text{DKN} = \begin{cases} 75;10^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \\ 37;35^\circ \text{ where 4 right angles} = 360^\circ. \end{cases}$$

And since ∠ DKN is an angle at the centre of the eccentre,

arc XM = 37;35°.[26]

But arc GX = ½ arc GXE = 84;42°. H399

Therefore, by subtraction [of (arc GX + arc XM) from 180°], the arc from the apogee to the third opposition,

arc GL = 57;43°.

But arc BG is given as 37;52°.

Therefore, by subtraction, the arc from the apogee to the second opposition,

arc LB = 19;51°.

Similarly, since arc AB is given as 75;43°,

by subtraction, the arc from the first opposition to the apogee,

arc AL = 55;52°.

Now again, since the epicycle centre is carried, not on this eccentre, but on that drawn with centre the point bisecting DK and with radius KL, we computed in due order, as we did for the other [planets], the differences in the apparent intervals [in true longitude] on the ecliptic which result from the above ratios (taking them to be approximately correct), if we transfer the epicycle's path to the eccentre in question, which produces the ecliptic anomaly [i.e. the equant].

Thus, let there be drawn [Fig. 11.14] the diagram for the first opposition, [similar to] the [previous] one in the same demonstration, but drawn in advance of the apogee L. Then, since the angle of the mean position in longitude,

$$\angle \text{NZX} (= \angle \text{DZH}) = \begin{cases} 55;52^\circ \text{ where 4 right angles} = 360^\circ \\ 111;44^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ}, \end{cases}$$

in the circle about right-angled triangle DZH,

arc DH = 111;44° H400

and arc ZH = 68;16° (supplement).

Therefore the corresponding chords

$$\left.\begin{aligned} \text{DH} &= 99;20^p \\ \text{and ZH} &= 67;20^p \end{aligned}\right\} \text{where hypotenuse DZ} = 120^p.$$

Therefore where the distance between the centres, DZ = $3;34^p$,

and the radius of the eccentre, DA = 60^p,

$$\text{DH} = 2;57^p$$

$$\text{and ZH} = 2;0^p.$$

And since $DA^2 - DH^2 = AH^2$,

AH = $59;56^p$ in the same units.

Similarly, since ZH = ΘH,

and ΘE = 2DH,

by addition, AΘ = $61;56^p$ where EΘ = $5;54^p$.

[26] The accumulation of small errors again leads to a significant difference between Ptolemy's result and the accurately computed value, 38;1°.

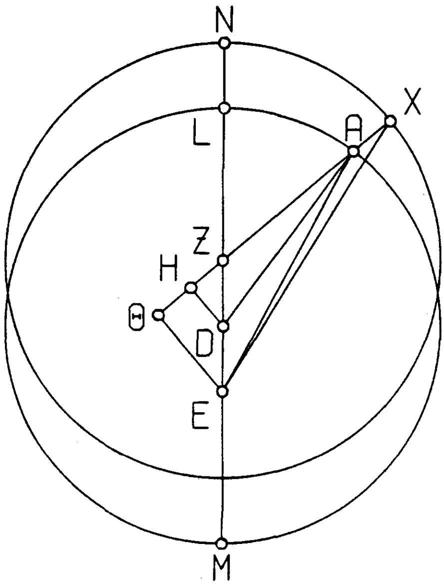

Fig. 11.14

Hence hypotenuse [of right-angled triangle ΘAE]

AE = 62;13^p in the same units.

Therefore, where hypotenuse AE = 120^p, EΘ = 11;21^p.[27]

and, in the circle about right-angled triangle AEΘ,

arc EΘ ≈ 10;51°

∴ ∠ EAΘ = 10;51°° where 2 right angles = 360°°.

H401 Furthermore, where EΘ = 5;54^p,

the radius of the eccentre, ZX = 60^p, and ZΘ = 4^p;

hence, by addition, ΘX, obviously, = 64^p,

and we get hypotenuse [of right-angled triangle EΘX]

EX as 64;16^p in the same units.

Therefore, where hypotenuse EX = 120^p, ΘE = 11;2^p,

and, in the circle about right-angled triangle EΘX,

arc ΘE = 10;33°.

∴ ∠ EXΘ = 10;33°° where 2 right angles = 360°°.

But we showed that ∠ EAΘ = 10;51°° in the same units.

Therefore, by subtraction, the angle of the required difference,

$$\angle \text{AEX} = \begin{cases} 0;18^{\circ\circ} & \text{where 2 right angles} = 360^{\circ\circ} \\ 0;9^{\circ} & \text{where 4 right angles} = 360^{\circ}. \end{cases}$$

But the planet at the first opposition, when viewed along line AE, had an apparent longitude of ♎ 1;13°. Thus it is clear that if the epicycle centre were carried, not on AL, but on NX, it would have been at point X [at the first

[27] I find 11;23^p, leading to arc EΘ = 10;53°.

opposition], and the planet would have been seen along line EX, 9′ in advance of its [actual] position at A, with a longitude of ♎ 1;4°.

Again, let there be drawn [Fig. 11.15] the diagram for the second opposition, [like that] in the same demonstration [previously], but drawn to the rear of the apogee. [Then,] since arc NX of the eccentre was shown to be 19;51°, H402

$$\angle \text{NZX} = \angle \text{DZH} \ \begin{matrix}\text{(vertically}\\ \text{opposite)}\end{matrix} = \begin{cases} 19;51^\circ & \text{where 4 right angles} = 360^\circ \\ 39;42^{\circ\circ} & \text{where 2 right angles} = 360^{\circ\circ}. \end{cases}$$

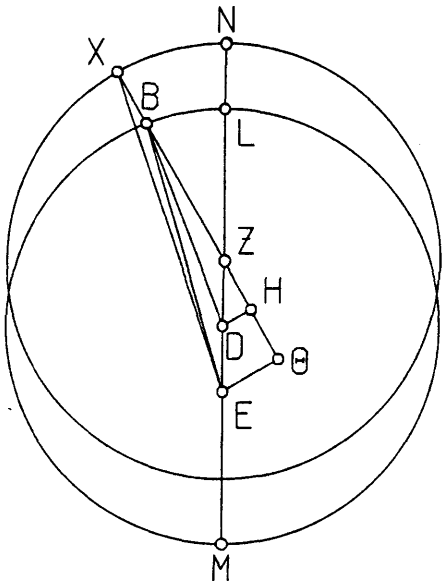

Fig. 11.15

Therefore, in the circle about right-angled triangle DZH,

arc DH = 39;42°

and arc ZH = 140;18° (supplement).

Therefore the corresponding chords

$$\left.\begin{aligned} \text{DH} &= \ \ 40;45^p \\ \text{and ZH} &= 112;52^p \end{aligned}\right\} \text{where hypotenuse DZ} = 120^p.$$

Therefore, where DZ = 3;34^p and the radius of the eccentre, DB = 60^p,

DH = 1;13^p

and ZH = 3;21^p.

And, since $\text{DB}^2 - \text{DH}^2 = \text{BH}^2$,

BH ≈ 59;59^p in the same units.

Similarly, since ZH = HΘ, and EΘ = 2DH,

by addition, BΘ = 63;20^p where EΘ = 2;26^p.

Hence hypotenuse [of right-angled triangle BEΘ]

EB = 63;23^p in the same units. H403

Therefore where hypotenuse BE = 120^p, EΘ = 4;36^p, and, in the circle about right-angled triangle BEΘ,

arc EΘ = 4;24°

∴ ∠ EBΘ = 4;24°° where 2 right angles = 360°°.

Likewise, where the radius of the eccentre, XZ = 60^p,

ZΘ is computed as $6;42^p$;

so, by addition, XΘ = $66;42^p$ where EΘ is given as $2;26^p$.

Hence we find hypotenuse [of right-angled triangle EΘX] EX as $66;45^p$ in the same units.

Therefore, where hypotenuse EX = 120^p, EΘ = $4;23^p$,

and, in the circle about right-angled triangle EΘX,

arc EΘ = 4;12°.

∴ ∠ EXΘ = 4;12°° where 2 right angles = 360°°.

But ∠ EBΘ was shown to be 4;24°° in the same units.

Therefore, by subtraction, ∠ BEX = $\begin{cases} 0;12^{\circ\circ} \text{ in the same units} \\ 0;6^{\circ} \quad \text{where 4 right angles} = 360^{\circ\circ}. \end{cases}$

Here too, then, it is clear, since the planet at the second opposition, when
H404 viewed along EB, had a longitude of ♐ 9;40°, that if, instead, it were viewed along EX, it would have a longitude of ♐ 9;46°. And we showed that at the first opposition it would, on the same hypothesis, have had a longitude of ♎ 1;4°. Hence it is clear that the interval in apparent [longitude] from the first to the second opposition, if it were taken with respect to the eccentre NX, would be 68;42° of the ecliptic.

Let the diagram for the third opposition be drawn [Fig. 11.16], with the same layout as that set out above for the second. [Then,] since we showed [p. 529] that arc NX = 57;43°,

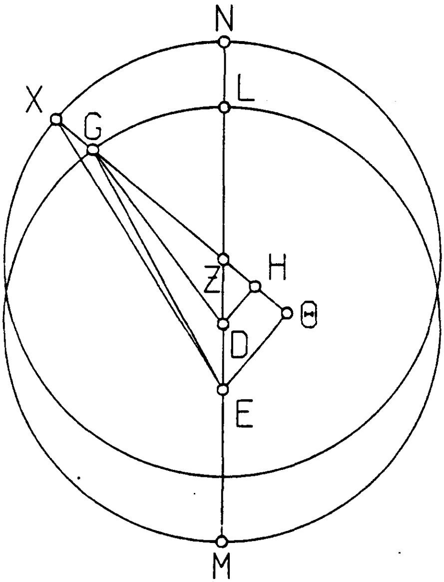

Fig. 11.16

$$\angle \mathrm{NZX}\ (= \angle \mathrm{DZH}) = \begin{cases} 57;43^\circ & \text{where 4 right angles} = 360^\circ \\ 115;26^{\circ\circ} & \text{where 2 right angles} = 360^{\circ\circ}. \end{cases}$$

Therefore, in the circle about right-angled triangle DZH,

arc DH = 115;26°

and arc ZH = 64;34° (supplement).

Therefore the corresponding chords H405

$$\left.\begin{aligned} \mathrm{DH} &= 101;27^p \\ \text{and } \mathrm{ZH} &= 64;6^p \end{aligned}\right\} \text{where hypotenuse DZ} = 120^p.$$

Therefore where DZ = 3;34^p and the radius of the eccentre, DG = 60^p,

DH = 3;1^p

and ZH = 1;54^p.

Again, since $\mathrm{DG}^2 - \mathrm{DH}^2 = \mathrm{GH}^2$,

GH = 59;56^p in the same units.

Similarly, since ZH = ΘH, and EΘ = 2DH,

by addition, GΘ = 61;50^p where EΘ is computed as 6;2^p;

Hence hypotenuse [of right-angled triangle GEΘ]

EG = 62;8^p in the same units.

Therefore, where hypotenuse GE = 120^p, EΘ = 11;39^p,

and, in the circle about right-angled triangle GEΘ,

arc EΘ ≈ 11;9°.

∴ ∠ EGΘ = 11;9°° where 2 right angles = 360°°.

Similarly, where the radius of the eccentre, XZ = 60^p,

ZΘ is computed as 3;48^p:

so, by addition, XΘ = 63;48^p where EΘ was found to be 6;2^p.

Hence hypotenuse [of right-angled triangle EΘX]

EX = 64;5^p in the same units.

Therefore, where hypotenuse EX = 120^p, EΘ = 11;18^p, H406

and, in the circle about right-angled triangle EΘX,

arc EΘ = 10;49°

∴ ∠ EXΘ = 10;49°° where 2 right angles = 360°°.

But we showed that ∠ EGΘ = 11;9°° in the same units.

$$\text{Therefore, by subtraction, } \angle \mathrm{GEX} = \begin{cases} 0;20^{\circ\circ} & \text{in the same units} \\ 0;10^\circ & \text{where 4 right angles} = 360^\circ. \end{cases}$$

Hence, since the planet at the third opposition, when viewed along line EG, had a longitude of ♑ 14;14°, it is clear that, if it had been on line EX, it would have had a longitude of ♑ 14;24°, and the interval from the second opposition to the third in apparent [longitude], taken with respect to eccentre NX, would have been [♑ 14;24° − ♐ 9;46° =] 34;38°.

Starting from these intervals, then, we follow through the same theorem, and find the distance between the centres of the ecliptic and the eccentre which produces the uniform motion of the epicycle (i.e. the distance equal to EZ [in Fig. 11.16]) as about 6;50^p where the diameter of the eccentre is 120^p, and [the following values] for the arcs of that same eccentre:

from the first opposition to the apogee	57;5°
from the apogee to the second opposition	18;38°
from the apogee to the third opposition	56;30°.

Here again, the above quantities have been accurately derived by this H407 method; for the differences in the ecliptic arcs computed from these arcs are very nearly the same as the previous set,[28] and the apparent intervals [in longitude] of the planet are found to be in agreement with those observed, as we shall show by a procedure similar [to the preceding ones for Jupiter and Mars].

Let the diagram for the first opposition be drawn [Fig. 11.17], with only the eccentre carrying the epicycle centre. Then since the angle subtending 57;5° of the eccentre [i.e. equant],

$\angle$ AZL = 57;5° where 4 right angles = 360°,

and $\angle$ AZL = $\angle$ DZH (vertically opposite) = 114;10°° where 2 right angles = 360°°,

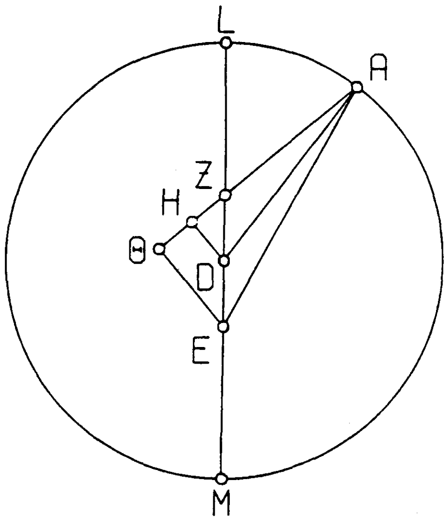

Fig. 11.17

in the circle about right-angled triangle DZH,

arc DH = 114;10°

and arc ZH = 65;50° (supplement).

Therefore the corresponding chords

$$\left.\begin{array}{r} \text{DH} = 100;44^p \\ \text{and ZH} = 65;13^p \end{array}\right\} \text{ where hypotenuse DZ} = 120^p.$$

H408 Therefore where the distance between the centres, DZ = 3;25^p, and the radius of the eccentre, DA = 60^p,

DH = 2;52^p

and ZH = 1;51^p.

Furthermore, since AD^2 − DH^2 = AH^2,

AH = 59;56^p in the same units.

Similarly, since ZH = HΘ, and EΘ = 2DH,

[28] Indeed, with one more iteration, one finds corrections of 0;9,28°, 0;5,36° and 0;9,40° (compare Ptolemy's 9′, 6′ and 10′), and a result for the eccentricity and apogee agreeing very closely with that adopted by Ptolemy.

by addition, AΘ = 61;47^p where EΘ is computed as 5;44^p.

Hence hypotenuse [of right-angled triangle AEΘ]

AE = 62;3^p in the same units.

Therefore, where hypotenuse AE = 120^p, EΘ = 11;5^p,

and, in the circle about right-angled triangle AEΘ,

arc EΘ = 10;36°.

$\therefore$ ∠ EAZ = 10;36°° where 2 right angles = 360°°.

But ∠ AZL was given as 114;10°° in the same units.

Therefore, by subtraction, ∠ AEL $= \begin{cases} 103;34^{\circ\circ} \text{ in the same units} \\ 51;47^{\circ} \text{ where 4 right angles} = 360^{\circ}. \end{cases}$

That [51;47°], then, was the amount by which the planet was in advance of the apogee at the first opposition.

Again, let the diagram for the second opposition be drawn in the same manner [Fig. 11.18]. [Then,] since

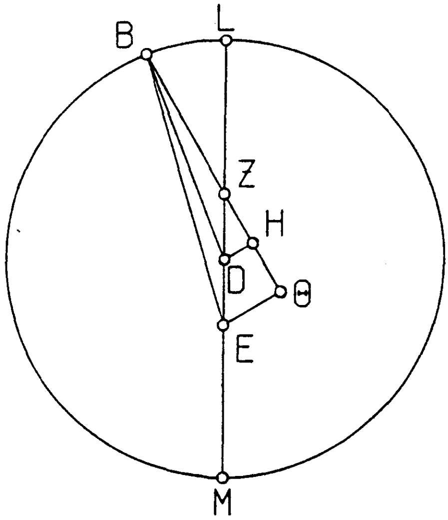

Fig. 11.18

∠ BZL was shown to be 18;38° where 4 right angles = 360°,

and ∠ BZL = ∠ DZH (vertically opposite) = 37;16°° where 2 right angles = 360°, H409

in the circle about right-angled triangle DZH,

arc DH = 37;16°

and arc ZH = 142;44° (supplement).

Therefore the corresponding chords

$\left.\begin{array}{r} \text{DH} = 38;20^p \\ \text{and ZH} = 113;43^p \end{array}\right\}$ where hypotenuse DZ = 120^p.

So where DZ = 3;25^p and the radius of the eccentre, DB = 60^p,

DH = 1;5^p

and ZH = 3;14^p.

And since $DB^2 - DH^2 = BH^2$,

$BH = 59;59^p$ in the same units.

Similarly, since $ZH = H\Theta$, and $E\Theta = 2DH$,

by addition, $B\Theta = 63;13^p$ where $E\Theta$ is computed as $2;10^p$.

Hence hypotenuse [of right-angled triangle $BE\Theta$]

$EB = 63;15^p$ in the same units.

Therefore, where hypotenuse $EB = 120^p$, $\Theta E = 4;7^p$,

H410 and, in the circle about right-angled triangle $BE\Theta$,

arc $\Theta E = 3;56°$.

$\therefore \angle EBZ = 3;56^{\circ\circ}$ where 2 right angles $= 360^{\circ\circ}$.

But $\angle BZL$ was given as $37;16^{\circ\circ}$ in the same units.

Therefore, by subtraction, $\angle BEL = \begin{cases} 33;20^{\circ\circ} \text{ in the same units} \\ 16;40° \text{ where 4 right angles} = 360°. \end{cases}$

Therefore at the second opposition the apparent position of the planet was 16;40° to the rear of the apogee. And we showed that at the first opposition it was 51;47° in advance of the same apogee. Therefore the interval in apparent [longitude] from the first opposition to the second is computed as the sum of the above amounts, 68;27°, in agreement with the distance found from the observations [p. 526].

Now let the diagram for the third opposition be drawn [Fig. 11.19]. [Then,] since

$\angle GZL$ was shown to be 56;30° where 4 right angles = 360°,

and $\angle GZL = \angle DZH$ (vertically opposite) $= 113;0^{\circ\circ}$ where 2 right angles $= 360^{\circ\circ}$,

in the circle about right-angled triangle DZH,

arc $DH = 113°$

H411 and arc $ZH = 67°$ (supplement).

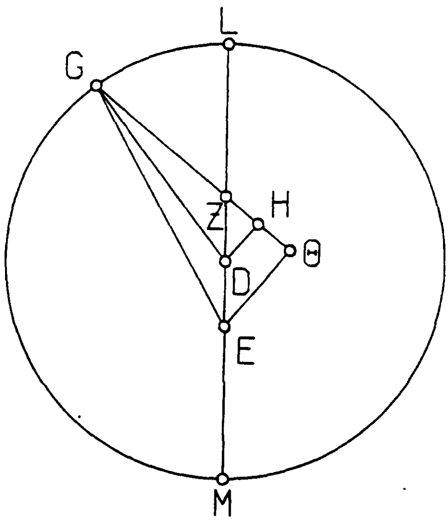

Fig. 11.19

Therefore the corresponding chords

$$\left.\begin{array}{r}DH = 100;4^p\\ \text{and } ZH = 66;14^p\end{array}\right\} \text{ where hypotenuse } DZ = 120^p.$$

Therefore, where $DZ = 3;25^p$, and the radius of the eccentre, $DG = 60^p$,

$$DH = 2;51^p$$
$$\text{and } ZH = 1;53^p.$$

Again, since $DG^2 - DH^2 = GH^2$,

$$GH = 59;56^p \text{ in the same units.}$$

Similarly, since $ZH = H\Theta$, and $E\Theta = 2DH$,

by addition, $G\Theta = 61;49^p$ where $E\Theta$ is computed as $5;42^p$;

hence hypotenuse [of right-angled triangle $GE\Theta$]

$$EG = 62;5^p \text{ in the same units.}$$

Therefore, where hypotenuse $GE = 120^p$, $E\Theta = 11;1^p$,[29]

and, in the circle about right-angled triangle $GE\Theta$,

$$\text{arc } E\Theta = 10;32^\circ$$
$$\therefore \angle EG\Theta = 10;32^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ}.$$

But $\angle GZL$ was given[30] as $113^{\circ\circ}$ in the same units. H412

$$\text{Therefore, by subtraction, } \angle GEL = \begin{cases} 102;28^{\circ\circ} & \text{in the same units} \\ 51;14^\circ & \text{where 4 right angles} = 360^\circ. \end{cases}$$

That [51;14°], then, is the amount by which the planet was to the rear of the apogee at the third opposition. And we showed that at the second opposition it was 16;40° to the rear of the same apogee. So the distance in apparent [longitude] from the second opposition to the third is computed as the difference [between 51;14° and 16;40°], 34;34°, which is, again, in agreement with that derived from the observations [p. 526].

It is immediately clear, since the planet at the third opposition had a longitude of ♑ 14;14°, and was shown to be 51;14° to the rear of the apogee, that the apogee of its eccentre had at that moment a longitude of ♏ 23°, while its perigee was diametrically opposite at ♉ 23°.

In the same way [as before], if we draw [Fig. 11.20] the epicycle $H\Theta$ about centre G, we immediately get the mean position of the epicycle in longitude from the apogee of the eccentre as 56;30° (as demonstrated [p. 533]), and arc ΘK of the epicycle as 5;16° (for $\angle$ EGZ was shown [above] to be $10;32^{\circ\circ}$ where 2 H413
right angles equal $360^{\circ\circ}$). Therefore, by subtraction [from 180°],

arc $H\Theta$, the arc from the apogee of the epicycle to the planet, is 174;44°.

Therefore at the moment of the third opposition, namely in the twentieth year of Hadrian, Mesore 24 in the Egyptian calendar, at noon, the planet Saturn had the following mean positions:

in longitude:	56;30° from the apogee of the eccentre
	(i.e. its [mean] longitude was ♑ 19;30°);
in anomaly:	174;44° from the apogee of the epicycle.

Q.E.D.

[29] Reading $\overline{\iota\alpha}\ \overline{\alpha}$ (with Ar) for $\overline{\iota\alpha}\ \overline{\iota}$ $(11;10^p)$ at H411,22. The reading is confirmed by the surrounding computations.

[30] Reading ὑπέκειτο, with D, for ὑπόκειται ('is given') at H412,1.

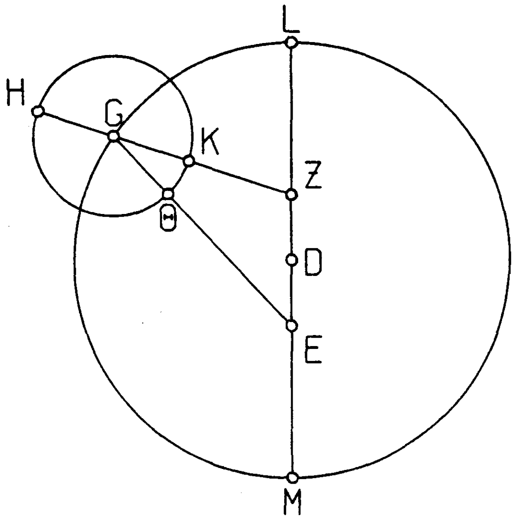

Fig. 11.20

H414 6. {*Demonstration of the size of Saturn's epicycle*}

Next, once again, in order to demonstrate the size of the epicycle, we took an observation which we made in the second year of Antoninus, Mechir [VI] 6/7 in the Egyptian calendar [138 Dec. 22/23]. It was 4 equinoctial hours before midnight, for according to the astrolabe the last degree of Aries was culminating, while the longitude of the mean sun was ♐ 28;41°. At that moment the planet Saturn, sighted with respect to the bright star in the Hyades [catalogue XXIII 14], was seen to have a longitude of ♒ $9\frac{1}{15}$°, and was about ½° to the rear of the centre of the moon (for that was its distance from the moon's northern horn). Now at that moment the moon's positions were as follows:

mean longitude ♒ 8;55°
anomaly 174;15° from the apogee of the epicycle
hence its true longitude must have been ♒ 9;40°
H415 and its apparent longitude at Alexandria ♒ 8;34°.[31]

Thus from these considerations too the planet Saturn must have had a longitude of ♒ $9\frac{1}{15}$° (since it was about ½° to the rear of the moon's centre).

[31] It is far from clear for what moment these amounts are computed. The equation of time with respect to epoch is about −13¾ minutes, and indeed the mean positions seem to be computed for 7;50 p.m. rather than 8 p.m.; but then Ptolemy's true longitude is much too big. I find:

	for 7;50 p.m.	for 8 p.m.	Ptolemy
$\bar{\lambda}$ ☾	308;52°	308,58°	308;55°
$\bar{\alpha}$ ☾	174;15°	174;20°	174;15°
λ ☾	309;29°	309,35°	309;40°.

Since the moon was almost on the horizon, the parallax was large: from Ptolemy's tables I find a longitudinal parallax of about − 1¼° (−1;6° text), leading to a discrepancy of about ¼° in the final result.

And its distance from the apogee of the eccentre (which was [in] the same [position as at the third opposition], since its shift over so short an interval is negligible), was 76;4°.

Now the interval from the third opposition to this observation is
2 Egyptian years 167 days 8 hours.

And the [mean] motions of Saturn over this interval, calculated roughly,[32] are

in longitude:	30;3°
in anomaly:	134;24°.

If we add the latter to the positions at the third opposition as found above [p. 537], we get, for the moment of the observation in question:

in [mean] longitude	86;33° from the apogee of the eccentre
in anomaly	309;8° from the apogee of the epicycle.

With the above as data, let us again draw the diagram [Fig. 11.21] as in the similar proof [for Mars and Jupiter, Figs. 10.17 and 11.10], but with the epicycle situated to the rear of the apogee of the eccentre, and the planet in advance of the apogee of the epicycle, in accordance with their given positions. [Then,] since

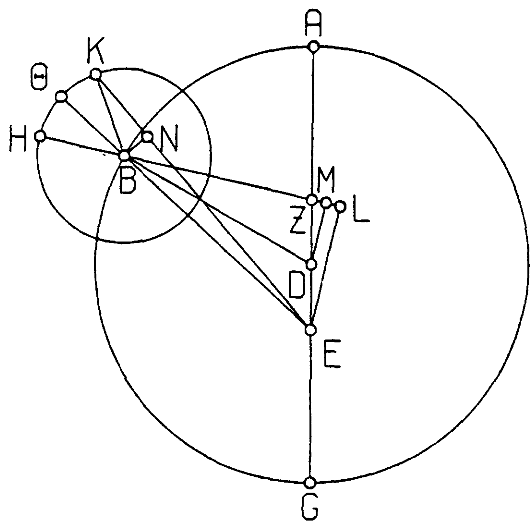

Fig. 11.21

$$\angle \text{AZB} (= \angle \text{DZM}) = \begin{cases} 86;33^\circ \text{ where 4 right angles} = 360^\circ \text{ (given)} \\ 173;6^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ}, \end{cases}$$ H416

in the circle about right-angled triangle DZM,

arc DM = 173;6°

and arc ZM = 6;54° (supplement).

Therefore the corresponding chords

$$\left.\begin{aligned} \text{DM} &= 119;47^p \\ \text{and ZM} &= 7;13^p \end{aligned}\right\} \text{ where hypotenuse DZ} = 120^p.$$

[32] These agree, to the nearest minute, with those found from the tables. Cf. p. 526 n.23.

Therefore, where the distance between the centres, DZ = $3;25^p$,
and the radius of the eccentre, DB = 60^p,

$$DM \approx 3;25^p$$
$$\text{and } ZM = 0;12^p.$$

H417 And since $DB^2 - DM^2 = BM^2$,

$$BM = 59;54^p \text{ in the same units.}$$

Similarly, since ZM = ML, and EL = 2DM,
by addition, BL = $60;6^p$ where EL is computed as $6;50^p$.

Hence hypotenuse [of right-angled triangle BEL]

$$EB = 60;29^p \text{ in the same units.}$$

Therefore, where hypotenuse EB = 120^p, EL = $13;33^p$,
and, in the circle about right-angled triangle BEL,

$$\text{arc } EL = 12;58^\circ$$
$$\therefore \angle EBZ = 12;58^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ}.$$

But $\angle$ AZB was given[33] as $173;6^{\circ\circ}$ in the same units.
Therefore, by subtraction, $\angle$ AEB = $160;8^{\circ\circ}$ in the same units.
But the angle representing the apparent distance of the planet from the apogee,

$$\angle AEK \text{ was given as} \begin{cases} 76;4^\circ \text{ where 4 right angles} = 360^\circ \\ 152;8^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ}. \end{cases}$$

Therefore, by subtraction, $\angle$ KEB = $8;0^{\circ\circ}$ in the same units.
Therefore, in the circle about right-angled triangle BEN,

$$\text{arc } BN = 8^\circ$$

H418 $$\text{and } BN = 8;22^p \text{ where hypotenuse } EB = 120^p.$$

Therefore, where EB = $60;29^p$, and the radius of the eccentre is 60^p,

$$BN = 4;13^p.$$

Furthermore, since the distance of the planet from H, the apogee of the epicycle, was $309;8^\circ$,
by subtraction [from 360°], arc HK = $50;52^\circ$.

$$\therefore \angle HBK = \begin{cases} 50;52^\circ \text{ where 4 right angles} = 360^\circ \\ 101;44^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ}. \end{cases}$$

But we found that $\angle$ EBZ (= $\angle$ HBΘ) = $12;58^{\circ\circ}$.
Therefore, by subtraction, $\angle$ ΘBK = $88;46^{\circ\circ}$ where $\angle$ KEB was shown to be $8^{\circ\circ}$.
Therefore, by subtraction, $\angle$ BKN = $80;46^{\circ\circ}$ in the same units.
Therefore, in the circle about right-angled triangle BKN,

$$\text{arc } BN = 80;46^\circ$$
$$\text{and } BN = 77;45^p \text{ where hypotenuse } BK = 120^p.$$

Therefore, where BN was found as $4;13^p$, and the radius of the eccentre is 60^p,
the radius of the epicycle, BK $\approx 6\frac{1}{2}^p$.

Thus we have computed the following:
round about the beginning of the reign of Antoninus the longitude of Saturn's
H419 apogee was ♏ 23°;
where the radius of the eccentre carrying the epicycle is 60^p,
the distance between the centres of the ecliptic and the eccentre which produces the uniform motion is $6;50^p$,
and the radius of the epicycle is $6;30^p$.

Q.E.D.

[33] Reading ὑπέκειτο (with D) for ὑπόκειται ('is given') at H417.13.

7. {*On the correction of Saturn's periodic motions*}

It remains to demonstrate the correction of the periodic motions. For this purpose we again selected one of the accurately recorded ancient observations. In this it is declared that in the 82nd year in the Chaldaean calendar, Xanthikos 5, in the evening, the planet Saturn was 2 digits [i.e. 10 minutes] below [the star on] the southern shoulder of Virgo.[34] Now that moment is in the 519th year from Nabonassar, Tybi [V] 14 in the Egyptian calendar [−228 Mar. 1], evening, at which time we find the longitude of the mean sun as ♓ 6;10°. But the fixed star on the southern shoulder of Virgo had a longitude at the time of our observation of ♍ $13\frac{1}{6}$°;[35] thus at the moment of the observation H420
in question, since to the intervening 366 years corresponds a motion of the fixed stars of about $3\frac{2}{3}$°, its longitude was, obviously, ♍ $9\frac{1}{2}$°. And the planet Saturn had the same longitude, since it was 2 digits to the south of the fixed star. By the same argument, since we showed that in our time its apogee was at ♏ 23°, at the observation in question it must have had a longitude of ♏ $19\frac{1}{3}$°. From this we conclude that at the above moment the apparent distance of the planet from the then apogee was 290;10° of the ecliptic, while the mean sun was 106;50° from the same apogee.

With the above as data, let there be drawn [Fig. 11.22] the diagram as for the same demonstration [for Mars and Jupiter, Figs. 10.18 and 11.11], [but] with the epicycle located in advance of the apogee of the eccentre, and the [mean] sun in advance of the perigee, with the radius from the epicycle centre to the

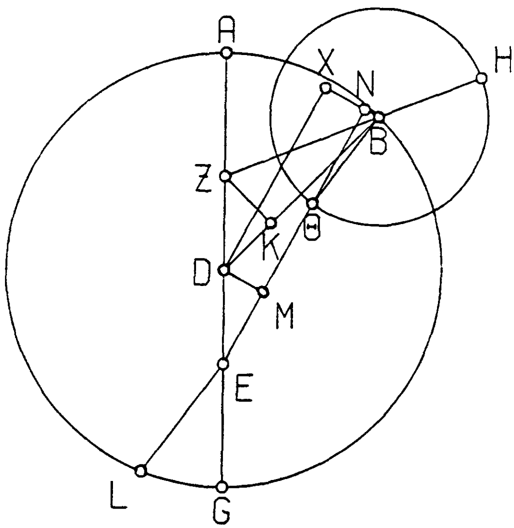

Fig. 11.22

[34] This is clearly a Babylonian observation: see Introduction p. 13. On the 'digit' see p. 322 n.5. The star in question, γ Vir, is one of the Babylonian 'normal stars' (cf. p. 453 n.70).
[35] Catalogue no. XXVII 7.

planet drawn parallel to [the line indicating] the sun's position. Then, since the apparent position of Saturn was in advance of the apogee by 69;50° (the difference [of 290;10°] from one revolution), the angle at the centre of the ecliptic,

H421 $$\angle\, AE\Theta = \begin{cases} 69;50^\circ \text{ where 4 right angles} = 360^\circ \\ 139;40^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ}. \end{cases}$$

And the angle of the sun's distance [from the apogee],

$$\angle AEL \text{ is given as} \begin{cases} 106;50^\circ \text{ where 4 right angles} = 360^\circ \\ 213;40^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ}. \end{cases}$$

Therefore, by addition, $\angle\, \Theta EL$ (= $\angle B\Theta E$, since $B\Theta$ is parallel to EL), is 353;20°° where 2 right angles = 360°°,

and, by subtraction [of $\angle$ $B\Theta E$ from 2 right angles]

$\angle$ $B\Theta N$ = 6;40°° in the same units.

Therefore, in the circle about right-angled triangle $B\Theta N$,

arc BN = 6;40°

and BN = 6;58^p where hypotenuse $B\Theta$ = 120^p.

H422 Therefore where the radius of the epicycle, $B\Theta$ = 6;30^p,

BN = 0;23^p.

Similarly, since $\angle$ $AE\Theta$ = 139;40°° where 2 right angles = 360°°

and $\angle$ EDM = 40;20°° in the same units [complement],

in the circle about right-angled triangle DEM,

arc DM = 139;40°

and DM = 112;39^p where hypotenuse ED = 120^p.

Therefore, where the distance between the centres, ED = 3;25^p,

and the radius of the eccentre, DB = 60^p,

DM (= XN) = 3;12^p,

and, by addition, BNX = 3;35^p where hypotenuse [of right-angled triangle BDX] DB = 60^p.

Therefore, where DB = 120^p, BX = 7;10^p,

and, in the circle about right-angled triangle BDX,

arc BX = 6;52°

∴ $\angle$ BDX = 6;52°° where 2 right angles = 360°°

and, by subtraction [from a right angle],

$\angle$ BDM = 173;8°° in the same units.

And, by addition [of $\angle$ EDM], $\angle$ BDE = 213;28°° in the same units,

and, by subtraction [from 2 right angles],

$\angle$ BDA = 146;32°° in the same units.

Therefore, in the circle about right-angled triangle DZK,

arc ZK = 146;32°

and arc DK = 33;28° (supplement).

Therefore the corresponding chords

H423 $$\left.\begin{array}{r} ZK = 114;55^p \\ \text{and } DK = 34;33^p \end{array}\right\} \text{where hypotenuse } DZ = 120^p.$$

Therefore, where the distance between the centres, DZ = 3;25^p,

and the radius of the eccentre, DB = 60^p,

ZK = 3;17^p

and DK = 0;59^p,

and, by subtraction [from DB], KB = 59;1^p where ZK = 3;17^p.

Hence hypotenuse [of right-angled triangle BZK]

ZB = 59;6^p in the same units.

Therefore, where hypotenuse ZB = 120^p, ZK = 6;40^p,
and, in the circle about right-angled triangle BZK,

arc ZK = 6;22°.

$\therefore \angle$ ZBK = 6;22°° where 2 right angles = 360°°.

But we found that $\angle$ ADB = 146;32°° in the same units.
Therefore, by addition, the angle representing the mean position in longitude,

$$\angle \text{AZB} = \begin{cases} 152;54^{\circ\circ} \text{ in the same units} \\ 76;27^\circ \text{ where 4 right angles} = 360^\circ. \end{cases}$$

Therefore at the moment of the above observation Saturn's distance from the apogee in mean longitudinal motion was 283;33°, i.e. its [mean] longitude was [♏ 19;20° + 283;33° =] ♍ 2;53°.

And since the sun's mean position is given as 106;50°, if we add the 360° of one revolution to the latter and from the resulting 466;50° subtract the 283;33° of H424
the longitude [from apogee], we get, for the anomaly at that moment,

183;17° from the apogee of the epicycle.[36]

So, since we have shown that at the moment of the above observation, which is in the 519th year from Nabonassar, Tybi [V] 14,[37] in the evening, [Saturn] was 183;17° [in anomaly] from the apogee of the epicycle, and at the moment of the third opposition, which was in the 883rd year from Nabonassar, Mesore [XII] 24, noon, it was 174;44°, it is clear that in the interval between the observations, which comprises

364 Egyptian years and 219¾ days,

the planet Saturn has moved

351;27° (beyond 351 complete revolutions in anomaly).

That is again almost the same increment as one derives from the [tables for] mean motions which we constructed. For it was from these very same elements that we derived the daily mean motion [in anomaly], by dividing the total in degrees computed from the number of complete revolutions plus the increment H425
by the total in days computed from the time [interval].[38]

8. {*On the epoch of Saturn's periodic motions*}

Now since the time interval from the first year of Nabonassar, Thoth 1, noon, to the above ancient observation is

518 Egyptian years 133¼ days,

and this interval comprises increments of

216;10° in longitude[39]

[36] Accurate computation gives 183;16° to the nearest minute.

[37] Reading ιδʹ for δʹ (4) at H424,6. The latter is found as the reading of the first hand in D, but is probably a misprint in Heiberg's text. Corrected by Manitius.

[38] On the actual derivation of Saturn's mean motion in anomaly see Appendix C.

[39] Reading $\overline{\sigma\iota\varsigma}\ \bar{\iota}$ (with GD¹, Ar) for $\overline{\sigma\iota\varsigma}\ \bar{\theta}$ (216;9°), which is Heiberg's correction (most Greek mss. have 216° or 216;0°). Heiberg was no doubt influenced by the fact that the mean motion, according

and 149;15° in anomaly,
if we subtract the latter from the [respective] positions at the observation, we get, for the same moment of epoch, the mean position of the planet Saturn as

in longitude: ♑ 26;43°
in anomaly: 34;2° from the apogee of the epicycle.

By the same computation [as before], we find the apogee of its eccentre in ♏ 14;10°.[40]

Q.E.D.

H426 9. {*How the true positions can be found geometrically from the periodic motions*}

Furthermore, conversely, given the arcs of the periodic [motions] on the eccentre which produces the uniform motion [i.e. the equant] and on the epicycle, one can readily obtain the apparent positions of the planets geometrically, as will become clear to us through the same [diagrams as above, e.g. Fig. 11.21].

For [see Fig. 11.23], in the simplified diagram containing [only] the eccentre and epicycle, we join ZBΘ and EBH. Then, if we are given the mean position in longitude, i.e. ∠ AZB, from what we proved previously, ∠ AEB will be given
H427 according to both hypotheses,[41] and so will ∠ EBZ, (which is the same as

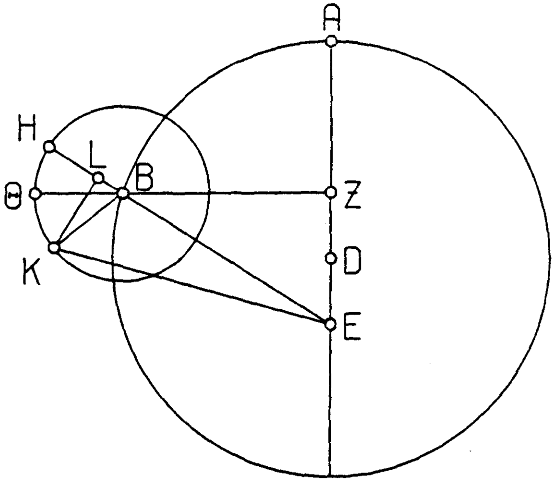

Fig. 11.23

to Ptolemy's table, is only 216;8,27°. But 216,10° is confirmed by the reading 26;43° below (in which all mss. agree here and in IX 4: Heiberg's correction to 26;44° must be rejected), and we must admit that Ptolemy made a small computing error. Cf. *HAMA* 182 n.15.

[40] The apogee was in ♏ 19;20° at the observation (p. 541). In 518½ʸ the movement in precession is 5;11°. Ptolemy, through inaccuracy or rounding, found 5;10°. The latter subtracted from ♏ 19;20° gives his result.

[41] I presume that by 'both hypotheses' Ptolemy means the simple eccentric model and the full, equant model. A possible alternative would be eccentric and epicyclic models, but since these are not discussed (for the planets) until Bk. XII, this seems unlikely.

∠ HBΘ), and also the ratio of line EB to the radius of the epicycle. And if we also suppose that the planet is located on the epicycle, e.g. at point K, and, when EK and BK are joined, arc ΘK is given, then, if instead of dropping the perpendicular from the epicycle centre B on to EK (as in the converse proof), we drop the perpendicular (here KL) from the planet K on to EB, then ∠ HBK will be given by addition [of the given angles ∠ ΘBK, ∠ HBΘ], and hence the ratio of KL and LB to BK and also, obviously, [their ratio] to EB.[42] Accordingly, the ratio of the whole line EBL to LK will be given.[43] Hence ∠ LEK will be given, and we will have computed the angle AEK which comprises the apparent distance of the planet from the apogee.

10. {*Method of constructing tables for the anomalies*}[44]

However, to avoid always computing the apparent positions geometrically (for although that method is the only one which provides a fully accurate solution to the problem, it is too cumbersome to be convenient for [astronomical] investigations), we have constructed for each of the five planets a table which is as easy to use as we could devise, while at the same time being very close to full accuracy. [Each table] contains the individually determined anomalies of the planets, so that we can use them as a ready means of computing any particular apparent position, once we are given the periodic motions from the respective apogees. H428

We have again arranged each of the tables in 45 lines for the sake of symmetry, and we have arranged each in 8 columns. The first 2 columns will contain the numbers of the mean positions arranged as for the sun and moon [III 6 and V 8]: in the first column the 180 degrees beginning from the apogee, from the top down, and in the second the remaining 180 degrees of the [other] semi-circle, from the bottom up, in such a way that the number '180' is in the last line in both columns, and the increment in the numbers is 6° in the top[45] 15 lines, but 3° in the 30 lines remaining below (for the differences between [successive] values for the anomalies remain almost constant for longer stretches near the apogee, whereas they change faster near the perigee). As for the next two columns, the third will contain the equations corresponding to the mean position in longitude (each to the arguments on the same line), computed for the greater eccentricity,[46] but under the simplifying assumption that the centre of the epicycle is carried on the eccentre which produces the mean motion [i.e. the equant]. The fourth column will contain the corrections to the equations due to the fact that the epicycle centre is carried, not on the above circle, but on another. The method by which each of these quantities [the equation and its correction], both in combination and separately, can be found geometrically has H429

[42] Euclid, *Data* Props. 40 and 8.
[43] Euclid, *Data* Props. 6 and 8.
[44] See *HAMA* 183–6, Pedersen 291–4.
[45] Reading ἄνωθεν (with D,Is) for ἄνωθεν πρώτων ('first top') at H428,18.
[46] I.e. the equations of center computed for the double eccentricity (ZE in Fig. 11.23, where the equation is ∠ ZBE).

already been made plain by numerous preceding theorems.[47] In this place, since this is a [scientific] treatise, it was appropriate to display this way of separating the zodiacal anomaly, and hence to tabulate it in two columns. However, for actual use, a single column formed by combining these two will suffice.[48]

Each of the next three columns will contain the equations due to the epicycle. These, again, are computed under a simplifying assumption, [namely] that the apogee or perigee of the epicycle is viewed along the line from the observer [to
H430 the epicycle centre].[49] The way in which this kind of demonstration is performed has also been made plain by the previous theorems. The midmost of these three columns (which is the sixth from the beginning) will contain the equations computed for the ratio [of epicycle radius to distance of epicycle centre] at mean distance; the fifth will contain, [for each argument], the difference between the equation at greatest distance [of the epicycle] and the equation for the same argument at mean distance; the seventh will contain the differences between the equations at least distance and the [corresponding] equations at mean distance. For we have shown that for the following epicycle sizes (from now on it would be best to list [the planets] in order from the outermost):

Saturn	Jupiter	Mars	Venus	Mercury
$6;30^{p}$	$11;30^{p}$	$39;30^{p}$	$43;10^{p}$	$22;30^{p}$,

the mean distance, i.e. the distance [equivalent] to the radius of the eccentre which carries the epicycle, is 60^{p} in all cases; and the greatest distances (with respect to the centre of the ecliptic), are:

	Saturn	Jupiter	Mars	Venus	Mercury
H431	$63;25^{p}$	$62;45^{p}$	66^{p}	$61;15^{p}$	69^{p}.

The least distances (defined similarly) are:

Saturn	Jupiter	Mars	Venus	Mercury
$56;35^{p}$	$57;15^{p}$	54^{p}	$58;45^{p}$	$55;34^{p}$.[50]

As for the remaining, eighth column, we provided it in order that one may find the applicable fraction of the above differences [in cols. 5 and 7] when the planet's epicycle is not exactly at mean, greatest or least distance, but in an intermediate position. The computation of this correction is based only on the maximum equation ([i.e.] that formed by the tangent from the observer to the epicycle) at each intermediate distance; for the [fraction] of the difference to be applied for any particular position [of the planet] on the epicycle is not significantly different from that for the greatest equation.

But in order to make our meaning clearer, and to explain the actual method of computing the [fractions] to be applied, let us draw [see Fig. 11.24] the line

[47] E.g. XI 5 pp. 529–37 and XI 9.

[48] The didactic purpose of the Almagest is made explicit here. '[scientific] treatise' is my translation of σύνταξις. For this meaning, which is typical of Hellenistic prose, but seems not to be classical, see LSJ s.v. 3. In the *Handy Tables* Ptolemy does indeed combine the two columns into one, and that is the pattern of all subsequent ancient and mediaeval astronomical tables.

[49] I.e. the equation of anomaly is computed as a function, not of the mean anomaly, but of the true, that is as counted from the true apogee of the epicycle.

[50] For this value for the least distance of the centre of the epicycle for Mercury see IX 9 p. 460 with n.89.

through both centres (the centre of the ecliptic and the centre of the eccentre producing the uniform motion of the epicycle), ABGD. Let the centre of the ecliptic be taken at G, and the centre of the epicycle's uniform motion [i.e. the equant point] at B. Produce line BEZ, describe the epicycle ZH about centre E, and draw the tangent to it from G, line GH. Join GE and perpendicular EH, and let us suppose, *exempli gratia*, that for each of the five planets the epicycle centre is 30° from the apogee of the eccentre in mean motion. H432

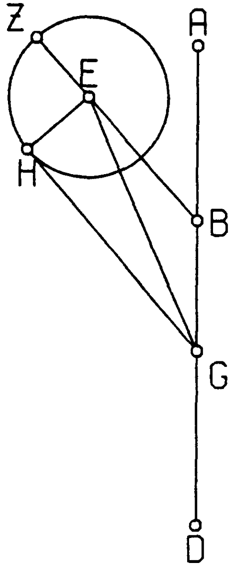

Fig. 11.24

Then (to avoid lengthening the computation by demonstrating the same thing over and over again), we have demonstrated at length in what preceded, both in the hypothesis for Mercury and in that for the other planets,[51] that if ∠ ABE is given, the ratio of GE to the radius of the epicycle (HE) is also given. Hence, by means of the computations for each particular planet, with ∠ ABE taken as 30°, this ratio comes to:

for Saturn	Jupiter	Mars	Venus	Mercury
63;2 : 6;30	62;26 : 11;30	65;24 : 39;30	61;6[52] : 43;10	66;35 : 22;30.

H433

Thus we will get for ∠ EGH, which comprises the maximum epicyclic equation at that point,

for Saturn	Jupiter	Mars	Venus	Mercury
5;55½°	10;36½°	37;9°	44;56½°	19;45°.

And we compute the greatest equations at the mean distance, according to the ratios set out just above, as (to avoid repetition, we [simply list them] in an order corresponding to the above order of the planets):

6;13°	11;3°	41;10°	46;0°	22;2°;

[51] Mercury, IX 9 pp. 457–60; other planets, X 2, X 8, XI 2, XI 6.

[52] Reading ξ̄α ς̄ (with AD,Ar) for ξ̄α κ̄ς (61;26) at H433,4. At H503,5 all mss. have 61;6. Corrected by Manitius.

those at the greatest distances as

5;53° 10;34° 36;45° 44;48° 19;2°;

and those at the least distances as

6;36° 11;35° 47;1° 47;17° 23;53°.

H434 Thus the differences between the equations at mean distance and those at greatest distance are

0;20° 0;29° 4;25° 1;12° 3;0°,

while the differences between [those at mean distance and] those at least distance are

0;23° 0;32° 5;51° 1;17° 1;51°.

Now the equations of the distances in question [for a mean longitude of 30° from the apogee] are less than those for mean distance, and differ from the latter by the following amounts:

0;17½° 0;26½° 4;1° 1;3½° 2;17°,

and the latter (expressed as sixtieths of the above total differences between [the equations for] mean and greatest distance)[53] are

for Saturn	Jupiter	Mars	Venus	Mercury
52;30	54;50	54;34	52;55	45;40.

So those are the values, in sixtieths, which we put in the 8th column of the appropriate table, on the line containing the number '30' for the mean motion in longitude.

H435 For those distances which have equations greater than those at mean distance, we again reduced the [resulting] differences to sixtieths, but in this case expressed as fractions, not of the [corresponding] equations at greatest distance, but of those at least distance. In the same way [as above], we performed the computation for all other positions [of the epicycle] at 6° intervals of mean longitude,[54] and tabulated the resulting fractions, expressed in sixtieths, opposite the appropriate arguments. As we said, the fraction of the difference to be applied is sensibly the same even when the position of a planet is not at the greatest epicyclic equation, but at some other point on the epicycle.

The layout of the five tables is as follows.

H436-45 11. {*Planetary equation tables*}[55]

[See pp. 549-53.]

[53] Thus, e.g., for Saturn 0;17½ : 0;20 = 52½ : 60.

[54] The statement that these values were computed at 6° intervals, even where the function is tabulated at 3° intervals, is easily verified by taking the differences between successive values in col. 8 for Mars.

[55] Corrections to Heiberg:

H441,49 Mars, arg. 174°, col. 6. Read ια ιε (with Ar) for ια ιθ (11;19°). Computed: 11;16°.
H442,17 Venus, arg. 66°, col. 6. Read κς νζ (with DL) for κς λζ (26;37°). 26;57° is the value I compute, and it also agrees with the value in col. 2 of the latitude table (XIII 5).
H443,34 arg. 129°, col. 3. Read α νδ (with Ar) for α να (1;51°). Corrected by Manitius.
H443,36 arg. 135°, col. 6. Read με νθ (with D,Ar) for με νε (45;35°). Computed: 46;0°.
H443,43 arg. 156°, col. 7. Read α μη (with D,Ar) for α νη (1;58°), which is obviously wrong since it is greater than the value for 159°. Computed: 1;47°. Corrected by Manitius.
H444,9 Mercury, arg. 18°, col. 5. Read ο κθ (with Ar) for ο κδ (0;24°). Computed: 0;29°.

SATURN
APOGEE: ♏ 14;10°

1 Common	2 Numbers	3 Equation in Longitude	4 Difference in Equation	5 Subtractive Difference	6 Equation of Anomaly	7 Additive Difference	8 Sixtieths
6	354	0 37	+0 2	0 2	0 36	0 2	−60 0
12	348	1 13	+0 4	0 4	1 11	0 4	−58 30
18	342	1 49	+0 6	0 5	1 45	0 7	−57 0
24	336	2 23	+0 8	0 7	2 18	0 9	−55 30
30	330	2 57	+0 9	0 8	2 50	0 11	−52 30
36	324	3 29	+0 10	0 10	3 20	0 13	−49 30
42	318	3 59	+0 11	0 11	3 49	0 15	−46 30
48	312	4 28	+0 11	0 12	4 17	0 17	−43 30
54	306	4 55	+0 10	0 14	4 42	0 19	−39 0
60	300	5 20	+0 9	0 15	5 4	0 20	−34 30
66	294	5 42	+0 8	0 17	5 25	0 20	−30 0
72	288	6 0	+0 7	0 18	5 42	0 21	−24 0
78	282	6 14	+0 5	0 18	5 55	0 21	−18 0
84	276	6 24	+0 3	0 19	6 5	0 22	−12 0
90	270	6 30	+0 1	0 19	6 12	0 22	− 4 30
93	267	6 31	+0 0	0 20	6 12	0 23	− 0 45
96	264	6 32	−0 2	0 20	6 13	0 23	+ 2 32
99	261	6 31	−0 3	0 20	6 12	0 24	+ 5 51
102	258	6 30	−0 4	0 21	6 12	0 24	+ 9 8
105	255	6 27	−0 5	0 21	6 9	0 24	+11 45
108	252	6 23	−0 6	0 20	6 5	0 25	+14 21
111	249	6 19	−0 7	0 20	6 0	0 25	+16 58
114	246	6 14	−0 8	0 20	5 55	0 24	+19 31
117	243	6 7	−0 9	0 19	5 48	0 24	+22 11
120	240	5 59	−0 10	0 19	5 40	0 23	+24 47
123	237	5 50	−0 10	0 19	5 31	0 23	+27 24
126	234	5 39	−0 11	0 18	5 21	0 22	+30 0
129	231	5 27	−0 11	0 18	5 10	0 22	+32 37
132	228	5 14	−0 12	0 17	4 58	0 21	+35 13
135	225	5 0	−0 12	0 17	4 45	0 20	+37 50
138	222	4 45	−0 12	0 16	4 31	0 19	+40 26
141	219	4 29	−0 12	0 15	4 16	0 18	+43 3
144	216	4 12	−0 12	0 14	4 0	0 17	+45 39
147	213	3 54	−0 12	0 14	3 43	0 15	+47 37
150	210	3 35	−0 11	0 12	3 25	0 14	+49 34
153	207	3 16	−0 11	0 11	3 7	0 13	+51 32
156	204	2 56	−0 10	0 10	2 48	0 12	+53 29
159	201	2 36	−0 9	0 9	2 29	0 11	+54 49
162	198	2 15	−0 8	0 7	2 9	0 10	+56 6
165	195	1 53	−0 7	0 6	1 48	0 8	+57 24
168	192	1 31	−0 6	0 5	1 27	0 7	+58 42
171	189	1 9	−0 5	0 5	1 6	0 5	+59 21
174	186	0 47	−0 3	0 4	0 45	0 4	+60 0
177	183	0 24	−0 2	0 2	0 23	0 2	+60 0
180	180	0 0	−0 0	0 0	0 0	0 0	+60 0

JUPITER
APOGEE: ♍ 2;9°

1	2	3	4	5	6	7	8
Common Numbers		Equation in Longitude	Difference in Equation	Subtractive Difference	Equation of Anomaly	Additive Difference	Sixtieths
6	354	0 30	+0 1	0 2	0 58	0 2	−60 0
12	348	1 0	+0 2	0 5	1 56	0 5	−58 58
18	342	1 30	+0 3	0 7	2 52	0 7	−57 56
24	336	1 58	+0 4	0 9	3 48	0 9	−56 54
30	330	2 26	+0 5	0 11	4 42	0 11	−54 50
36	324	2 52	+0 6	0 13	5 34	0 13	−51 43
42	318	3 17	+0 7	0 15	6 25	0 15	−47 35
48	312	3 40	+0 7	0 17	7 12	0 18	−43 27
54	306	4 1	+0 7	0 19	7 57	0 20	−39 19
60	300	4 20	+0 6	0 21	8 37	0 22	−35 8
66	294	4 37	+0 5	0 23	9 14	0 24	−28 58
72	288	4 51	+0 4	0 24	9 46	0 26	−22 45
78	282	5 2	+0 3	0 25	10 13	0 28	−17 35
84	276	5 9	+0 2	0 26	10 35	0 30	−11 23
90	270	5 14	+0 1	0 26	10 51	0 31	− 4 40
93	267	5 15	+0 0	0 27	10 57	0 31	− 1 8
96	264	5 16	−0 1	0 27	11 0	0 32	+ 1 52
99	261	5 15	−0 1	0 27	11 2	0 32	+ 5 9
102	258	5 14	−0 2	0 28	11 3	0 32	+ 8 26
105	255	5 12	−0 2	0 28	11 1	0 33	+11 43
108	252	5 9	−0 3	0 29	10 59	0 33	+15 0
111	249	5 5	−0 4	0 29	10 53	0 33	+17 49
114	246	5 0	−0 5	0 30	10 45	0 34	+20 37
117	243	4 54	−0 5	0 30	10 35	0 34	+23 26
120	240	4 47	−0 6	0 30	10 24	0 34	+26 15
123	237	4 39	−0 6	0 29	10 10	0 33	+29 4
126	234	4 30	−0 7	0 29	9 54	0 33	+31 52
129	231	4 20	−0 7	0 28	9 36	0 32	+34 41
132	228	4 9	−0 8	0 28	9 16	0 32	+37 30
135	225	3 58	−0 8	0 27	8 54	0 31	+40 19
138	222	3 46	−0 8	0 26	8 30	0 30	+43 7
141	219	3 33	−0 8	0 25	8 4	0 28	+45 28
144	216	3 20	−0 7	0 23	7 36	0 26	+47 49
147	213	3 6	−0 7	0 22	7 6	0 25	+49 42
150	210	2 51	−0 6	0 21	6 34	0 23	+51 31
153	207	2 36	−0 6	0 19	6 0	0 21	+52 58
156	204	2 20	−0 5	0 17	5 24	0 19	+54 22
159	201	2 4	−0 5	0 15	4 47	0 17	+55 47
162	198	1 47	−0 4	0 13	4 9	0 15	+57 11
165	195	1 30	−0 3	0 11	3 29	0 13	+57 40
168	192	1 13	−0 2	0 9	2 49	0 10	+58 13
171	189	0 55	−0 2	0 7	2 7	0 8	+58 40
174	186	0 37	−0 1	0 5	1 25	0 5	+59 4
177	183	0 18	−0 1	0 3	0 43	0 3	+59 32
180	180	0 0	−0 0	0 0	0 0	0 0	+60 0

MARS
APOGEE: ♋ 16;40°

1	2	3 Equation in Longitude	4 Difference in Equation	5 Subtractive Difference	6 Equation of Anomaly	7 Additive Difference	8 Sixtieths
Common Numbers							
6	354	1 0	+0 5	0 8	2 24	0 9	−59 53
12	348	2 0	+0 10	0 16	4 46	0 18	−58 59
18	342	2 58	+0 15	0 24	7 8	0 28	−57 51
24	336	3 56	+0 20	0 33	9 30	0 37	−56 36
30	330	4 52	+0 24	0 42	11 51	0 46	−54 34
36	324	5 46	+0 27	0 51	14 11	0 56	−52 11
42	318	6 39	+0 28	1 0	16 29	1 6	−49 28
48	312	7 28	+0 29	1 9	18 46	1 16	−46 17
54	306	8 14	+0 28	1 18	21 0	1 28	−42 38
60	300	8 57	+0 27	1 27	23 13	1 40	−38 8
66	294	9 36	+0 24	1 37	25 22	1 53	−33 26
72	288	10 9	+0 20	1 49	27 29	2 6	−28 20
78	282	10 38	+0 15	2 1	29 32	2 19	−22 47
84	276	11 2	+0 10	2 14	31 30	2 33	−16 33
90	270	11 19	+0 4	2 28	33 22	2 45	−10 5
93	267	11 25	+0 0	2 35	34 15	2 57	− 6 34
96	264	11 29	−0 4	2 42	35 6	3 6	− 3 3
99	261	11 32	−0 8	2 49	35 56	3 15	+ 0 5
102	258	11 32	−0 12	2 56	36 43	3 25	+ 3 13
105	255	11 31	−0 16	3 4	37 27	3 36	+ 6 1
108	252	11 28	−0 19	3 13	38 9	3 47	+ 8 49
111	249	11 22	−0 22	3 22	38 48	3 58	+11 44
114	246	11 14	−0 25	3 32	39 24	4 9	+14 38
117	243	11 5	−0 28	3 43	39 56	4 21	+17 33
120	240	10 53	−0 31	3 54	40 23	4 35	+20 27
123	237	10 39	−0 33	4 4	40 44	4 50	+23 35
126	234	10 23	−0 35	4 14	40 59	5 5	+26 42
129	231	10 4	−0 37	4 24	41 7	5 21	+29 31
132	228	9 44	−0 39	4 35	41 9	5 37	+32 20
135	225	9 21	−0 40	4 45	41 2	5 55	+35 9
138	222	8 55	−0 41	4 56	40 45	6 14	+37 58
141	219	8 27	−0 41	5 7	40 16	6 34	+40 35
144	216	7 59	−0 41	5 18	39 37	6 53	+43 12
147	213	7 27	−0 41	5 28	38 40	7 12	+45 26
150	210	6 54	−0 38	5 34	37 25	7 30	+47 39
153	207	6 19	−0 36	5 38	35 52	7 45	+49 50
156	204	5 41	−0 33	5 38	33 53	7 58	+52 1
159	201	5 3	−0 30	5 34	31 30	8 3	+53 47
162	198	4 22	−0 27	5 18	28 35	7 58	+55 32
165	195	3 41	−0 23	4 52	25 3	7 47	+56 44
168	192	2 58	−0 19	4 18	21 0	7 6	+57 55
171	189	2 14	−0 15	3 32	16 25	5 59	+58 49
174	186	1 30	−0 10	2 27	11 15	4 26	+59 43
177	183	0 45	−0 5	1 16	5 45	2 20	+59 52
180	180	0 0	−0 0	0 0	0 0	0 0	+60 0

VENUS
APOGEE: ♉ 16;10°

1 Common	2 Numbers	3 Equation in Longitude	4 Difference in Equation	5 Subtractive Difference	6 Equation of Anomaly	7 Additive Difference	8 Sixtieths
6	354	0 14	+0 1	0 1	2 31	0 2	−59 10
12	348	0 28	+0 1	0 3	5 1	0 4	−57 55
18	342	0 42	+0 1	0 5	7 31	0 6	−56 40
24	336	0 56	+0 2	0 7	10 1	0 8	−55 0
30	330	1 9	+0 2	0 9	12 30	0 10	−52 55
36	324	1 21	+0 2	0 11	14 58	0 12	−49 35
42	318	1 32	+0 3	0 13	17 25	0 14	−45 50
48	312	1 43	+0 3	0 15	19 51	0 16	−42 5
54	306	1 53	+0 3	0 18	22 15	0 18	−37 5
60	300	2 1	+0 2	0 20	24 38	0 20	−31 40
66	294	2 8	+0 2	0 22	26 57	0 23	−26 15
72	288	2 14	+0 2	0 24	29 14	0 25	−20 25
78	282	2 18	+0 1	0 27	31 27	0 28	−14 35
84	276	2 21	+0 1	0 29	33 38	0 30	− 8 20
90	270	2 23	+0 1	0 31	35 44	0 33	− 1 40
93	267	2 23	−0 0	0 33	36 40	0 36	+ 1 31
96	264	2 23	−0 1	0 35	37 43	0 38	+ 4 42
99	261	2 22	−0 1	0 38	38 40	0 40	+ 7 39
102	258	2 21	−0 1	0 40	39 35	0 43	+10 35
105	255	2 20	−0 1	0 42	40 29	0 45	+13 32
108	252	2 18	−0 1	0 45	41 20	0 47	+16 28
111	249	2 16	−0 1	0 47	42 9	0 50	+19 25
114	246	2 13	−0 2	0 49	42 54	0 52	+22 21
117	243	2 10	−0 2	0 52	43 35	0 55	+25 18
120	240	2 6	−0 2	0 54	44 12	0 58	+28 14
123	237	2 2	−0 2	0 57	44 45	1 1	+31 0
126	234	1 58	−0 2	1 0	45 14	1 4	+33 44
129	231	1 54	−0 2	1 3	45 36	1 8	+36 18
132	228	1 49	−0 3	1 6	45 51	1 11	+38 50
135	225	1 44	−0 3	1 10	45 59	1 14	+41 11
138	222	1 39	−0 3	1 14	45 57	1 18	+43 32
141	219	1 33	−0 3	1 19	45 45	1 22	+45 42
144	216	1 27	−0 2	1 24	45 20	1 27	+47 51
147	213	1 21	−0 2	1 29	44 40	1 32	+49 37
150	210	1 14	−0 2	1 33	43 39	1 38	+51 23
153	207	1 7	−0 2	1 37	42 18	1 43	+52 46
156	204	1 0	−0 2	1 39	40 28	1 48	+54 8
159	201	0 53	−0 2	1 41	38 7	1 51	+55 18
162	198	0 46	−0 1	1 42	35 7	1 52	+56 26
165	195	0 39	−0 1	1 38	31 24	1 50	+57 28
168	192	0 32	−0 1	1 31	26 46	1 43	+58 26
171	189	0 24	−0 1	1 19	21 15	1 27	+59 1
174	186	0 16	−0 1	0 58	14 47	1 5	+59 36
177	183	0 8	−0 1	0 31	7 38	0 35	+59 58
180	180	0 0	−0 0	0 0	0 0	0 0	+60 0

MERCURY
APOGEE: ♎ 1;10°

1 Common Numbers	2	3 Equation in Longitude	4 Difference in Equation	5 Subtractive Difference	6 Equation of Anomaly	7 Additive Difference	8 Sixtieths
6	354	0 18	−0 1	0 10	1 38	0 5	−59 20
12	348	0 34	−0 2	0 20	3 16	0 11	−57 20
18	342	0 51	−0 4	0 29	4 53	0 17	−54 40
24	336	1 7	−0 5	0 39	6 29	0 23	−50 40
30	330	1 22	−0 5	0 49	8 4	0 28	−45 40
36	324	1 37	−0 4	0 59	9 36	0 34	−39 40
42	318	1 51	−0 4	1 8	11 6	0 40	−33 0
48	312	2 4	−0 3	1 18	12 33	0 45	−25 40
54	306	2 15	−0 1	1 28	13 58	0 50	−18 0
60	300	2 25	−0 0	1 39	15 18	0 56	−10 20
66	294	2 34	+0 2	1 49	16 33	1 4	− 2 20
72	288	2 41	+0 4	1 59	17 43	1 11	+ 9 14
78	282	2 46	+0 6	2 9	18 47	1 17	+20 0
84	276	2 50	+0 7	2 19	19 44	1 23	+29 44
90	270	2 52	+0 9	2 29	20 33	1 29	+39 28
93	267	2 52	+0 10	2 34	20 54	1 32	+43 31
96	264	2 52	+0 10	2 39	21 14	1 35	+47 34
99	261	2 51	+0 11	2 44	21 29	1 38	+50 0
102	258	2 50	+0 10	2 48	21 42	1 41	+52 26
105	255	2 48	+0 10	2 53	21 52	1 44	+54 52
108	252	2 46	+0 10	2 58	21 59	1 46	+57 18
111	249	2 44	+0 9	3 2	22 2	1 49	+58 23
114	246	2 41	+0 9	3 4	22 1	1 52	+59 28
117	243	2 37	+0 9	3 6	21 56	1 55	+59 44
120	240	2 33	+0 8	3 8	21 47	1 57	+60 0
123	237	2 28	+0 7	3 9	21 33	1 59	+59 44
126	234	2 23	+0 7	3 10	21 15	2 0	+59 23
129	231	2 18	+0 6	3 12	20 53	2 0	+58 39
132	228	2 12	+0 6	3 12	20 25	2 1	+57 50
135	225	2 6	+0 5	3 9	19 50	2 1	+56 46
138	222	2 0	+0 4	3 6	19 10	2 0	+55 41
141	219	1 53	+0 4	3 2	18 24	2 0	+54 3
144	216	1 46	+0 3	2 57	17 32	1 58	+52 26
147	213	1 38	+0 3	2 51	16 35	1 53	+50 48
150	210	1 30	+0 2	2 42	15 31	1 47	+49 11
153	207	1 22	+0 2	2 32	14 20	1 41	+47 34
156	204	1 13	+0 2	2 21	13 3	1 34	+45 57
159	201	1 5	+0 1	2 9	11 41	1 26	+44 36
162	198	0 56	+0 1	1 55	10 13	1 17	+43 15
165	195	0 46	+0 1	1 38	8 40	1 7	+42 26
168	192	0 38	+0 0	1 19	7 1	0 56	+41 37
171	189	0 28	+0 0	1 1	5 19	0 43	+40 48
174	186	0 19	+0 0	0 42	3 35	0 28	+40 0
177	183	0 9	+0 0	0 21	1 48	0 14	+39 44
180	180	0 0	+0 0	0 0	0 0	0 0	+39 28

H446 12. {*On the computation of the longitude of the 5 planets*}[56]

So when we want to determine the apparent position of any one of the planets from the periodic motions in longitude and anomaly, by employing the above [tables], we carry out the numerical computation (which is one and the same for all five planets) in the following way.

From the tables for mean motion we compute the mean positions in longitude and anomaly for the moment required (by addition, and casting out complete revolutions). Then, taking as argument the distance from the apogee of the eccentre at that moment to the mean position in longitude, we enter the anomaly table belonging to the planet in question, and take the value for the longitudinal correction corresponding to that argument in the third column, together with the value (in minutes) in the fourth column (which has to be added or subtracted). We subtract the result from the [mean] longitude and add it to the anomaly if the above-mentioned argument for the longitude [i.e.
H447 the mean centrum] falls in the first column, but if it falls in the second column, we add the result to the longitude and subtract it from the anomaly, to get both positions corrected.

Then we enter with the corrected anomaly [counted] from the [epicyclic] apogee into [one of] the first two columns, take the corresponding amount in the sixth column (the equation for mean distance), and write it down separately. Similarly, we enter with the amount for the mean longitude [i.e. mean centrum] (which we used as argument at the beginning) into the same argument [columns]; then, if [that argument] falls in the upper lines, which are closer to the apogee than that for mean distance (this will be clear from the entries in the eighth column),[57] we take the corresponding number of sixtieths in the eighth column, take, from the fifth column (for the [difference at] greatest distance), the entry on the same line as that for the equation at mean distance which was written down separately, form the fraction of that [entry for the] difference corresponding to the above number of sixtieths, and subtract the result from the amount which we wrote down separately. But if the argument of the above longitude [i.e. the mean centrum] falls in the lower lines, which are closer to the perigee than that for mean distance, we take the corresponding number of sixtieths in the eighth column, as before, take, from the seventh column (for the [difference at] least distance), the entry corresponding to the equation for mean [distance] which was written down separately, form the fraction of that difference corresponding to the above number of sixtieths, and
H448 add the result to the number we wrote down separately. The result will be the corrected equation [of anomaly]. If the corrected anomaly is in the first column, we add that corrected equation to the amount for the corrected longitude, but we subtract it if the corrected anomaly is in the second column. Using the result to count from the apogee of the planet at that moment, we reach its apparent position.

[56] See *HAMA* 186–7 and Appendix A, Example 14.

[57] I.e. if the entry in the eighth column is subtractive, the epicycle centre is closer to apogee than to mean distance; if additive, closer to perigee (for Mercury, to least distance) than to mean distance.

Book XII

1. {*On the preliminaries for the retrogradations*}[1] H450

Now that we have demonstrated the above, the appropriate sequel would be to examine the greatest and least retrogradations associated with each of the 5 planets, and to show that the sizes of these, [as computed] from the above hypotheses, are in as close agreement as possible with those found from observations.

In the definition of this kind of problem, there is a preliminary lemma demonstrated (for a single anomaly, that related to the sun) by a number of mathematicians, notably Apollonius of Perge, to the following effect.

[1] If [the synodic anomaly] is represented by the epicyclic hypothesis, in which the epicycle performs the [mean] motion in longitude on the circle concentric with the ecliptic towards the rear [i.e. in the order] of the signs, and the planet performs the motion in anomaly on the epicycle [uniformly] with respect to its centre, towards the rear along the arc near the apogee, and if a line is drawn from our point of view intersecting the epicycle in such a way that the ratio of half that segment of the line intercepted within the epicycle to that segment intercepted between the observer and the point where the line intersects the epicycle nearer its perigee is equal to the ratio of the speed of the epicycle to the H451
speed of the planet, then the point on the arc of the epicycle nearer the perigee determined by the line so drawn is the boundary between forward motion and retrogradation, so that when the planet reaches that point it creates the appearance of station.

[2] If the anomaly related to the sun is represented by the eccentric hypothesis (which is a viable hypothesis only for the three [outer] planets which can reach any elongation from the sun),[2] in which the centre of the eccentre moves [uniformly] about the centre of the ecliptic with the speed of the [mean] sun towards the rear [i.e. in the order] of the signs, while the planet moves on the eccentre in advance [i.e. in the reverse order] of the signs with a speed [uniform] with respect to the centre of the eccentre and equal to the [mean] motion in anomaly, and if a line is drawn in the eccentre through the centre of the ecliptic (i.e. the observer) in such a way that the ratio of half the whole line to the smaller of the two segments of the line formed by [the position of] the observer is

[1] On chs. 1-6 see *HAMA* 190-201, Pedersen 331-49.

[2] This type of eccentric model is in fact applicable to the inner planets as well, provided that, for the speed of the centre of the eccentre, one uses, not the speed of the mean sun, but the sum of the speeds of the mean sun and the planet's anomaly (which sum is the same as the modern heliocentric mean motion). I do not understand why Ptolemy does not recognise this.

equal to the ratio of the speed of the eccentre to the speed of the planet, then when the planet arrives at the point in which the above line cuts the arc of the eccentre near the perigee, it will produce the appearance of station.

H452 We too shall achieve the required result by a method which, though summary, is none the less more convenient: we employ a proof which contains both hypotheses combined in a common [figure], to demonstrate their agreement and similarity in these ratios of theirs too.[3]

Let [Fig. 12.1] the epicycle be ABGD on centre E and diameter AEG, which is produced to Z, the centre of the ecliptic (i.e. our point of view). Cut off equal arcs, GH, GΘ, on either side of the perigee G, and draw ZHB and ZΘD from Z through points H and Θ. Join DH and BΘ to intersect each other at point K, which will, obviously, lie on diameter AG.

We say, first, that

$$AZ:ZG = AK:KG.$$

[Proof:] Join AD, DG, and draw LGM through G parallel to AD. Then LGM

H453 will, obviously, be perpendicular to DG (for ∠ ADG is right).

Then, since ∠ GDH = ∠ GDΘ [on equal arcs, Euclid III 27],

GL = GM [triangles LDG, MDG congruent].

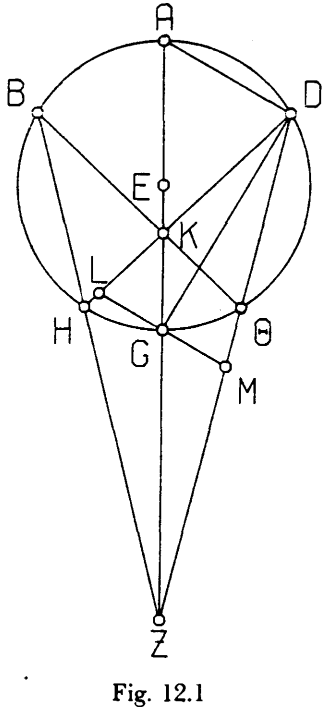

Fig. 12.1

[3] 'in these . . . too' refers to the earlier demonstrations of the equivalence of the hypotheses in III 3 and IV 5. Note that Ptolemy opposes his proof (ἡμεῖς δὲ) to that of the earlier mathematicians, notably Apollonius (προαποδεικνύουσι μὲν, H450.9). This counts against Neugebauer's supposition (*HAMA* 264) that Ptolemy has taken this elegant equivalence theorem from Apollonius, despite its relationship to *Conics* III 37-40 and to *Plane Loci* II 8 ('Circle of Apollonius').

∴ AD:GL = AD:GM.
But AD:GM = AZ:ZG [triangle ADZ ||| triangle GMZ]
and AD:LG = AK:KG [triangle ADK ||| triangle GLK].
∴ AZ:ZG = AK:KG.

So, if we imagine epicycle ABGD to be the actual eccentre in the eccentric hypothesis, the point K will be the centre of the ecliptic, and diameter AG will be divided by it in the same ratio as [the corresponding amounts] in the epicyclic hypothesis. For we have shown that the ratio of the greatest distance in the epicyclic [hypothesis], AZ, to the least distance, ZG, is the same as the greatest distance in the eccentric [hypothesis], AK, to the least distance, KG.

We also say, [secondly], that

DZ:ZΘ = BK:KΘ.

[Proof:] In the similar diagram [Fig. 12.2] join the line BND (obviously, this will be perpendicular to diameter AG), and draw ΘX parallel to it from Θ. Then, since H454

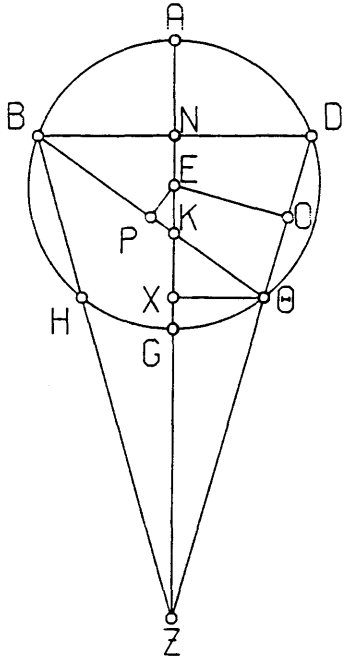

Fig. 12.2

BN = ND,
BN:XΘ = ND:XΘ.
But ND:XΘ = DZ:ZΘ [triangle ZND ||| triangle ZXΘ]
and BN:XΘ = BK:KΘ [triangle BNK ||| triangle ΘXK].
∴ DZ:ZΘ = BK:KΘ.

So, *componendo*,

(DZ+ZΘ):ZΘ = BΘ:ΘK.

And, dropping perpendiculars EO and EP, and *dividendo*, [we get],

$$OZ:Z\Theta = P\Theta:K\Theta.$$ [4]

And, *dividendo* once again,

$$O\Theta:Z\Theta = PK:K\Theta.$$

Therefore, if, in the epicyclic hypothesis, DZ is drawn in such a way that the ratio of OΘ to ZΘ equals the ratio of the speed of the epicycle to the speed of the
H455 planet, in the eccentric hypothesis PK:KΘ will have that same ratio.

The reason that in this case [i.e. in the eccentric hypothesis] we do not use this ratio obtained *dividendo* (namely PK:KΘ) to get the stations, but rather the undivided ratio (namely PΘ:KΘ), is that the epicycle's speed is in the same ratio to the planet's as the [mean] motion in longitude (alone) to the [mean] motion in anomaly, whereas the ratio of the eccentre's speed to the planet's is the same as that of the sun's mean motion (i.e. the sum of the planet's [mean] motions in longitude and anomaly) to the motion in anomaly. Thus, e.g. for Mars,

speed of epicycle : speed of planet ≈ 42:37

(for that, approximately, is the ratio which, as we demonstrated, holds between the [mean] motions in longitude and anomaly).[5]

Hence that is also the ratio of OΘ:ΘZ.

But speed of eccentre : speed of planet ≈ [42 + 37 =] 79:37,

i.e. this is the same as the ratio obtained *componendo*, PΘ:ΘK,
H456 since we found that the divided ratio, PK:KΘ, is equal to OΘ:ΘZ (i.e. 42:37).

Let the above suffice us as preliminary theorems. It remains to prove that when one takes lines [corresponding to ZD, BΘ] divided in the ratio described, then in both hypotheses H and Θ represent the points in which station appears to take place, and [thus] arc HGΘ must be retrograde, and the remainder [of the circle] possessing forward motion. [For this purpose] Apollonius proposes the following preliminary lemma.

[See Fig. 12.3.] In triangle ABG, in which

$$BG > AG,$$

if we cut off [from GB] GD ≥ AG,[6] then

$$GD:BD > \angle ABG:\angle BGA.$$

H457 His proof is as follows.

Complete the parallelogram ADGE (he says), and let BA and GE be produced to meet at Z. Then, since

$$AE\ [= GD] \geq AG,$$

the circle drawn on centre A with radius AE will either pass through G or beyond G. Let it be drawn to pass through G, as HEG. Then, since

$$\text{triangle } AEZ > \text{sector } AEH$$

$$\text{and triangle } AEG < \text{sector } AEG,$$

$$\text{triangle } AEZ : \text{triangle } AEG > \text{sector } AEH : \text{sector } AEG.$$

But sector AEH : sector AEG = ∠ EAZ:∠ EAG

[4] For DZ + ZΘ = 2OZ, and BΘ = 2PΘ (Euclid III 3). ∴ 2OZ:ZΘ = 2PΘ:ΘK. ∴ OZ:ZΘ = PΘ:ΘK. It is this last step which is described as *dividendo* (διελόντι). See Introduction pp. 17–18 for the two senses of this term.

[5] IX 3 p. 424. 37 returns in anomaly correspond to about 42 revolutions in longitude and 79 years.

[6] Literally 'not less than AG'.

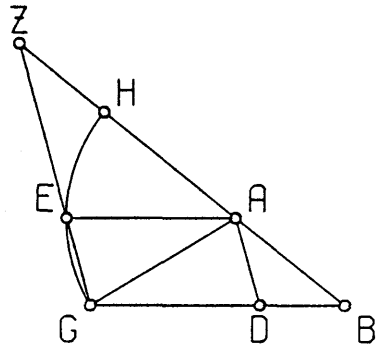

Fig. 12.3

and triangle AEZ : triangle AEG = ZE:EG (bases).[7]

$\therefore$ ZE:EG $>\angle$ ZAE:$\angle$ EAG.

But ZE:EG = [ZA:AB =] GD:DB.

And $\angle$ ZAE = $\angle$ ABG

and $\angle$ EAG = $\angle$ BGA.

$\therefore$ GD:DB $>\angle$ ABG:$\angle$ AGB.

And it is obvious that if GD (= AE) is supposed, not equal to AG, but greater, H458
the difference in the ratios will be even greater.

Now that we have established this preliminary lemma, let [Fig. 12.4] the epicycle be ABGD on centre E and diameter AEG. Produce AEG to Z, [representing] our point of view, so that

EG:GZ $>$ speed of epicycle : speed of planet.[8]

Thus it will be possible to draw a line ZHB[9] in such a way that

$\frac{1}{2}$BH:HZ = speed of epicycle : speed of planet.

Then, by what we proved previously, if we cut off arc AD equal to arc AB, and join DΘH, point Θ will represent our point of view in the eccentric hypothesis, and

$\frac{1}{2}$DH:ΘH = speed of eccentre : speed of planet.

We say, then, that in either hypothesis, when the planet reaches point H, it H459
will produce the appearance of station, and if we cut off arcs, however small, on either side of H, we will find that the arc intercepted towards the apogee will be an arc of forward motion, and the arc towards the perigee will be retrograde. [Proof:] First, cut off an arbitrary arc towards the apogee, KH, draw ZKL and KΘM, and join BK, DK and also EK and EH.

Then since, in triangle BKZ,

BH $>$ BK,[10]

BH:HZ $>\angle$ HZK:$\angle$ HBK [cf. above].

[7] Euclid VI 1: triangles with the same height are in proportion to their bases.

[8] The situation where EG:GZ = speed of epicycle : speed of planet is the limiting situation for retrogradation to occur: see p. 561.

[9] Because of Euclid III 8, which proves that of all lines drawn to a circle from a point outside it, that through the centre is the least.

[10] Euclid III 15.

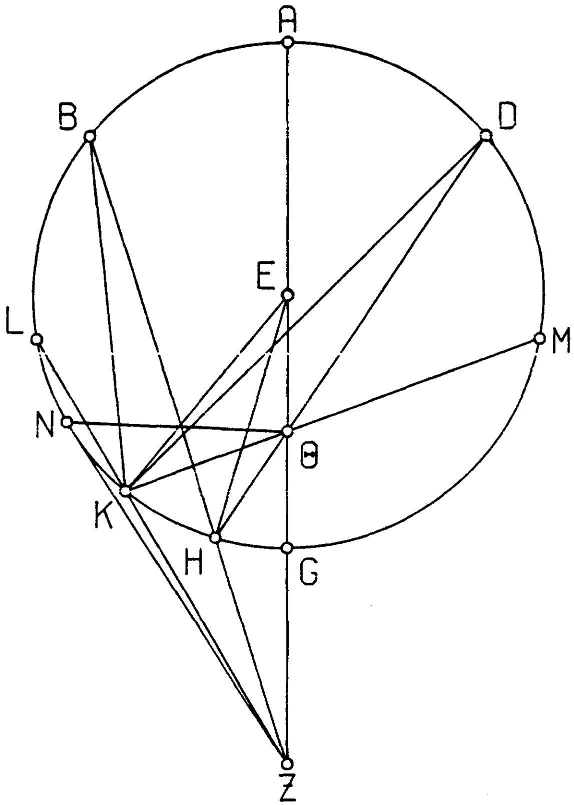

Fig. 12.4

$$\therefore\ \tfrac{1}{2}BH:HZ > \angle HZK:2\angle KBH = \angle HZK:\angle KEH$$

$$\text{But } \tfrac{1}{2}BH:HZ = \text{speed of epicycle : speed of planet.}$$

$$\therefore\ \angle HZK:\angle KEH < \text{speed of epicycle : speed of planet.}$$

Therefore the angle which has the same ratio to ∠ KEH as the ratio (speed of
H460 epicycle : speed of planet) is greater than ∠ HZK. Let that angle be ∠ HZN. Then, in the time that the planet takes to travel arc KH of the epicycle, the epicycle centre has moved in the opposite direction by an amount equal to the [angular] distance from ZH to ZN. So it is clear that arc KH of the epicycle has moved the planet in advance through an angle at our eye (∠ HZK) which is less than the angle (∠ HZN) through which [the motion of] the epicycle itself has moved it towards the rear during the same space of time. Thus the planet has undergone a forward motion [of the amount] of ∠ KZN.

Similarly, to carry out the reasoning as if the circle [ABGD] were an eccentre:[11]

[11] Reading ὡς ἐπὶ ἐκκέντρου τοῦ κύκλου (with C^2D) for ὡς ἐπὶ τοῦ ἐκκέντρου κύκλου ('as on the eccentric circle') at H460,13.

since BH:HZ $> \angle$ HZK:$\angle$ HBK,

componendo, BZ:ZH $> [\angle$ HZK + $\angle$ HBK =] $\angle$ BKL:$\angle$ HBK.

But BZ:ZH = DΘ:ΘH.[12]

And $\angle$ BKL = $\angle$ DKM[13]

and $\angle$ HBK = $\angle$ HDK.

$\therefore$ DΘ:ΘH $> \angle$ DKM:$\angle$ HDK. H461

So, *componendo*, DH:HΘ $> [\angle$ DKM + $\angle$ HDK =] $\angle$ HΘK:$\angle$ HDK.

Therefore, *dividendo*, $\frac{1}{2}$DH:HΘ $> \angle$ HΘK:2 $\angle$ HDK = $\angle$ HΘK:$\angle$ HEK.

But $\frac{1}{2}$ DH:ΘH = speed of eccentre : speed of planet.

$\therefore \angle$ HΘK:$\angle$ HEK $<$ speed of eccentre : speed of planet.

Therefore the angle which bears the same ratio to $\angle$ HEK as the speed of the eccentre bears to the speed of the planet is greater than $\angle$ HΘK. Let it, again, be $\angle$ HΘN. So, since the planet, in its own motion along KH, has travelled in advance through $\angle$ KEH, and in the same space of time has been carried by the motion of the eccentre towards the rear through $\angle$ HΘN, which is greater than $\angle$ KΘH, it is clear that, by this [hypothesis] too, the planet will appear to have undergone a forward motion [of the amount] of $\angle$ KΘN.

It is easy to see that the same method can be used to prove the opposite case,[14] H462
if in the same figure [Fig. 12.5] we suppose that

$\frac{1}{2}$LK:KZ = speed of epicycle : speed of planet

and hence $\frac{1}{2}$MK:ΘK = speed of eccentre : speed of planet;

and imagine arc KH cut off towards the perigee side of line LZ.

For, if we join LH to produce the triangle LZH, in which there is cut off ZK $>$ ZH, then

LK:KZ $< \angle$ HZK:$\angle$ HLK.

$\therefore \frac{1}{2}$LK:KZ $< \angle$ HZK:2 $\angle$ HLK = $\angle$ HZK:$\angle$ KEH,

which is the opposite of what was proved above.[15]

And, by the same reasoning, one will come to a conclusion opposite [to the above, namely] that

$\angle$ KEH:$\angle$ HZK $<$ speed of planet : speed of epicycle H463

and $\angle$ KEH:$\angle$ HΘK $<$ speed of planet : speed of eccentre.

So the angle which has the same ratio [to $\angle$ HZK or $\angle$ HΘK as the speed of the planet has to the speed of the epicycle or eccentre] turns out to be greater than $\angle$ KEH, and the resulting retrograde [component of] motion is greater than the forward.

Furthermore, it is clear that for distances at which

EG:GZ $\leq$ speed of epicycle : speed of planet

it will be impossible to draw another line [to the circle which will be cut] in a ratio equal to that [of the speeds of epicycle and planet], and the planet will not appear stationary or retrograde.

[12] This was proven p. 557 (in Fig. 12.2 DZ:ZΘ = BK:KΘ).

[13] Euclid III 27: angles standing on equal arcs are equal. I.e. Ptolemy assumes that arc BL = arc DM. This follows from the fact that Θ is a fixed point for given Z (cf. *HAMA* 264–5). Cf. p. 556, where it is shown that AZ:ZG = AK:KG, hence K (corresponding to Θ here) is a fixed point.

[14] I.e. that the planet will be retrograde on the other side of the point defined by the ratio of the speeds.

[15] p. 560, where $\frac{1}{2}$ BH:HZ $> \angle$ HZK:$\angle$ KEH.

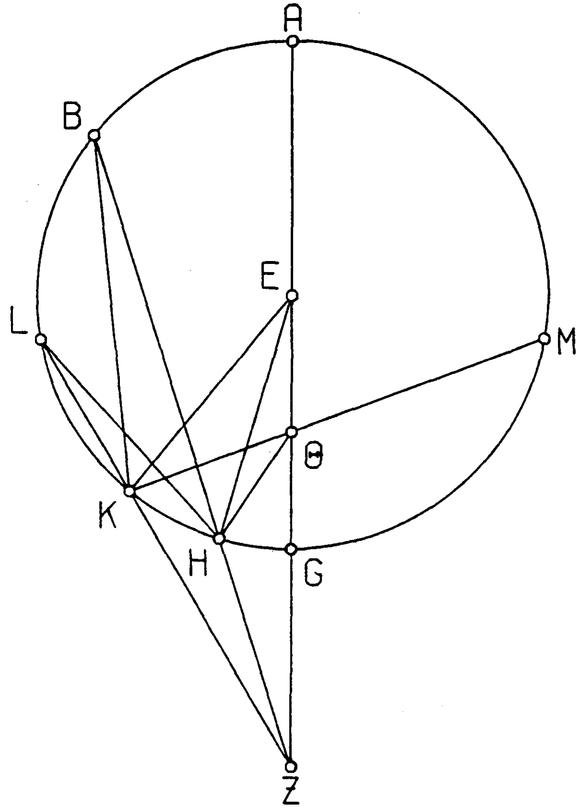

Fig. 12.5

For since, in triangle EKZ, EG has been cut off and is [equal to, i.e.] not less than EK,

$$\angle GZK{:}\angle GEK < EG{:}GZ.$$

But $EG{:}GZ \leq$ speed of epicycle : speed of planet.

$\therefore \angle GZK{:}\angle GEK <$ speed of epicycle : speed of planet.

H464 Hence, since we have shown [p. 560] that, where this occurs, the planet has undergone a forward motion, we shall find no arc either on epicycle or on eccentre on which it will appear retrograde.

2. {*Demonstration of the retrogradations of Saturn*}

That being established, we shall next set out the calculations of the retrogradations for each of the planets, in accordance with the hypotheses [previously] demonstrated, beginning with Saturn. The method is as follows.

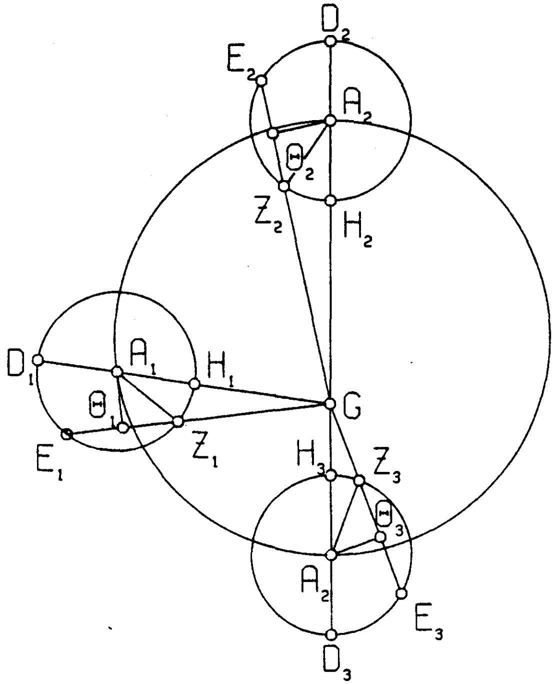

Fig. Q

[See Fig. 12.6.][16] Let the circle carrying the epicycle centre be AB on diameter AGB, on which G represents the centre of the ecliptic, i.e. our point of view. Describe the epicycle DEZH on centre A, and draw line GZE in such a way that, when perpendicular AΘ is dropped on to it, the ratio of half EZ (i.e. ΘZ) to ZG is that of the speed of the epicycle to the speed of the planet. Let us suppose, first, that the epicycle is situated at mean distance: thus the mean motions in longitude and anomaly are very nearly the same as the motions [in longitude and anomaly] taken with respect to the centre of the ecliptic.[17]

Now for Saturn, as we demonstrated [XI 6], where the mean distance GA is H465
60^p, the epicycle radius AD = $6\frac{1}{2}^p$.

Thus, by addition, DG = $66;30^p$,

and, by subtraction, GH = $53;30^p$ in the same units.

[16] Ptolemy uses an identical simplified figure (Figs. 12.6 – 12.12), in which the observer, G, is represented as the centre of the circle, for all situations. The actual situation is depicted in Fig. Q (copied from Manitius), where the subscripts 1, 2 and 3 represent the situations at mean, greatest and least distances respectively.

[17] I.e. because the epicycle centre is the same distance from the observer as it would be in the simple model treated in ch. 1, one can assimilate the situation to that, and use the mean motions unmodified. As Ptolemy says, this involves an approximation, since the *centre* of motion is not the observer, but the equant point. However, for small eccentricities this is negligible.

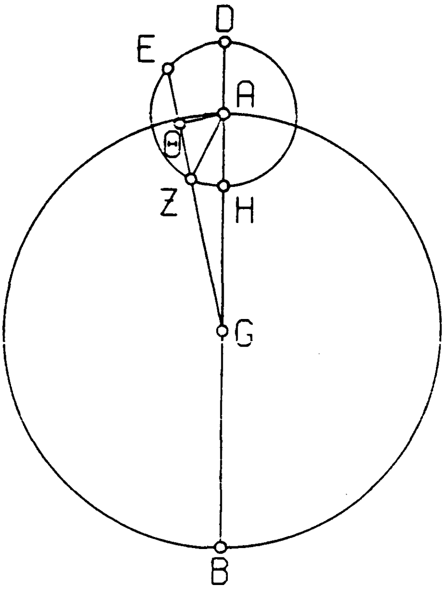

Fig. 12.6

Thus their product[18] is 3557;45^p.

But DG.GH = EG.GZ,

so EG.GZ = 3557;45^p in the same units.

Furthermore (in accordance with the mean motions), where the speed of the epicycle (i.e. ΘZ) is 1^p, the speed of the planet (i.e. ZG) is about 28;25,46^p.[19]

Therefore, by addition, EG [= ZG + 2ΘZ] = 30;25,46^p,

and EG.GZ = 865;5,32^p in the same units.

H466 So if we divide[20] 3557;45 by 865;5,32, which gives a quotient of 4;6,45, take the square root of the latter, 2;1,40, and multiply this factor into ΘZ (= 1^p) and ZG (= 28;25,46^p) separately, we get

$$\left.\begin{aligned} \Theta Z &= 2;1,40^p \\ \text{and } ZG &= 57;38,55^p \end{aligned}\right\} \text{ where } (EG.GZ) = 3557;45^p.$$

Then if we join AZ, where AZ = 6;30^p,

ZΘ = 2;1,40^p,

so where AZ = 120^p, ZΘ = 37;26,9^p.

Therefore, in the circle about right-angled triangle AZΘ,

arc ΘZ = 36;21,15°,[21]

$$\text{so } \angle\, Z A\Theta \left\{\begin{aligned} &= 36;21;15^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \\ &\approx 18;10,38^{\circ} \text{ where 4 right angles} = 360^{\circ}. \end{aligned}\right.$$

[18] Literally 'the rectangle contained by them'.

[19] Taking the mean daily motions tabulated in IX 4 one finds the ratio of longitude to anomaly as 1 : 28;25,*55* . . . Ptolemy may have taken the rounded numbers 0;57,7,43°/d and 0;2,0,34°/d, which lead to 28;25,*48*.

[20] παραβάλωμεν παρὰ, literally 'measure it by laying alongside'.

[21] Accurately, 36;21,20°.

Furthermore, where hypotenuse [of right-angled triangle AGΘ] GHA = 60^p,
by addition, GZΘ [= $57;38,55^p + 2;1,40^p$] = $59;40,35^p$,
so where GHA = 120^p, GZΘ = $119;21,10^p$.

So, in the circle about right-angled triangle AGΘ,
arc GΘ = 168;5,39°.

$$\therefore \angle \text{GA}\Theta \begin{cases} = 168;5,39^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \\ \approx \ 84;2,50^{\circ} \text{ where 4 right angles} = 360^{\circ}. \end{cases}$$

Hence we get ∠ AGΘ = 5;57,10° (complement), H467
and ∠ ZAH = ∠ GAΘ − ∠ ZAΘ = 65;52,12°.

So, since the planet is seen along line GZ at first station, and along GH at [mean] opposition, it is clear that, if the epicycle centre had no motion towards the rear [during this interval], arc ZH of the epicycle, comprising 65;52,12°, would produce a retrograde motion of the amount of ∠ AGZ, 5;57,10°. But since, according to the above ratio of the speed of the epicycle to the speed of the planet, to this anomaly of 65;52,12° correspond approximately 2;19° in longitude,[22] we get a retrograde motion of:

from either station to opposition 3;38,10° and 69^d[23]

(the latter is approximately the time the planet takes to move 2;19° in mean longitude),

and a total retrogradation of 7;16,20° and 138^d.

Next we will investigate the [corresponding] quantities near the greatest distance under the same conditions, namely when the opposition halfway between the [two] stations brings the epicycle centre precisely to the apogee of the eccentre, and, obviously, brings each of the two stations to a distance in corrected longitude from the opposition (i.e. from the apogee)[24] which is close to H468
the 2;19° which was derived [above] from the ratio between the mean [motions]. In this situation AG, which represents the distance at that moment, is negligibly different from the greatest distance,[25] and hence is obtained via the theorems previously developed, and to 1° of longitude corresponds an equation of about 6;30′.[26] Therefore the ratio of the corrected [motion in] longitude to the corrected [motion in] anomaly, i.e. of the apparent speed of the epicycle at that moment to the apparent speed of the planet, is 0;53,30 : 28;32,16.[27]

Then, repeating the same figure [Fig. 12.7], where the radius of the epicycle DA is $6;30^p$, GA (which is negligibly different from the greatest distance) is $63;25^p$.

Hence, by addition, DG is computed as $69;55^p$,
and, by subtraction, GH = $56;55^p$.
And DG.GH (= EG.EZ) = $3979;25,25^p$.

[22] 65;52,12/28;25,46 = 2;19,1.

[23] 5;57,10° − 2;19° = 3;38,10°. In 69 days the planet moves 2;18,39° in longitude, i.e. here (and throughout) Ptolemy rounds to the nearest day or convenient fraction of a day.

[24] Since this must be the meaning, one has to correct Heiberg's punctuation at H468,3, deleting the comma after μήκους, and inserting a comma after ἀπογείου.

[25] Since the epicycle centre is in the apogee of the eccentre halfway between the stations, at the actual stations the epicycle is a little before or after apogee: hence 'negligibly different'.

[26] In the anomaly table for Saturn (XI 11), to 6° corresponds an equation of centre of 39′: hence to 1′ corresponds exactly 6½′.

[27] I.e. 1° − 0;6,30° and 28;25,46° + 0;6,30° (cf. p. 564 n.19). On the rationale for this procedure see *HAMA* 193–4.

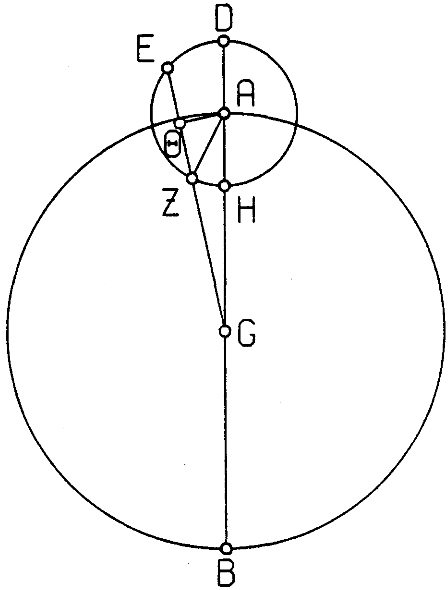

Fig. 12.7

And, by hypothesis, where ZΘ (representing the speed of the epicycle) is
H469 0;53,30^p, GZ (representing the speed of the planet) is 28;32,16^p;
so, by addition, EG [= GZ + 2ZΘ] = 30;19,16^p,
and EG.GZ = 865;17,50^p.

So, again, dividing 3979;25,25 by 865;17,50, which gives 4;35,56, taking the square root of the latter, 2;8,40, and multiplying this factor into ΘZ (= 0;53,30^p) and ZG (= 28;32,16^p) separately, we get

$$\left.\begin{aligned} \Theta Z &= 1;54,44^p \\ \text{and } GZ &= 61;11,52^p \end{aligned}\right\} \text{ where } AZ = 6;30^p \text{ and } AG = 63;25^p.$$

And, by addition, GΘ = 63;6,36^p in the same units.

Therefore where hypotenuse AZ [of right-angled triangle AZΘ] = 120^p,

$$\Theta Z = 35;18,9^p,$$

and where hypotenuse GA [of right-angled triangle AGΘ] = 120^p,

$$G\Theta = 119;25,11^p.$$

Therefore, in the circle about right-angled triangle AZΘ,

$$\text{arc } \Theta Z = 34;13,4°,$$

and, in the circle about right-angled triangle AGΘ,

$$\text{arc } G\Theta = 168;43,38°.$$

H470

$$\left.\begin{aligned} \therefore \angle Z A \Theta &= 34;13,4^{\circ\circ} \\ \text{and } \angle G A \Theta &= 168;43,38^{\circ\circ} \end{aligned}\right\} \text{ where 2 right angles} = 360^{\circ\circ}.$$

$$\left.\begin{aligned} \therefore \angle Z A \Theta &= 17;6,32° \\ \text{and } \angle G A \Theta &= 84;21,49° \end{aligned}\right\} \text{ where 4 right angles} = 360°.$$

Hence, by subtraction [from 90°], ∠ AGΘ (which represents the amount of retrogradation[28] which there would be between either of the stations and opposition, if the epicycle had no[29] forward motion) is 5;38,11°,

and, by subtraction [of ∠ ZAΘ from ∠ GAΘ], ∠ ZAH (which represents the apparent motion on the epicycle[30] at the same [unchanging] distance) is 67;15,17°.

Now, according to the ratio of the speeds at the apogee, to the latter amount correspond 2;6,6° in corrected longitude;[31] so we get, for half of the total retrogradation,

[5;38,11° − 2;6,6° =] 3;32,5° and $70\frac{1}{3}^{d}$

(the latter is approximately the time the planet takes to travel 2;21,25° in mean longitude, which is the amount corresponding to the above 2;6,6° in corrected longitude);

and, for the total retrogradation,

7;4,10° and $140\frac{2}{3}^{d}$.

Again, we will investigate the [corresponding] quantities near the least distance, using the same figure [Fig. 12.8] and under similar conditions, i.e.

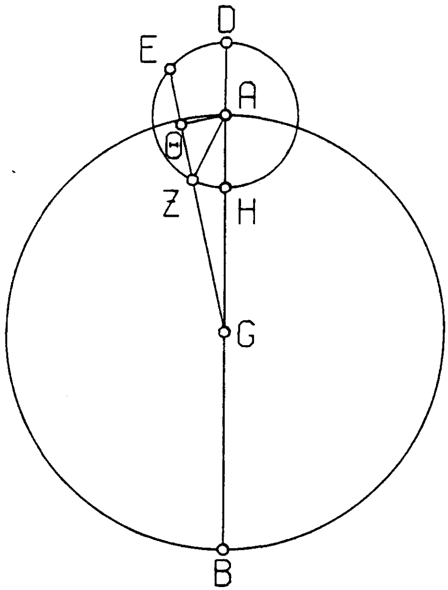

Fig. 12.8

[28] Reading τῆς (with C²D) for τοῦ at H470,6. Cf. H473,1. Corrected by Manitius.

[29] Reading μηδὲν at H470,8 for μηδενός. There is no ms. authority for my correction, but it is necessary for the sense. As a consequence of the corruption of τῆς to τοῦ just above, it was assumed that προηγήσεως was connected with ὑπελείπετο, hence μηδὲν was changed to μηδενός to agree with it.

[30] By 'apparent motion' Ptolemy means 'as counted from the true [and not the mean] epicyclic perigee'.

[31] One might suppose from what he says here that Ptolemy computes 67;15,17° × 0;53,30/28;32,16. This leads to 2;6,5°. The actual method of computation is explained at the end of XII 6 (p. 582). It is as follows: 67;15,17° × 1/28;32,16 = 2;21,24°. To the latter corresponds an equation of 0;15,19°, which, subtracted from 67;15,17°, gives about 67°. Then 67° × 1/28;25,46 = 2;21,25°. 2;21,25° − 0;15,19° = 2;6,6°.

H471 when the opposition halfway between the [two] stations is precisely at the perigee of the eccentre, and both stations are the above [ca. 2;19°] distance in longitude from the opposition (i.e. from the perigee).

In this situation the distance at that moment, AG, is found in the same way [as at greatest distance], since it is negligibly different from the least distance. And to 1° of longitude corresponds an equation of about 7;20 minutes.[32] So here

apparent speed of epicycle : apparent speed of planet = 1;7,20 : 28;18,26.[33]

Hence, where ΘZ = 1;7,20^{p}, GZ = 28;18,26^{p},

and, by addition, EG = 30;33,6^{p},[34]

and EG.GZ = 864;49,58^{p}.[35]

But where the epicycle radius, DA = 6;30^{p},

AG (which is negligibly different from the least distance) is 56;35^{p};

hence, by addition, DG = 63;5^{p},

and, by subtraction, GH = 50;5^{p},

H472 and DG.GH (= EG.GZ) = 3159;25,25^{p}.

Therefore if, as before, we divide 3159;25,25 by 864;49,58, which gives 3;39,12, take the square root of that, 1;54,41,[36] and multiply the latter factor into ΘZ (= 1;7,20^{p}) and ZG (= 28;18,26^{p}) separately, we get

ΘZ = 2;8,43^{p}

where the epicycle radius, AZ = 6;30^{p}, and the distance at that moment, AG = 56;35^{p};

and GZ = 54;6,22^{p} in the same units.

Hence, by addition, GΘ = 56;15,5^{p} in the same units.

Therefore, where hypotenuse AZ = 120^{p}, ΘZ = 39;36,18^{p},

and, where hypotenuse GA = 120^{p}, GΘ = 119;17,46^{p}.[37]

Hence, in the circle about right-angled triangle AZΘ,

arc ZΘ = 38;32,34°,

and, in the circle about right-angled triangle AGΘ,

arc GΘ = 167;34,54°.

$$\left.\begin{aligned}\therefore \angle\, Z A\Theta &= 38;32,34^{\circ\circ}\\ \text{and } \angle\, G A\Theta &= 167;34,54^{\circ\circ}\end{aligned}\right\}\ \text{where 2 right angles} = 360^{\circ\circ}.$$

$$\left.\begin{aligned}\text{And } \angle\, Z A\Theta &= 19;16,17^{\circ}\\ \text{and } \angle\, G A\Theta &= 83;47,27^{\circ}\end{aligned}\right\}\ \text{where 4 right angles} = 360^{\circ}.$$

H473 Therefore, by subtraction [from 90°], we get ∠ AGΘ, which represents the retrogradation (due to the planet's speed) between either of the stations and opposition, as 6;12,33°,

[32] To an argument of 177° (= 180° − 3°) corresponds (Table XI 11) an equation of centre of 0;22°. Hence to 1° near perigee corresponds 0;7,20°.

[33] I.e. 1 + 0;7,20 and 28;25,46 − 0;7,20.

[34] Deleting τοιούτων at H471,18–19 (with D,Ar).

[35] Reading $\overline{\nu\eta}$ for $\overline{\nu\ }$ (misprint in Heiberg) at H471,20.

[36] Reading $\overline{\mu\alpha}$ at H472,5 for $\overline{\mu\beta}$ (1;54,42). The latter has no ms. authority, but is Heiberg's correction for the $\overline{\mu\varepsilon}$ (45) or $\overline{\mu\theta}$ (49) of the Greek mss. '41' is the reading of Ger (all other Arabic mss. I have seen have '49'), and is shown to be correct not only because it is the square root of 3;39,12 (accurate to two sexagesimal places), but because (below) 1;54,41 × 28;18,26 ≈ 54;6,22 (in agreement with the text), whereas 1;54,42 × 28;18,26 ≈ 54;6,50.

[37] 119;17,45^{p} would be a more accurate result, and corresponds better to the arc 167;34,54° given below. But in the absence of any ms. authority I hesitate to change it.

and, again by subtraction [of ∠ ZAΘ from ∠ GAΘ], ∠ ZAH, which represents the apparent motion on the epicycle at the same [unchanging] distance, as 64;31,10°.

According to the ratio of the speeds at the perigee, to the latter amount correspond 2;33,28° in corrected longitude.[38] Hence we get for half the total retrogradation,

[6;12,33° − 2;33,28° =] 3;39,5° and 68^d

(the latter is approximately the time taken by the planet to travel, at mean speed, 2;16,45°, which is the amount in mean longitude corresponding to the above 2;33,28° of corrected longitude).

[Thus] the total retrogradation is

7;18,10° and 136^d.

3. {*Demonstration of the retrogradations of Jupiter*}

For Jupiter [see Fig. 12.9], according to our calculations for mean distance,

ΘZ:GZ = 1 : 10;51,29,[39]

and EG:ZG = 12;51,29 : 10;51,29,

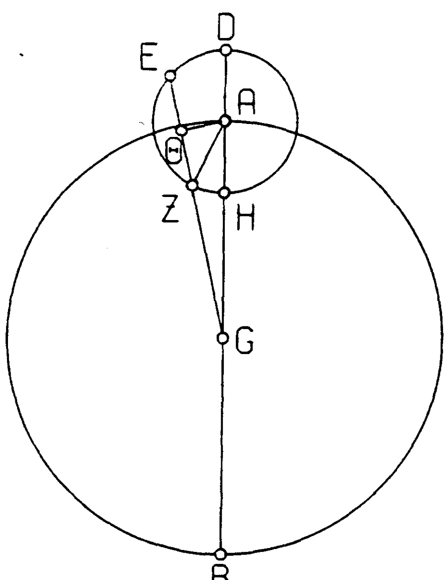

Fig. 12.9

[38] Cf. p. 567 n.31. Computation: 64;31,10° × 1/28;18,26 = 2;16,45°. Equation for 180°−2;16,45° is 0;16.43°. 64;31,10° + 0;16,43° = 64;47,53°. The latter multiplied by 1/28;25,46 gives 2;16,45°, and 2;16,45° + 0;16.43° = 2;33.28°.

[39] Taking the first three places (rounded) of the mean daily motions from IX 4 (cf. p. 564 n.19), one gets 0;54,9,3 : 0;4,59,14 = 10;51,28,29 . . .

so EG.ZG = 139;37,39.[40]

H474 Furthermore GA:AD = 60 : 11;30

and GD:GH = 71;30 : 48:30,

so GD.GH = 3467;45.

Dividing [3467;45 by 139;37,39] we get 24;50,9, the square root of which, 4;59,1, we multiply into the above ratio of ΘZ:GZ, and get, in terms of the given sizes of GA and AZ [i.e. 60 and 11;30],

$\Theta Z = 4;59,1^p$

and $GZ = 54;6,44^p$ in the same units.

and, by addition, $G\Theta = 59;5,45^p$.

Hence, expressed in units where hypotenuses AZ and AG [respectively] are 120^p,

$\Theta Z = 52;0,10^p$

and $G\Theta = 118;11,30^p$,

and the corresponding[41] arcs are:

arc ZΘ = 51;21,41°

and arc GΘ = 160;4,55°.

Accordingly we compute ∠ ZAΘ ≈ 25;40,50°

H475 and ∠ GAΘ ≈ 80;2,28°,

and, by subtraction [of ∠ GAΘ from 90°], ∠ ZGA, which represents the retrogradation due to the planet's speed, is 9;57,32°, and ∠ ZAH, which represents the apparent [motion in] anomaly, is [∠ GAΘ − ∠ ZAΘ =] 54;21,38°. To the latter correspond 5;1,24° in longitudinal motion, according to the above ratio [of 1 : 10;51,29].[42] Thus half the retrogradation is

4;56,8° and about $60\frac{1}{2}^d$,

and the total retrogradation is

$9;52,16°$ and 121^d.

The distance at an elongation of about 5° from apogee or perigee is [respectively] negligibly smaller than the greatest distance and negligibly larger than the least distance.

According to our calculations for greatest distance, the equation [corresponding to 1°] for correcting [the speeds] is $5\frac{1}{6}$ minutes.[43] Hence

ΘZ:GZ = 0;54,50 : 10;56,39

and EG:GZ = 12;46,19 : 10;56,39,

and EG.GZ = 139;46,42.

[40] Ptolemy has made a computing error: correct is 139;36,48, and this is indeed found in Ger, derived no doubt from the kind of marginal correction found in D^2 (139;36,48,32). That the error is Ptolemy's is shown by the subsequent calculations (at H474,5 Ger reads 24;50,17, again in agreement with D^2 and the above amount, but the square root should be 4;59,2, whereas the whole tradition agrees on 4;59,1, which is confirmed by the following computations).

[41] Reading ἐπ' αὐταῖς at H474,16 (with all mss.) for Heiberg's correction ἐπ' αὐτῶν. Although the genitive is normal in the Almagest in expressions of the type ἡ ἐπὶ τῆς ZΘ περιφερείας, the dative after ἐπί is perfectly good Greek, and is explicable here as avoiding the ambiguity of two genitive plurals referring to different things. I have restored the mss.' reading in the similar passages H476,9 and H477,18.

[42] In fact 54;21,38/10;51,29 = 5;0,23°. But the number in the text is confirmed by the following computations.

[43] Reading ε ς' (with L,Ger) at H475,14 for ε ζ (5;6). The correction was made by Manitius, who notes that, in the table of anomaly, to an argument of 6° corresponds an equation of centre of 0;31°, hence, to 1°, 0;5,10°.

Furthermore, GA:AD = 62;45 : 11;30,
DG:GH = 74;15 : 51;15,
and DG.GH = 3805;18,45.

Dividing [3805;18,45 by 139;46,42], we get 27;13,26, the square root of which, 5;13,4, when multiplied into the above ratio of ΘZ:GZ, gives, in terms of the given sizes of GA and AZ [i.e. 62;45 and 11;30] H476

ZΘ = 4;46,6^p,
GZ = 57;6,19^p,[44]
and, by addition, GΘ = 61;52,25^p.

Hence, expressed in units where hypotenuses AZ and AG [respectively] are 120^p,

ZΘ = 49;45,23^p
and GΘ = 118;19,27^p,

and the corresponding arcs are:

arc ZΘ = 48;59,34°
arc GΘ = 160;49,36°.
Accordingly, ∠ ZAΘ = 24;29,47°
and ∠ GAΘ = 80;24,48°.

And, by subtraction, ∠ ZGA, which represents the retrogradation due to the planet's speed, is [90° − ∠ GAΘ =] 9;35,12°, and ∠ ZAH, which represents the apparent [motion in] anomaly, is [∠ GAΘ − ∠ ZAΘ =] 55;55,1°. To the latter correspond 4;40,35° in corrected longitudinal motion,[45] and 5;6,35° in mean [longitudinal] motion, according to the ratio [of speeds] at the apogee. Thus half the retrogradation is

[9;35,12°− 4;40,35°=] 4;54,37° and about 61½^d,

and the total retrogradation

9;49,14° and 123^d.

According to our calculations for least distance, the equation [corresponding to 1°] for correcting [the speeds] is found to be 5⅔ minutes.[46] Hence H477

ΘZ:ZG = 1;5,40 : 10;45,49,
EG:ZG = 12;57,9 : 10;45,49,
and EG.ZG = 139;24,56.
Furthermore, GA:AD = 57;15 : 11;30,
DG:GH = 68;45 : 45;45,
and DG.GH = 3145;18,45.

Dividing [the latter by 139;24,56], we get 22;33,39, the square root of which, 4;45, multiplied into the above ratio of ΘZ:GZ, gives, in terms of the above sizes of GA and AZ [i.e. 57;15 and 11;30],

ΘZ = 5;11,55^p,
ZG = 51;7,38^p,
and, by addition, GΘ = 56;19,33^p.

[44] More accurate would be 57;6,15, which is the reading of D and is given as an alternative in ABC. But the text is guaranteed by the following computations.

[45] Cf. p. 567 n.31. Computation: 55;55,1° × 1/10;56,39 = 5;6,33°, to which corresponds an equation of 0;26,24° ≈ 26′. 55;55,1° − 0;26° = 55;29,1°. This multiplied by 1/10;51,29 = 5;6,35° [so text; accurately 5;6,36]. 5;6,35° − 0;26° = 4,40;35°.

[46] In the table of anomaly, to an argument of [180° − 3° =] 177° corresponds an equation of 0;17°, hence to 1° near perigee corresponds 5⅔′.

Hence, expressed in units where hypotenuses ZA and AG [respectively] are 120^p,

$$Z\Theta = 54;14,47^p$$
$$\text{and } G\Theta = 118;3,46^p,$$

and the corresponding arcs

$$\text{arc } Z\Theta = 53;45,4°$$
$$\text{and arc } G\Theta = 159;22,40°.$$
$$\text{Accordingly } \angle Z A\Theta = 26;52,32°$$

H478
$$\text{and } \angle GA\Theta = 79;41,20°.$$

And, by subtraction, $\angle$ ZGA, which represents the retrogradation due to the planet's speed, is [90° − $\angle$ GAΘ =] 10;18,40°, and $\angle$ ZAH, which represents the apparent [motion in] anomaly, is [$\angle$ GAΘ − $\angle$ ZAΘ =] 52;48,48°. To the latter correspond 5;21,20° in corrected longitudinal motion,[47] and 4;54,20° in mean [longitudinal] motion, according to the ratio [of speeds] at the perigee. Thus half the retrogradation is

[10;18,40° − 5;21,20° =] 4;57,20° and about 59^d,

and the total retrogradation is

9;54,40° and 118^d.

4. {*Demonstration of the retrogradations of Mars*}

Again, in the case of Mars [see Fig. 12.10], according to our calculations for near mean distance,

$$\Theta Z:ZG = 1 : 0;52,51,\text{[48]}$$
$$\text{and } EG:GZ = 2;52,51 : 0;52,51,$$
$$\text{so } EG.GZ = 2;32,15.$$
$$\text{Furthermore, } GA:AH = 60 : 39;30,$$
$$\text{and } DG:GH = 99;30 : 20;30,$$
$$\text{so } DG.GH = 2039;45.$$

H479 Dividing [2039;45 by 2;32,15], we get 803;50,50,[49] the square root of which, 28;21,8, multiplied into the above ratio of ΘZ:ZG, gives, in terms of the above sizes of GA and AZ [i.e. 60 and 39;30],

$$\Theta Z = 28;21,8^p,$$
$$GZ = 24;58,25^p \text{ in the same units,}$$
$$\text{and, by addition, } G\Theta = 53;19,33^p.$$

Hence, in units where hypotenuses AZ and AG are each [respectively] 120^p,

$$Z\Theta = 86;8,0^p$$
$$\text{and } G\Theta = 106;39,6^p.$$

[47] Cf. p. 567 n.31. Computation: 52;48,48° × 1/10;45,49 = 4;54,24°, to which corresponds an equation of 27′ [so text: accurate would be 29′]. 52;48,48° + 0;27° = 53;15,48°, which multiplied by 1/10;51,29 gives 4;54,20° [accurately 4;54,19°]. 4;54,20° + 0;27° = 5;21,20°.

[48] From the mean daily motions (IX 4) : 0;27,41,40/0;31,26,36 = 0;52,50,47 . . .

[49] Accurate would be 803,50,33, which is found as the reading of the second hand in D. Ger has 803;50,32, T 803,50,30. The variation has no further consequences, since the square root of all (to the nearest second) is 28;21,8.

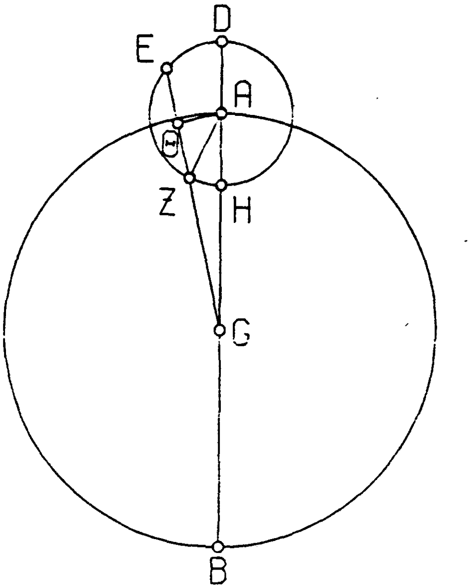

Fig. 12.10

The corresponding arcs are

$$\text{arc } Z\Theta = 91;44,34^\circ$$
$$\text{and arc } G\Theta = 125;26,10^\circ.$$
$$\text{Accordingly } \angle\, ZA\Theta = 45;52,17^\circ$$
$$\text{and } \angle\, GA\Theta = 62;43,5^\circ.$$

And, by subtraction, ∠ ZGA, which represents the retrogradation due to the planet's speed, is [90° − ∠ GAΘ =] 27;16,55°, and ∠ ZAH, which represents the [motion in] anomaly, is [∠ GAΘ − ∠ ZAΘ =] 16;50,48°. To the latter amount correspond 19;7,33° in [mean] longitudinal motion, according to the above ratio [of speeds, of 1 : 0;52,51]. Thus half the retrogradation is

[27;16,55° − 19;7,33° =] 8;9,22° and about $36\frac{1}{2}^{d}$.

And the total retrogradation is

$$16;18,44^\circ \text{ and } 73^{d}.$$

[Hence] the distance at the elongation of the stations from apogee and perigee is [respectively] about $0;20^{p}$ of the mean distance [i.e. 60^{p}] less than the greatest distance, and about the same amount greater than the least distance.[50] H480

According to our calculations for near greatest distance, the equation corresponding to an argument of 1° for correcting [the speeds] is found to be $10\frac{1}{3}'$.[51] Hence

[50] For a true centrum (κ) of 19;7,33°, the distance of the centre of the epicycle, $\rho = 65;38,12^{p} \approx 66^{p} - 22'$. For $\kappa = 160;52,27^\circ$, $\rho = 54;17,56^{p} \approx 54^{p} + 18'$, i.e. 20′ is a reasonable mean.

[51] In the anomaly table for Mars (XI 11), to an argument of 18° corresponds an equation of 3;13° and to 24°, 4;16°; hence, as Manitius notes, the correct amount corresponding to 1° should be $(4;16 - 3;13)/6 = 10\frac{1}{2}'$.

$$\Theta Z:ZG = 0;49,40 : 1;3,11,$$
$$EG:GZ = 2;42,31 : 1;3,11,$$
$$\text{and } EG.GZ = 2;51,8.$$
$$\text{Furthermore, } GA:AH = 65;40 : 39;30,$$
$$DG:GH = 105;10 : 26;10,$$
$$\text{and } DG.GH = 2751;51,40.$$

And, when we divide [2751;51.40 by 2;51,8], we get 964;48,47, the square root of which, 31;3,41, multiplied into the above ratio of ΘZ:ZG, gives, in terms of the above sizes of GA and AZ [i.e. 65;40 and 39;30],

$$\Theta Z = 25;42,43^p,$$
$$GZ = 32;42,34^p,$$

H481 $$\text{and, by addition, } G\Theta = 58;25,17^p.$$

Hence, expressed in units where hypotenuses AZ and AG are each [respectively] 120^p,

$$Z\Theta = 78;6,44^p$$
$$\text{and } G\Theta = 106;45,36^p.$$

The corresponding arcs are

$$\text{arc } Z\Theta = 81;13,8^{\circ}$$ [52]
$$\text{and arc } G\Theta = 125;39,46^{\circ}.$$
$$\text{Accordingly } \angle Z A\Theta = 40;36,34^{\circ}$$
$$\text{and } \angle GA\Theta = 62;49,53^{\circ}.$$

And, by subtraction, ∠ ZGA, which represents the retrogradation due to the planet's speed, is [90° − ∠ GAΘ =] 27;10.7°, while ∠ ZAH, which represents the [motion in] apparent anomaly, is [∠ GAΘ − ∠ ZAΘ =] 22;13,19°. To the latter correspond [motions in] corrected longitude of 17;13,21°, and in mean [longitude] of 20;58,21°,[53] according to the ratios [of the speeds] at the apogee. Thus half the retrogradation is

[27;10.7° − 17;13,21° =] 9;56,46° and about 40^d,

and the total retrogradation is

19;53,32° and 80^d.

According to our calculations for near least distance, the equation [corresponding to an argument of 1°] for correcting [the speeds] is found to be $12\frac{2}{3}'$.[54] Hence

$$\Theta Z:ZG = 1;12,40 : 0;40,11,$$
$$EG:GZ = 3;5,31 : 0;40,11,$$
$$\text{and } EG.GZ = 2;4,14.$$

H482 $$\text{Furthermore, } GA:AH = 54;20 : 39;30,$$
$$DG:GH = 93;50 : 14;50,$$
$$\text{and } DG.GH = 1391;51,40.$$

Dividing [1391;51,40 by 2;4,14], we get 672;13, the square root of which,

[52] Correct would be 81;13,28°, and this is the reading of BCL,Ger. However, all mss. agree in the reading for the half of this, 40;36,34°, which would seem to confirm Heiberg's reading here. It is possible, however, that Ptolemy made an error in halving, and that the reading '8' in AD is due to scribal correction.

[53] Ptolemy gives the computation for this at XII 6 p. 582.

[54] In the anomaly table for Mars (XI 11), to an argument of 162° corresponds an equation of 3;55°, and to 159°, 4;33°. Therefore to 1°, at about 20° from perigee, corresponds (4;33 − 3;55)/3 = $12\frac{2}{3}'$.

25;55,38, multiplied into the above ratio of $\Theta Z{:}ZG$, gives, in terms of the above sizes of GA and AZ [i.e. 54;20 and 39;30],

$$\begin{aligned} \Theta Z &= 31;24,3^p, \\ GZ &= 17;21,51^p \text{ in the same units,} \\ \text{and, by addition, } G\Theta &= 48;45,54^p. \end{aligned}$$

Hence, where the hypotenuses AZ and AG are each [respectively] 120^p,

$$\begin{aligned} Z\Theta &= 95;23,42^p \\ \text{and } G\Theta &= 107;42,7^p. \end{aligned}$$

The corresponding arcs are

$$\begin{aligned} \text{arc } Z\Theta &= 105;18,10^\circ \\ \text{and arc } G\Theta &= 127;40,22^\circ.\text{[55]} \\ \text{Accordingly } \angle ZA\Theta &= 52;39,5^\circ \\ \text{and } \angle GA\Theta &= 63;50,11^\circ. \end{aligned}$$

And, by subtraction, $\angle ZGA$, which represents the [amount of] retrogradation due to the planet's speed, is [$90^\circ - \angle GA\Theta =$] $26;9,49^\circ$, *while* $\angle ZAH$, *which* represents the [motion in] apparent anomaly, is [$\angle GA\Theta - \angle ZA\Theta =$] $11;11,6^\circ$. To the latter correspond [motions in] corrected longitude of $20;33,42^\circ$, and in mean longitude of $16;52,52^\circ$, according to the ratios [of the speeds] at the perigee.[56] So half the retrogradation comes out as H483

[$26;9,49^\circ - 20;33,42^\circ =$] $5;36,7^\circ$ and about $32\frac{1}{4}^d$,

and the total retrogradation is

$$11;12,14^\circ \text{ and } 64\tfrac{1}{2}^d.$$

5. {*Demonstration of the retrogradations of Venus*}

Again, in the case of the planet Venus [see Fig. 12.11], according to our calculations for mean distance,

$$\begin{aligned} \Theta Z{:}ZG &= 1 : 0;37,31,\text{[57]} \\ EG{:}GZ &= 2;37,31 : 0;37,31, \\ \text{and } EG.GZ &= 1;38,30. \\ \text{Furthermore, } GA{:}AH &= 60 : 43;10, \\ DG{:}GH &= 103;10 : 16;50, \\ \text{and } DG.GH &= 1736;38,20. \end{aligned}$$

Dividing [1736;38,20 by 1;38,30], we get 1057;51,[58] the square root of which,

[55] Accurately, $127;40,3^\circ$.

[56] Cf. p. 567 n.31. Computation: $11;11,6^\circ \times 1/0;40,11 = 16;42,3^\circ$, to which corresponds an equation of $3;40,50^\circ$ [accurately $3;38,59^\circ$: it appears as if Ptolemy took the equation of ($180^\circ - 16;51^\circ$)]. $11;11,6^\circ + 3;40,50^\circ = 14;51,56^\circ$, which multiplied by $1/0;52,51$ gives $16;52,52^\circ$ [accurately $16;52,36^\circ$]. $16;52,52^\circ + 3;40,50^\circ = 20;33,42^\circ$.

[57] However one computes, 0;37,32 would be more accurate. From the relationship (IX 3 p. 424) 5 revolutions in anomaly correspond to 8 revolutions in longitude less $2\frac{1}{4}^\circ$, one finds 0;37,31,45 . . ., and the same from the mean daily motion carried to three places. Even taking only two places (0;36,59/0; 59,8), one gets 0;37,31,31 . . .

[58] Reading $\overline{\nu\alpha}$ (with C^2) for $\overline{\nu}\,\varsigma$ (1057;50,6) at H483,22. The latter is Heiberg's emendation for the reading of most mss., $\overline{\nu\varsigma}$ (1057;56), which I take to be a scribal corruption of $\overline{\nu\alpha}$. Correct to two fractional places is 1057;51,4, and that Ptolemy did not make a computing error is indicated by the amount given for the square root. The reading of D,Ar (1057;50,56) is also consistent with the square root, but seems to be a conjectural (and baseless) correction of the corruption 1057,56.

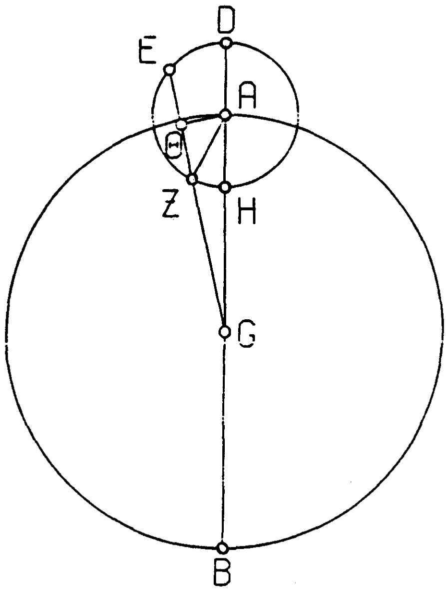

Fig. 12.11

H484 32;31,29, multiplied into the above ratio of ΘZ:ZG, gives, in terms of the above sizes of GA and AZ [i.e. 60 and 43;10],

$$\Theta Z = 32;31,29^{p},$$

$$GZ = 20;20,11^{p} \text{ in the same units,}$$

$$\text{and, by addition, } G\Theta = 52;51,40^{p}.$$

Hence, where hypotenuses AZ and AG are each [respectively] 120^{p},

$$Z\Theta = 90;24,58^{p}$$

$$\text{and } G\Theta = 105;43,20^{p}.$$

The [corresponding] arcs are:

$$\text{arc } Z\Theta = 97;47,0^{\circ}$$

$$\text{and arc } G\Theta = 123;31,49^{\circ}.$$

$$\text{Accordingly } \angle Z A\Theta = 48;53,30^{\circ}$$

$$\text{and } \angle GA\Theta \approx 61;45,54^{\circ}.$$

And, by subtraction, ∠ ZGA, which represents the [amount of] retrogradation due to the planet's speed, is [90° − ∠ GAΘ =] 28;14,6°, while ∠ ZAH, which represents the [motion in mean] anomaly, is [∠ GAΘ − ∠ ZAΘ =] 12;52,24°. To the latter corresponds a motion in [mean] longitude of 20;35,19°,[59] according to the above mean ratio [of the speeds], and half the retrogradation is computed to be

[28;14,6° − 20;35,19° $\doteq$] 7;38,47° and about $20\frac{5}{6}^{d}$.

The total retrogradation is

$$15;17,34^{\circ} \text{ and } 41\tfrac{2}{3}^{d}.$$

[59] 12;52,24/0;37,31 is, accurately, 20;35,17.

[Hence] the distance at the elongation of the stations from apogee and perigee is [respectively] about 0;5^p of the mean distance [i.e. 60^p] less than the greatest distance, and about the same amount greater than the least distance.[60]

According to our calculations for near greatest distance, the equation [corresponding to 1°] for correcting [the speeds] is found to be 2⅓′.[51] Hence H485

ΘZ:ZG = 0;57,40 : 0;39,51,
EG:GZ = 2;35,11 : 0;39,51,
and EG.GZ = 1;43,4.
Furthermore GA:AH = 61;10 : 43;10,
DG:HG = 104;20 : 18;0,
and DG.HG = 1878;0.

Dividing [1878 by 1;43,4], we get 1093;16,23, the square root of which, 33;3,53, multiplied into the above ratio of ΘZ:ZG, gives, in terms of the above sizes of GA and AZ [i.e. 61;10 and 43;10],

ΘZ = 31;46,44^p,
GZ = 21;57,38^p in the same units,
and, by addition, GΘ = 53;44,22^p.

Hence, where hypotenuses AZ and AG are each [respectively] 120^p,

ZΘ = 88;20,34^p
and GΘ = 105;25,44^p.

The [corresponding] arcs are:

arc ZΘ = 94;48,54°
and arc GΘ = 122;56,27°.
Accordingly ∠ ZAΘ = 47;24,27° H486
and ∠ GAΘ = 61;28,14°.

And, by subtraction, ∠ ZGA, which represents the [amount of] retrogradation due to the planet's speed, is [90° − ∠ GAΘ =] 28;31,46°, while ∠ ZAH, which represents the [motion in] apparent anomaly, is [∠ GAΘ − ∠ ZAΘ =] 14;3,47°. To the latter correspond [motions of] 20;19,3° in corrected longitude and 21;9,3° in mean longitude, according to the ratios [of the speeds] at apogee.[62] Thus half of the retrogradation comes to

[28;31,46° − 20;19,3° =] 8;12,43° and about 21½^d.

The total retrogradation is

16;25,26° and 43^d.

According to our calculations for near least distance, the equation [corresponding to an argument of 1°] for correcting [the speeds] is found to be the same amount, 2⅓′.[63] Hence

[60] For a true centrum (κ) of 20;35,19 the distance of the centre of the epicycle is 61;10,6^p (≈ 61;15^p − 5), and for κ = 180° − 20;35,19° the distance is 58;49,41^p ≈ 58;45^p + 5′.

[61] The increment between successive values of the equation in the anomaly table for Venus (XI 11) is 14′ *for* 6° of argument *near* the apogee, hence 2⅓′ for 1°. However, *one should take the* increment between 18° and 24°, which is 15′, leading to 2½′ for 1°.

[62] Cf. p. 567 n.31. Computation: 14;3,47° × 1/0;39,51 ≈ 21° [accurately 21;10,26°], to which corresponds an equation of 0;50° [accurately 0;50,30°]. 14;3,47° − 0;50° = 13;13,47° ≈ 13;13½°. 13;13½° × 1/0;37,31 = 21;9,3°, and 21;9,3° − 0;50° = 20;19,3°.

[63] This corresponds to an increment of 7′ for an increment of 3° in the argument. In the anomaly table for Venus (XI 11), near perigee, the increment is 7′ between 165° and 162° and between 159° and 156°, but between 162° and 159°, which is the proper interval (κ ≈ 20°), it is only 6′.

$Z\Theta : ZG = 1;2,20 : 0;35,11,$
$EG : GZ = 2;39,51 : 0;35,11,$
and $EG.GZ = 1;33,44.$
Furthermore $GA : AD = 58;50 : 43;10,$
$DG : GH = 102;0 : 15;40,$
and $DG.GH = 1598;0.$

H487 Dividing [1598 by 1;33,44], we get 1022;54,7, the square root of which, 31;58;58, multiplied into the above ratio of ΘZ:ZG, gives, in terms of the above sizes of GA and AZ [i.e. 58;50 and 43;10],

$\Theta Z = 33;13,36^{p},$
$GZ = 18;45,16^{p}$ in the same units,
and, by addition, $G\Theta = 51;58,52^{p}.$

Hence, where hypotenuses AZ and AG are each [respectively] 120^{p},

$Z\Theta = 92;22,3^{p}$
and $G\Theta = 106;1,23^{p}$.[64]

The [corresponding] arcs are:

arc $Z\Theta = 100;39,34°$
and arc $G\Theta = 124;8,22°.$
Accordingly $\angle Z A \Theta = 50;19,47°$
and $\angle G A \Theta = 62;4,11°.$

And, by subtraction, ∠ ZGA, which represents the [amount of] retrogradation due to the planet's speed, is [90° − ∠ GAΘ =] 27;55,49°, while ∠ ZAH, which represents the [motion in] apparent anomaly, is [∠ GAΘ − ∠ ZAΘ =] 11;44,24°. To the latter correspond [motions of] 20;53,30° in corrected longitude, and 20;4,30° in mean longitude, according to the ratios [of the speeds] at perigee.[65] Accordingly half of the retrogradation comes to

[27;55,49° − 20;53,30° =] 7;2,19° and about $20\frac{1}{3}^{d}$.

The total retrogradation is

14;4,38° and $40\frac{2}{3}^{d}$.

H488 6. {*Demonstration of the retrogradations of Mercury*}

Again, in the case of Mercury [see Fig. 12.12], according to our calculations for mean distance,

$\Theta Z : ZG = 1 : 3;9,8,$[66]
$EG : GZ = 5;9,8 : 3;9,8,$
and $EG.GZ = 16;14,27.$
Furthermore, $GA : AH = 60 : 22\frac{1}{2},$
$DG : GH = 82;30 : 37;30,$
and $DG.GH = 3093;45.$

[64] Calculation gives $106;1,26^{p}$, and perhaps one should correct to that, which is the reading of Is. However, an arc of 124;8,22° agrees better with a chord of $106;1,23^{p}$.

[65] Cf. p. 567 n.31. Computation: 11;44,24° × 1/0;35,11 = 20;1,15° ≈ 20°. To (180° − 20°) corresponds an equation of 0;49°. 11;44,24° + 0;49° = 12;33,24° ≈ 12;33°. 12;33° × 1/0;37,31 ≈ 20;4½° [accurately 20;4,16°]. 20;4½° + 0;49° = 20;53,30°.

[66] From the mean daily motions taken to 2 sexagesimal places (IX 4), 3;6,24/0;59,8 = 3;9,7,54 ≈ 3;9,8.

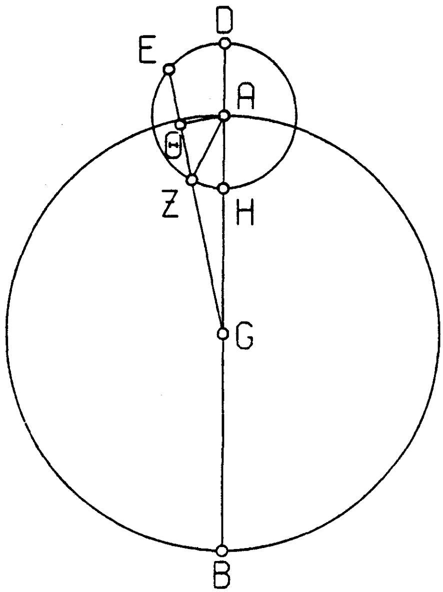

Fig. 12.12

Dividing [3093;45 by 16;14,27], we get 190;29,31, the square root of which, 13;48,7, multiplied into the above ratio of lines ΘZ:ZG, gives, in terms of the above sizes of GA and AZ [i.e. 60 and 22;30],

ΘZ = 13;48,7ᵖ,
ZG = 43;30,24ᵖ,
and, by addition, GΘ = 57;18,31ᵖ.

Hence, where hypotenuses AZ and AG are each [respectively] 120ᵖ,

ZΘ = 73;36,37ᵖ, H489
and GΘ = 114;37,2ᵖ.

The corresponding arcs are:

arc ZΘ = 75;40,28°
and arc GΘ = 145;32,52°.
Accordingly ∠ ZAΘ = 37;50.14°
and ∠ ΘAG = 72;46.26°.

And, by subtraction, ∠ ZGA, which represents the [amount of] retrogradation due to the planet's speed, is [90° − ∠ ΘAG =] 17;13,34°, while ∠ ZAH, which represents the [motion in mean] anomaly, is [∠ ΘAG − ∠ ZAΘ =] 34;56,12°. To the latter corresponds a motion in [mean] longitude of 11;4,59°, according to the above ratio [of the speeds],[67] and half the retrogradation is found by subtraction as

[17;13,34° − 11;4,59° =] 6;8,35° and about $11\frac{1}{4}^{d}$.

The total retrogradation is computed as

12;17,10° and $22\frac{1}{2}^{d}$.

[67] 34;56,12/3;9,8 is indeed 11;4,59 (accurate to two places).

According to our calculations for near greatest distance, i.e. when the corrected longitude is about 11° from apogee (corresponding to a mean longitude of about $11\frac{1}{2}$°), the equation for correcting [the speeds] corresponding to 1° [of anomaly] is about $2\frac{1}{3}$′.[68] Hence

$$\Theta Z{:}ZG = 0;57,40 : 3;11,28,$$

H490

$$EG{:}GZ = 5;6,48 : 3;11,28,$$

$$\text{and } EG.GZ = 16;19,2.$$

$$\text{Furthermore, } GA{:}AH = 68;36 : 22;30,^{69}$$

$$DG{:}GH = 91;6 : 46;6,$$

$$\text{and } DG.GH = 4199;42,36.$$

Dividing [4199;42,36 by 16;19,2], we get 257;22,44, the square root of which, 16;2,35, multiplied into the above ratio of ΘZ:ZG, gives, in terms of the above sizes of GA and AZ [i.e. 68;36 and 22;30],

$$\Theta Z = 15;25,9^p$$

$$ZG = 51;11,43^p \text{ in the same units,}$$

$$\text{and, by addition, } G\Theta = 66;36,52^p.$$

Hence, where hypotenuses ZA and AG are each [respectively] 120^p,

$$Z\Theta = 82;14,8^p$$

$$\text{and } G\Theta = 116;31,36^p.$$

The corresponding arcs are:

$$\text{arc } Z\Theta = 86;31,4°$$

$$\text{and arc } \Theta G = 152;27,56°.^{70}$$

$$\text{Accordingly } \angle ZA\Theta = 43;15,32°$$

H491

$$\text{and } \angle \Theta AG = 76;13,58°.$$

And, by subtraction, ∠ ZGA, which represents the [amount of] retrogradation due to the planet's speed, is [90° − ∠ ΘAG =] 13;46,2°, while ∠ ZAH, which represents the [motion in] apparent anomaly, is [∠ ΘAG − ∠ ZAΘ =] 32;52;26°.[71] To the latter correspond [motions of] 9;48,51° in corrected longitude and 10;16,51° in mean [longitude], according to the ratios [of the speeds] at the apogee.[72] Thus half the retrogradation is found by subtraction as

[13;46,2° − 9;48,51° =] 3;57,11° and about $10\frac{1}{2}^d$.

The total retrogradation is

$$7;54,22° \text{ and } 21^d.$$

According to our calculations for near least distance (which occurs near the

[68] In the table of anomaly for Mercury (XI 11), to an argument of 6° corresponds an equation of 17′, and to 12°, 32′. Thus to an increment of 6° corresponds an increment of 15′, or, to 1°, $2\frac{1}{2}$′. I have no explanation for the discrepancy.

[69] The distance at apogee is 69^p; hence Ptolemy assumes that the distance at the given situation is 24′ less. For $\bar{\kappa} = 11\frac{1}{2}°$, the distance (ρ) is in fact $68;37^p$. It is about $68;36^p$ for $\bar{\kappa} = 11;40°$.

[70] Ptolemy has committed a considerable computing error here: the arc of the chord $116;31,36^p$ should be about 152;22°.

[71] As noted by Heiberg and Manitius, 76;13,58 − 43;15,32 in fact equals 32;58,26. But Ptolemy's erroneous number is confirmed by the following calculations and by H500,23. It is worth noting that had Ptolemy used the correct arc of the chord $116;31,36^p$ (cf. n.70), he would have found ∠ Θ AG ≈ 76;11° and ∠ ZAH ≈ 32;55°, which is closer to the text, but still not in perfect agreement.

[72] Cf. p. 567 n.31. Computation: 32;52,26° × 1/3;11,28 ≈ 10;18°, to which corresponds an equation of 0;28° [accurately 0;27,45°]. 32;52,26° − 0;28° = 32;24,26°, which divided by 3;9,8 gives 10;16,51°. 10;16,51° − 0;28° = 9;48,51°.

elongations of 120° in mean motion from the apogee), the equation for correcting [the speeds], derived from entering [the table] at around 11° either side of the perigees is approximately $1\frac{1}{2}'$.[73] Hence

$$\Theta Z : ZG = 1;1,30 : 3;7,38,$$
$$EG : GZ = 5;10,38 : 3;7,38,$$
$$\text{and } EG.GZ = 16;11,25.$$
$$\text{Furthermore, } GA : AH \approx 55;42 : 22;30,^{74}$$
$$DG : GH = 78;12 : 33;12,$$ H492
$$\text{and } DG.GH = 2596;14,24.$$

Dividing [2596;14,24 by 16;11,25], we get 160;21,29, the square root of which, 12;39,48, multiplied into each member of the above ratio of ΘZ:ZG, gives, in terms of the above sizes of GA and AZ [i.e. 55;42 and 22;30],

$$\Theta Z = 12;58,47^p$$
$$ZG = 39;36,4^p \text{ in the same units,}$$
$$\text{and, by addition, } G\Theta = 52;34,51^p.$$

Hence, where hypotenuses AZ and AG are each [respectively] 120^p,

$$\Theta Z = 69;13,31^p$$
$$\text{and } \Theta G = 113;16,48^p.$$

The corresponding arcs are:

$$\text{arc } \Theta Z = 70;27,44°$$
$$\text{and arc } \Theta G = 141;28,14°.$$
$$\text{Accordingly } \angle \Theta AZ = 35;13,52°$$
$$\text{and } \angle \Theta AG = 70;44,7°.$$

And, by subtraction, ∠ ZGA, which represents the [amount of] retrogradation due to the planet's speed, is [90° − ∠ ΘAG =] 19;15,53°, while ∠ ZAH, which represents the [motion in] apparent anomaly, is [∠ ΘAG − ∠ ΘAZ =] 35;30,15°. To the latter correspond [motions of] 11;39,30° in corrected longitude, and 11;21,30° in mean [longitude], according to the above ratios [of the speeds near the perigee].[75] Thus half of the retrogradation is found by subtraction as

[19;15,53° − 11;39,30° =] 7;36,23° and about $11\frac{1}{2}^d$. H493

The total retrogradation is

$$15;12,46° \text{ and } 23^d.$$

The amounts [of the retrogradations] we have demonstrated agree very closely with those derived from the actual phenomena associated with each planet.

[73] From the table of anomaly for Mercury (XI 11) it can be seen that $1\frac{1}{2}'$ is a compromise between the two values derived on either side of the perigee: to $\bar{\kappa} = 108°$ corresponds an equation of 2;56°, and to $\bar{\kappa} = 111°$, 2;53°. Here, then, an increment of 1° produces 1′. For $\bar{\kappa} = 129°$ and 132° one finds 2;24° and 2;18° respectively, and hence, for an increment of 1°, 2′.

[74] Cf. p. 580 n.69. Here, for a distance of $11\frac{1}{2}°$ in mean motion from 'perigee' (at $\bar{\kappa} = 120°$), one finds, for $\bar{\kappa} = 131\frac{1}{2}°$, $\rho = 55;41,58^p$ (text $55;42^p$). On the other side of the perigee, however, for $\bar{\kappa} = 108\frac{1}{2}°$, $\rho = 55;45,50^p$.

[75] Cf. p. 567 n.31. Computation: 35;30,15° × 1/3;7,38 = 11;21,11°, to which corresponds an equation of 18′ [in fact 11;21,11° *before* the perigee leads to an equation of +15′, and 11;21,11° *after* it to −23′, i.e. 18′ is, again, a compromise]. 35;30,15° + 0;18° = 35;48,15°, which divided by 3;9,8 gives 11;21,30°. 11;21,30° + 0;18° = 11;39,30°.

We used the following method to find the motions in longitude at greatest and least distances.[76]

For example, in the case of Mars [XII 4 p. 574], we showed that, near the greatest distance,[77] the apparent arc of the epicycle from either of the stations to opposition (i.e. the arc as viewed from the centre of the ecliptic) is 22;13,19°. To the latter corresponds (according to the ratio 1 : 1;3,11) a motion in mean longitude of about 21;10°.[78] But the latter does not represent [the actual mean motion] accurately, since the ratios of the speeds which we set out for the

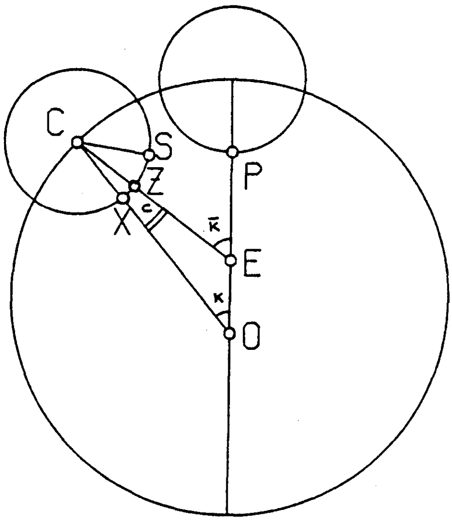

Fig. R

stations do not remain unchanged throughout the whole period of retrogradation. However, it is close enough to the truth so that the equation corresponding to it (which is about 3;45°)[79] is not significantly different [from the
H494 true equation]. So we subtracted that [3;45°] from the 22;13,19° of the epicycle (since at greatest distance the apparent motion on the epicycle is greater than the mean motion), and [thus] found that the corresponding mean motion in anomaly from either of the stations to opposition is 18;28,19°. To this, according to the ratio of the mean motions [0;52,51 : 1] corresponds a motion in mean longitude of 20;58,21°.[80] So we adopted that as the accurate value instead of the

[76] There is no need to assume, with Neugebauer (note in Manitius, revised edition, p. 301) that the following passage has been displaced in antiquity from its rightful place in XII 4. For the method applies to all planets, not just Mars. It is quite in Ptolemy's manner to attach an explanation or justification of a particular method as an appendix at the end of his general treatment. Cf. V 19 pp. 267–73 and VI 4 p. 282.

[77] See Fig. R. The planet is at opposition (P) when the epicycle is at apogee, and at second station (S) when the epicycle is at a mean centrum $\bar{\kappa}$ from apogee. Then 'the apparent arc of (motion on) the epicycle' is XS, and 'the mean motion on the epicycle' (which differs from it by the equation c) is ZS.

[78] Accurately 21;6,8°.

[79] Accurately 3;46,15°.

[80] Accurately 20;58,15°.

[previous] 21;10°, and subtracted from it the 3;45° of the equation (which remains very nearly unchanged for this position). [We subtracted] because at greatest distance the apparent motion in longitude is less than the mean. Thus we found the apparent motion in longitude as 17;13,21°, the interval set out above.

7. {*Construction of a table for the stations*}[81]

Furthermore, to enable us to investigate conveniently at what point on the epicycle each planet is when it produces the appearance of being stationary, for distances in the interval between mean distance and greatest or least distance as well, we have constructed[82] for this purpose a table with 31 lines and 12 columns. H495 The first two of these columns will contain the numbers of the mean longitude at intervals of 6° (corresponding to the arrangement of the other tables). The following 10 columns will contain the distances in corrected anomaly from the apparent apogee of the epicycle for each of the 5 planets: in each case the first column [of the pair for that planet] will contain the amount for first station, and the second column the amount for second station. We obtained the amounts for these [entries] too from the [numbers] demonstrated above for mean, least and greatest distances, and from the increments at distances in between these, which we happen to have determined already in [our computations of] the minutes to be tabulated in the eighth column of the tables for anomaly.[83] For in demonstrating the amount of the maximum equation of anomaly corresponding to each entry in mean motion, one simultaneously demonstrates the distance of the epicycle, which is the principal factor affecting the difference in [the position of] the stations.

But first, since the retrogradations which we demonstrated for near apogee and perigee represent, not the stations which occur when the centre of the epicycle is precisely at apogee and perigee, but those when it is a certain H496 specified distance [from them], we used the latter to determine, for each planet, the amount corresponding to the actual apogee and perigee, as follows.

In the case of Saturn and Jupiter, since the distances of the epicycle at actual apogee and perigee do not differ significantly from those at the elongations from apogee and perigee used above, we entered the amounts of anomaly (counted from apparent apogee of the epicycle) derived for those elongations on the appropriate lines, i.e. we entered the amount for apogee on the line with the argument '360', and the amount for perigee on the line with the argument '180'. We showed that for Saturn [XII 2, pp. 567–9] the distance [in anomaly] from the perigee of the epicycle at apogee of the eccentre is about 67;15°, and at perigee of the eccentre about 64;31°; and that for Jupiter [XII 3, pp. 571–2] it is 55;55° at apogee and 52;49° at perigee. For convenience in use, we entered the

[81] See *HAMA* 202-06, Pedersen 349–51.

[82] Reading μεθωδεύσαμεν (with D,Ar) at H494,20 for μεθοδεύομεν ('we construct').

[83] Cf. XI 10 p. 547. It was necessary for Ptolemy to compute the distances of the centre of the epicycle all round the orbit in order to calculate the 'minutes of interpolation' in the planetary anomaly tables.

amounts [in anomaly] corresponding to these, counted from the apogee of the
H497 epicycle, on the appropriate lines in the 4 columns following the [argument columns of] longitude: on the line with the argument '360' (for the apogee) [we entered], in the third column, '112;45°' for the first station of Saturn, and, in the fourth column, '247;15°' for its second station; similarly, in the fifth column, '124;5°' for Jupiter's first station, and, in the sixth column, '235;55°' for its second station. And on the line with the argument '180' (for the perigee) [we entered], following the same order, '115;29°' and '244;31°', and similarly '127;11°' and '232;49°'.

In the case of Mars, we showed [XII 4, pp. 573–4] that when the epicycle centre is 20;58° in mean [longitude] from the apogee of the eccentre, the planet performs its stations at a distance of 22;13° [in anomaly] from the apparent perigee of the epicycle; and the [corresponding] amount [of anomaly] at mean distance is 16;51°, so that the difference is 5;22°. Furthermore, where the mean distance is 60^p, the greatest distance is 66^p and the difference between greatest and mean is 6^p, while at the above distance from the apogee [of 20;58°] the distance is $65;40^p$[84] and the difference between this and the mean is $5;40^p$. So,
H498 multiplying 6 into 5;22 and dividing the result by 5;40, we find that the difference with respect to the mean distance at the actual apogee is about 5;41°. Thus we calculate the distance [in anomaly] from the apparent perigee of the epicycle as [16;51° + 5;41° =] 22;32°, and from the apogee as, for the first station, 157;28°, which we enter in the seventh column on the line with '360', and, for the second station, 202;32°, which we enter in the eighth column on the same line.

Similarly [see p. 575], when the epicycle centre is 16;53° in mean [longitude] from the perigee [of the eccentre], [Mars] performs its stations at a distance of 11;11° [in anomaly] from the apparent perigee of the epicycle, so that the difference [in anomaly] from that for mean distance is [16;51° − 11;11° =] 5;40°. And, in the same units [as before], the least distance is 54^p (with a difference from the mean of 6^p), and at the above elongation from the perigee of the eccentre it is $54;20^p$, with a difference from the mean of $5;40^p$. Thus at the actual perigee we get the total difference [in anomaly from the mean] as [5;40° × 6 ÷ 5;40 =] 6°. Hence the amount [of anomaly] from apparent perigee of the epicycle is [16;51° − 6° =] 10;51°, and from the apogee, for the first
H499 station, 169;9°, and for the second 190;51°, which we enter in the appropriate columns on the line with '180'.

In the case of Venus, we showed [XII 5, pp. 576–7] that when it is 21;9° in mean longitude from the apogee [of the eccentre], the planet performs its stations at a distance of 14;4° [in anomaly] from the apparent perigee of the epicycle, while the [corresponding] amount at mean distance is 12;52°, so that the difference is 1;12°. And, where the mean distance is 60^p, the greatest distance is $61;15^p$, and the difference from the mean $1;15^p$, while at the above elongation from the apogee the distance is $61;10^p$ and the difference from the mean $1;10^p$. So, again, multiplying 1;15 into 1;12 and dividing the result by

[84] Cf. p. 573 with n.50. One should probably read ἀπὸ τοῦ ἀπογείου (with D) at H497,21 (cf. H499,11) Corrected by Manitius.

1;10, we find the difference [in anomaly] at the actual apogee with respect to that for the mean distance as 1;17°. Thus we calculate the distance [in anomaly] from the apparent perigee of the epicycle as [12;52° + 1;17° =] 14;9°, and from the apogee as, for the first station, 165;51°, which we enter in the ninth column on the line with '360', and, for the second station, 194;9°, which we enter in the tenth column on the same line.

Similarly [see p. 578], when the epicycle is about 20° in mean longitude from perigee of the eccentre, [Venus] performs its stations at a distance [in anomaly] of 11;44° from the apparent perigee of the epicycle, so that the difference with respect to [that for] mean distance is [12;52° − 11;44° =] 1;8°. And the least distance is 58;45^p where the mean is 60^p, and their difference is 1;15^p, while the distance at the above elongation from the perigee is 58;50^p in the same units, and the difference from the mean 1;10^p. So, multiplying 1;15 into 1;8 and dividing the result by 1;10, we find the difference [in anomaly] at the actual perigee with respect to the mean distance as 1;13°. Hence the amount of anomaly from the apparent perigee of the epicycle is [12;52° − 1;13° =] 11;39°, and from the apogee, for the first station, 168;21°, and, for the second station, 191;39°, which we enter in the same columns [i.e. the ninth and tenth respectively] opposite the number[85] '180'. H500

In the case of the planet Mercury, we showed [XII 6, pp. 579–80] that when the epicycle is 10;17° in mean longitude from the apogee of the eccentre, the planet performs its stations at a distance [in anomaly] from the apparent perigee of the epicycle of 32;52°, while the [corresponding] amount at mean distance is 34;56°, so that the difference is 2;4°. Furthermore, where the mean distance is 60^p, the greatest distance is 69^p and the difference between them 9^p, while at the above elongation from the apogee the distance is 68;36^p,[86] and the difference from the mean 8;36^p. By the same procedure as before, multiplying 9 into 2;4 and dividing the result by 8;36, we find the difference [in anomaly] at the actual apogee with respect to that for the mean distance as about 2;10°. Thus we calculate the distance [in anomaly] from apparent perigee of the epicycle as [34;56° − 2;10° =] 32;46°, and from the apogee as, for the first station, 147;14°, which we enter in the eleventh column opposite the number '360', and for the second station 212;46°, which we enter in the twelfth column on the same line. H501

Similarly [see p. 581], when the epicycle is 11;22° in mean [longitude] from the perigee, the planet performs its stations at a distance [in anomaly] from the apparent perigee of the epicycle of 35;30°, so that the difference from that for mean distance is [35;30° − 34;56° =] 34′. And the least distance is 55;34^p where the mean is 60^p, and their difference is 4;26^p, while at the above elongation from the perigee the distance is about 55;42^p, and the difference from the mean 4;18^p. So, again, multiplying 4;26 into 0;34 and dividing the result by 4;18, we find the difference [in anomaly] at the actual perigee with respect to that for the mean distance as 0;35°. Hence the distance in anomaly from the apparent perigee of H502

[85] κατὰ τὸν τῶν ρ̄π̄ ἀριθμόν. One would expect κατὰ τοῦ τῶν ρ̄π̄ στίχου (cf. e.g. H499, 1–2, 22), and that occurs (at least as an alternative reading) in L,Ger. But the same expression occurs at H501,14 and 502,12.

[86] Cf. p. 580 with n.69.

the epicycle is [34;56° + 0;35° =] 35;31°, and from the apogee, for the first station, 144;29°, and for the second station 215;31°. We enter the latter in the same [i.e. eleventh and twelfth] columns, in this case, however, not opposite the number '180' of longitude, but opposite '120' and '240', since we have shown that the points of the planet Mercury's eccentre closest to the earth are at those positions.

Now that the above has been set out, the increments for the positions in between [apogee and perigee] can be obtained using the same methods.

To take an example, let us set ourselves the task of finding the entries (in apparent anomaly) for first station when the mean position in longitude is 30°
H503 from the apogee. At this situation the distance of the epicycle, for a mean distance in every case of 60^p, calculated by the methods explained previously, is (as we stated before)[87] as follows:

Saturn	Jupiter	Mars	Venus	Mercury
$63;2^p$	$62;26^p$	$65;24^p$	$61;6^p$	$66;35^p$.

Hence the differences of each with respect to the mean (using the above order, to avoid repetition) are

$3;2^p$	$2;26^p$	$5;24^p$	$1;6^p$	$6;35^p$

But the differences between the distance at actual apogee and the mean, since the above amounts for the distance are in all cases greater than the mean, are, in the same units,

$3;25^p$	$2;45^p$	$6;0^p$	$1;15^p$	$9;0^p$.

Now the total differences in apparent anomaly between apogee and mean distance come to (using the same order)[88]

1;23°	1;33°	5;41°	1;17°	2;10°

We multiply each of the latter in turn into the difference between the distance at that point and the mean for the planet in question (e.g. [for Saturn we multiply] 1;23 into 3;2), and divide the result by the difference between greatest distance
H504 [and mean], (e.g. [for Saturn] by 3;25), and thus get for the above position in longitude, for each planet, the following amounts of difference in anomaly with respect to that for mean distance:

1;14°	1;22°	5;7°	1;8°	1;35°.

The distances [in anomaly] from the apparent apogee of the epicycle at the mean distances are:[89]

114;8°	125;38°	163;9°	167;8°	145;4°.

The [corresponding amount] at greatest distance is greater than the above for Mercury, but less for the other planets. So for Mercury we add the difference which we found for the distance in question to that for the mean distance, but for the other planets we subtract it, and get the following amounts, in apparent

[87] XI 10 p. 547. See that chapter for the method of calculating these quantities.

[88] Saturn (p. 567) Apogee 67;15°, mean 65,52°, difference 1;23°. Jupiter (p. 571) Apogee 55;55° mean 54;22°, difference 1;33°. For the other amounts see pp. 584, 585, and 585. Although Ptolemy does not explicitly say so, logic demands, and the tables confirm, that for positions of the epicycle between mean distance and perigee one takes the corresponding differences in anomaly between mean distance and perigee (namely 1;21, 1;33, 6;0, 1;13 and 0;35) and interpolates accordingly. Cf. *HAMA* 204 bottom.

[89] For the following amounts see ∠ ZAH on pp. 565, 570, 573, 576, and 579, where in each case the supplements (i.e. the distances from apparent perigee) are given.

anomaly from the apogee of the epicycle, which are entered in the columns for first station opposite 30° of mean longitude:

Saturn	Jupiter	Mars	Venus	Mercury
112;54°	124;16°	158;2°	166;0°	146;39°.

We can immediately complete the columns for second station, by entering, for each [planet],[90] the difference from 360° of the amount for first station, [putting the result] in the column for second station on the same line. Thus at the above H505 position [we enter]

247;6°	235;44°	201;58°	194;0°	213;21°.

It is easy to see that if, for the sake of greater convenience, we should choose to enter, not the anomaly, taken with respect to the apparent apogee of the epicycle, but the uncorrected anomaly, taken with respect to the mean [epicyclic apogee], we can immediately derive this too, by taking in the table of anomaly the equation (combined [from the 3rd and 4th columns]) corresponding to each argument of mean longitude, and subtracting it from the amount we found for the apparent anomaly on the 180° of the eccentre counted from apogee, but adding it for [longitudes from apogee] of more than 180°.

The layout of the table is as follows.

8. {*Table of Stations*}[91] H506–7

[See p. 588.]

9. {*Demonstration of the greatest elongations from the sun of Venus and Mercury*}[92] H508

Now that we have gone through the theorems concerning retrogradations, next in the logical sequence is to demonstrate the greatest elongations of the planets Venus and Mercury from the sun, in each of the zodiacal signs, as derived from the above hypotheses. In setting out [the tables] for these, we have taken [the elongations] with respect to the apparent position of the sun, and assumed that the actual planets are at the beginning of the [respective] signs, and that the positions of their apogees with respect to the solstitial and equinoctial points are those which obtain in our time, namely, for Venus, in ♉ 25°, and, for Mercury, in ♎ 10°. It will be easy for those who come after us to correct for the change in the greatest distances due to the shift in the apogees, using the same methods, and in any case the change remains negligible for a very long time.

In order to make it easy to understand the method of our approach [to this problem], by way of example we must demonstrate, for Venus first, the greatest

[90] Deleting the word στίχου at H504,20. If kept, this would mean 'on each line'. But, first, Ptolemy does not use ἐπί in this sense, but κατά; secondly, it is hideously clumsy to follow ἐφ' ἑκάστου στίχου by κατὰ τῶν αὐτῶν στιχῶν; and thirdly one needs a reference to each planet (exactly as at H504,1). This is an ancient interpolation, since it is in all mss.

[91] For Mars, argument 138° (H507,28), D,Ar have the readings 167;10° (also A') and 192;50°, which are more correct than the 167;8°, 192;52° adopted by Heiberg, and should perhaps be preferred. However, errors of as much as 2′ occur elsewhere in the Mars table.

[92] See *HAMA* 230–4, Pedersen 351–4.

[TABLE OF STATIONS]
(AMOUNTS IN CORRECTED ANOMALY)

Common Numbers		SATURN First Station	SATURN Second Station	JUPITER First Station	JUPITER Second Station	MARS First Station	MARS Second Station	VENUS First Station	VENUS Second Station	MERCURY First Station	MERCURY Second Station
0	360	112 45	247 15	124 5	235 55	157 28	202 32	165 51	194 9	147 14	212 46
6	354	112 45	247 15	124 6	235 54	157 29	202 31	165 52	194 8	147 13	212 47
12	348	112 46	247 14	124 7	235 53	157 34	202 26	165 53	194 7	147 8	212 52
18	342	112 48	247 12	124 9	235 51	157 41	202 19	165 55	194 5	147 1	212 59
24	336	112 51	247 9	124 12	235 48	157 50	202 10	165 57	194 3	146 51	213 9
30	330	112 54	247 6	124 16	235 44	158 2	201 58	166 0	194 0	146 39	213 21
36	324	112 58	247 2	124 21	235 39	158 18	201 42	166 4	193 56	146 25	213 35
42	318	113 3	246 57	124 26	235 34	158 34	201 26	166 9	193 51	146 11	213 49
48	312	113 8	246 52	124 32	235 28	158 55	201 5	166 15	193 45	145 55	214 5
54	306	113 15	246 45	124 39	235 21	159 17	200 43	166 22	193 38	145 39	214 21
60	300	113 22	246 38	124 47	235 13	159 42	200 18	166 29	193 31	145 23	214 37
66	294	113 29	246 31	124 55	235 5	160 10	199 50	166 35	193 25	145 8	214 52
72	288	113 36	246 24	125 3	234 57	160 39	199 21	166 42	193 18	144 58	215 2
78	282	113 44	246 16	125 12	234 48	161 10	198 50	166 50	193 10	144 52	215 8
84	276	113 53	246 7	125 22	234 38	161 44	198 16	166 58	193 2	144 46	215 14
90	270	114 1	245 59	125 32	234 28	162 18	197 42	167 7	192 53	144 40	215 20
96	264	114 10	245 50	125 41	234 19	162 54	197 6	167 14	192 46	144 36	215 24
102	258	114 18	245 42	125 51	234 9	163 31	196 29	167 21	192 39	144 33	215 27
108	252	114 27	245 33	126 0	234 0	164 9	195 51	167 28	192 32	144 30	215 30
114	246	114 35	245 25	126 10	233 50	164 47	195 13	167 35	192 25	144 30	215 30
120	240	114 43	245 17	126 19	233 41	165 25	194 35	167 43	192 17	144 29	215 31
126	234	114 51	245 9	126 28	233 32	166 3	193 57	167 50	192 10	144 29	215 31
132	228	114 58	245 2	126 36	233 24	166 37	193 23	167 56	192 4	144 30	215 30
138	222	115 5	244 55	126 44	233 16	167 8	192 52	168 1	191 59	144 31	215 29
144	216	115 11	244 49	126 51	233 9	167 39	192 21	168 6	191 54	144 33	215 27
150	210	115 16	244 44	126 57	233 3	168 4	191 56	168 10	191 50	144 35	215 25
156	204	115 21	244 39	127 2	232 58	168 28	191 32	168 14	191 46	144 37	215 23
162	198	115 25	244 35	127 6	232 54	168 46	191 14	168 17	191 43	144 38	215 22
168	192	115 27	244 33	127 8	232 52	168 59	191 1	168 19	191 41	144 39	215 21
174	186	115 29	244 31	127 10	232 50	169 8	190 52	168 20	191 40	144 40	215 20
180	180	115 29	244 31	127 11	232 49	169 9	190 51	168 21	191 39	144 40	215 20

H509 morning and evening elongations (as defined above) when the planet is at the spring equinox, [namely] at the beginning of Aries.

Let [Fig. 12.13] the line through A, the apogee of the eccentre, be ABGDE, on which B is taken as the centre of uniform motion, G as the centre of the eccentre carrying the epicycle, and D as the centre of the ecliptic. Draw GZ as radius of the eccentre, describe the epicycle HΘ about Z, and from D draw DΘ as tangent on the side of the epicycle which represents morning [visibility] and is in advance of it[s centre]. Join BZH and ZΘ, and drop perpendiculars GK, GL and BM.

Then, since DA points towards ♉ 25° and DΘ towards the beginning of Aries,

H510

$$\angle\ AD\Theta = \begin{cases} 55^\circ \text{ where 4 right angles} = 360^\circ \\ 110^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ}; \end{cases}$$

and $\angle\ DGK = 70^{\circ\circ}$ (complement).

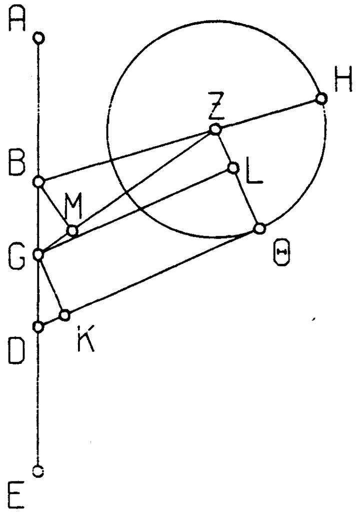

Fig. 12.13

Therefore, in the circle about right-angled triangle GDK,

arc GK = 110°

and GK = $98;18^p$ where hypotenuse GD = 120^p.

Therefore where GD = $1;15^p$ and the radius of the epicycle, ZΘ = $43;10^p$

GK (= LΘ) = $1;1^p$,

and, by subtraction [of LΘ from ZΘ], ZL = $42;9^p$,
where GZ, the radius of the eccentre, is taken as 60^p.
Therefore where hypotenuse GZ = 120^p, ZL = $84;18^p$,
and, in the circle about right-angled triangle GZL,

arc ZL = 89;16°.

$\therefore \angle$ ZGL = $89;16^{\circ\circ}$ where 2 right angles = $360^{\circ\circ}$.

But $\angle$ DGK = $70^{\circ\circ}$ in the same units, and $\angle$ LGK is right.

Therefore, by addition, $\angle$ ZGD is found to be [89;16 + 70 + 180] = $339;16^{\circ\circ}$,
and, by subtraction [from 2 right angles], $\angle$ AGZ = $20;44^{\circ\circ}$.
Therefore, in the circle about right-angled triangle BGM,

arc BM = 20;44°

and arc GM = 159;16° (supplement).

Therefore the corresponding chords

$$\left.\begin{array}{r} \text{BM} = 21;35^p \\ \text{and GM} = 118;2^p \end{array}\right\} \text{ where hypotenuse BG} = 120^p.$$ H511

Therefore where BG = $1;15^p$, and GZ, the radius of the eccentre, is 60^p,

BM = $0;13^p$,

GM = $1;14^p$,

and, by subtraction [of GM from GZ], MZ = $58;46^p$.

Hence hypotenuse BZ [= $\sqrt{BM^2 + MZ^2}$] = $58;46^p$ in the same units.

Therefore, where BZ = 120^p, BM = $0;27^p$, and, in the circle about right-angled triangle BZM,

arc BM = 0;26°.

∴ ∠ BZG = 0;26°° where 2 right angles = 360°°.

And we showed that ∠ AGZ = 20;44°° in the same units.

Therefore, by addition, ∠ ABZ, which represents the mean motion in longitude,

$$\text{is}\begin{cases} 21;10^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \\ 10;35^{\circ} \text{ where 4 right angles} = 360^{\circ}. \end{cases}$$

Therefore the mean position of the sun will be 10;35° in advance of the apogee at A, and, obviously, will be in ♉ 14;25°.

And the true position of the sun will be in ♉ 15;14°. Therefore the planet, when it is at the beginning of Aries, will have a maximum morning elongation from the true sun of 45;14°.

Again, let there be drawn next [Fig. 12.14] the diagram with the tangent to the side of the epicycle which represents evening [visibility] and is towards the

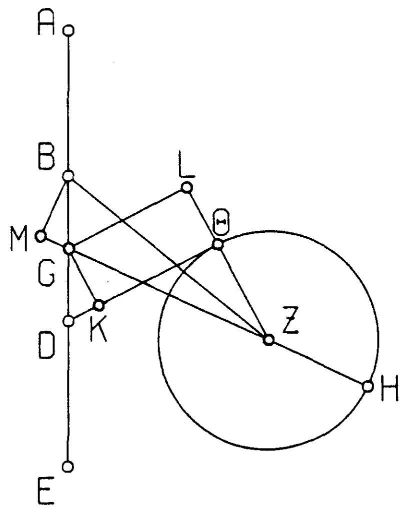

Fig. 12.14

rear of the epicycle [centre], while the planet, as before, is taken as being at the

H512 beginning of Aries.

By what was shown above, ∠ ADΘ will remain the same,

and ∠ DGK = 70°° where 2 right angles = 360°°,

and GK = LΘ = 1;1^p

where GZ, the radius of the eccentre, is 60^p,

and ZΘ, the radius of the epicycle, is 43;10^p.

Therefore, by addition, ZL[= ZΘ + LΘ] = 44;11^p in the same units.

And it is obvious that, where hypotenuse [of triangle GZL] GZ = 120^p,

ZL = 88;22^p,

and, in the circle about right-angled triangle GZL,

arc ZL = 94;51°.

∴ ∠ ZGL = 94;51°° where 2 right angles = 360°°,

and ∠ ZGK = 85;9°° (complement).

So, by addition, ∠ ZGD (= ∠ BGM) [= ∠ DGK + ∠ ZGK] = 155;9°° in the same units.

Hence, in the circle about right-angled triangle BGM,

arc BM = 155;9°

and arc GM = 24;51° (supplement). H513

Therefore the corresponding chords

$$\left.\begin{array}{r} \text{BM} = 117;11^p \\ \text{and GM} = 25;49^p \end{array}\right\} \text{where hypotenuse BG} = 120^p.$$

Therefore, where BG = $1;15^p$,

BM = $1;13^p$,

MG = $0;16^p$,

and, by addition, MZ = $60;16^p$.

Hence hypotenuse BZ [$= \sqrt{\text{BM}^2 + \text{MZ}^2}$] = $60;17^p$ in the same units.

Therefore, where BZ = 120^p, BM = $2;25^p$.

and, in the circle about right-angled triangle BZM,

arc BM = 2;19°.

∴ ∠ BZM = 2;19°° where 2 right angles = 360°°.

And ∠ BGZ = 204;51°° in the same units.

since ∠ DGZ was shown to be 155;9°° in those units.

Therefore, by addition, ∠ ABZ, which represents the mean motion in longitude,[93]

$$\text{comes to} \begin{cases} 207;10^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \\ 103;35^\circ \text{ where 4 right angles} = 360^\circ. \end{cases}$$

Therefore the sun's mean position will be at [♉ 25° − 103;35° =] ♒ 11;25°

and its true position at ♒ 13;38°.

Thus the greatest evening elongation of the planet from the true sun, when, as before, it is at the beginning of Aries, will be 46;22°.

In the case of the planet Mercury, in order to have a more convenient approach to the demonstrations of its missing phases which we shall give further on,[94] let us set ourselves the task of finding the maximum elongation of the planet from the true sun, as evening star when it is at the beginning of Scorpius, and as morning star when it is at the beginning of Taurus. H514

Now, according to our hypothesis for Mercury, when the apparent position of the planet is given, the mean position in longitude cannot be found, since line GZ does not remain the same constant length,[95] always equal to the radius of the eccentre (as it does in the hypothesis for the other [planets]). But if the mean position in longitude is given, the apparent position can be demonstrated. So we assume, for each [zodiacal] sign, two positions in [mean] longitude which can bring the planet [at greatest elongation] near the beginning of the sign in

[93] Reading τῆς ὁμαλῆς κατὰ μῆκος παρόδου (with D^{1}G, Ar) at H513.15–16 for the nonsensical τῆς ὁμαλῆς καὶ κατὰ μῆκος παρόδου. Corrected by Manitius.

[94] The reference is to XIII 8 (p. 644).

[95] For the other planets (e.g. Venus, Fig. 12.14) this denotes the distance from the centre of the eccentre to the centre of the epicycle, but for Mercury Ptolemy seems to be referring to a figure such as Fig. 9.9, where it denotes the distance from the *equant* point to the centre of the epicycle. These two amounts are indeed trigonometrically comparable. Ptolemy is correct in stating that, for Mercury, one cannot find the mean position from the true, at least by Euclidean geometry.

question, the first in advance [of the beginning of the sign], and the second to the rear [of it]; we compute the greatest elongations at the chosen positions, and thence[96] find the greatest elongation which occurs at the actual beginning of the sign. This will be easily comprehensible from the [particular] problems we have set ourselves to solve: and first for the greatest evening elongation at the beginning of Scorpius.

H515 Let [Fig. 12.15] the diameter through the apogee A be ABGD, on which G is taken as the centre of the ecliptic, and B as the centre of the epicycle's uniform motion. First let the epicycle centre be imagined as being precisely at the

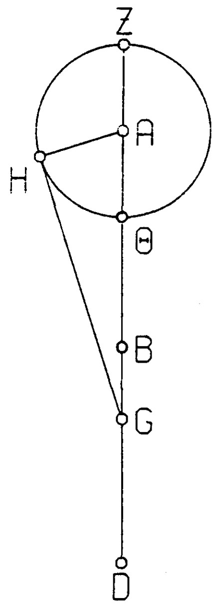

Fig. 12.15

apogee, so that the mean position in longitude of the sun will be ♎ 10°, and its true longitude ♎ 8°. On centre A describe the epicycle ZH, draw GH as tangent to the side of it representing evening, and drop perpendicular AH.

Then, since in our previous treatment [IX 9, p. 459] it was shown that where GA, the greatest distance, is 69^p, AH, the epicycle radius, is $22\frac{1}{2}^p$,

where hypotenuse [of right-angled triangle AGH] AG = 120^p,

$$AH = 39;8^p,$$

and, in the circle about right-angled triangle AGH,

$$\text{arc } AH = 38;4°,$$

$$\text{and } \angle\, AGH = \begin{cases} 38;4^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \\ 19;2° \text{ where 4 right angles} = 360°. \end{cases}$$

H516 And GA is at ♎ 10°.

Therefore the planet will have a position of ♎ 29;2°, its maximum elongation from the true sun being 21;2°.

[96] By linear interpolation.

Again, let [Fig. 12.16] the distance in mean longitude from the apogee be 3°: thus the mean sun will be at ≏ 13°, and the true sun at ≏ 11;4°. Draw BE and on centre E describe the epicycle ZH. As before, draw the tangent GH, and join EG, EH. Then at the situation in question, i.e. with ∠ ABE taken as 3°, by our previous methods one can show that the angle corrected for the eccentricity,[97]

∠ AGE = 2;52°,

and the distance of the epicycle in that situation,[98]

EG ≈ 68;58ᵖ where EH, the radius of the epicycle, is 22;30ᵖ.

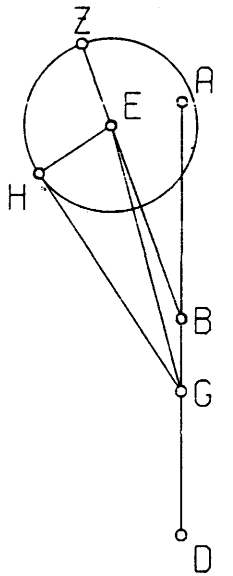

Fig. 12.16

Therefore, where hypotenuse EG = 120ᵖ, EH = 39;9ᵖ.

Therefore, in the circle about right-angled triangle GEH,

arc EH = 38;5°,

$$\text{and } \angle\ \text{EGH} = \begin{cases} 38;5^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \\ 19;3^{\circ}\text{, approximately, where 4 right angles} = 360^{\circ}. \end{cases}$$ H517

Hence, by addition, ∠ AGH = 21;55° in the same units.

So when the planet is at ♏ 1;55°, its greatest elongation from the true sun will be [♏ 1;55° − ≏ 11;4° =] 20;51°.

And we showed that when it is at ≏ 29;2°, its greatest elongation from the true sun will be 21;2°.

Thus the difference between the longitudes is 2;53°, and the difference between the greatest elongations is 11′, and so to the 0;58° from the first position

[97] If the text is to be trusted here, this must be the meaning of τῆς παρὰ τὴν ἐκκεντρότητα διαφορᾶς. But the normal reference of such an expression would be to the *equation* (of centre) itself, not to the angle corrected by the equation. I strongly suspect that the phrase is interpolated (it is in the whole ms. tradition).

[98] By trigonometrical calculation, EG = 68;58,25ᵖ, ∠ AGE = 2;52,10°.

to the beginning of Scorpius corresponds [a decrement in greatest elongation of] about 4′, which we subtract from 21;2° to get the greatest evening elongation from the true sun [when the planet is] precisely at the beginning of Scorpius as 20;58°.

Next, to find the greatest morning elongation at the beginning of Taurus, let us suppose first that the mean position in longitude is 39° towards the rear from the perigee. Thus the mean sun is at ♉ 19°, and the true sun at ♉ 19;38°. Let there be drawn [Fig. 12.17] a figure similar [to the preceding], in which the
H518 epicycle is described to the rear of the perigee, and the tangent is drawn to the morning side of the epicycle.

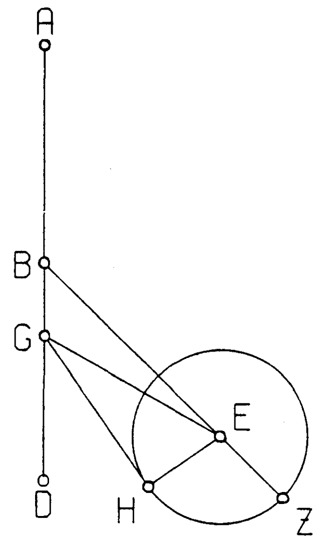

Fig. 12.17

Then at the position in question, i.e. with ∠ DBZ taken as 39°, by the method previously described one can show that

$$\angle \text{ DGE} = 40;57°,$$[99]

and that the distance at that moment,

$$\text{GE} = 55;59^p \text{ where the radius of the epicycle, EH} = 22;30^p.$$

Therefore where hypotenuse [of right-angled triangle GEH] GE = 120^p,

$$\text{EH} = 48;14^p$$

and, in the circle about right-angled triangle GEH,

$$\text{arc EH} = 47;24°.$$

H519

$$\therefore \angle \text{ EGH} = \begin{cases} 47;24^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \\ 23;42° \text{ where 4 right angles} = 360°. \end{cases}$$

And, by subtraction [from ∠ DGE], ∠ HGD = 17;15° in the same units. Therefore when the planet Mercury has a longitude of ♈ 27;15°, its greatest morning elongation from the true sun will be [♉ 19;38° − ♈ 27;15° =] 22;23°.

[99] For $\bar{\kappa} = 219°$, $\rho = 55;59,1^p$, and $\kappa = 220;55,57°$, hence ∠ DGE ≈ 40;56°.

Again, let it be assumed to have a distance in mean longitude from the perigee, on the same side, of 42°. Thus the sun will have a mean longitude of ♉ 22° and a true longitude of ♉ 22;31°.

Then at this position, i.e. with ∠ DBZ taken as 42°, one can show that

$$\angle \text{DGE} = 44;4°,$$

and that the distance at that moment,

$$\text{GE} = 55;53^{p}\text{[100] where the radius of the epicycle, EH} = 22;30^{p}.$$

Therefore, where hypotenuse EG = 120^{p}, EH = $48;19^{p}$, H520

and, in the circle about right-angled triangle EGH,

$$\text{arc EH} = 47;30°.$$

$$\therefore \angle \text{EGH} = \begin{cases} 47;30^{\circ\circ} & \text{where 2 right angles} = 360^{\circ\circ} \\ 23;45° & \text{where 4 right angles} = 360°, \end{cases}$$

and, by subtraction [from ∠ DGE], ∠ HGD = 20;19° in the same units.

Therefore when the planet Mercury has a longitude of ♉ 0;19°,[101] its greatest morning elongation from the true sun will be [♉ 22;31° − ♉ 0;19° =] 22;12°.

And we showed that when it has a longitude of ♈ 27;15°, its greatest elongation (similarly defined) will be 22;23°.

So, again, since the difference between the longitudes is 3;4°, and the difference between the greatest elongations is 11′, to the 2;45° from the longitude at the first position to the beginning of Taurus correspond approximately 10′. So, subtracting the latter from the 22;23°, we get the greatest morning elongation from the true sun [when the planet is] at the beginning of Taurus as 22;13°.

Q.E.D.

In the same way we computed the greatest morning and evening elongations for both planets by calculation at [the beginning of] the other signs, and H521
constructed a small table for them, with 12 lines (equal in number [to the signs]) and 5 columns. At the beginning we put, in the first column, the first points of the signs, starting with Aries. In the following 4 columns we put the corresponding computed greatest elongations from the true sun: the second contains the morning elongations of the planet Venus, the third its evening elongations, the fourth the morning elongations of Mercury, and the fifth its evening elongations. The table is as follows.

[100] Reading $\overline{\nu\varepsilon}\ \overline{\nu\gamma}$ at H519,13 for $\overline{\nu\varepsilon}\ \overline{\nu}$ ($55;50^{p}$). Calculation (for $\bar{\kappa} = 222°$) gives $\rho = 55;52,58^{p}$. Although Ptolemy is capable of a computing error of this amount, he did not in fact make it, for the following calculations are consistent with $55;53^{p}$ and not with $55;50^{p}$ (thus $22;30 \times 120/55;50 = 48;21\frac{1}{2}$, whereas 55;53 leads to 48;19, as the text). The error, though scribal, is old, since it is shared by all mss.

[101] Literally 'of 19′ of the *first* degree of Taurus'.

H522 10. {*Greatest elongations with respect to the true sun*}[102]

Beginning of the Sign	VENUS As Morning Star	VENUS As Evening Star	MERCURY As Morning Star	MERCURY As Evening Star
♈	45 14	46 22	24 14	19 36
♉	45 17	45 31	22 13	21 7
♊	45 34	44 49	20 18	23 41
♋	45 56	44 25	18 17	26 16
♌	46 20	44 31	16 35	27 37
♍	46 38	44 55	16 8	26 17
♎	46 45	45 41	17 46	23 31
♏	46 47	46 30	21 32	20 58
♐	46 30	47 13	26 9	19 28
♑	46 7	47 35	28 37	19 14
♒	45 41	47 34	28 17	18 51
♓	45 20	47 7	26 24	19 0

[102] Correction to Heiberg: omit (with G,Ar) the column of argument before the entries for Mercury. Ptolemy's own description indicates that it was not in the original.

There are occasional computing errors of up to 5′ in the entries. For Venus, ♒, evening, the printed version of the *Handy Tables* (Halma III p. 32) has 47;37 (computed 47;39), but this greater accuracy seems coincidental, as the version in *Vat. Gr.* 1291, f. 90r, agrees with the *Almagest*. For Mercury, ♑, evening, there is a serious computing error, as noted *HAMA* 234 n.10. I find 18;53, but all mss. known to me agree in 19;14.

Book XIII

1. {*On the hypotheses for the positions in latitude of the 5 planets*}[1]

The following two topics still remain to [complete] the treatment of the 5 planets: their position in latitude with respect to the ecliptic, and the discussion of their elongations at their first and last visibilities with respect to the sun. For the second topic the latitudinal distances of each must also be taken into account first, since some considerable differences in the first and last visibilities occur due to that factor. So we shall again first set out the hypotheses which we assign to the inclination of the circles of all [five] in common.

Now [first], just as each [planet] appears to perform a twofold anomaly in longitude, each exhibits a twofold difference in latitude, one [varying] with respect to the parts of the ecliptic, and due to the eccentre, the other with respect to [its elongation from] the sun, and due to the epicycle. Therefore in every case we suppose that the eccentre is inclined to the plane of the ecliptic, and that the H525 epicycle is inclined to the plane of the eccentre. However, as we said [IX 6, p. 443], no noticeable difference occurs in the longitudinal position or the demonstrations of the anomalies on account of such small inclinations, as we shall show later.[2] [Secondly,] from individual observations of every planet, [we see that] the planets appear exactly in the plane of the ecliptic when the corrected longitude is approximately a quadrant from the northern or southern limit of the eccentre, and at the same time the corrected anomaly is approximately a quadrant from its own apogee.[3] So we suppose the inclinations of the eccentres to take place at the centre of the ecliptic (just as for the moon), and with respect to the diameters through the northern and[4] southern limits; and [we suppose] that the inclinations of the epicycles take place with respect to that diameter of the epicycle which points towards the centre of the ecliptic, on which its apparent apogee and perigee are observed.

Moreover, in the case of the 3 planets Saturn, Jupiter and Mars, we have observed that when their longitudinal positions are in the section of the eccentre farther from the earth they are always[5] north of the ecliptic, and are more H526

[1] On chs. 1 and 2 see *HAMA* 206–7, Pedersen 355–61.

[2] See XIII 4 pp. 608–22.

[3] I.e. from the true apogee of the epicycle.

[4] One would expect καὶ (text ἤ), and καὶ was apparently read by al-Ḥajjāj. If one keeps the text, one has to understand 'through [the centre of the ecliptic] and the northern or southern limits'.

[5] Excising τὸ πλεῖστον at H525,23, with Ar. It is a gloss ('for the most part') put in by a commentator to qualify ἀεὶ: since the northern limit does not quite coincide with the apogee (except for Mars), the planets are not *always* north of the ecliptic when on the semi-circle containing the apogee.

northerly for positions at the perigee of the epicycle than for those at the apogee;[6] but that when their longitudinal positions are in the section of the eccentre nearer the earth, quite the opposite, they appear south of the ecliptic. And [we have observed] that the northern limit of the eccentre is, for Saturn and Jupiter, around the beginning of the sign of Libra, and, for Mars, around the end of Cancer, almost exactly at its apogee. From these [observations] we conclude that the parts of their eccentres in the above-mentioned regions of the zodiac are inclined towards the north, and the diametrically opposite parts [depressed] by an equal amount towards the south, and that the parts of the epicycle nearer the earth are always inclined in the same direction as the eccentre,[7] while the diameter [of the epicycle] at right angles to the diameter through its apogee always remains parallel to the plane of the ecliptic.

In the case of Venus and Mercury, however, we have observed that [firstly], when their longitudinal positions are at the apogee or perigee of the eccentre, then positions at the perigee of the epicycle do not differ at all in latitude from
H527 positions at the apogee [of the epicycle]: rather they are either north or south of the ecliptic by an equal amount, always north for Venus, always south, on the contrary, for Mercury; whereas their positions at the greatest elongations differ [in latitude] from each other by the greatest amount (that is, the morning greatest elongations differ from the evening greatest elongations), while they differ from the positions at apogee and perigee of the epicycle (i.e. from the difference [in latitude] due to the eccentre)[8] by an equal amount, [but] in opposite directions: the greatest elongation which is towards the rear [of the epicycle centre] and in the evening is, for Venus, more northerly [than the morning one] at the apogee of the eccentre and more southerly at the perigee, while for Mercury the opposite is true, it is more southerly at the apogee [of the eccentre] and more northerly at the perigee. [Secondly, we have observed that,] when their corrected longitudinal positions are at the nodes, then a distance of a quadrant on either side of apogee or perigee of the epicycle brings [the planet] into the plane of the ecliptic, whereas positions at the perigee [of the epicycle] have the greatest difference [in latitude] from positions at the apogee: for Venus this inclination is towards the south at the node on the semi-circle on which the
H528 equation is subtractive,[9] and towards the north at the opposite [node]; for Mercury the opposite is again true: at the node on the subtractive semi-circle the inclination is towards the north, at the opposite one towards the south. From this too, then, we conclude that the inclination of the eccentre is also variable, and that its variation has the same period as the epicycle [on the

[6] Excising τῷ πλείστῳ τότε at H526,1. This would have to mean 'the amount by which they are more northerly for apogee positions than for perigee positions is greatest at that point', where τότε refers to the apogee of the eccentre. But in fact the point where this occurs is not the apogee, but the northern limit, and in any case this refinement is simply not appropriate here.

[7] I.e. if the eccentre is north of the ecliptic, the perigee of the epicycle is north of the eccentre, and if it is south, south.

[8] At the positions in question (at apogee or perigee of the eccentre) the diameter of the epicycle through apogee and perigee of the epicycle lies in the plane of the eccentre, hence the latitudinal effect comes entirely from the inclination of the eccentre.

[9] This nomenclature is used, rather than 'ascending' and 'descending' (as for the moon and the outer planets), because the *effect* of the inclination of the eccentre is always in one direction (north for Venus and south for Mercury). Cf. Manitius p. 328 n.a) and Pedersen 376.

eccentre]: when the epicycle is in the nodes, the eccentre is in the same plane as the ecliptic, but when [the epicyle] is at apogee or perigee, the eccentre produces the greatest difference in the epicycle's latitude, making it most northerly for Venus and most southerly for Mercury. [We also conclude that] the epicycle brings about two variations [in latitude]: it produces the greatest inclination of the diameter through the apparent apogee at the nodes of the eccentre, and the greatest 'slant' (let us use this term to distinguish this kind of angular variation) of the diameter at right angles to the former at the apogee and perigee of the eccentre. Contrariwise, it brings the first [diameter] into the plane of the eccentre at its [the eccentre's] apogee and perigee, and brings the second diameter into the plane of the ecliptic at the above-mentioned nodes.

2. {*On the type of the motions in inclination and slant according to the hypotheses*}[10] H529

The general structure of the hypotheses, then, which we infer is as follows. The eccentric circles of [all] 5 planets are inclined to the plane of the ecliptic about the centre of the ecliptic. But in the case of the 3 planets Saturn, Jupiter and Mars the eccentre has a fixed inclination, so that diametrically opposite positions of the epicycle have opposite directions in latitude, whereas in the case of Venus and Mercury the eccentre moves together with the epicycle in the same latitudinal direction, for Venus always to the north, for Mercury always to the south. The epicycle [for all 5 planets] has the diameter through its apparent apogee moved from a starting-point in the plane of the eccentre, by a small circle which we may suppose attached to the end [of the diameter] nearer the earth. This circle is of a size corresponding to the appropriate [maximum] deviation in latitude, is perpendicular to and centred in the plane of the eccentre, and revolves with uniform motion, with a period equal to that of the motion in longitude, from one end of the intersection of its own plane and the H530
plane of the epicycle towards the north (by hypothesis), carrying with it the plane of the epicycle: in its revolution through the first quadrant it carries the epicycle's plane, obviously, to the northern limit, in the second back to the plane of the eccentre, in the third to the southern limit, and in its return to [the end of] the remaining quadrant back to the original plane. We also [infer] that the origin and point of return of this revolution is for Saturn, Jupiter and Mars the ascending node, for Venus the perigee of the eccentre, and for Mercury the apogee of the eccentre.[11] The diameter [of the epicycle] at right angles to the aforementioned, in the case of the 3 [superior] planets, as we said [p. 598], always remains parallel to the plane of the ecliptic, or at any rate is not inclined to it by a significant amount, but in the case of Venus and Mercury it too is carried from a starting-point in the plane of the ecliptic by a small circle which we may suppose attached to the rearward end, which is again of a size corresponding to the appropriate [maximum] deviation in latitude,

[10] On the mechanism imagined by Ptolemy (and in particular the 'small circles') the best discussion is by Riddell, 'Latitudes of Venus and Mercury', despite occasional inaccuracies due to his use of Taliaferro's faulty translation.

[11] It is essential to change Heiberg's punctuation from a comma to a full stop at H530,13.

perpendicular to the plane of the ecliptic, and centred on the diameter[12] parallel to the ecliptic. This circle revolves, with a speed equal to that of the
H531 other [small circle], from one end of the intersection of its plane and the plane of the epicycle towards the north, again by hypothesis, and carries with it the evening [i.e. rearward][13] end of the aforementioned diameter in the same order as before. For this too the origin and point of return of the similar type of revolution is, in the case of Venus, at the node in the additive semi-circle, and, in the case of Mercury, at the node in the subtractive semi-circle.

However, we must make the following assumption concerning those small circles which produce the motions in latitude of the epicycles: they too are, indeed, bisected by the planes about which we declare that the variations in latitude take place; for that is the only way in which it can come about that their [the epicycles'] motions in latitude are equal on both sides [of the planes]. However, their revolution in uniform motion takes place, not about their own centres, but about some other point which will produce in the small circle an eccentricity corresponding to [the eccentricity] of the planet in longitude in the ecliptic. For since the times of revolution on the ecliptic and the small circle are, by hypothesis, equal, and the arrivals at the quadrants in both [circles] also
H532 correspond to each other, according to the [observational] phenomena, if the [uniform] revolution of the small circle were to take place about its own centre, the desired result would not be achieved; since [in that case] each of the quadrants of the small circle would be traversed in an equal time, while the quadrants of the ecliptic traversed by the epicycle would not be, because of the eccentricity assumed for each planet. But if [the uniform revolution of the small circle takes place] about a point placed similarly to the [centre of uniform motion] in the eccentre, the returns in the inclinations will also traverse the corresponding quadrants of the ecliptic and the small circle in equal times.[14]

Now let no one, considering the complicated nature of our devices, judge such hypotheses to be over-elaborated. For it is not appropriate to compare human [constructions] with divine, nor to form one's beliefs about such great things on the basis of very dissimilar analogies. For what [could one compare] more dissimilar than the eternal and unchanging with the ever-changing, or that which can be hindered by anything with that which cannot be hindered even by itself?[15] Rather, one should try, as far as possible, to fit the simpler hypotheses to the heavenly motions,[16] but if this does not succeed, [one should apply hypotheses] which do fit. For provided that each of the phenomena is
H533 duly saved by the hypotheses, why should anyone think it strange that such

[12] Cf. Manitius' note p. 331 b). If 'diameter' is to make any sense here, it must be a diameter of the *epicycle* which is parallel to the ecliptic (at a certain point in the orbit), and notionally remaining there all the time, even when the epicycle is 'slanted'. Cf. *HAMA* 1279 Fig. 219a (where the line through A is parallel to the ecliptic), and Riddell Fig. 4 and p. 101.

[13] πρὸς ἑσπέραν, literally 'toward evening', which one would expect to mean 'western'. But the sense demands 'eastern', and, if the text is correct, one must interpret it, with the Arabic translators, as 'the side of the epicycle where the planet appears as evening star', cf. H511,22, τὰ ἑσπέρια καὶ ἑπόμενα τοῦ ἐπικύκλου.

[14] It is essential to correct Heiberg's punctuation of this passage by deleting the comma after τεταρτημορίων (H532,9) and inserting one after ἐκκέντρου (H532,8).

[15] I.e. the substance of the heavenly bodies, the 'fifth essence'. Cf. p. 36 n.8.

[16] On this principle of simplicity see p. 136 n.17.

complications can characterise the motions of the heavens when their nature is such as to afford no hindrance, but of a kind to yield and give way to the natural motions of each part, even if [the motions] are opposed to one another? Thus, quite simply, all the elements can easily pass through and be seen through all other elements, and this ease of transit applies not only to the individual circles, but to the spheres themselves and the axes of revolution. We see that in the models constructed on earth the fitting together of these [elements] to represent the different motions is laborious, and difficult to achieve in such a way that the motions do not hinder each other, while in the heavens no obstruction whatever is caused by such combinations. Rather, we should not judge 'simplicity' in heavenly things from what appears to be simple on earth, especially when the same thing is not equally simple for all even here. For if we were to judge by those criteria, nothing that occurs in the heavens would appear simple, not even the unchanging nature of the first motion, since this very quality of eternal unchangingness is for us not [merely] difficult, but completely impossible. H534
Instead [we should judge 'simplicity'] from the unchangingness of the nature of things in the heaven and their motions. In this way all [motions] will appear simple, and more so than what is thought 'simple' on earth, since one can conceive of no labour or difficulty attached to their revolutions.

3. {*On the amount of the inclination and slant for each [planet]*}[17]

From the above considerations one may infer the general situation and arrangement of the inclinations of the [various] circles. But [concerning] the actual size for each planet of the arc cut off by the inclination on the great circle drawn perpendicular to the plane of the ecliptic through the poles of the inclined circle[18] (with respect to which [great circle] the positions in latitude are measured), this is readily calculated in the case of Venus and Mercury from the apparent positions in latitude at the situations described.

For when their motion in longitude brings them to apogee or perigee of the eccentre, if the planet's position is at perigee or apogee of the epicycle, they H535
appear, as we said, (operating from nearby observations),[19] an equal amount either north or south of the ecliptic: Venus always about $\frac{1}{6}^{\circ}$ north, and Mercury always $\frac{3}{4}^{\circ}$ south. Hence [we conclude that] the inclinations of the eccentres are of that size for each. But if they are at a greatest elongation from the sun, both planets appear about 5° (in the mean) farther north or south than at the opposite greatest elongation: for Venus has an apparent difference in latitude of the kind mentioned [i.e. between greatest morning and evening elongations] of negligibly less than 5° at the apogee of the eccentre, and negligibly greater than 5° at the perigee, while Mercury has about $\frac{1}{2}^{\circ}$ [less and greater than 5° in

[17] On chs. 3 and 4 see *HAMA* 208–16, Pedersen 361–85, Riddell, 'Latitudes of Venus and Mercury'.

[18] 'inclined circle': deferent or epicycle as the case may be.

[19] From 'nearby observations' because the planets are invisible when precisely at apogee or perigee of the epicycle. Correct Heiberg's punctuation by inserting a comma after ὡς ἔφαμεν ('as we said'), which cannot refer to the use of nearby observations, but only to the fact that the planet is north or south etc. (as p. 599).

latitudinal difference at apogee and 180° from apogee respectively]. Hence the slant of the epicycle to either side of the plane of the eccentre subtends about $2\frac{1}{2}$°, in the mean, of the [great] circle orthogonal to the ecliptic. From this [quantity] the size of the angles formed by the slant of the epicycle to the plane of the eccentre [for each planet] can be found, as will become clear in our proofs
H536 concerning them in what follows [XIII 4, p. 625] (so as not to fragment, at this point, our discussion of the inclinations, which will treat the five planets in common).

But when their corrected motion in longitude brings them to the nodes and [hence] very nearly to mean distance: then Venus, when its position is near the apogee of the epicycle, appears 1° north or south[20] of the ecliptic, and, when its position is near the perigee, about $6\frac{1}{3}$°: hence the inclination of its epicycle too cuts off $2\frac{1}{2}$° of the great circle drawn through its poles in the way described; for we find from the [table for] epicyclic anomaly that at mean distance that amount [$2\frac{1}{2}$°] subtends at the observer's eye an angle of 1;2° for [the planet at] the apogee of the epicycle, and 6;22° for [the planet at] the perigee.[21] As for Mercury, when its position is near the apogee of the epicycle, as one can calculate from the phases nearest to it, it is north or south of the ecliptic[22] by $1\frac{3}{4}$°, and, when near the perigee, about 4°; hence the inclination of its epicycle comes to $6\frac{1}{4}$°. For
H537 again we find from the [table for] epicyclic anomaly that at the distances of greatest inclination, that is when the corrected longitude is a quadrant from apogee, that amount [$6\frac{1}{4}$°] subtends, at the observer's eye, 1;46° for [the planet at] the apogee of the epicycle, and 4;5° for [the planet at] the perigee.[23]

In the case of the other planets, Saturn, Jupiter and Mars, there is no method for finding the sizes of the inclination immediately [from the observational data], since both inclinations, that connected with the eccentre and that connected with the epicycle, are always intermingled: however, once again, from the latitudinal positions observed at perigee and apogee of eccentre and epicycle, we determine each inclination separately in the following manner.

[See Fig. 13.1.] In the plane orthogonal to the ecliptic let the intersection with it of the plane of the ecliptic be AB, and of the plane of the eccentre, GD. Let point E be the centre of the ecliptic, and at the intersection of the planes, [that of the eccentre and that orthogonal to the ecliptic], draw,[24] in the defined plane, about G, the apogee of the eccentre, and about D, the perigee, equal circles
H538 ZHΘK and LMNX to represent the circles through the poles of the epicycles. On these circles let the planes of the epicycles [be drawn] on lines HGK and MDX, inclined, obviously, at equal angles at G and D. From E, the centre of

[20] See *HAMA* 1279 Fig. 219b: Venus at apogee is north for $\kappa_0 = 90°$, south for $\kappa_0 = 270°$.

[21] For the rationale of this calculation see *HAMA* 215. From Table XI 11, col. 6, to an argument of $2\frac{1}{2}$° near apogee corresponds an equation of anomaly of $2;31° \times 2\frac{1}{2}/6 \approx 1;2°$, while to $2\frac{1}{2}$° near perigee corresponds $7;38° \times 2\frac{1}{2}/3 \approx 6;22°$.

[22] See *HAMA* 1280 Fig. 221: Mercury at apogee is north for $\kappa_0 = 270°$ and south for $\kappa_0 = 90°$.

[23] See *HAMA* 216 (which has several small errors). The corrected longitude for Mercury is exactly 90° from apogee when the mean longitude is 92;52°. From Table XI 11, cols. 6–8, one finds, for $\bar{\kappa} = 92;52°$ and $\alpha = 6\frac{1}{4}°$, an equation of 1;45,51°, and for $\bar{\kappa} = 92;52°$ and $\alpha = 173\frac{3}{4}°$, an equation of 4;4,47°, confirming Ptolemy's calculations.

[24] Excising τε after γεγράφθωσαν at H537,20 (with D). Alternatively one could put a strong stop after ἐπιπέδων at H537,19 (with A,Is), and translate 'Let point E be the centre of the ecliptic and at the intersection of [all three] planes. Then draw. . . .'

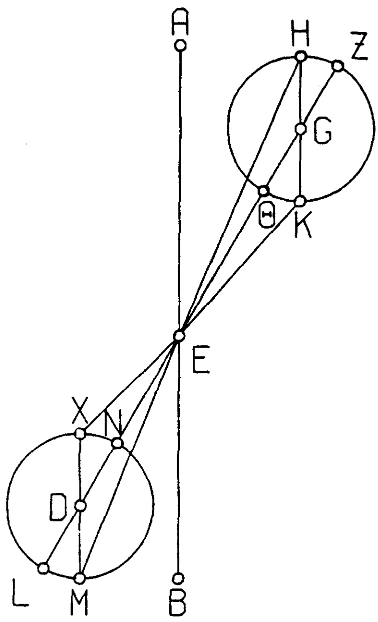

Fig. 13.1

the ecliptic, at which the observer's eye is, draw straight lines joining it to the apogees and perigees of the epicycles, EH and EM to the apogees, and EK and EX to the perigees. It is clear that points K and X will represent the positions at opposition, and H and M those at conjunction.

For Mars, then, we obtained the positions in latitude round about the oppositions occurring at the apogee of the eccentre (that is, round about point K of the epicycle), and also round about the oppositions occurring at the perigee of the eccentre (that is, round about point X of the epicycle), since the H539
difference between them is quite noticeable. At the oppositions near the apogee it is $4\frac{1}{3}°$ to the north of the ecliptic, and at those near the perigee about 7° to the south. Thus

$$\left.\begin{aligned} \angle\ \mathrm{AEK} &= 4\tfrac{1}{3}° \\ \text{and } \angle\ \mathrm{BEX} &= 7° \end{aligned}\right\}\ \text{where 4 right angles} = 360°.$$

With that as data, we find the angle formed by the inclination of the eccentre, namely ∠ AEG, and that formed by the inclination of the epicycle, namely ∠ HGZ, in the following manner.

Since it is easy to see from our demonstrations of the anomalies of Mars that, if one considers the angles subtended at the observer's eye by equal arcs of the epicycle near its perigee, those for positions near the apogee of the eccentre bear to those for positions near the perigee [of the eccentre] a ratio of approximately 5:9,[25] and since

$$\text{arc } \Theta\mathrm{K} = \text{arc } \mathrm{NX},$$

[25] For the derivation of this ratio from the anomaly table see *HAMA* 209–10, Pedersen 363 (with the correction Toomer [3], 141).

it follows that ∠ GEK:∠ DEX = 5:9.

H540 So, since angles AEK and BEX are given,
and the ratio of ∠ GEK:∠ DEX is given,
and ∠ AEG = ∠ BED,
if we form the difference between the magnitudes of the whole [angles, i.e. ∠ AEK and ∠ BEX], and the difference between [the members of] the ratio [i.e. 5 and 9], take the fraction which the first [difference] is of the second, and take that fraction of each [member of the] ratio, we will get the magnitude corresponding to each part of the ratio. This can be proven by means of an arithmetical lemma.[26]

So, since the magnitudes are $4\frac{1}{3}$ and 7, and their difference $2\frac{2}{3}$,
and the ratio is 5:9 and the difference 4.
and $2\frac{2}{3}$ is two-thirds of 4,
we take two-thirds of 5 and 9 [respectively], and get
∠ GEK = $3\frac{1}{3}$° and ∠ DEX = 6°.

Accordingly, by subtraction,
∠ AEG = ∠ BED = 1°, the inclination of the eccentre.

Hence arc ΘK, representing the inclination of the epicycle, is $2\frac{1}{4}$°, for from the table of anomaly we find that that amount [$2\frac{1}{4}$°] corresponds approximately to the quantities we found for the angles GEK and DEX.[27]

In the case of Saturn and Jupiter, we find that the [latitudinal] positions occurring near the apogee of the eccentre are not sensibly different from those
H541 diametrically opposite, near the perigee. So we computed the required results in another way, by comparing the [latitudinal positions] near apogee of the epicycle with those near perigee. It has become clear to us from individual observations that at positions near first and last visibilities the maximum deviation to north and south is about 2° for Saturn and 1° for Jupiter, while for positions near opposition [the maximum latitudinal deviation] is about 3° for Saturn and 2° for Jupiter. Now for these planets too it is obvious from the [table for] anomaly that, if one considers the angles subtended at the observer's eye by equal arcs near apogee and perigee of the epicycle, the angles subtended by arcs near apogee bear a ratio to those subtended by arcs near perigee of 18:23 for Saturn, and 29:43 for Jupiter;[28]
and arcs ZH and ΘK of the epicycle are equal.

$$\text{So } \angle \text{ZEH}: \angle \text{ZEK} = \begin{cases} 18:23 \text{ for Saturn} \\ 29:43 \text{ for Jupiter.} \end{cases}$$

But ∠ HEK, which is the difference between the two latitudes [at apogee and

[26] Given two magnitudes A, B, and the ratio l:m of two other magnitudes, C, D such that A = x + C, B = x + D, the lemma states that
C = l × (B − A)/(m − l), D = m × (B − A)/(m − l).
Proof: Since D/C = m/l, (D − C)/C = (m − l)/l.
But D − C = B − A
∴ C = l × (B − A)/(m − l);
D = C × m/l = m × (B − A)/(m − l).

[27] For the method see p. 602 n.21. Here, from Table XI 11, cols. 5–7, for argument α = (180° − $2\frac{1}{4}$°) at greatest and least distance respectively, one finds (5;45 − 1;16) × $2\frac{1}{4}$/3 ≈ 3;22° and (5;45 + 2;20) × $2\frac{1}{4}$/3 ≈ 6;4° (text $3\frac{1}{3}$° and 6°).

[28] See *HAMA* 211, where however one should change to $\frac{c_6\ (183)}{c_6\ (3)} = \frac{23}{18}$ for Saturn and $\frac{43}{29}$ for Jupiter, in exact agreement with Ptolemy.

perigee of the epicycle], is, by subtraction, 1° for both planets. Therefore, if we H542
divide that 1° in the above ratios, we get

$$\angle \text{ZEH} = \begin{cases} 0;26^\circ \text{ for Saturn} \\ 0;24^\circ \text{ for Jupiter,} \end{cases}$$

$$\text{and } \angle \text{ZEK} = \begin{cases} 0;34^\circ \text{ for Saturn} \\ 0;36^\circ \text{ for Jupiter.} \end{cases}$$

So, by subtraction [from ∠ AEK], the inclination of the eccentre

$$\angle \text{AEG} = \begin{cases} 2;26^\circ \text{ for Saturn} \\ 1;24^\circ \text{ for Jupiter.} \end{cases}$$

Instead of these, to achieve greater symmetry, we have adopted the round numbers $2\frac{1}{2}°$ and $1\frac{1}{2}°$. Then arc ΘK, representing the inclination of the epicycle, can immediately be computed as $4\frac{1}{2}°$ for Saturn and $2\frac{1}{2}°$ for Jupiter. For again, in the tables of anomaly for each planet, those were the amounts which correspond approximately to the quantities we found for angles ZEH and ZEK.[29]

Q.E.D.

4. {*Construction of tables for the individual positions in latitude*}

From the above, then, we established the generally applicable quantities of the greatest inclinations of eccentres and epicycles. But in order that we may be able to conveniently and systematically find the positions in latitude for a given moment for the individual distances [from apogee] as well, we constructed 5 tables for the 5 planets. Each contains the same number of lines as the tables for H543
anomaly [i.e. 45], and 5 columns. The first 2 of these columns comprise the arguments, in the same way as in those [tables for anomaly]; the third column contains the latitudinal distances from the ecliptic corresponding to the particular degrees of [motion on] the epicycle, under the assumption of greatest inclination – for Venus and Mercury this is the inclination at the nodes of the eccentre, and for the other 3 planets the inclination at the northern limit of the eccentre. For the latter the fourth column will contain the similar corresponding amounts at the southern limit, and in the case of these 3 planets the maximum deviation to north and south of the eccentres too has also been included in the computation. The way in which we determined these quantities for Venus and Mercury again rested on a single theorem [for both], as follows.

[See Fig. 13.2] In the plane orthogonal to the ecliptic let ABG be the intersection with it of the plane of the ecliptic, and DBE the intersection [with it] of the plane of the epicycle. Let A be the centre of the ecliptic, B the centre of H544
the epicycle, and AB the distance of the epicycle at the greatest inclination. About B describe the epicycle DZEH,[30] and draw diameter ZBH perpendicular

[29] See p. 602 n.21. Here, from Table XI 11, col. 6 for Saturn, $0;36 \times 4\frac{1}{2}/6 = 0;27°$ (text 0;26°), and $0;23 \times 4\frac{1}{2}/3 = 0;34,30°$ (text 0;34°); for Jupiter $0;58 \times 2\frac{1}{2}/6 = 0;24,10°$ (text 0;24°); $0;43 \times 2\frac{1}{2}/3 =$ 0;35,50 (text 0;36).

[30] Note that G is not a point on the epicycle, as might appear from Fig. 13.2 and from the corresponding figure for Mercury, Fig. 13.4. To make the various planes in this three-dimensional figure clearer it has been redrawn as Fig. S.

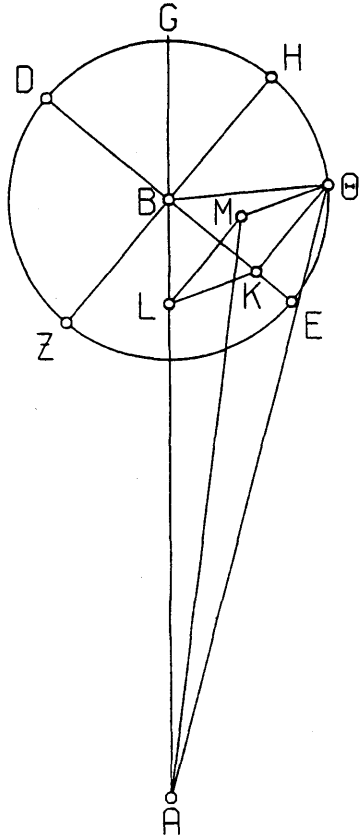

Fig. 13.2

to DE. Let the plane of the epicycle too be taken as perpendicular to the assumed plane [that orthogonal to the plane of the ecliptic], so that when lines are drawn in it perpendicular to DE, all will be parallel to the plane of the ecliptic, excepting only ZH, which will lie in the plane of the ecliptic.

Then let the problem be, given the ratio of AB to BE, and the amount of the inclination (i.e. of ∠ ABE), to find the positions of the planets in latitude when (to take an example) they are at a distance of 45° (where [the circumference of] the epicycle is 360°) from the perigee of the epicycle, E. [We choose 45°] because we intend to demonstrate at the same time the differences in the positions in
H545 longitude produced by these [maximum] inclinations, and these differences ought to reach their maximum at about halfway between the perigee E and the positions Z and H, since at those points [the longitudes so computed] are identical with the longitudes produced by neglecting the inclination.

So let arc EΘ be cut off in the above amount of 45°, and drop ΘK perpendicular to BE, and KL, ΘM perpendicular to the plane of the ecliptic. Join ΘB, LM, AM and AΘ.

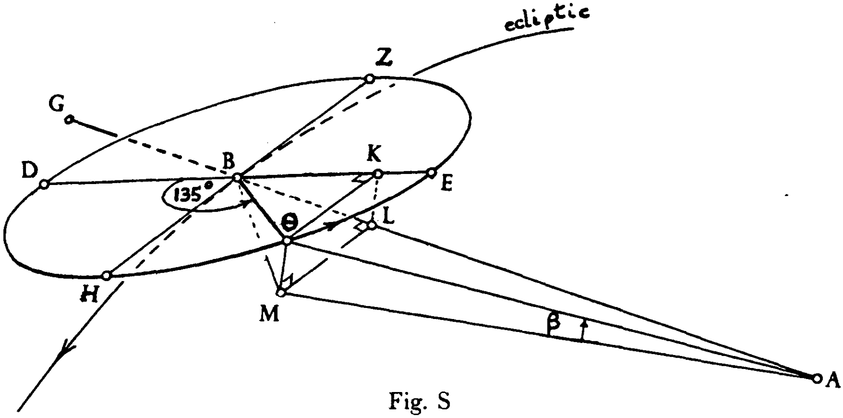

Fig. S

It is immediately obvious that
[1] quadrilateral LKΘM has parallel sides and right angles
(since KΘ is parallel to the plane of the ecliptic); and
[2] the equation in longitude is comprised by ∠ LAM, and
[3] the position in latitude is comprised by ∠ ΘAM
(since angles ALM and AMΘ too turn out to be right angles, because line AM lies in the plane of the ecliptic).[31]

But now we must demonstrate the numerical amounts of the required positions to be computed for each of the above planets, and first for Venus.

Since arc EΘ = 45° where [the circumference of] the epicycle is 360°,

∠ EBΘ (since it is at the centre of the epicycle) $= \begin{cases} 45° \text{ where 4 right angles} = 360° \\ 90^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ}. \end{cases}$ H546

Therefore, in the circle about right-angled triangle BΘK,

arc BK = arc KΘ = 90°.

So the corresponding chords

BK = KΘ = 84;52^p where hypotenuse BΘ = 120^p.

Therefore where BΘ, the radius of the epicycle, is 43;10^p,
and AB, the mean distance, is 60^p
(for the greatest inclination of the epicycle occurs at approximately that point),

BK = KΘ = 30;32^p.

Again, since the angle of inclination,

∠ ABE is taken as $\begin{cases} 2;30° \text{ where 4 right angles} = 360° \\ 5^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ}, \end{cases}$

in the circle about right-angled triangle BLK,

arc LK = 5°
and arc BL = 175° (supplement).

So the corresponding chords

$\left.\begin{array}{r} \text{KL} = 5;14^p \\ \text{and BL} = 119;53^p \end{array}\right\}$ where hypotenuse BK = 120^p.

[31] See Fig. S, which makes most of Ptolemy's statements obvious. In particular, since M is in the ecliptic (by construction) and ∠ AMΘ is constructed as a right angle, LM, KΘ and BH are all parallel, so ∠ ALM is a right angle.

Therefore, where hypotenuse BK $= 30;32^p$, and AB $= 60^p$,

KL $= 1;20^p$,

BL $= 30;30^p$,

and, by subtraction [of BL from AB], AL $= 29;30^p$.

H547 But, in the same units, LM $=$ KΘ $= 30;32^p$.

Therefore hypotenuse AM $[= \sqrt{AL^2 + LM^2}] = 42;27^p$ in the same units.

Therefore, where hypotenuse AM $= 120^p$, LM $= 86;19^p$,

and the equation in longitude at that point,

$$\angle LAM = \begin{cases} 92;0^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \\ 46;0^{\circ} \text{ where 4 right angles} = 360^{\circ}. \end{cases}$$

Similarly, where AM $= 42;27^p$,

ΘM $=$ KL $= 1;20^p$;

and $\Theta M^2 + AM^2 = A\Theta^2$,

so AΘ $= 42;29^p$ in the same units.

Therefore, where hypotenuse AΘ $= 120^p$,

ΘM $= 3;46^p$,

and the angle of the deviation in latitude,

$$\angle \Theta AM = \begin{cases} 3;36^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \\ 1;48^{\circ} \text{ where 4 right angles} = 360^{\circ}. \end{cases}$$

That [1;48°] is what we shall put in the third column of the table for Venus on the line containing '135°'.

In order to make a comparison of the difference in the equation of longitude which results [from the above computation], let there be drawn [Fig. 13.3] the corresponding figure without any inclination of the epicycle. Then we showed that

BK $=$ KΘ $= 30;32^p$ where AB $= 60^p$,

so, by subtraction, AK $= 29;28^p$;

H548 and $AK^2 + K\Theta^2 = A\Theta^2$,

so AΘ $= 42;26^p$ in the same units.

Therefore, where hypotenuse AΘ $= 120^p$, KΘ $= 86;21^p$,

and the angle of the equation in longitude,

$$\angle \Theta AK = \begin{cases} 92;3^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \\ 46;2^{\circ}, \text{ approximately, where 4 right angles} = 360^{\circ}. \end{cases}$$

And with the inclination it was shown to be 46°.

Therefore the equation in longitude, computed according to the inclination, was less by 2′.

Q.E.D.[32]

Again, to enable us to demonstrate the [latitudinal] positions for Mercury too, let there be drawn a figure [Fig. 13.4] similar to the one before the last, with arc EΘ taken as the same size, 45°. Hence again

BK $=$ KΘ $= 84;52^p$ where hypotenuse BΘ $= 120^p$.

H549 Therefore, where the radius of the epicycle, BΘ $= 22;30^p$,

[32] Accurately, one finds 45;59° (to the nearest minute) with the inclination, and 46;0° without it. Ptolemy's inaccuracy here is mysterious, since for the table of anomaly (XI 11), argument 135° at mean distance, he found (presumably by an identical computation) the better value 45;59°.

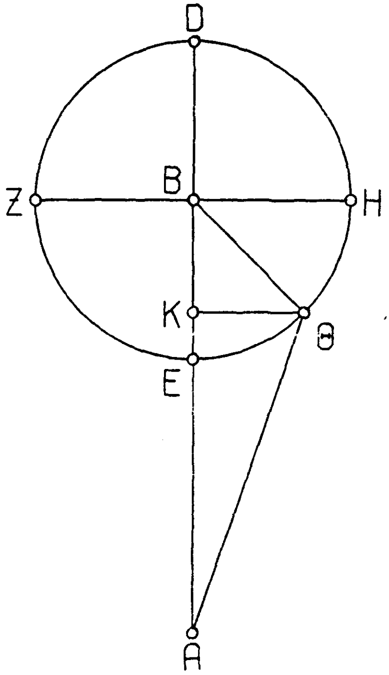

Fig. 13.3

and AB, the distance at which the greatest inclinations occur, is $56;40^p$ (all of which we have previously demonstrated),[33]

$$BK = K\Theta = 15;55^p \text{ in the same units.}$$

Again, since by hypothesis the angle of the inclination of the epicycle,

$$\angle ABE = \begin{cases} 6;15^\circ & \text{where 4 right angles} = 360^\circ \\ 12;30^{\circ\circ} & \text{where 2 right angles} = 360^{\circ\circ}, \end{cases}$$

in the circle about right-angled triangle BKL,

$$\text{arc } LK = 12;30^\circ$$
$$\text{and arc } BL = 167;30^\circ \text{ (supplement).}$$

So the corresponding chords

$$\left.\begin{aligned} KL &= 13;4^p \\ \text{and } BL &= 119;17^p \end{aligned}\right\} \text{ where hypotenuse } BK = 120^p.$$

Therefore where BK, as we showed, is $15;55^p$,
and AB, by hypothesis, is $56;40^p$,

$$KL = 1;44^p,$$
$$BL = 15;49^p,$$

[33] This last number is not, in fact, previously attested. However, Ptolemy must have computed the distances all the way round the orbit in order to construct the table of anomaly, and no doubt found this value by interpolation. Neugebauer (*HAMA* 221) found $56;37^p$ from a cubic equation. However, from a computer program I find, for $\bar{\kappa} = 93;1,41^\circ$, $\kappa_0 = 90;0,0^\circ$, $\rho = 56;43,9^p$.

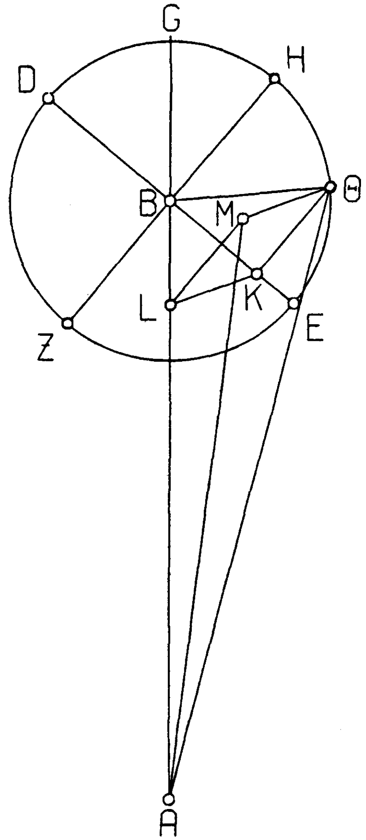

Fig. 13.4

and, by subtraction [from AB], AL = 40;51^p in the same units.

And LM = KΘ = 15;55^p.

H550 And since $AL^2 + LM^2 = AM^2$,

AM = 43;50^p where line LM = 15;55^p.

Therefore, where hypotenuse AM = 120^p, LM = 43;34^p,

and the angle of the equation in longitude,

$$\angle \text{LAM} = \begin{cases} 42;34^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \\ 21;17^{\circ} \text{ where 4 right angles} = 360^{\circ}. \end{cases}$$

Similarly, where AM = 43;50^p,

ΘM = KL = 1;44^p;

and $AM^2 + \Theta M^2 = A\Theta^2$,

so AΘ = 43;52^p in the same units.

Therefore, where hypotenuse AΘ = 120^p,

ΘM = 4;44^p,

and the angle of the deviation in latitude,

$$\angle \Theta\text{AM} = \begin{cases} 4;32^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \\ 2;16^{\circ} \text{ where 4 right angles} = 360^{\circ}. \end{cases}$$

That [2;16°] is what we shall enter in the third column of the table for Mercury on the same line, namely that containing the argument '135°'.

In order again to make a comparison of the equation, let there be drawn [Fig. 13.5] the figure without the inclination [of the epicycle]. Then we showed that, where line AB = 56;40^p,

$$\Theta K = KB = 15;55^p,$$ H551

and, by subtraction, obviously, AK = 40;45^p in the same units;

$$\text{and } AK^2 + K\Theta^2 = A\Theta^2,$$

$$\text{so } A\Theta = 43;45^p \text{ where } \Theta K = 15;55^p.$$

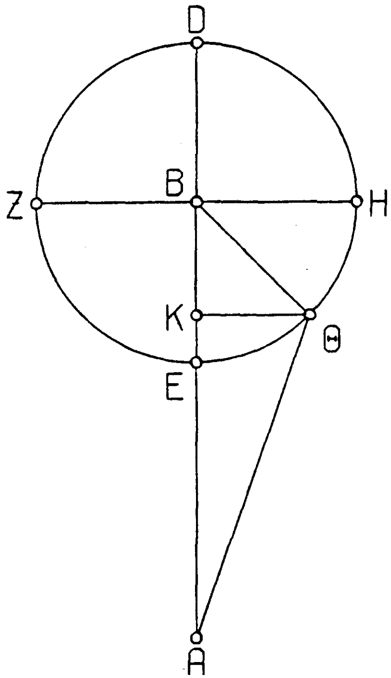

Fig. 13.5

Therefore, where hypotenuse AΘ = 120^p, ΘK = 43;39^p,

and the angle of the equation in longitude,

$$\angle KA\Theta = \begin{cases} 42;40^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \\ 21;20^{\circ} \text{ where 4 right angles} = 360^{\circ}. \end{cases}$$

But we showed that with the inclination it was 21;17°.

Therefore here too the equation in longitude computed according to the inclination was less, by 3′.

Q.E.D.

Such, then, is the method by which we computed the positions in latitude at the greatest inclinations for these two planets. For the greatest inclinations occur when the eccentre is in the same plane as the ecliptic. For the remaining 3 planets, however, we computed [those positions] by means of a theorem which H552 requires a different diagram, since [for these] the greatest inclination of the epicycle occurs when the inclination of the eccentre is also at a maximum, and it would benefit us to have the positions in latitude resulting from both inclinations computed together.

[See Fig. 13.6 and cf. Fig. T.] In the plane orthogonal to the ecliptic, again, let the intersection with it of the plane of the ecliptic be AB, the intersection of the plane of the eccentre AG, and the intersection of the plane of the epicycle DGE. Let A be taken as the centre of the ecliptic, and G as the centre of the epicycle, and let the epicycle DZEH be described about G in such a way, again,

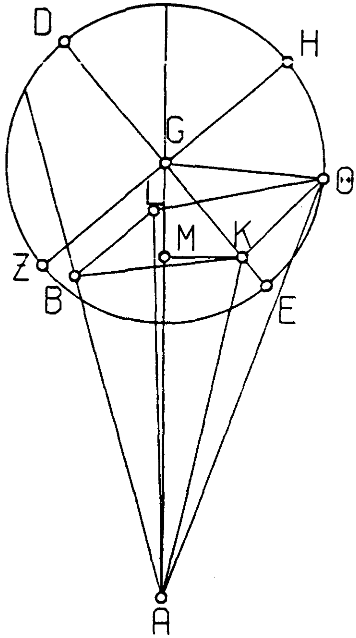

Fig. 13.6

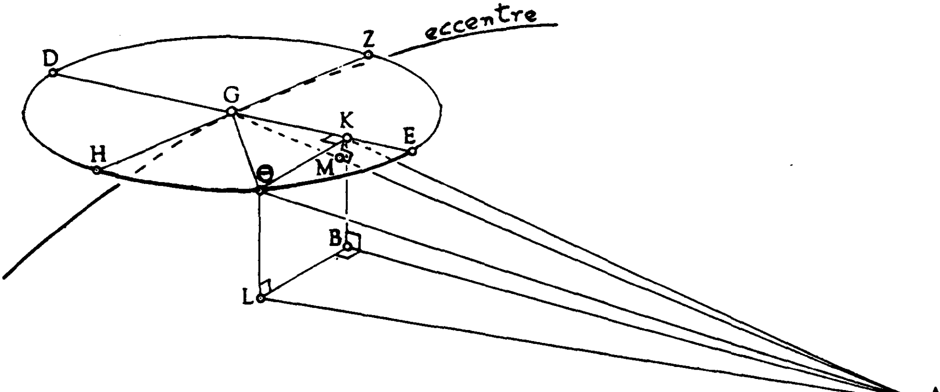

Fig. T

that when lines are drawn perpendicular to DE, diameter ZGH lies in the plane of the eccentre and parallel to the plane of the ecliptic, while the other [perpendiculars] are parallel to both the above planes. Similarly, let arc EΘ be cut off in the same amount of 45°, and drop perpendicular ΘK from Θ (the point at which the planet is located), and also drop perpendiculars ΘL, KB from points Θ and K to the plane of the ecliptic. Join BL and AL. Then let the problem be, to find the equation in longitude, represented by ∠ BAL, and the position in latitude, represented by ∠ LAΘ. H553

So draw perpendicular KM from K to AG, and join GΘ, AK and AΘ. Let us again take it as given, from what was proved before, that

$$GK = K\Theta = 84;52^p \text{ where hypotenuse } G\Theta = 120^p.$$

Then first, for Saturn:

Since we showed that the radius of the epicycle is 6;30ᵖ where the mean distance is 60ᵖ,

$$GK = K\Theta = 4;36^p \text{ where hypotenuse } G\Theta = 6;30^p.$$

And since, by hypothesis, the angle of the inclination of the epicycle,

$$\angle AGE = \begin{cases} 4;30^\circ \text{ where 4 right angles} = 360^\circ \\ 9^{\circ\circ} \text{ where 2 right angles} = 360^\circ, \end{cases}$$

in the circle about right-angled triangle GKM,

$$\text{arc } KM = 9^\circ$$
$$\text{and arc } GM = 171^\circ \text{ (supplement).}$$

So the corresponding chords

$$\left.\begin{aligned} KM &= 9;25^p \\ \text{and } GM &= 119;38^p \end{aligned}\right\} \text{ where hypotenuse } GK = 120^p.$$

Therefore, where $GK = 4;36^p$, H554

$$KM = 0;22^p$$
$$\text{and } GM = 4;35^p.$$

Now at the greatest inclination on the semi-circle containing the apogee, AG, representing the distance [when the epicycle is] near the beginning of Libra,[34] is computed, by means of the theorems which we went through before, in treating the anomalies, as 62;10ᵖ in the same units.[35] Hence, by subtraction [of GM from AG],

$$AM = 57;35^p \text{ where line } MK = 0;22^p;$$

hence hypotenuse AK $[= \sqrt{AM^2 + MK^2}] = 57;35^p$ in the same units.

Therefore, where hypotenuse $AK = 120^p$, $KM = 0;46^p$,

and ∠ KAM[36] = 0;44°° where 2 right angles = 360°°.

But, by hypothesis, the angle of the inclination of the eccentre,

$$\angle BAG = \begin{cases} 2;30^\circ \text{ where 4 right angles} = 360^\circ \\ 5^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ}. \end{cases}$$

Therefore, by addition, ∠ BAK = 5;44°° where 2 right angles = 360°°.

Therefore, in the circle about right-angled triangle BAK,

$$\text{arc } BK = 5;44^\circ$$
$$\text{and arc } AB = 174;16^\circ \text{ (supplement).}$$

[34] Cf. XIII 1 p. 598.

[35] Accurately, 62;8,21ᵖ when the centre of the epicycle is at a true longitude of ♎ 0° (the apogee being in ♏ 20°, cf. XIII 6 p. 635).

[36] Reading KAM for KΛM (misprint in Heiberg's text) at H554,11. Corrected by Manitius.

So the corresponding chords

$$\left.\begin{aligned} BK &= 6;0^p \\ \text{and } AB &= 119;51^p \end{aligned}\right\}\text{ where hypotenuse } AK = 120^p.$$

Therefore, where line $AK = 57;35^p$,

H555 $BK = 2;53^p$,

$AB = 57;31^p$,

and $BL = K\Theta = 4;36^p$ [p. 613].

And since $AB^2 + BL^2 = AL^2$,

$AL = 57;42^p$ in the same units.

Similarly, since $L\Theta = BK = 2;53^p$ in the same units,

and $AL^2 + L\Theta^2 = A\Theta^2$,

$A\Theta = 57;46^p$.

Therefore, where hypotenuse $A\Theta = 120^p$, $\Theta L = 5;59^p$,

and the angle of the deviation in latitude,

$$\angle\, \Theta AL = \begin{cases} 5;44^{\circ\circ} & \text{where 2 right angles} = 360^{\circ\circ} \\ 2;52^{\circ} & \text{where 4 right angles} = 360^{\circ}. \end{cases}$$

That [2;52°] is what we shall enter in the third column of the table for Saturn opposite '135°'.

But at the greatest inclination on the semi-circle containing the perigee, since AG, representing the distance [when the epicycle is] near the beginning of Aries, is computed as $57;40^p$,[37]

where, as we demonstrated [p. 613], $KM = 0;22^p$ and $GM = 4;35^p$,

hence, by subtraction, $AM = 53;5^p$.

And hypotenuse $AK = 53;5^p$ in the same units, since it is negligibly greater than line AM.

Therefore, where hypotenuse $AK = 120^p$,

$KM = 0;50^p$,

H556 and $\angle\, KAM = 0;48^{\circ\circ}$ where 2 right angles $= 360^{\circ\circ}$.

But, by hypothesis, $\angle\, BAG = 5^{\circ\circ}$ in the same units.

So, by addition, $\angle\, BAK = 5;48^{\circ\circ}$ where 2 right angles $= 360^{\circ\circ}$.

Therefore, in the circle about right-angled triangle BAK,

arc $BK = 5;48^{\circ}$

and arc $AB = 174;12^{\circ}$ (supplement).

So the corresponding chords

$$\left.\begin{aligned} BK &= 6;4^p \\ \text{and } AB &= 119;51^p \end{aligned}\right\}\text{ where hypotenuse } AK = 120^p.$$

Therefore, where line $AK = 53;5^p$,

$BK = 2;41^p$

and $AB = 53;1^p$.

And since $AB^2 + BL^2 = AL^2$,

and BL was shown to be $4;36^p$ in the same units,

$AL = 53;13^p$ in the same units.

[37] Accurately, $57;44,48^p$ when the centre of the epicycle is at a true longitude of ♈ 0°. Precisely opposite a distance of $\rho = 62;10^p$ is the distance $(63;25 \times 56;35/62;10 =)\ 57;43^p$. It is obvious that Ptolemy has rounded to the nearest convenient number, whatever method of computation he used.

Therefore, where hypotenuse AL = 120^p, BL = $10;23^p$,
and the angle of the equation in longitude,

$$\angle \text{BAL} = \begin{cases} 9;56^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \\ 4;58^\circ \text{ where 4 right angles} = 360^\circ. \end{cases}$$

Again, where line AL = $53;13^p$,
ΘL = KB = $2;41^p$,
and $AL^2 + \Theta L^2 = A\Theta^2$,
so AΘ = $53;17^p$.

Therefore where hypotenuse AΘ = 120^p, ΘL = $6;3^p$, H557
and the angle of the deviation in latitude,

$$\angle \Theta\text{AL} = \begin{cases} 5;46^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \\ 2;53^\circ \text{ where 4 right angles} = 360^\circ. \end{cases}$$

That [2;53°] is what we shall enter in the fourth column of the table opposite '135°'.

Then in order to compare the equations in longitude for the inclination nearer the perigee, let the diagram with no inclination be drawn again [Fig. 13.7]. Then, where the distance at that point,

AG = $57;40^p$,
GK (= KΘ) is given as $4;36^p$;
and, by subtraction, AK = $53;4^p$ in the same units;
but $AK^2 + K\Theta^2 = A\Theta^2$,
so AΘ = $53;16^p$.

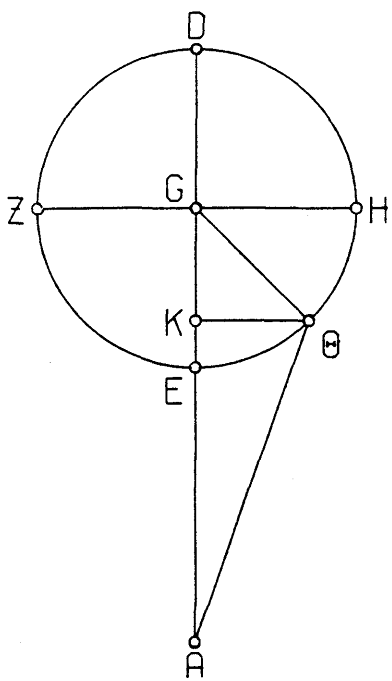

Fig. 13.7

Therefore, where hypotenuse $A\Theta = 120^p$, $K\Theta = 10;22^p$,
and the angle of the equation in longitude,

H558 $$\angle \Theta AK = \begin{cases} 9;54^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \\ 4;57^{\circ} \text{ where 4 right angles} = 360^{\circ}. \end{cases}$$

But when the inclinations [of eccentre and epicycle] were taken into account it was shown to be 4;58°. So the equation in longitude computed according to both inclinations was 1′ greater.

Q.E.D.

Let there again be drawn [Fig. 13.8], first, the diagram for the inclinations, representing the ratios established for Jupiter.

Hence, where the radius of the epicycle, $G\Theta = 11;30^p$,
GK (= KΘ) is computed as [84;52 × 11;30/120 =] $8;8^p$.

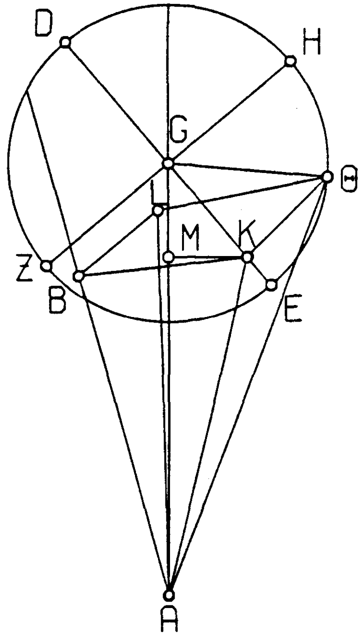

Fig. 13.8

Then, since the angle of the inclination of the epicycle,

$$\angle AGE = \begin{cases} 2;30^{\circ} \text{ where 4 right angles} = 360^{\circ} \\ 5^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ}, \end{cases}$$

in the circle about right-angled triangle GKM,

arc KM = 5°

H559 and arc GM = 175° (supplement).

So the corresponding chords

$$\left.\begin{aligned} KM &= 5;14^p \\ \text{and } GM &= 119;53^p \end{aligned}\right\} \text{ where hypotenuse } GK = 120^p.$$

Therefore, where line GK = $8;8^p$,
and AG, the distance near the beginning of Libra, is $62;30^p$,[38]

KM = $0;21^p$,
GM = $8;8^p$,
and, by subtraction, MA = $54;22^p$.

Hence hypotenuse AK, being negligibly greater than MA, is $54;22^p$ in the same units.

Therefore, where hypotenuse AK = 120^p, KM = $0;46^p$,
and $\angle$ KAM = $0;44^{\circ\circ}$ where 2 right angles $\doteq 360^{\circ\circ}$.

But, by hypothesis, the angle of the inclination of the eccentre,

$$\angle \text{BAG} = \begin{cases} 1;30^\circ \text{ where 4 right angles} = 360^{\circ\circ}. \\ 3^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ}. \end{cases}$$

Therefore, by addition, $\angle$BAK = $3;44^{\circ\circ}$ where 2 right angles = $360^{\circ\circ}$.

Therefore, in the circle about right-angled triangle BAK,
arc KB = $3;44^\circ$
and arc AB = $176;16^\circ$ (supplement).

So the corresponding chords

$$\left.\begin{array}{r} \text{KB} = 3;54^p \\ \text{and AB} = 119;56^p \end{array}\right\} \text{where hypotenuse AK} = 120^p.$$

Therefore, where line AK = $54;22^p$,
KB = $1;46^p$
and AB = $54;20^p$.

And, from what was demonstrated previously, BL = $8;8^p$ in the same units. H560

And since $AB^2 + BL^2 = AL^2$,
AL = $54;56^p$ in the same units.

Similarly, since LΘ [= KB] = $1;46^p$ in the same units,
and $AL^2 + L\Theta^2 = A\Theta^2$,
AΘ = $54;58^p$ in the same units.

Hence, where hypotenuse AΘ = 120^p, LΘ = $3;52^p$,
and the angle of the deviation in latitude,

$$\angle \Theta\text{AL} = \begin{cases} 3;42^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \\ 1;51^\circ \text{ where 4 right angles} = 360^\circ. \end{cases}$$

That [1;51°] is what we shall enter in the third column of the table for Jupiter opposite '135°'.

In the same way, AG, when it represents the distance at the beginning of Aries, is computed as $57;30^p$,[39] where, as we demonstrated, KM = $0;21^p$ and GM = $8;8^p$;

hence, by subtraction, AM(= AK which is negligibly greater) is $49;22^p$ in the same units.

Therefore, where hypotenuse AK = 120^p, KM = $0;51^p$,
and $\angle$ KAM = $0;49^{\circ\circ}$ where 2 right angles = $360^{\circ\circ}$.

[38] Accurately, $62;34,36^p$ when the centre of the epicycle is at a true longitude of ♎ 0° (the apogee being in ♍ 10°, cf. XIII 6 p. 635).

[39] Accurately $57;24,31^p$. The values of Ptolemy for both distances (cf. n.38) would fit better an elongation from the apogee of −24½° and (180° − 24½°), rather than the −20° which he specifies in XIII 6. But if one were to take the precise position of the apogee in his time, ♍ 11°, this would give −19° with even worse agreement with the text.

Therefore, by addition, $\angle$ BAK [= $\angle$ KAM + $3^{\circ\circ}$] = $3;49^{\circ\circ}$ in the same units.

Therefore, in the circle about right-angled triangle AKB,

arc KB = 3;49°

and arc AB = 176;11° (supplement).

H561 So the corresponding chords

$$\left.\begin{array}{r} BK = 3;59^p \\ \text{and } AB = 119;56^p \end{array}\right\} \text{where hypotenuse } AK = 120^p.$$

Therefore, where line AK = $49;22^p$,

KB = $1;39^p$

and AB = $49;20^p$.

Hence, since BL = $8;8^p$ in the same units,

and $AB^2 + BL^2 = AL^2$,

AL = $50;0^p$ in the same units.

Therefore, where hypotenuse AL = 120^p, BL = $19;31^p$,

and the angle of the equation in longitude,

$$\angle BAL = \begin{cases} 18;44^{\circ\circ} & \text{where 2 right angles} = 360^{\circ\circ} \\ 9;22^{\circ} & \text{where 4 right angles} = 360^{\circ}. \end{cases}$$

Again, where line AL = $50;0^p$,

ΘL [= KB] = $1;39^p$,

and $AL^2 + \Theta L^2 = A\Theta^2$,

so AΘ = $50;2^p$.

Therefore, where hypotenuse AΘ = 120^p, LΘ = $3;57^p$,

and the angle of the deviation in latitude,

$$\angle \Theta AL = \begin{cases} 3;46^{\circ\circ} & \text{where 2 right angles} = 360^{\circ\circ} \\ 1;53^{\circ} & \text{where 4 right angles} = 360^{\circ}. \end{cases}$$

That [1;53°] is what we shall enter in the fourth column of the table opposite the same '135°'.

In order to compare the equations in longitude, let the diagram with no inclinations be drawn again [Fig. 13.9]. Then at the distance in question,

where ΘK = GK = $8;8^p$,

H562 the whole line AG = $57;30^p$,

and, by subtraction, AK = $49;22^p$ in the same units.

But $AK^2 + K\Theta^2 = A\Theta^2$,

so AΘ = $50;2^p$ in the same units.

Therefore, where hypotenuse AΘ = 120^p, ΘK = $19;30^p$,

and the angle of the equation in longitude,

$$\angle \Theta AK = \begin{cases} 18;42^{\circ\circ} & \text{where 2 right angles} = 360^{\circ\circ} \\ 9;21^{\circ} & \text{where 4 right angles} = 360^{\circ}. \end{cases}$$

And when the inclinations were taken into account it was shown to be 9;22°. So the equation in longitude computed according to both inclinations was, again, greater by only a single minute.)

Q.E.D.

Next, to determine the quantities for Mars, let there be drawn, first, the diagram for the inclinations [Fig. 13.10], and let GK (= KΘ) be computed as [84;52 × 39;30/120 =] $27;56^p$, where the radius of the epicycle, GΘ = $39;30^p$.

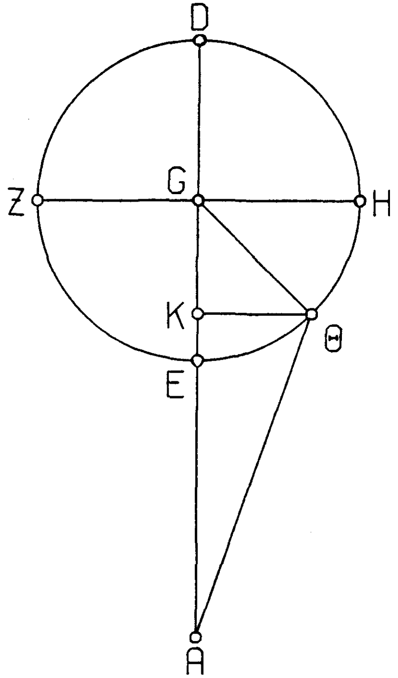

Fig. 13.9

Then, since the angle of the inclination of the epicycle,

$$\angle \text{AGE} = \begin{cases} 2;15^\circ \text{ where 4 right angles} = 360^\circ \\ 4;30^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ}, \end{cases}$$ H563

in the circle about right-angled triangle GMK,

arc KM = 4;30°

and arc GM = 175;30° (supplement).

So the corresponding chords

$$\left.\begin{aligned} \text{KM} &= 4;43^p \\ \text{and GM} &= 119;54^p \end{aligned}\right\} \text{where hypotenuse GK} = 120^p.$$

Therefore, where line GK = 27;56ᵖ,

and AG, the greatest distance, is 66ᵖ,[40]

KM = 1;6ᵖ

and GM = 27;54ᵖ,

and, by subtraction, AM = 38;6ᵖ.

Hence hypotenuse AK [$= \sqrt{\text{AM}^2 + \text{KM}^2}$] = 38;7ᵖ in the same units.

Therefore, where hypotenuse AK = 120ᵖ,

KM = 3;28ᵖ,

and ∠ KAM = 3;19°° where 2 right angles = 360°°.

But, by hypothesis, the angle of the eccentre's inclination, H564

[40] I.e. the northpoint is taken as coinciding with the apogee, both being placed in the (rounded) ♌ 0°.

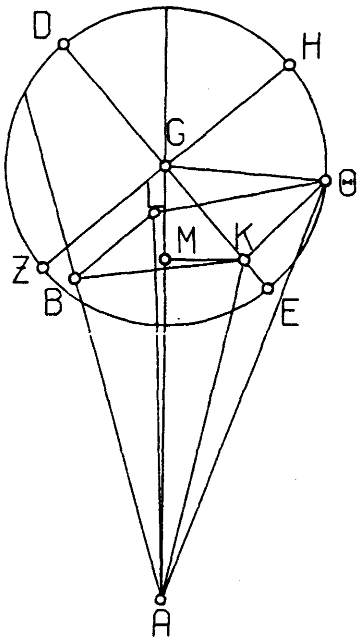

Fig. 13.10

$$\angle \text{ BAG} = \begin{cases} 1^\circ \text{ where 4 right angles} = 360^\circ \\ 2^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ}. \end{cases}$$

Therefore, by addition, ∠ BAK = 5;19°° where 2 right angles = 360°°.
So, in the circle about right-angled triangle BAK,

arc KB = 5;19°
and arc AB = 174;41° (complement).

So the corresponding chords

$$\left.\begin{array}{r} \text{BK} = 5;34^p \\ \text{and AB} = 119;52^p \end{array}\right\} \text{ where hypotenuse AK} = 120^p.$$

Therefore, where line AK = 38;7ᵖ,
KB = 1;46ᵖ
and AB = 38;5ᵖ.

But line BL [= KΘ = GK] = 27;56ᵖ in the same units.
And, since $AB^2 + BL^2 = AL^2$,
AL = 47;14ᵖ.
Similarly, since ΘL = 1;46ᵖ in the same units,
and $AL^2 + L\Theta^2 = A\Theta^2$,
AΘ = 47;16ᵖ in the same units.

Therefore, where hypotenuse AΘ = 120ᵖ, ΘL = 4;29ᵖ,
and the angle of the deviation in latitude,

$$\angle \Theta\text{AL} = \begin{cases} 4;18^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \\ 2;9^\circ \text{ where 4 right angles} = 360^\circ. \end{cases}$$

That [2;9°] is what we shall enter in the third column of the table for Mars opposite '135°'.

In the same way, for the inclinations at least distance:

$AG = 54^p$ where, as was shown,

$KM = 1;6^p$ H565

and $GM = 27;54^p$.

Thus, by subtraction, $AM = 26;6^p$,

and hypotenuse AK $[= \sqrt{KM^2 + AM^2}] = 26;7^p$ in the same units.

Therefore, where hypotenuse $AK = 120^p$, $KM = 5;3^p$,

and $\angle KAM = 4;49^{\circ\circ}$ where 2 right angles $= 360^{\circ\circ}$.

Hence, by addition, $\angle BAK = 6;49^{\circ\circ}$ in the same units.

Therefore, in the circle about right-angled triangle ABK,

arc $BK = 6;49°$

and arc $AB = 173;11°$ (supplement).

So the corresponding chords

$$\left.\begin{aligned} BK &= 7;8^p \\ \text{and } AB &= 119;47^p \end{aligned}\right\} \text{where hypotenuse } AK = 120^p.$$

Therefore, where line $AK = 26;7^p$,

$BK = 1;33^p$

and $AB = 26;4^p$.

And line BL is, again, $27;56^p$ in the same units.

And, since $AB^2 + BL^2 = AL^2$,

$AL = 38;12^p$.

Therefore, where hypotenuse $AL = 120^p$, $BL = 87;45^p$,

and the angle of the equation in longitude,

$$\angle BAL = \begin{cases} 94^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \\ 47° \text{ where 4 right angles} = 360°. \end{cases}$$

Similarly, where line $AL = 38;12^p$, $L\Theta$ $[= BK] = 1;33^p$,

and $AL^2 + L\Theta^2 = A\Theta^2$,

so $A\Theta = 38;14^p$.

Therefore, where hypotenuse $A\Theta = 120^p$, $L\Theta = 4;52^p$, H566

and the angle of the deviation in latitude,

$$\angle \Theta AL = \begin{cases} 4;40^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \\ 2;20° \text{ where 4 right angles} = 360°. \end{cases}$$

That [2;20°] is what we shall enter in the fourth column of the table opposite the same '135°'.

Again, if, in order to compare the equations in longitude, we set out the diagram without the inclinations [Fig. 13.11], at the least distance (where the difference must necessarily become most noticeable),

$AG:GK$ $(= K\Theta) = 54 : 27;56$.

hence, by subtraction, $AK = 26;4^p$,

and hypotenuse $A\Theta$ $[=\sqrt{AK^2 + K\Theta^2}] = 38;12^p$ in the same units.

Hence, where hypotenuse $A\Theta = 120^p$,

$\Theta K = 87;45^p$ again [as BL in the previous computation],

and the angle of the equation in longitude,

$$\angle \Theta AK = \begin{cases} 94^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \\ 47° \text{ where 4 right angles} = 360°. \end{cases}$$

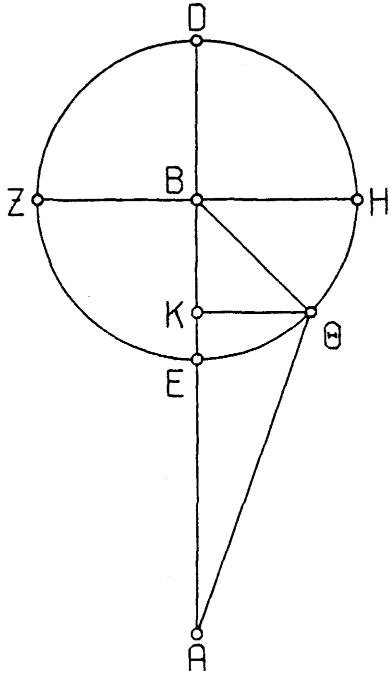

Fig. 13.11

H567 But that is the same size as was demonstrated by means of the calculations including the inclinations. Therefore the equation in longitude for Mars computed according to the inclinations of the circles [of epicycle and eccentre] did not differ at all.

Q.E.D.

The fourth column in the two tables for Venus and Mercury will contain the positions in latitude produced by the greatest slants of their epicycles, which occur at the apogee and perigee of the eccentre. However, we have computed these separately, without the effect due to the inclination of the eccentre, since that would have required a greater number of tables and a more complicated method of calculation [from the tables]: for the [corresponding latitudinal] positions as morning-star and evening-star are not going to be equal to each other, and not even always on the same side [i.e. north or south] of the ecliptic; and in any case, since the inclination of the eccentre is not constant, the differences in the amount to be diminished with respect to the greatest inclination [of the epicycle] would not correspond to the differences in the amount to be diminished with respect to the greatest slant.[41] However, if we
H568 separate the effects, we can determine each element in a more convenient way, as will become clear from the actual procedure which we shall adduce.

[41] Ptolemy means that one could not use a single coefficient column (c_5 in *HAMA*) to compute the diminution with respect to maximum of both inclination and slant as a function of the planet's position on the epicycle.

Let AB [see Fig. 13.12] be the intersection of the planes of the ecliptic and the epicycle. Let point A be taken as the centre of the ecliptic, and B as the centre of the epicycle, and let the epicycle GDEZH be described about it slanted to the plane of the ecliptic,[42] i.e. so that straight lines drawn in the [two planes] perpendicular to the common section GH all form equal angles at the points on GH. Draw AE tangent to the epicycle, and AZD intersecting the epicycle at an arbitrary point, and drop from points D, E and Z perpendiculars DΘ, EK and ZL to GH, and perpendiculars DM, EN and ZX to the plane of the ecliptic. Join H569
ΘM, KN, LX, and also AN and AXM (for AXM will be a straight line, since the three points [A, X, M all] lie in two planes, the plane of the ecliptic and the plane through AZD perpendicular to the ecliptic.

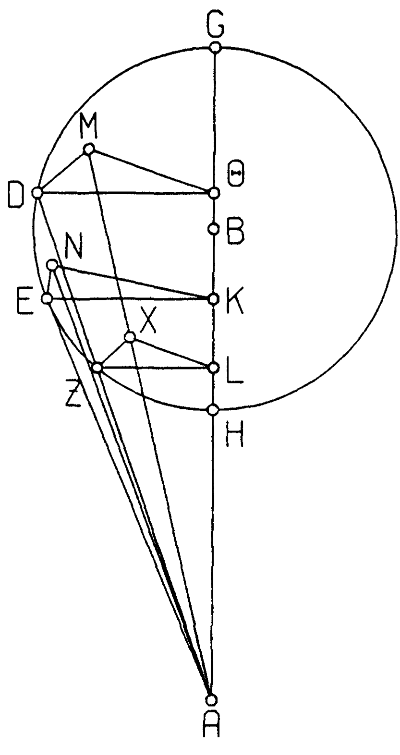

Fig. 13.12

It is obvious that, with the slant as depicted, the equations in longitude of the planet [at D and E respectively] will be represented by angles ΘAM and KAN, and the [positions] in latitude by angles DAM and EAN. We must demonstrate, first, that the position in latitude at the tangent point, ∠ EAN, is the maximum, just as the equation in longitude [is maximum at that point].

[42] See Fig. U for a redrawing of this three-dimensional figure. Note that Ptolemy's figure is an artificial one, since when the intersection of the planes of ecliptic and epicycle passes through the centre of the epicycle, the 'slant' is zero. But it is justified by the 'separation of the effects'.

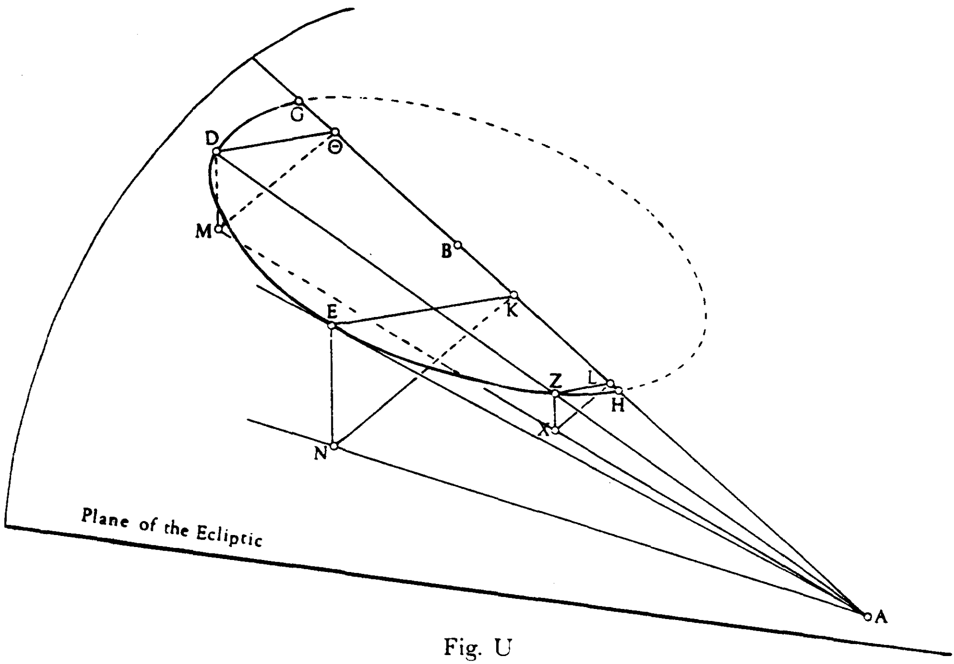

Fig. U

[Proof:] Since ∠ EAK is the maximum,

KE:EA > ΘD:DA = LZ:ZA.

But EK:EN = ΘD:DM = LZ:ZX.

for, as we said, the triangles formed by them [EKN, DΘM and ZLX] have equal angles [at GH] and right angles at M, N and X.

∴ NE:EA > MD:DA = XZ:ZA.

H570 And, again, the angles DMA, ENA and ZXA are right.

Therefore ∠ EAN > ∠ DAM, and hence, obviously,

∠ EAN is greater than any angle so formed.

It is immediately obvious that, when one considers the effect on the equations in longitude caused by the slant, the maximum difference is produced at the greatest deviations in latitude at E. For the differences [in the equation caused by the slant] are represented by the angles subtended by (ΘD − ΘM), (KE − KN) and (LZ − LX) [when the planet is at D, E and Z respectively], and since the ratios of these lines [ΘD:ΘM etc.] to each other and to the difference between them [(ΘD − ΘM) etc.] remains the same, it follows that

(EK − KN) : EA > (ΘD − ΘM) : AD, etc.[43]

And it is also immediately clear that, whatever the ratio between the greatest equation in longitude and the greatest deviation in latitude [due to the slant], that ratio holds between the equation in longitude for any position [of the planet] on the epicycle and the [corresponding] position in latitude.

[43] Ptolemy's argument here is fallacious, as pointed out by Pedersen 382. He seems to have been misled by his figure, which substitutes straight lines for arcs.

For KE:EN = LZ:ZX = ΘD:DM,

and so on for the other points [on the epicycle].[44]

Q.E.D.

Having established these preliminary points, let us first examine the size of the angle which is contained by the slant of the planes for each of the two planets. We take for granted what was noted at the beginning [of the discussion, p. 601], that both planets, when halfway between greatest and least distances, display a maximum difference [in latitude] between opposite positions on the epicycle of 5° to north or south: for Venus appears to [so] vary by slightly more than 5° at perigee and slightly less than 5° at apogee, while Mercury varies by about $\frac{1}{2}$° [more and less than 5° at 180° from apogee and apogee respectively]. H571

So let [Fig. 13.13] ABG again be the intersection of ecliptic and epicycle. Describe the epicycle GDE about centre B, slanting to the plane of the ecliptic[45] in the way described. From A, the centre of the ecliptic, draw AD tangent to the epicycle, and from D drop perpendicular DZ on to GBE, and perpendicular DH on to the plane of the ecliptic. Join BD, ZH and AH, and let ∠ DAH be taken as comprising half the above deviation in latitude for each of the two

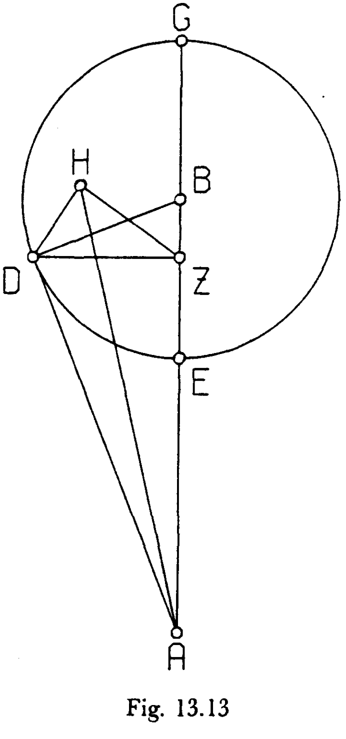

Fig. 13.13

[44] This too is fallacious, since Ptolemy has substituted chords for arcs (in modern terminology, has treated a relationship between the sines of angles as a relationship between the angles). See Pedersen 380–1. However, if one treats it as an approximation, it is a very reasonable one: see my remark on Pedersen, Toomer [3] 145.

[45] Cf. p. 623 n.42.

H572 planets (thus it is $2\frac{1}{2}°$). Let our problem be, to find for each the amount of the slant between the planes, namely the size of $\angle$ DZH.

For Venus, since, where the radius of the epicycle is $43;10^p$, the greatest distance is $61;15^p$, the least $58;45^p$, and the mean between them 60^p,

$$AB:BD = 60 : 43;10.$$

And since $AB^2 - BD^2 = AD^2$,

$$AD = 41;40^p \text{ in the same units.}$$

Similarly, since $BA:AD = BD:DZ$,

$$DZ = 29;58^p \text{ in the same units.}$$

Furthermore, since, by hypothesis,

$$\angle DAH = \begin{cases} 2;30° \text{ where 4 right angles} = 360° \\ 5^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ}, \end{cases}$$

in the circle about right-angled triangle ADH,

H573 arc DH = 5°

and the corresponding chord $DH = 5;14^p$ where hypotenuse $AD = 120^p$.

Therefore, where line $AD = 41;40^p$, $DH = 1;50^p$.

And DZ was shown to be $29;58^p$ in the same units.

Therefore, where hypotenuse $DZ = 120^p$, $DH = 7;20^p$.

and the angle of the slant,

$$\angle DZH = \begin{cases} 7^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \\ 3;30° \text{ where 4 right angles} = 360°.\text{[46]} \end{cases}$$

But since the amount by which $\angle$ DAZ exceeds $\angle$ HAZ represents the resulting difference in the equation in longitude, we must immediately compute this too, by finding the amounts of these angles. For we showed that, where line $DH = 1;50^p$, hypotenuse $AD = 41;40^p$ and $DZ = 29;58^p$:

$$\text{and } AD^2 - DH^2 = AH^2$$
$$\text{while } ZD^2 - DH^2 = HZ^2;$$
$$\text{so } AH = 41;37^p$$
$$\text{and } HZ = 29;55^p \text{ in the same units.}$$

Therefore, where hypotenuse $AH = 120^p$, $ZH = 86;16^p$,

$$\text{and } \angle ZAH = \begin{cases} 91;56^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \\ 45;58° \text{ where 4 right angles} = 360°. \end{cases}$$

H574 Similarly, since $DZ = 86;18^p$ where hypotenuse $AD = 120^p$,

$$\angle DAZ = \begin{cases} 91;58^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \\ 45;59° \text{ where 4 right angles} = 360°. \end{cases}$$

Thus the equation in longitude computed according to the slant was less by one minute.

For Mercury [see Fig. 13.14], where the radius of the epicycle is $22;30^p$, the greatest distance, as we demonstrated, is 69^p, and the distance diametrically opposite to that 57^p; the mean between these two is calculated as 63^p in the same units.

$$\text{So } AB:BD = 63 : 22;30.$$

And since $AB^2 - DB^2 = AD^2$,

$$AD = 58;51^p.$$

[46] This neat result is achieved only by some devious rounding: computing accurately one finds $3;28\frac{1}{2}°$.

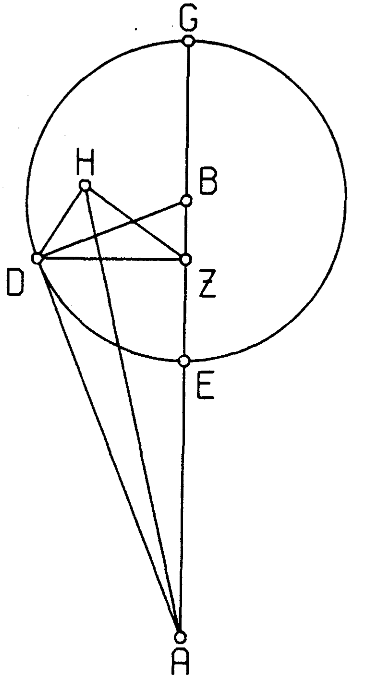

Fig. 13.14

Similarly, since AB:AD = BD:DZ,

DZ = $21;1^p$ in the same units.

Again, since, by hypothesis,

$\angle$ DAH = $5^{\circ\circ}$ where 2 right angles = $360^{\circ\circ}$, H575

in the circle about right-angled triangle ADH,

arc DH = 5°,

and the corresponding chord DH = $5;14^p$ where hypotenuse AD = 120^p.

Therefore, where line AD = $58;51^p$, DH = $2;34^p$.

But we showed that DZ = $21;1^p$ in the same units.

Therefore, where hypotenuse DZ = 120^p, DH = $14;40^p$,

and the angle of the slant,

$$\angle \text{DZH} = \begin{cases} 14^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \\ 7^{\circ} \text{ where 4 right angles} = 360^{\circ} \end{cases}$$ [47].

In the same way [as for Venus], in order to compare the angles of the equation [in longitude]:

again, where DH = $2;34^p$, we showed that

hypotenuse AD = $58;51^p$ and DZ = $21;1^p$.

And $DA^2 - DH^2 = AH^2$,

$DZ^2 - DH^2 = HZ^2$,

so AH = $58;47^p$

and ZH = $20;53^p$ in the same units.

[47] Accurately, $7;1^{\circ}$.

Therefore, where hypotenuse $AH = 120^p$, $HZ = 42;38^p$,

$$\text{and } \angle ZAH = \begin{cases} 41;38^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \\ 20;49^{\circ} \text{ where 4 right angles} = 360^{\circ}. \end{cases}$$

In the same way, where hypotenuse $AD = 120^p$, DZ is calculated as $42;50^p$,

H576
$$\text{and } \angle DAZ = \begin{cases} 41;50^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \\ 20;55^{\circ} \text{ where 4 right angles} = 360^{\circ}. \end{cases}$$

So in this case the equation in longitude due to the slant was less by 6′.[48]

Q.E.D.

Next let us examine whether, if we take the above amounts of the slant as given, we find the greatest latitudes at the greatest and least distances [derived from them] to agree with those derived from our observations. In the same figure [Fig. 13.15], let us now take as basis the greatest distance of Venus, i.e.

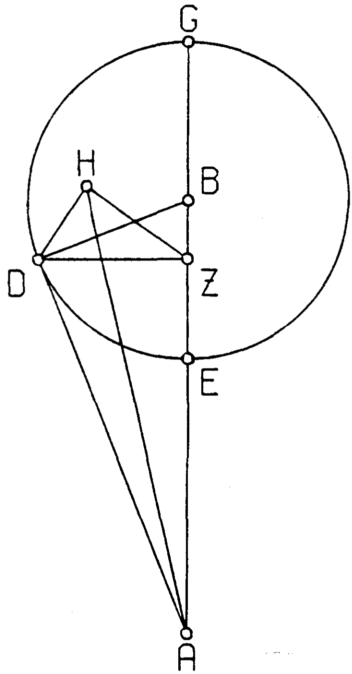

Fig. 13.15

$$AB:BD = 61;15 : 43;10.$$

Hence, since $AB^2 - BD^2 = AD^2$,

$$AD = 43;27^p.$$

But $AB:AD = BD:DZ$.

So $DZ = 30;37^p$ in the same units.

Again, since, by hypothesis, the angle of the slant,

H577
$$\angle DZH = 7^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ}$$

[48] Ptolemy has fudged the calculations a little to get this result. Accurate computation gives $\angle ZAH = 41;33,58^{\circ\circ}$, $\angle DAZ = 41;50,50^{\circ\circ}$, with a difference of $0;16,52^{\circ\circ}$, or about $8\frac{1}{2}'$.

and [hence] DH = $7;20^p$ where hypotenuse DZ = 120^p,

therefore, where line DZ = $30;37^p$, and AD = $43;27^p$,

DH = $1;52^p$.

So where hypotenuse AD = 120^p,

DH = $5;9^p$.

and the greatest deviation in latitude,

$$\angle \mathrm{DAH} = \begin{cases} 4;54^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \\ 2;27^{\circ} \text{ where 4 right angles} = 360^{\circ}. \end{cases}$$

But at the least distance, where the radius of the epicycle,

BD = $43;10^p$,

AB is given as $58;45^p$.

And $AB^2 - DB^2 = AD^2$.

so AD = $39;51^p$ in the same units.

Similarly, since AB:AD = BD:DZ,

DZ = $29;17^p$ in the same units.

But DZ:DH is given as 120 : 7;20.

Therefore, where DZ = $29;17^p$ and AD = $39;51^p$,

DH = $1;47^p$.

Therefore, where hypotenuse AD = 120^p, DH = $5;22^p$,

and the greatest deviation in latitude,

$$\angle \mathrm{DAH} = \begin{cases} 5;8^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \\ 2;34^{\circ} \text{ where 4 right angles} = 360^{\circ}. \end{cases}$$

Thus [the greatest latitude] differs from the $2\frac{1}{2}°$ of [greatest] deviation in latitude assumed for the mean, being less at the apogee and greater at the perigee, but [in both cases] by an amount which is negligible to the senses; for at the greatest distance it was only three minutes less, and at the least distance four minutes more. Such [small differences] could not be at all easily detected from the observations. H578

Next [see Fig. 13.16] let us take the greatest distance of Mercury as basis, namely

AB:BD = 69 : 22;30.

Hence, by the same procedure as above,

AD [= $\sqrt{AB^2 - BD^2}$] = $65;14^p$,

and DZ [= AD × BD/AB] = $21;16^p$ in the same units.

But in this case the angle of slant,

∠ DZH is given as $14^{\circ\circ}$ where 2 right angles = $360^{\circ\circ}$.

Hence we have DH = $14;40^p$[49] where hypotenuse DZ = 120^p.

Therefore, where line DZ = $21;16^p$, and AD = $65;14^p$,

DH = $2;36^p$.

Therefore, where hypotenuse AD = 120^p, DH = $4;47^p$,

and the greatest deviation in latitude,

$$\angle \mathrm{DAH} = \begin{cases} 4;34^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \\ 2;17^{\circ} \text{ where 4 right angles} = 360^{\circ}. \end{cases}$$ H579

[49] The chord of 14° (I 11) is $14;37,27^p$. But Ptolemy's $14;40^p$ is justified by p. 627, where the 7° of the slant is derived from that value.

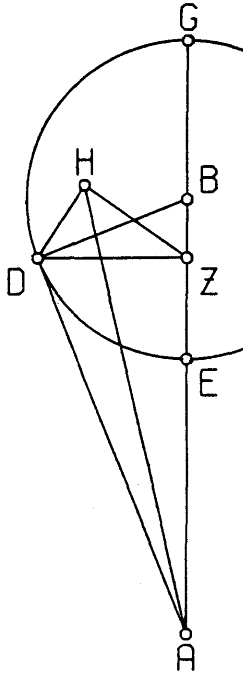

Fig. 13.16

But at the least distance,[50]
AB:BD is given as 57 : 22;30,
and so, by the same procedure again,

$$AD = 52;22^p \text{ in the same units}$$
$$\text{and } DZ = 20;40^p.$$

And the slant is the same as before,
and hence ZD:DH is given as 120 : 14;40,

$$\text{so where } DZ = 20;40^p \text{ and } AD = 52;22^p,$$
$$DH = 2;32^p.$$

Therefore, where hypotenuse AD = 120^p, DH = $5;48^p$,

$$\text{and } \angle\ DAH = \begin{cases} 5;32^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ} \\ 2;46^{\circ} \text{ where 4 right angles} = 360^{\circ}. \end{cases}$$

Thus the difference from the maximum deviation in latitude at the mean (which was taken as $2\frac{1}{2}^\circ$ here too) was 13′ in the negative direction at apogee and 16′ in the positive direction at perigee. To represent these, we shall use a
H580 correction of $\frac{1}{4}^\circ$ with respect to the mean in the calculations [from the table], in accordance with the perceptible difference derived from the observations.

Now that we have demonstrated the above, and also demonstrated that the ratio between the greatest equation in longitude and the greatest deviation in

[50] Ptolemy is speaking loosely here. 57^p represents, not the least distance, (which is *c.* $55;34^p$ at 120° from apogee, IX 9 p. 460), but the distance at the point opposite the greatest distance, i.e. strictly analogous to the situation for Venus. Cf. the use of 'perigee' below.

latitude also holds good at other points on the epicycle for the ratio between the individual equations in longitude and the [corresponding] individual positions in latitude,[51] we immediately have a convenient method for computing the positions in latitude due to the slant to be entered in the fourth column of the tables for Venus and Mercury. However, as we mentioned, these positions are based only on the slant of the epicycles at mean distance: the difference due to the inclination of the eccentres, and also the difference due to [the approach towards] apogee or perigee for Mercury, will be found by means of a correction procedure in the computation [from the tables], for convenience of calculation.

For, at the mean distances as set out above, the greatest deviation due to the slant was shown to be 2;30° on either side of the ecliptic for both planets; and the greatest equation in longitude is approximately 46° for Venus and 22° for Mercury;[52] and we already have, set out in the tables for anomaly of these planets, the equations corresponding to the individual positions on the epicycle. H581
So we form the ratios between the latter and the greatest equation, take the same proportion of $2\frac{1}{2}$°, separately for each planet, and enter the results in the fourth column of the tables of latitude opposite the corresponding arguments.

We have produced the fifth column [in each table] in order to correct the positions in latitude for other positions [of the epicycle] on the eccentre, by using the sixtieths entered [in that column]. For since, as we said, the increase and decrease in the inclination and slant of the epicycle, through the action of the attached small circles, have a period precisely corresponding to the period of return on the eccentre, and since the amounts of all the inclinations and slants is not very different from that associated with the moon's inclined orbit, and the individual deviations in latitude, for such small inclinations, are, again, almost proportional, and since we already have the corresponding entries for the moon computed geometrically, we multiplied each of the entries in that table by 12 (because the maximum there is about 5°, and here we are making the maximum 60), and entered the results opposite the appropriate argument in the fifth column of each table.

The layout of the tables is as follows.

5. {*Layout of the tables for the computations in latitude*}[53] H582-6

[See pp. 632-4.]

[51] See p. 625 n.44.

[52] These numbers are simply rounded from the maxima in col. 6 of the tables of anomaly (XI 11), 45;59° for Venus and 22;2° for Mercury. Heiberg mistakenly refers to XII 9, which gives nothing to compare, since it refers to true, not mean elongations.

[53] As Manitius (p. 428) notes, there are a number of entries in col. 5 (the 'sixtieths') which are derived, not from the corresponding values in col. 7 of the lunar table (V8), but from a value 1′ less. Most (those for 24°, 36°, 42°, 72°, 111°, 153°, 155°) are less accurate, but some (those for 12°, 78°, 99°) are more accurate. Since there is no doubt that Ptolemy did, as he says, obtain the values in col. 5 simply by multipying by 12, this may be a remnant of an earlier stage in the computation of the lunar table.

INCLINATIONS OF SATURN					INCLINATIONS OF JUPITER				
Argument [in Distance] from Apogee		Northern Limit	Southern Limit	Sixtieths	Argument [in Distance] from Apogee		Northern Limit	Southern Limit	Sixtieths
6	354	2 4	2 2	59 36	6	354	1 7	1 5	59 36
12	348	2 5	2 3	58 36	12	348	1 8	1 6	58 36
18	342	2 6	2 3	57 0	18	342	1 8	1 6	57 0
24	336	2 7	2 4	54 36	24	336	1 9	1 7	54 36
30	330	2 8	2 5	52 0	30	330	1 10	1 8	52 0
36	324	2 10	2 7	48 24	36	324	1 11	1 9	48 24
42	318	2 11	2 8	44 24	42	318	1 12	1 10	44 24
48	312	2 12	2 10	40 0	48	312	1 13	1 11	40 0
54	306	2 14	2 12	35 12	54	306	1 14	1 13	35 12
60	300	2 16	2 15	30 0	60	300	1 16	1 16	30 0
66	294	2 18	2 18	24 24	66	294	1 18	1 18	24 24
72	288	2 21	2 21	18 24	72	288	1 21	1 21	18 24
78	282	2 24	2 24	12 24	78	282	1 24	1 24	12 24
84	276	2 27	2 27	6 24	84	276	1 27	1 27	6 24
90	270	2 30	2 30	0 0	90	270	1 30	1 30	0 0
93	267	2 31	2 31	3 12	93	267	1 31	1 31	3 12
96	264	2 33	2 33	6 24	96	264	1 33	1 33	6 24
99	261	2 34	2 34	9 24	99	261	1 34	1 34	9 24
102	258	2 36	2 36	12 24	102	258	1 36	1 36	12 24
105	255	2 37	2 37	15 24	105	255	1 37	1 37	15 24
108	252	2 39	2 39	18 24	108	252	1 39	1 39	18 24
111	249	2 40	2 40	21 24	111	249	1 40	1 40	21 24
114	246	2 42	2 42	24 24	114	246	1 42	1 42	24 24
117	243	2 43	2 43	27 12	117	243	1 43	1 43	27 12
120	240	2 45	2 45	30 0	120	240	1 45	1 45	30 0
123	237	2 46	2 46	32 36	123	237	1 46	1 46	32 36
126	234	2 47	2 48	35 12	126	234	1 47	1 48	35 12
129	231	2 49	2 49	37 36	129	231	1 49	1 49	37 36
132	228	2 50	2 51	40 0	132	228	1 50	1 51	40 0
135	225	2 52	2 53	42 12	135	225	1 51	1 53	42 12
138	222	2 53	2 54	44 24	138	222	1 52	1 54	44 24
141	219	2 54	2 55	46 36	141	219	1 53	1 55	46 36
144	216	2 55	2 56	48 24	144	216	1 55	1 57	48 24
147	213	2 56	2 57	50 12	147	213	1 56	1 59	50 12
150	210	2 57	2 58	52 0	150	210	1 58	2 0	52 0
153	207	2 58	2 59	53 12	153	207	1 59	2 1	53 12
156	204	2 59	3 0	54 36	156	204	2 0	2 3	54 36
159	201	2 59	3 1	56 0	159	201	2 1	2 4	56 0
162	198	3 0	3 2	57 0	162	198	2 2	2 5	57 0
165	195	3 0	3 2	57 48	165	195	2 2	2 6	57 48
168	192	3 1	3 3	58 36	168	192	2 3	2 6	58 36
171	189	3 1	3 3	59 12	171	189	2 3	2 7	59 12
174	186	3 2	3 4	59 36	174	186	2 4	2 7	59 36
177	183	3 2	3 4	59 48	177	183	2 4	2 8	59 48
180	180	3 2	3 5	60 0	180	180	2 4	2 8	60 0

INCLINATIONS OF MARS					INCLINATIONS OF VENUS				
Argument [in Distance] from Apogee		Northern Limit	Southern Limit	Sixtieths	Argument [in Distance] from Apogee		Inclination	Slant	Sixtieths
6	354	0 8	0 4	59 36	6	354	1 2	0 8	59 36
12	348	0 9	0 4	58 36	12	348	1 1	0 16	58 36
18	342	0 11	0 5	57 0	18	342	1 0	0 25	57 0
24	336	0 13	0 6	54 36	24	336	0 59	0 33	54 36
30	330	0 14	0 7	52 0	30	330	0 57	0 41	52 0
36	324	0 15	0 9	48 24	36	324	0 55	0 49	48 24
42	318	0 18	0 12	44 24	42	318	0 51	0 57	44 24
48	312	0 21	0 15	40 0	48	312	0 46	1 5	40 0
54	306	0 24	0 18	35 12	54	306	0 41	1 13	35 12
60	300	0 28	0 22	30 0	60	300	0 35	1 20	30 0
66	294	0 32	0 26	24 24	66	294	0 29	1 28	24 24
72	288	0 36	0 30	18 24	72	288	0 23	1 35	18 24
78	282	0 41	0 36	12 24	78	282	0 16	1 42	12 24
84	276	0 46	0 42	6 24	84	276	0 8	1 50	6 24
90	270	0 52	0 49	0 0	90	270	0 0	1 57	0 0
93	267	0 55	0 52	3 12	93	267	0 5	2 0	3 12
96	264	0 59	0 56	6 24	96	264	0 10	2 3	6 24
99	261	1 3	1 0	9 24	99	261	0 15	2 6	9 24
102	258	1 6	1 4	12 24	102	258	0 20	2 9	12 24
105	255	1 10	1 8	15 24	105	255	0 26	2 12	15 24
108	252	1 14	1 13	18 24	108	252	0 32	2 15	18 24
111	249	1 18	1 18	21 24	111	249	0 38	2 17	21 24
114	246	1 23	1 24	24 24	114	246	0 44	2 20	24 24
117	243	1 28	1 30	27 12	117	243	0 50	2 22	27 12
120	240	1 34	1 37	30 0	120	240	0 59	2 24	30 0
123	237	1 41	1 44	32 36	123	237	1 8	2 26	32 36
126	234	1 48	1 51	35 12	126	234	1 18	2 27	35 12
129	231	1 54	2 0	37 36	129	231	1 28	2 29	37 36
132	228	2 1	2 10	40 0	132	228	1 38	2 30	40 0
135	225	2 9	2 20	42 12	135	225	1 48	2 30	42 12
138	222	2 16	2 32	44 24	138	222	1 59	2 30	44 24
141	219	2 25	2 44	46 36	141	219	2 11	2 29	46 36
144	216	2 34	2 56	48 24	144	216	2 23	2 28	48 24
147	213	2 44	3 12	50 12	147	213	2 43	2 26	50 12
150	210	2 54	3 29	52 0	150	210	3 3	2 22	52 0
153	207	3 5	3 46	53 12	153	207	3 23	2 18	53 12
156	204	3 16	4 9	54 36	156	204	3 44	2 12	54 36
159	201	3 27	4 32	56 0	159	201	4 5	2 4	56 0
162	198	3 38	4 55	57 0	162	198	4 26	1 55	57 0
165	195	3 49	5 24	57 48	165	195	4 49	1 42	57 48
168	192	4 0	5 53	58 36	168	192	5 13	1 27	58 36
171	189	4 10	6 21	59 12	171	189	5 36	1 9	59 12
174	186	4 14	6 36	59 36	174	186	5 52	0 48	59 36
177	183	4 18	6 51	59 48	177	183	6 7	0 25	59 48
180	180	4 21	7 7	60 0	180	180	6 22	0 0	60 0

INCLINATIONS OF MERCURY				
Argument [in Distance] from Apogee		Inclination	Slant	Sixtieths
6	354	1 45	0 11	59 36
12	348	1 44	0 22	58 36
18	342	1 43	0 33	57 0
24	336	1 40	0 44	54 36
30	330	1 36	0 55	52 0
36	324	1 30	1 6	48 24
42	318	1 23	1 16	44 24
48	312	1 16	1 26	40 0
54	306	1 8	1 35	35 12
60	300	0 59	1 44	30 0
66	294	0 49	1 52	24 24
72	288	0 38	2 0	18 24
78	282	0 26	2 7	12 24
84	276	0 16	2 14	6 24
90	270	0 0	2 20	0 0
93	267	0 8	2 23	3 12
96	264	0 15	2 25	6 24
99	261	0 23	2 27	9 24
102	258	0 31	2 28	12 24
105	255	0 40	2 29	15 24
108	252	0 48	2 29	18 24
111	249	0 57	2 30	21 24
114	246	1 6	2 30	24 24
117	243	1 16	2 30	27 12
120	240	1 25	2 29	30 0
123	237	1 35	2 28	32 36
126	234	1 45	2 26	35 12
129	231	1 55	2 23	37 36
132	228	2 6	2 20	40 0
135	225	2 16	2 16	42 12
138	222	2 27	2 11	44 24
141	219	2 37	2 6	46 36
144	216	2 47	2 0	48 24
147	213	2 57	1 53	50 12
150	210	3 7	1 46	52 0
153	207	3 17	1 38	53 12
156	204	3 26	1 29	54 36
159	201	3 34	1 20	56 0
162	198	3 42	1 10	57 0
165	195	3 48	0 59	57 48
168	192	3 54	0 48	58 36
171	189	3 58	0 36	59 12
174	186	4 2	0 24	59 36
177	183	4 4	0 12	59 48
180	180	4 5	0 0	60 0

6. {*Computation of the deviation in latitude for the 5 planets*}[54] H587

Those [tables] thus established, we carry out the latitude computation for the 5 planets as follows.

For the 3 planets Saturn, Jupiter and Mars, we take the corrected longitude (for Mars just as it is, for Jupiter subtracting 20° and for Saturn adding 50°),[55] and entering the argument [columns] of the appropriate table, find the sixtieths corresponding to it in the fifth column of the latitude, and write that down separately. Similarly, we enter the same argument [columns] with the corrected amount of the anomaly,[56] and take the difference in latitude corresponding to it, in the third column if the corrected longitude falls within the first 15 lines, but in the fourth column if it falls within the lines after [the 15th]. We multiply this by the sixtieths we wrote down, and the result will give us the amount by which the planet is north of the ecliptic, if we took the difference in latitude from the third column, or south of it, if we took it from the fourth.

For Venus and Mercury we first enter with the corrected amount of the anomaly into the argument [columns] of the appropriate table, take the H588
corresponding amounts in the third and fourth columns of the latitude, and write them down separately; we take them unchanged from all columns except the fourth column for Mercury, but for that, if the corrected longitude falls within the first 15 lines, we subtract a tenth part of the amount, whereas if the corrected longitude falls within the lines below [the 15th], we add a tenth part.[57] Then we add to the corrected longitude, for Venus always 90°, and for Mercury always 270°, subtract [the 360° of] a circle if it comes to that [i.e. to more than 360°], enter with the result into the same argument [columns], and take the corresponding number of sixtieths in the fifth column. We multiply the latter into the amount we wrote down from the third column, and set out the result. The direction of this will be:

[A] if the longitude (with the addition as detailed above) falls within the first 15 lines, and
  - [1] the amount of the corrected anomaly falls within the first 15 lines: southerly
  - [2] the anomaly falls within the lines following [the 15th]: northerly;

[B] if the above-mentioned longitude falls within the lines below the 15, and
  - [1] the amount of the above-mentioned anomaly falls within the first 15 lines: northerly
  - [2] the anomaly falls within the lines following [the 15th]: southerly.

Next we again take the corrected longitude, just as it is for Venus, but with the addition of 180° for Mercury, enter with it into the same [columns of] H589

[54] See *HAMA* 219-20, 222-6, and Appendix A, Example 15.

[55] The 'corrected longitude' means 'the distance of the epicycle centre from apogee, as seen from the observer (i.e. corrected by the equation of centre)'. The amounts to be applied to it represent the (rounded) distance between apogee and northpoint of the inclined orbit.

[56] I.e. the true anomaly α, corrected for equation of centre.

[57] The 'tenth part' represents the ratio $\frac{1}{4}°$: $2\frac{1}{2}°$. Cf. XIII 4 p. 630.

argument; take the sixtieths corresponding to this in the fifth column, multiply them into the amount we wrote down from the fourth column, and set out the result. The direction of this will be:

[A] if the longitude we entered with (as described above) falls within the first 15 lines, and
    [1] the corrected anomaly is 180° or less: northerly
    [2] the anomaly is greater than 180°: southerly;

[B] if the longitude falls within the lines below the 15, and
    [1] the anomaly is 180° or less: southerly
    [2] the anomaly is greater than 180°: northerly.

Then we take these same sixtieths which were found by the second entry with the longitude, calculate the amount which is the same fraction of them as they are of 60, and, for Venus, take $\frac{1}{6}$th of this and set it out too, always with a northerly direction; but for Mercury we take $\frac{3}{4}$ of the amount and set it out, always in a southerly direction.[58]

Thus, by combining the 3 quantities set out, we determine the apparent position in latitude with respect to the ecliptic of these [two planets].

H590 ## 7. {*On the first and last visibilities of the 5 planets*}[59]

Now that we have dealt with the basic problem of the deviations in latitude of the 5 planets, there remains the supplementary topic of the requisite theorems for their first and last visibilities with respect to the sun. For, as we explained in the treatise on the fixed stars [VIII 6, p. 413], it turns out that their distances from the sun along the ecliptic are variously unequal, for both first and last visibilities, for a number of reasons: the first of these is due to the fact that they are of unequal size, the second due to the variation of the inclination of the ecliptic to the horizon, and the third due to their positions in latitude.

For if we again imagine [see Fig. 13.17] segments of great circles, AB of the horizon, and GD of the ecliptic,[60] and take point E as their intersection at rising or setting, points G and A in the direction of south [i.e. the meridian],[61] and point D as the sun's centre, and we draw through D and the pole of the horizon
H591 another great circle segment DBZ, and suppose the planet to rise or set along the horizon AEB (when it is situated on the ecliptic, it will do so, obviously, at E; when it is north of the ecliptic, at H, and when it is south, at Θ), and drop perpendiculars HK and ΘL on to the ecliptic from points H and Θ, then we will again[62] have, in BD, an arc which is equal to the amount which the sun must always be below the earth in order for the same [given] planet to be first or last visible. For it is on a great circle so drawn [i.e. perpendicular to the horizon]

[58] For an explanation of this procedure see *HAMA* 224.

[59] See *HAMA* 234-8, Pedersen 386–8, with the correction Toomer [3], 145.

[60] Reading κύκλου (with D,Ar) for μεγίστου κύκλου ('the great circle of the ecliptic') at H490,18. Corrected by Manitius.

[61] Ger adds 'and points Θ and H in the direction of south and north', which makes good sense.

[62] 'again' refers back to the similar situation with the fixed stars, VIII 6 p. 413.

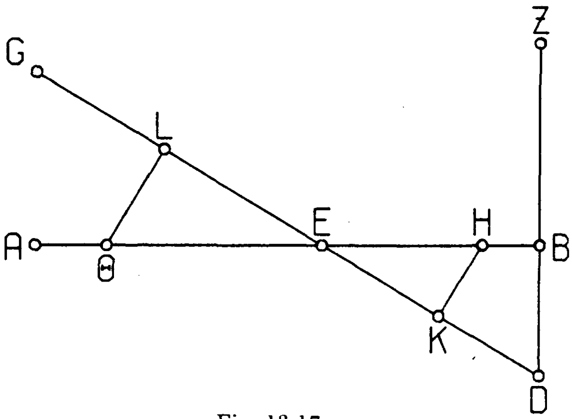

Fig. 13.17

that equal intervals below the earth must be taken in order for the identical obscuring effect of the sun's rays to take place.

First, then, this arc [BD] is, naturally, unequal for the various planets, which are unequal [in size], so, even if all other factors remain the same, the arc of the H592
ecliptic subtending the right angle, i.e. the interval corresponding to ED, must vary, being, obviously, smaller for the larger planets, and greater for the smaller planets.

Similarly, even if BD remains the same for the same [given] planet, but the angle of inclination of the ecliptic, BED, varies either because there is a different zodiacal sign [crossing the horizon] or [the latitude of] the location is different, the arc of the [sun's] distance, ED, will again vary, and will become greater as the angle in question decreases and lesser as it increases.

In the same way, even if we join to the above condition [of BD being constant] the further condition that the inclination remains the same, but the planet does not lie on the ecliptic, but is either north of it at H or south of it at Θ, its first and last visibility will no longer take place at a distance [from the sun] of arc DE, but when it is north of the ecliptic, at the lesser distance DK, and when it is south, at the greater distance DEL.

Therefore, for our investigations of the particular cases, it is essential that there first be given, for each of the 5 planets, the universally applicable size of the arc corresponding to BD, from the more reliable observations of the phases. H593
These would be those made in summer, round about Cancer, since at that season the atmosphere is thin and clear, and the inclination of the ecliptic to the horizon is symmetrical [at eastern and western horizons].[63] We find, then, by examining observations of [first] risings of this kind,[64] that near the beginning of Cancer, in general,

[63] This is Neugebauer's interpretation of 'symmetrical' (*HAMA* 235), and it is confirmed by p. 639, 'when the beginning of Cancer is setting, it forms the same angle and inclination to the horizon as before [at rising]'.

[64] For Saturn at least, these could hardly have been Ptolemy's own observations, as the requirement of a longitude near ♋ 0° takes us back to about the year 120, much earlier than any of Ptolemy's quoted observations. This is confirmed by the references to the Babylonians.

Saturn rises [i.e. is first visible] at a distance from the true sun of 14°
Jupiter at $12\frac{3}{4}$°
Mars at $14\frac{1}{2}$°
Venus as evening star at $5\frac{2}{3}$°, and
Mercury as evening star at $11\frac{1}{2}$°.

With these data given, let the diagram of the preceding figure be drawn [Fig. 13.18]. (For such small arcs it will make no difference if, for convenience' sake, we substitute in our calculations the corresponding chords which are not sensibly different from them). Let point E be the intersection of ecliptic and horizon at the above-mentioned phases, at the beginning of Cancer, and rising

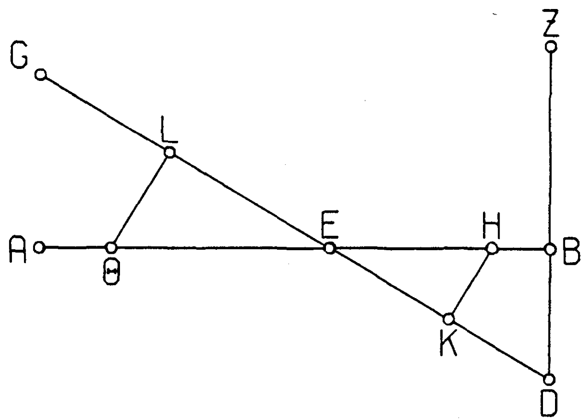

Fig. 13.18

for the 3 morning-star planets, Saturn, Jupiter and Mars, but, obviously, setting for the evening-stars, Venus and Mercury. Let us take as geographical
H594 latitude the parallel through Phoenicia, where the longest day is $14\frac{1}{4}$ hours, since it is mainly on this parallel or round about it that the majority and most reliable of the observations of the phases have been made, those of the Babylonians almost on it, and those in Greece and Egypt round about it.[65]

Now we find, by means of the procedure for angles [between ecliptic and horizon] previously demonstrated [II 11], that when the beginning of Cancer is rising at the latitude in question,

$$\angle\ BED = 103^{\circ\circ} \text{ where 2 right angles} = 360^{\circ\circ},$$[66]

and hence the ratio of the sides about the right angle,[67]

$$[BD:BE] \approx 94 : 75 \text{ where the hypotenuse } [DE] = 120^{p}.$$

[65] According to the *Geography* Babylon has a latitude of 35° (which corresponds closely to the standard Babylonian daylight ratio M:m = 3 : 2). In fact its latitude is about $32\frac{1}{2}$°. The parallel with $M = 14\frac{1}{4}^{h}$ (and $\varphi = 33;18°$) is halfway between the climata of Lower Egypt (14^{h} and 30;22°) and Rhodes ($14\frac{1}{2}^{h}$ and 36°).

[66] How Ptolemy got this angle remains mysterious: whether he used interpolation in the tables II 13 (cf. *HAMA* 236) or direct computation, he should have found (in round numbers) $53° = 106^{\circ\circ}$. On the general problem of the angles between ecliptic and horizon in this chapter see *HAMA* 245–50.

[67] The text has 'right angles', 'hypotenuses' etc. because it is true for *each* planet.

By means of the procedure for the [planetary] latitude, we find that (considering now just the 3 [outer] planets), when they [first] rise near the beginning of Cancer, that is, when they are near the apogee of the epicycle, then at any distance from the apogee not exceeding $\frac{1}{12}$th [of the epicycle circumference],[68] with no sensible error Saturn and Jupiter are practically on the ecliptic, while Mars is about $\frac{1}{5}$° north of the ecliptic.[69] H595

Therefore their distance from the sun along the ecliptic will be represented by DE for Saturn and Jupiter, and by DK for Mars, since it is north [of the ecliptic] by KH, of the amount 12′.

And since KH:KE = 94 : 75,

KE ≈ 10′ in the same units.

But DK is given for Mars as $14\frac{1}{2}$°,

so, by addition, DE = 14;40°.

And for Saturn it is 14°

and for Jupiter $12\frac{3}{4}$°.

So, since ED:DB = 120 : 94,

we get, approximately, for DB, the arc of the great circle drawn through the poles of the horizon,

11° for Saturn

10° for Jupiter

and $11\frac{1}{2}$° for Mars.

Similarly, for Venus and Mercury, when the beginning of Cancer is setting, it forms the same angle and inclination to the horizon as before; and we are given that, when these planets have their first visibility as evening-star in this part of the ecliptic, the distance of Venus from the true sun is $5\frac{2}{3}$°, while Mercury's is $11\frac{1}{2}$°. Therefore at their [first] risings the true sun will have a longitude of

♊ $24\frac{1}{3}$° for Venus H596

and ♊ $18\frac{1}{2}$° for Mercury,

while the longitude of the mean sun will be about

♊ 25° for Venus

and ♊ 19° for Mercury.

Therefore the planets will have these positions in mean longitude. And when, with these [mean] longitudes, the planets have apparent positions at the beginning of Cancer, we find that their distances from the apogee are about

14° for Venus

and 32° for Mercury.

(This kind of computation can be carried out by means of the theorems on their anomaly which we set out before).[70] Accordingly, at these positions, we find that

[68] At apogee of the epicycle the planet is at mean conjunction. So Ptolemy is considering elongations from the mean sun of up to one zodiacal sign.

[69] See *HAMA* 235,237.

[70] From the anomaly tables, XI 11, given, for Venus, $\bar{\lambda} = 85°$, $\bar{a} = 14°$ and the apogee in ♉ 25°, then $\bar{\kappa} = 30°$, leading to an equation of centre of 1;11°, so $\alpha = 15;11°$, which leads to an equation of anomaly of $+6;6\frac{1}{2}°$, so $\lambda = 85° - 1;11° + 6;6\frac{1}{2}° = 89;56\frac{1}{2}° \approx$ ♋ 0°. For Mercury, with $\bar{\lambda} = 79°$, $\bar{a} = 32°$ and the apogee in ♎ 10°, $\bar{\kappa} = 249°$, leading to an equation of centre of 2;53°, so $\alpha = 29;7°$, which leads to an equation of anomaly of 8;16°, hence $\lambda = 79° + 2;53° + 8;16° = 90;9° \approx$ ♋ 0°.

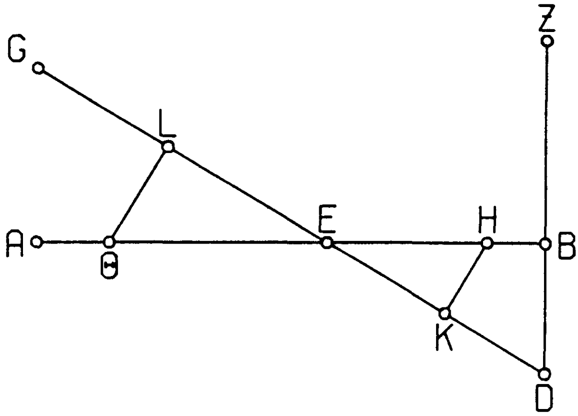

Fig. 13.19

Venus is about 1° north of the ecliptic,
and Mercury about $1\frac{2}{3}$° north.[71]
These, obviously, are the amounts of KH [in Fig. 13.19].

So, since KH:EK = 94 : 75,

$$\text{and } 94 : 75 \approx \begin{cases} 1 : \frac{3}{4} \\ 1\frac{2}{3} : 1\frac{1}{3}, \end{cases}$$

$$\text{EK} = \begin{cases} \frac{3}{4}^\circ \text{ for Venus} \\ 1\frac{1}{3}^\circ \text{ for Mercury.} \end{cases}$$

And in the same units, by hypothesis, the apparent distance of the planet from the sun,

$$\text{DK} = \begin{cases} 5\frac{2}{3}^\circ \text{ for Venus} \\ 11\frac{1}{2}^\circ \text{ for Mercury.} \end{cases}$$

H597 Therefore, by addition, $\text{DKE} \approx \begin{cases} 6\frac{2}{5}^\circ \text{ for Venus} \\ 12\frac{5}{6}^\circ \text{ for Mercury.} \end{cases}$

So, since ED:BD is again 120 : 94,
and that ratio is about the same as $6\frac{2}{5} : 5$
and $12\frac{5}{6} : 10$,
we get for DB, the size of the normal distance,
5° for Venus
and 10° for Mercury.

Q.E.D.

8. {*That the peculiar characteristics of the phases of Venus and Mercury are also in accordance with the hypotheses*}[72]

Furthermore, it is in accordance with the hypotheses detailed above that the strange characteristics of the first and last visibilities of Venus and Mercury take

[71] For the calculations confirming this see *HAMA* 237–8.
[72] See *HAMA* 239–42. There is a reference to this in Proclus, *Hypotyposis* I 17 (ed. Manitius p. 10).

place: namely that, for Venus, the interval from evening setting to morning rising is about 2 days round about the beginning of Pisces, but about 16 days round about the beginning of Virgo; and, for the planet Mercury, the phases as evening-star are missing, when one would expect it to appear round about the beginning of Scorpius, and the phases as morning-star, when round about the H598
beginning of Taurus. We can come to understand that as follows; and first for Venus.

Let there be drawn a diagram [Fig. 13.20] similar to the preceding figure for the phases, and let point E represent, first, the point on the ecliptic at the beginning of Pisces (at this point Venus, when it is near the perigee of the

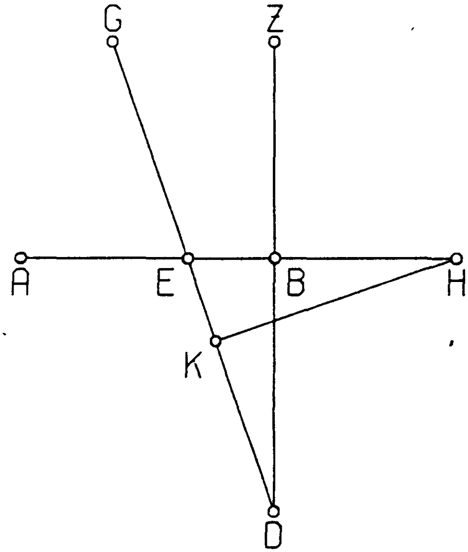

Fig. 13.20

epicycle, is about $6\frac{1}{3}°$ north of the ecliptic).[73] Let the diagram represent the *evening setting* [i.e. last visibility as evening-star]. In this ∠ BED, at the terrestrial latitude in question, is calculated as $154^{\circ\circ}$ where 2 right angles equal $360^{\circ\circ}$.[74]

And [in the right-angled triangles BED,KEH], where the hypotenuse is 120^p, the greater of the sides about the right angle,

$$[BD \text{ or } KH] \approx 117^p,$$

and the lesser, $[BE \text{ or } KE] \approx 27^p$.

Hence, where the normal distance, $DB = 5°$,

$$DE = 5;8°.$$

[73] See *HAMA* 239, and cf. XIII 3 p. 602; when Venus is in the node and near the perigee of the epicycle its latitude is $6\frac{1}{3}°$. Since Venus' apogee is taken as ♉ 25°, for a position of ♓ 0° it is 275° from apogee or 5° from the node.

[74] On the angles between ecliptic and horizon given by Ptolemy see *HAMA* 245–50. The (rounded) value here, 77°, can be found from the tables II 13, taking the values for ♓ 0° at Clima III and Clima IV, 10;5° and 15;53°, taking the mean, 12;59°, and taking its complement, 77;1°. The other values given by Ptolemy, however, cannot be so derived.

H599 But since the planet is 6⅓° north of the ecliptic (which amount is represented by arc KH),

and the ratio 117 : 27 ≈ 6⅓ : 1½,

KE = 1½°,

and, by subtraction, KD, which represents the distance of the planet towards the rear from the sun at its evening setting, is

[5;8 − 1;30 =] 3;38°.

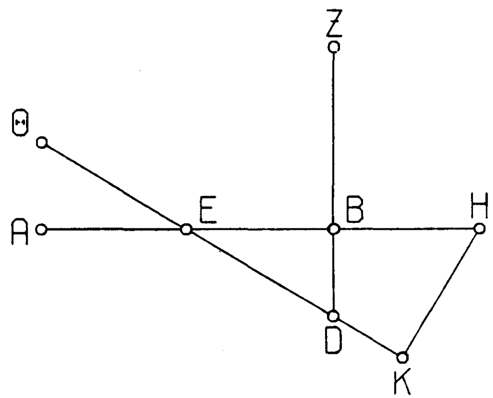

Fig. 13.21

Again, on the similar diagram [Fig. 13.21], since at the morning rising [i.e. at the planet's first visibility as morning-star]

∠ BED = 69°° where 2 right angles = 360°°,

and hence, where the hypotenuse [of the right-angled triangles] is 120ᵖ, the lesser of the sides about the right angle, [BD or KH] ≈ 68ᵖ,

and the greater, [BE or KE] ≈ 99ᵖ;

and we calculate that 68 : 120 = 5 : 8;49

and that 68 : 99 = 6⅓ : 9;13,

so we get DE = 8;49° in the same units,

and the difference [in longitude] due to the latitude,

KE = 9;13°;

and, by subtraction, DK, [the planet's distance] from the sun, towards the rear (obviously), is 0;24°.

H600 And at its evening setting its distance, likewise towards the rear, was 3;38°. Therefore during the interval from evening setting to morning rising it has moved a distance which is less than the sun's motion (that is, approximately,[75] its own motion in [mean] longitude) by 3;14°, which is due to its motion in advance on the epicycle. Now it is easy to determine from the table of anomaly that a motion in advance of that amount [3;14°] is produced by a motion on the epicycle near its perigee of 1¼°:[76] and the planet traverses 1¼° in mean motion [in

[75] 'approximately', because the sun's motion is that of the true sun, while the planet's mean motion in longitude is equal to that of the mean sun.

[76] From the table of anomaly, XI 11, Venus has an equation of anomaly of 7;38° for α = 177° (= 180° − 3°); hence to 3;14° corresponds 3;14 × 3/7;38 = 1;16,14° ≈ 1¼°. Similarly, (below pp. 643–4), for α = 172½° we find an equation of 18;1° (text 18;2°), and for α = 177½° an equation of 6;21° (text 6;38°).

anomaly] in about 2 days. Hence it is clear that that [2 days] is the period of the above interval, in agreement with the phenomena.

Again, on the similar diagram [Fig. 13.22], let point E be taken as the beginning of Virgo (at this point, when Venus is at the perigee of the epicycle, it is south of the ecliptic by about the same amount, $6\frac{1}{3}$°).[77] Let us consider, first, the evening setting, when

∠ BED = 69°° where 2 right angles = 360°°.

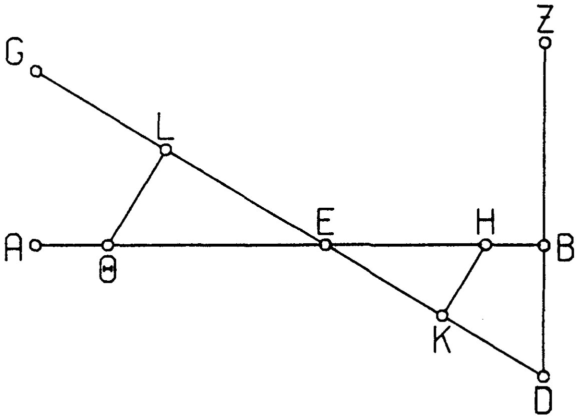

Fig. 13.22

Thus where the hypotenuse [of right-angled triangle BED] is 120ᵖ,
the lesser of the sides about the right angle, [BD] ≈ 68ᵖ,
and the greater, [BE] ≈ 99ᵖ. H601

Thus since the ratios [of BD:BE:DE] are the same as for the morning rising in Pisces, and the difference due to the latitude is equal [to its amount there], we get

arc ED = 8;49°,

the difference [in longitude] due to the latitude, LE = 9;13°,
and, by addition, DL, the planet's distance from the sun towards the rear, is 18;2°.

From the table of anomaly, as mentioned before, [the motion in anomaly] near the perigee of the epicycle corresponding to that amount [18;2°] of retrogradation with respect to the mean motion in longitude of sun and planet is about $7\frac{1}{2}$°

Similarly, at the morning rising at the beginning of Virgo, when

∠ BED = 154°° where 2 right angles = 360°°,

and [hence], where the hypotenuse [of right-angled triangle BED] is 120ᵖ,
the greater of the sides about the right angle, [BD] = 117ᵖ,
and the lesser, [BE] = 27ᵖ;

and one again finds the same ratios as those set out for the evening setting in Pisces, so we get

DE = 5;8°,

[77] Cf. p. 641 n.73.

the difference [in longitude] due to the latitude, EL = 1;30°,
and, by addition, DL, the planet's distance from the sun in advance, is 6;38°. To
H602 this amount corresponds, in the same way as above, about $2\frac{1}{2}$° of [motion in anomaly] near the perigee of the epicycle.

Therefore the total amount of motion on the epicycle which the planet Venus will perform from evening setting to morning rising is 10°; and it traverses that amount in about 16 days, which, as stated above, is the amount agreeing with the phenomena.

Having demonstrated the above, we must apply our theory to the facts concerning the missing phases of Mercury,[78] and [show], first, that at the beginning of Scorpius, even if it reaches its greatest elongation towards the rear from the sun,[79] it cannot become visible as evening-star.

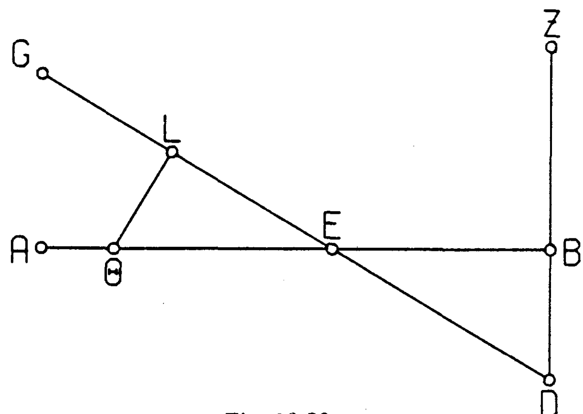

Fig. 13.23

[Proof:] Let the diagram for the phases [Fig. 13.23] be drawn, with point E taken as the point on the ecliptic at the beginning of Scorpius at a [terrestrial latitude] such that at setting

$\angle$ BED = 69°° where 2 right angles = 360°°,

and [thus] where the hypotenuse [of right-angled triangle BED] is 120^p,

the lesser of the sides about the right angle, [BD] = 68^p,

and the greater, [BE] = 99^p.

H603 Therefore where the amount of the normal distance, BD = 10°,

DE = 17;39°.

But when the planet is in the above situation, it is about 3° south of the ecliptic.[80]

So, according to the above ratios.

where LΘ, the amount of the latitude, is 3°,

LE = 4;22°,

and, by addition, DEL [= 17;39° + 4;22°] ≈ 22°.

[78] A similar phrase is used of Mercury as early as Aristotle (*Meteorologica* 342b34) διὰ γὰρ τὸ μικρὸν ἐπαναβαίνειν πολλὰς ἐκλείπει φάσεις 'because it rises only a little above [the horizon] it misses many phases (appearances)'.

[79] At XII 9 Ptolemy has calculated the maximum elongations for Mercury at ♏ 0° and ♉ 0°, in preparation, as he says (p. 591) for this problem.

[80] For a computation of this see *HAMA* 241 n.11.

Hence the planet must have that elongation [22°] from the true sun in order to have its first visibility. But since its maximum elongation from the true sun when it is at the beginning of Scorpius is only 20;58°, as we demonstrated previously [XII 9, p. 594] in our treatment of the greatest elongations, it is obvious that it is natural for phases of this kind to be missing.

Again, if we set out the same diagram for the phases [Fig. 13.24] and take point E as the beginning of Taurus at morning rising, when the planet, in accordance with the positions in question, is about $3\frac{1}{6}°$ south of the ecliptic,[81] and the ratios of the sides [of triangles BED, LEΘ] about the right angles are the same as those above,

then DE = 17;39°
and, where the latitude ΘL = 3;10°,
LE = 4;37°.
Thus, by addition, DEL = 22;16°.

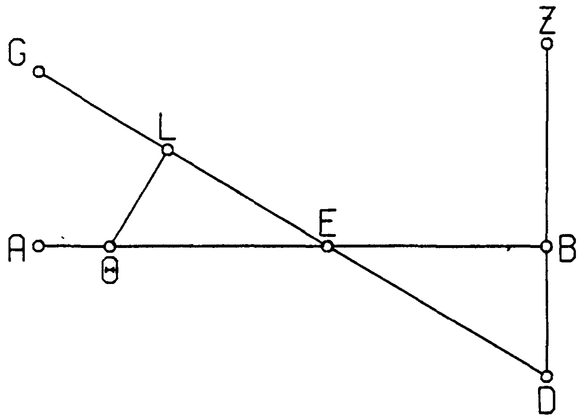

Fig. 13.24

Thus here too the planet must have an elongation of that amount [22;16°] from the true sun in order to have its first visibility. But since its maximum elongation H604
[in this situation] does not exceed 22;13°, as we demonstrated previously [p. 595], naturally, this kind of phase too is missing. Thus we have shown that the facts in question are in agreement with the hypotheses we set out as well as with the phenomena.

9. {*Method of determining the individual elongations from the sun of the first and last visibilities*}[82]

It is immediately obvious [see Fig. 13.25] that if we take as fixed, for each planet, the normal arc [*arcus visionis*] BD, and are given the beginning of [each of] the [zodiacal] signs at the intersection E, and hence angle BED, there will also be given DE and the position in latitude of the planet at that elongation [i.e. DE],

[81] See *HAMA* 241 n.11.
[82] See *HAMA* 242-56.

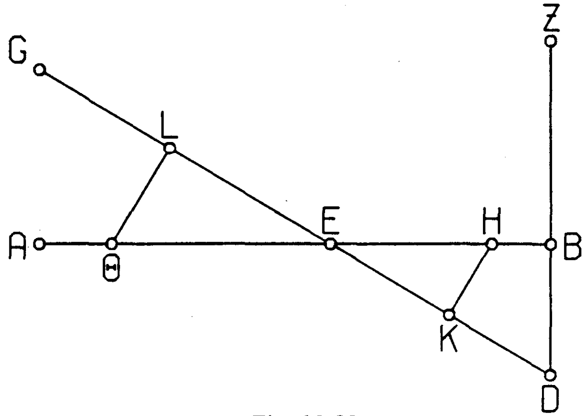

Fig. 13.25

namely KH or ΘL; thence will be given KE or EL [respectively], and also the [corresponding] apparent distance, DK or DL. In this way, (to avoid lengthening our discussion), we computed, for all the signs and for each of the 5 planets, but for only one [terrestrial latitude], the intermediate parallel used
H605 above, since that is sufficient in itself, the apparent distance from the true sun of the risings and settings [i.e. first and last visibilities], on the assumption that the planets themselves were located at the beginning of the signs. We have set these out below, putting them, too, for the user's convenience, in 5 tables, [one] for [each of] the 5 planets, each containing 12 lines. The first 3 tables, for Saturn, Jupiter and Mars, are arranged in 3 columns: the first column contains the beginnings of the signs, the second the elongations at morning rising, and the third those at evening setting. The next 2 tables, for Venus and Mercury, are arranged in 5 columns: the first, as before, contains the beginnings of the signs, the second the elongations at evening rising, the third those at evening setting, and the fourth, again, those at morning rising, and the fifth those at morning setting. The layout of the tables is as follows.

H606–H607 10. {*Layout of the tables containing the first and last visibilities of the 5 planets*}[83]

[See p. 647.]

[83] The basis of computation of these tables is in part unclear (see *HAMA* 242–56), hence I have not been able to recompute them to check the numbers. However, from Neugebauer's computations, the following corrections to Heiberg have been made:
H606.6 Saturn, Morning Rising, Aries, κγ λ (with DK,Is) for κγ α (23;1°) (*HAMA* 248, n.11).
H606,7 Mars, Morning Rising, Taurus, κ ις (with DHKL) for κ η (20;8°) (*HAMA* 248 n. 9 suggests 20;19°).
See also *HAMA* 255 for a suggestion to emend Venus, Morning Rising, Aries, to 2;0° from 3;0°.

TABLES FOR FIRST AND LAST VISIBILITIES OF THE 5 PLANETS

Beginning of Sign	SATURN Morning Rising	SATURN Evening Setting	JUPITER Morning Rising	JUPITER Evening Setting	MARS Morning Rising	MARS Evening Setting
Aries	23 30	11 28	20 10	10 19	21 12	11 40
Taurus	21 57	11 41	19 6	10 29	20 16	11 48
Gemini	17 52	12 26	15 51	11 10	17 21	12 30
Cancer	14 2	14 2	12 46	12 46	14 33	14 33
Leo	11 34	15 34	10 40	14 31	12 28	17 19
Virgo	10 53	16 53	10 1	16 12	11 46	20 5
Libra	10 48	17 6	9 57	16 34	11 38	21 1
Scorpius	10 53	16 53	10 1	16 12	11 48	20 19
Sagittarius	11 34	15 34	10 40	14 31	12 34	17 32
Capricornus	14 2	14 2	12 46	12 46	14 45	14 45
Aquarius	17 52	12 26	15 51	11 10	17 35	12 36
Pisces	21 57	11 41	19 6	10 29	20 26	11 49

Beginning of Sign	VENUS Evening Rising	VENUS Evening Setting	VENUS Morning Rising	VENUS Morning Setting	MERCURY Evening Rising	MERCURY Evening Setting	MERCURY Morning Rising	MERCURY Morning Setting
Aries	5 10	4 9	3 0	10 28	9 58	9 43	23 58	23 38
Taurus	5 8	4 16	6 16	9 40	10 4	10 15	22 15	22 15
Gemini	5 12	5 7	9 15	7 36	10 18	11 47	18 0	16 44
Cancer	5 36	8 23	9 50	5 59	12 22	15 34	14 4	12 30
Leo	6 16	13 3	8 2	5 5	13 43	19 59	11 25	10 21
Virgo	7 22	18 2	6 38	4 54	18 1	23 13	10 21	9 59
Libra	7 53	17 43	5 41	4 54	22 49	23 16	9 51	10 0
Scorpius	8 20	13 47	5 28	4 55	20 1	22 1	9 44	10 19
Sagittarius	7 49	8 1	4 39	5 16	18 11	17 25	9 25	11 19
Capricornus	6 52	4 8	2 43	6 35	13 54	12 10	9 36	14 5
Aquarius	5 51	3 16	0 30	8 33	11 10	9 50	12 27	17 50
Pisces	5 22	3 38	0 24	10 16	10 11	9 43	19 15	21 46

11. {*Epilogue of the treatise*} H608

We have now completed these additional topics, Syrus, and have shown the way to deal with almost all the topics which should, at least to my mind, be subjected to theory for the purposes of this kind of treatise, at any rate as far as the time up to our own days[84] contributed to greater accuracy in our discoveries or in corrections [of earlier discoveries], and as far as was suggested by a memorandum[85] directed only toward scientific usefulness, and not towards ostentation. So at this point our present discussion can be terminated at an appropriate place and at the right length.

[84] Cf. p. 37 n.11.
[85] Cf. p. 37 n.12.

Appendix A

Examples of Computations

1 (*a*). II 4 p. 80. Given the terrestrial latitude (φ), compute the distance of the sun from the summer solstice as measured along the ecliptic ($\Delta\lambda$).
Example: $\varphi = 4;15°$ (cf. II 6, second parallel, p. 83).

From Table I 15	λ	δ
	10°	4;1,38°
	11°	4;25,32°.

Hence to a declination of 4;15° corresponds a longitude (counted from equinox) of 10;33,33°.
Therefore the distance from solstice, $\Delta\lambda = (90° - 10;33,33°) = 79;26,27°$ (text: $79\frac{1}{2}°$).

1 (*b*). II 6 p. 89. Find the terrestrial latitude (φ) at which the sun does not set for a given period of time.
Example: Period of one month. Taking a month as 30 days, and assuming the sun to move $1°/_d$ in the ecliptic, we find that the parallel in question cuts off 30° of the ecliptic, or 15° either side of the summer solstice.

From Table I 15	λ	δ
	$90° - 15° = 75°$	22;59,41°.

Hence $\varphi = 90° - \delta = 67;0,19°$ (text: 67°).

2. II 9 p. 99. Given the longitude of the sun ($\lambda_{\odot}$) and the terrestrial latitude (i.e. the 'clima'), find the length of day or night and the length of the seasonal hour.
Example: $\lambda_{\odot}$ = ♐ 28;18°. Place: Babylon (cf. IV 11 p. 212). What is the length of night?
We use the rising-time table (II 8) for Rhodes ($M = 14\frac{1}{2}^h$).
(*a*) First method.
Since it is night, we take the degree opposite the sun, ♊ 28;18°.

From the table:	ρ (♊ 28;18°):	69;27°
	ρ (♐ 28;18°):	286;50°

Difference (in order of signs), Δ: 217;23°.
Length of night in equinoctial hours is $\Delta/15$: $14;29^h$ (text: $14\frac{2}{3}^h$).
Length of 1 seasonal night-hour in time-degrees is $\Delta/12$: 18;7° (text: 18°)
(hence length of 1 seasonal hour in equinoctial hours: $1;12,28^h$).

(*b*) Second method.

From rising-time table (II 8) at *sphaera recta*:	α (♊ 28;18°):	88;9°
as above	ρ (♊ 28;18°):	69;27°
	Difference (Δ):	18;42°
	Δ/6:	3;7°
Since Gemini is north of the ecliptic, add 15°:		18;7°

This is the length of 1 seasonal night-hour in time-degrees.

3. II 9 p. 104. Given the length of a seasonal hour in time-degrees, convert the time in seasonal hours to the time in equinoctial hours.
From Example 2 (q.v.), length of 1 seasonal night-hour: 18;7°.
What is 5½ seasonal hours after midnight in equinoctial hours?
5½ × 18;7/15 = 6;38, so the time is 6;38 a.m.
Ptolemy (l.c.) multiplies by $\frac{18}{15}$ and gets 6⅗ equinoctial hours after midnight.

4. II 9 p. 104. Given the longitude of the sun (λ☉), the terrestrial latitude, and the time in seasonal hours, find the point of the ecliptic which is rising (the 'horoscope').
Example (cf. VII 3 p. 336). λ☉: ♏ 13;17° (text, 'about the middle of ♏')
Place: Alexandria. Time: 2¼ seasonal hours after midnight.[1]

Length of 1 night-hour (λ☉ = ♏ 13;17°, M = 14^h, cf. Example 2):	16;38°
Time from sunset: 8¼ seasonal hours. 8¼ × 16;38:	137;14°
From Table II 8 for Clima III: ρ (♉ 13;17°):	31;4°
(we take the point *opposite* the sun, since it is night) Sum	168;18°.

168;18° is the rising-time (at Clima III) of the horoscope: ρ (♍ 19;51°)
(text: 'about ♍ 22½°).

5. II 9 p. 104. Given the same data as in Example 4, find the point of upper culmination.
Total of seasonal hours from last midday: 6 day-hours plus 8¼ night-hours.
Length of 1 day-hour: 13;22°
Length of 1 night-hour: 16;38°

6 × 13;22° + 8¼ × 16;38° = 80;12° + 137;14° =	217;26°
Rising-time at *sphaera recta* of sun's degree: α (♏ 13;17°)	220;46°
Sum:	78;12°

78;12° = α (♊ 19;11°) (text: ♊ 22½°).

[1] Ptolemy (l.c.) gives 2½ equinoctial hours, which is approximately the same.

6. II 9 p. 104. Given the longitude of the horoscope at a given place, find the point of upper culmination.

Example: same data as in Example 4.

Rising-time of horoscope at Clima III: ρ (♍ 19;51°): 168;18°
− 90;0°

78;18°

78;18° = α (♊ 19;16°) (text: ♊ $22\frac{1}{2}$°).

The discrepancy from the result of Example 5 is due to the rounding to minutes of the tables and at every step of the computation.

7. III 8 p. 169. Given the date, compute the position of the sun.

Example (Cf. IV 11 p. 214). Date: Nabonassar 548, Mechir [VI] 9/10, $1\frac{1}{3}$ equinoctial hours after midnight.

From mean motion table, III 2:

	$\Delta\bar{\lambda}_{\odot}$
540^y	228;42,48°
7^y	358;17,53°
150^d	147;50,43°
8^d	7;53,6°
13^h	0;32,2°
$0;20^h$	0;0,49°
Sum 547^y 158^d $13\frac{1}{3}^h$	743;17,21°→ 23;17,21°

$\bar{\kappa}$ (epoch): + 265;15°

$\bar{\kappa}$: 288;32,21°

From Table III 6, for argument 288;32°, we find (by interpolation) the equation as 2;13,28°. This is additive, since κ falls in the second column. 288;32,21°

longitude of apogee: + 65;30°

$\bar{\lambda}$: 354;2,21°
θ: + 2;13,28°

λ: 356;15,49°,

or about ♓ 26;16° (text: ♓ 26;17°).

8. III 9 p. 171. Computation of the 'equation of time', E (given an interval in true solar days, find the interval in mean solar days).

Example (cf. IV 6 p. 198):

t_1: Hadrian 17 (Nabonassar 880) Pauni [X] 20/21, 11;15 p.m.

t_2: Hadrian 19 (Nabonassar 882) Choiak [IV] 2/3, 11 p.m.

From the solar tables (cf. Manitius I p. 437):

$\bar{\lambda}$ (t_1): 42;21°, λ (t_1): ♉ 13;15°

$\bar{\lambda}$ (t_2): 206;42° λ (t_2): ♎ 25;10°.

Hence, from Table II 8 (rising-times at *sphaera recta*):

$$\alpha\ (t_1):\quad 40;44^\circ$$
$$\alpha\ (t_2):\quad 203;17^\circ.$$

$$\left.\begin{array}{l}\Delta\bar{\lambda} = \bar{\lambda}\ (t_2) - \bar{\lambda}\ (t_1) = 164;21^\circ \\ \Delta\alpha = \alpha\ (t_2) - \alpha\ (t_2) = 162;33^\circ\end{array}\right\} E = 1;48^\circ = 7\tfrac{1}{5} \text{ mins.}$$

Since $\Delta\bar{\lambda} > \Delta\alpha$, we subtract E from the 'simple' interval, $1^y\ 166^d\ 23;45^h$, to get, for the interval in mean solar days, $1^y\ 166^d\ 23;37,48^h$ (text: $23\tfrac{5}{8}^h$ = $23;37,30^h$).

9. V 9 p. 239. Computation of the moon's latitude and longitude from the tables for a given date.

Example: Nabonassar 466, Thoth [I] 7/8, 2 equinoctial hours after midnight (cf. VII 3 p. 336).

From the mean motion tables, IV 4:

	$\bar{\lambda}$	$\bar{\alpha}$	$\bar{\omega}$	$\bar{\eta}$
epoch value		268;49°	354;15°	70;37°
450^y	260;46,44°	323;26,5°	320;54,6°	10;11,3°
15^y	140;41,33°	250;46,52°	70;41,48°	144;20,22°
6^d	79;3,30°	78;23,24°	79;22,34°	73;8,40°
14^h	7;41,10°	7;37,16°	7;43,2°	7;6,41°
Sum	488;12,57°	929;2,37°	832;56,30°	305;23,46°.

$\Delta\bar{\lambda} = 128;13^\circ \quad \bar{\alpha} = 209;3^\circ \quad \bar{\omega} = 112;56^\circ \quad 2\bar{\eta} = 250;48^\circ.$

From anomaly table, V 8.

col. 3: $c_3(2\bar{\eta}) = -13;4^\circ$

true anomaly $\alpha = \bar{\alpha} + c_3 = 209;3 - 13;4^\circ = 195;59^\circ$

col. 4: $c_4(\alpha) = 1;30^\circ$

col. 5: $c_5(\alpha) = 0;55^\circ$

col. 6: $c_6(2\bar{\eta}) = ;36,52$

equation $c = c_4 + c_5 . c_6 = +(1;30^\circ + 0;55^\circ \times 0;36,52) = +2;4^\circ$

longitude $= \Delta\bar{\lambda} + c + \lambda_{epoch} = 128;13^\circ + 2;4^\circ + 41;22^\circ = 171;39^\circ$ (text: 171;30°).

$\omega = \bar{\omega} + c = 112;56^\circ + 2;4^\circ = 115;0^\circ$.

col. 7: latitude $\beta(\omega) = -2;7^\circ$ (text: $-2\tfrac{1}{6}^\circ$).

10. V 19 p. 264. Computation of the parallax of the moon for a given time, place, solar longitude and lunar longitude, latitude and elongation, from the tables.

Example: time, $2\tfrac{1}{2}$ equinoctial hours after midnight (true local time Alexandria); $\lambda_\odot$: ♏ 13;17°; $\lambda_☾$: ♍ 21;30°, $\beta_☾$: $-2\tfrac{1}{6}^\circ$ (cf. VII 3 p. 336 and Example 9).

From solar longitude and local time: culminating point: ♊ 19;11° (cf. Example 5).

Distance of moon from meridian: α (♍ 21;30°) – α (♊ 19;11°)
= 172;12° – 78;12° = 94°
= $6;16^h$ east.

From Table II 13 (Clima III), arguments $6;16^h$ (vertical) and ♍ 21;30° (horizontal), by interpolation in tables for Virgo and Libra:

arc 90°

east angle 172;30°.

Correction to arc and angle for moon's latitude (cf. V 19 p. 272):

$\text{Crd}\,(2 \times (180° - 172;30°)) = \text{Crd}\ 15° = 15;40^p$

$\text{Crd}\,(180° - 15°) = \text{Crd}\ 165° = 118;58^p$.

Multiplying β by each of these and dividing by 120, we get 0;17° and 2;9° respectively. Then the corrected arc is given by

$\sqrt{(90° + 0;16°)^2 + (2;9°)^2} \approx 90;18°$,

and the corresponding angle of correction from: $2;9 \times \dfrac{120}{90;18} = 2;51^p$, which is the chord of ca. 2;44°, half of which is 1;22°.

Therefore the corrected angle is 172;30° − 1;22° = 171;8°.

We take the arc as exactly 90° (since otherwise the moon would be below the horizon).

Computation of total parallax.

From Table V 18, argument ζ = 90°.

Lunar parallax ($\alpha_☾$ = 195;59°, $\bar{\eta}$ = 305;24°, cf. Example 9):

col. 3	col. 4	col. 5	col. 6
0;53,34	0;10,17	1;19,0	0;25,0

with argument $(360° - \alpha)/2$ ($\approx$ 82°), from col. 7: minutes: 58,39

from col. 8: minutes: 58,31.

Parallax at syzygy: 0;53,34 + 0;10,17 × 0;58,39 = 1;3,37°

Parallax at quadrature: 1;19,0 + 0;25,0 × 0;58,31 = 1;43,23°

Δ = 0;39,46°

with argument $(360° - \bar{\eta})$ = 54;36, from col. 9: minutes: 42,35.

Parallax: 1;3,37 + 0;39,46 × 0;42,35 $\approx$ 1;32°.

Determination of longitudinal and latitudinal components of parallax.

Angle between hour-circle and ecliptic (see above): 171;8°.

This is greater than 90°, so we take the supplement, 8;52°.

Twice this is 17;44°, and the supplement of the latter 162;16°.

The chords of these angles are $18;30^p$ and $118;34^p$ respectively.

Latitudinal parallax: 1;32 × 17;44/120 $\approx$ 0;13½°.

Longitudinal parallax: 1;32 × 118;34/120 $\approx$ 1;31°.

Latitudinal parallax is southwards (zenith to the north of the culminating point).

Since latitudinal parallax is southwards and the angle greater than 90°, longitudinal parallax is positive.

Result: parallax in latitude: −0;13½° (text: −0;5°)

parallax in longitude: +1;31° (text: +1;0°).

11. VI 9. Given year and month, compute lunar eclipse.
Example: Date, Nabonassar 28, Thoth (cf. IV 6 pp. 191–2).
From Table VI 3, compute mean opposition:

	Days of Thoth	$\overline{\kappa}$	$\overline{\alpha}$	$\overline{\omega}$
Period: 26	9;55,35	267;58,12°	83;24,29°	230;10,5°
Year: 2	8;15,53	7;39,36°	285;25,4°	46;45,54°
Year: 28	18;11,28^d	275;37,48°	8;49,33°	276;55,59°.

Time of mean opposition: 18;11,28^d = Thoth 18/19, 4;35 p.m.
$\overline{\omega}$ lies within ecliptic limits for lunar eclipse, which is therefore possible.
Computation of true opposition.
From Table III 6, c($\overline{\kappa}$): +2;21° solar equation
From Table IV 10, c($\overline{\alpha}$): −0;42° lunar equation.
True position in latitude: $\omega = \overline{\omega} + c(\alpha)$ = 276;14° at mean opposition.
$\Delta\lambda$ = 2;21° + 0;42° = 3;3°.
Moon's true hourly motion in longitude: 0;32,56 − 0;32,40 × $4\frac{2}{3}'$ = 0;30,24°.
$\Delta t = 3;3 \times \frac{13}{12} \div 0;30,24 = 6;31^h$.
True longitude of moon at mean syzygy is less than true longitude of sun (minus 180°). So we add Δt to the time of mean opposition to get the time of true opposition as 11;6 p.m. (text: 11;10 p.m.).
Motion over Δt: $3;3 \times \frac{13}{12}$ = 3;18°.
We *add* this to the position in latitude: ω = 279;32° at true opposition.
In 6;31^h motion in anomaly is 3;33°, so at true opposition $\overline{\alpha}$ = 12;22°.
Computation of circumstances of eclipse.
From Table VI 8, II, argument 279;32°.

At greatest distance		At least distance	
Magnitude	Duration	Magnitude	Duration
2;32 digits	0;26,22°	4;42 digits	0;39,35°

Δ: 2;10 digits and 0;13,13°.
From III, argument 12;22°: sixtieths: 0;43.
Magnitude: 2;32 + 2;10 × 0;0,43 = 2;34 digits (text: 3 digits observed).
Duration: 0;26,22 + 0;13,13 × 0;0,43 = 0;26,31°.
To get time from beginning to middle of eclipse, we divide the duration, increased by a twelfth, by the moon's true hourly motion:
$0;26,31 \times \frac{13}{12} \div 0;30,24 = 0;57^h$.

Beginning of eclipse (Alexandria)	10;9 p.m.
Eclipse middle	11;6 p.m.
End of eclipse	12;3 a.m.
Magnitude	ca. $2\frac{1}{2}$ digits.

12. VI 10. Given year, month and place, compute solar eclipse.
There is no example of a solar eclipse in the Almagest, so I have selected the eclipse of 364, June 16, which Theon observed at Alexandria, and gave as the example of computation in his commentary on the Almagest, first according to the Almagest, and again according to the Handy Tables (Basel edition pp. 332–339, cf. Rome [6]). A somewhat different calculation of the same eclipse also

appears in some mss. of Theon's small commentary on the Handy Tables, and has been published *in extenso* by Tihon, 'Calcul de l'éclipse'.
Example: Nabonassar 1112, Thoth, Alexandria.
From Table VI 3 compute mean conjunction:

	Days of Thoth	$\bar{\kappa}$	$\bar{\alpha}$	$\bar{\omega}$
Period: 1101	22;41,45	19;11,56°	222;53,32°	65;41,57°
Year: 11	1;9,39	358;28,11°	271;4,19°	211;12,3°
Year 1112	23;51,24^d	17;40,7°	133;57,51°	276;54,0°.

Time of mean conjunction: 23;51,24^d = Thoth 24, 8;34 a.m.
$\bar{\omega}$ lies within ecliptic limits for solar eclipse, which is therefore possible.
Computation of true conjunction.

From Table III 6, $c(\bar{\kappa})$: −0;41° solar equation
From Table IV 10, $c(\bar{\alpha})$: −3;50° lunar equation.

True position in latitude: $\omega = \bar{\omega} + c(\bar{\alpha})$ = 273;4° at mean conjunction.
$\Delta\lambda$ = −0;41° + 3;50° = 3;9°.
Moon's true hourly motion in longitude: 0;32,56° + 0;32,40 × 3$\frac{5}{6}$′ = 0;34,56° (Theon: 0;34,56°).
Δt = 3;9 × $\frac{13}{12}$ ÷ 0;34,56 = 5;52^h.
Time of true conjunction: 8;34 a.m. + 5;52^h = 2;26 p.m. (Theon: 2 + $\frac{1}{3}$ + $\frac{1}{10}$ hours after noon).
Motion over Δt: 3;9 × $\frac{13}{12}$ ≈ 3;25°.
We *add* this to the position in latitude: ω = 276;29° at true conjunction.
In 5;52^h mean motion in anomaly is 3;12°, so at true conjunction $\bar{\alpha}$ = 137;10°.
To find time of apparent conjunction at Alexandria we have first to find true local time, i.e. apply equation of time.
True longitude of sun at mean conjunction: $\bar{\kappa} + \bar{\lambda}_A + c(\bar{\kappa})$ =
17;40° + 65;30° − 0;41° = 82;29°.
Motion of sun from mean to true conjunction: $\Delta\lambda$/12 = 0;16°.
True longitude of sun at true conjunction: 82;45°.
Hence equation of time with respect to era Nabonassar (cf. Example 8 for method): +24 mins.
Time of true conjunction with respect to noon at Alexandria: 2;50 p.m.
Calculation of apparent conjunction.
(1) Parallax computation (cf. Example 10).
From Table II 13, Clima III, λ = ♊ 22;45°, 2;50 p.m.:
zenith distance: 38;28° angle: 17;35°.
From Table V 18, ζ = 38;28°, α = 137;10° (latitude of moon neglected):

total parallax of sun: 0;1,45°
total parallax of moon: 0;39,35° (from cols. 3 and 4 only)

difference in parallax: 0;37,50°.

Longitudinal parallax (for angle 17;35°): p_λ = 0;36°.
Time from true to apparent conjunction is found by dividing the above by the true hourly velocity of the moon: 0;36 ÷ 0;34,56 ≈ 1;2^h.
Hence time of apparent conjunction (first approximation): 3;52 p.m.
(2) Second parallax computation, for corrected time.

From Table II 13, Clima III, λ = ♊ 22;45°, 3;52 p.m.:
zenith distance: 51;48°   angle: 18;32°.
In 1;2^h motion in anomaly is about 0;33°, hence α for corrected time is 137;10° + 0;33° = 137;43°.
Neglecting lunar latitude, as before, from Table V 18, ζ = 51;48°, α = 137;43°:

total parallax of sun:	0;2,15°
total parallax of moon:	0;49,47°
difference in parallax:	0;47,32°.

Longitudinal parallax (for angle 18;32°): p'_λ = 0;45°.
Computation of the 'epiparallax':
Difference between first and second longitudinal parallaxes,

$$d = p'_\lambda - p_\lambda = 0;45° - 0;36° = 0;9°.$$

Further increment, f, is found by f:d = d:p, hence f = 0;9 × 0;9 ÷ 0;36 ≈ 0;2, and epiparallax = d + f = 0;11°.
Final parallax in longitude: 0;36° + 0;11° = 0;47°.
To account for sun's motion add $\frac{1}{12}$th to this: $\frac{13}{12}$ × 0;47° ≈ 0;51°.
Time from true to apparent conjunction: 0;51 ÷ 0;34,56 ≈ 1;28^h.
Hence time of apparent conjunction: 2;50^h + 1;28^h = 4;18 p.m. (Theon: $4\frac{1}{3}^h$ p.m.)
Position of moon at this time:

λ: ♊22;45° + 0;51° = ♊23;36°
ω: 276;29° + 0;51° = 277;20°
α: 137;10° + 0;51° = 138;1°

Computation of circumstances of eclipse.
Computation of latitudinal parallax.
From Table II 13, Clima III, λ = ♊ 23;36°, 4;18 p.m.:
zenith distance: 57;18°   angle: 19;46°.
From Table V 18, with ζ = 57;18°, α = 138;1°:

total parallax of sun:	0;2,24°
total parallax of moon:	0;53,2°
difference in parallax:	0;50,38°.

Latitudinal parallax (cf. Example 10) for angle 19;46°: p_β = 0;17°.
We convert this to a distance along the moon's orbit by multiplying it by 12: $\Delta\omega$ = 12.p_β = 3;24° (Theon uses the factor $11\frac{1}{2}$ and gets 3;19°).
Since ω is 277;20°, the moon is just past the *ascending* node. The effect of the parallax is southwards, therefore its effect on ω is negative.
Final position of moon on orbit: 277;20° − 3;24° = 273;56°, apparent argument of latitude.
From Table VI 8, I, argument 273;56°:

At greatest distance		At least distance	
Magnitude	Duration	Magnitude	Duration
4;8 digits	23;44,28 minutes of travel	4;56 digits	26;18,52 minutes of travel

Δ: 0;48 digits and 2;34,24 minutes.
From III, argument α = 138;1°: sixtieths: 51,39.
Magnitude: 4;8 + 0;48 × 0;51,39 = 4;49 digits.
Duration: 23;44,28 + 2;34,24 × 0;51,39 = 25;57 minutes of travel.

We increase the latter by $\frac{1}{12}$th, to account for the sun's motion: 28;7′, and divide by the moon's hourly velocity, 0;34,56°, to get half-duration of the eclipse: 0;28,7 ÷ 0;34,56 ≈ $0;48,18^h$ (Theon: $\frac{1}{2} + \frac{1}{4} + \frac{1}{20} = 0;48^h$).

Thus circumstances of eclipse (neglecting variation of zenith distance during the eclipse):

Magnitude:	4;49 digits	(Theon: 4;39,18 digits)
Beginning of eclipse, Alexandria:	3;30 p.m.	(Theon: 3;32 p.m.)
mid-eclipse, Alexandria:	4;18 p.m.	(Theon: 4;20 p.m.)
end of eclipse, Alexandria:	5;6 p.m.	(Theon: 5;8 p.m.).

(Theon goes on to calculate the differences in beginning and end of eclipse because of the variation in the zenith distance, cf. Almagest VI 10 pp. 312–13. These amount to 12 minutes earlier and 7 minutes later respectively, verifying Ptolemy's statements about the effect on the intervals).

Using modern tables (those in P. V. Neugebauer, *Astronomische Chronologie*), I find:

maximum phase at Alexandria:	5.6 digits	
times of phases at Alexandria:	beginning:	$15;18^h$
	middle:	$16;28^h$
	end:	$17;24^h$.

13. VI 13 p. 319. Given the circumstances of an eclipse (magnitude and times of principal phases), compute the 'inclination' (πρόσνευσις, i.e. point on the horizon towards which the line joining the centres points).
We take as example the solar eclipse of Example 12 (364 June 16 = Nabonassar 1112, Thoth 24), beginning of eclipse (first contact).
Given: time at Alexandria, 3;30 p.m.; magnitude, 4;49 digits.
First, find the rising-point of the ecliptic (cf. Example 4).
The longitude of the sun is ♊ 22;45° (Example 12 p. 655).
Time in seasonal hours at Alexandria (cf. Example 2): 3^h after noon.
Hence rising-point of ecliptic: ♏ 10°; and setting-point is therefore ♉ 10°.
From Fig. 6.7, azimuth of ♉ 10° at Clima III:

	♉ 0°	13;33° N. of W.
	♊ 0°	23;53° N. of W.
Hence	♉ 10°	is 17° N. of W.

From Table VI 12, col. 2 argument 4;49 digits: 37;41°.
Moon is north of ecliptic (ω is somewhat more than 270° in Example 12).
Hence this angle is set off to the north of the setting-point.
So point of 'inclination' on the horizon is 17° + 37;41° = 54;41° N. of W.

14. XI 12 p. 554. Compute the longitude of a planet from the tables for a given time.
Example: Mars, Nabonassar 886, Epiphi [XI] 15/16, 9 p.m. (cf. X 8, where Mars is observed for this moment).

From mean motion tables, IX 4, find mean longitude and mean anomaly:

	$\overline{\lambda}$	$\overline{\alpha}$
epoch	3;32°	327;13°
810^{y}	138;15,13°	24;48,59°
72^{y}	92;17,21°	250;12,21°
3^{y}	213;50,43°	145;25,31°
10^{m} (300^{d})	157;13,4°	138;28,21°
14^{d}	7;20,13°	6;27,43°
9^{h}	0;11,47°	0;10,23°
885^{y} 314^{d} 9^{h}	612;40,21°	892;46,18°

hence $\overline{\lambda}$ = 252;40° $\qquad$ $\overline{\alpha}$ = 172;46° (as X 8 p. 500).

Apogee position at epoch: ♋ 16;40°
motion of apogee in 886^{y} (at 1° in 100^{y}): 8;52°
hence apogee position at date: 115;32°.
Mean centrum ($\overline{\kappa}$): 252;40° − 115;32° = 137;8° (X 8: 137;11°).
From anomaly table (XI 11):
with argument $\overline{\kappa}$, find equation of centre from col. 3 and col. 4:
137;8°→ 9;3 − 0;41 = 8;22° (cf. X 8, ∠ ZBE = 16;44°°).
Since $\overline{\kappa}$ is in the first column (less than 180°), we subtract the latter from $\overline{\lambda}$ and add it to $\overline{\alpha}$:
λ' = 252;40 − 8;22 = 244;18°, α = 172;46 + 8;22 = 181;8°.
With argument α, take the equation from col. 6: c_6(181;8°) = 2;10°.
With argument $\overline{\kappa}$, take the 'sixtieths' from col. 8: c_8(137;8°) = 37,9
Since $\overline{\kappa}$ is between mean distance and perigee (c_8 positive), we take the increment from col. 7: c_7(181;8°) = 0;53°.
Then equation of anomaly $c = c_6 + c_8.c_7$ = 2;10° + 0;53° × 0;37,9 = 2;43°. (cf. X 8, ∠ BEX = 5;26°°).
Since α is greater than 180° (in second column of argument), this equation is negative.
Therefore $\lambda = \lambda' - c$ = 244;18° − 2;43° = 241;35° (X 8: observed: ♐ 1$\frac{3}{5}$°).

15. XIII 6. Compute latitude of planet, given 'corrected longitude' (see p. 635 n.55: distance of epicycle center from apogee, κ_0) and 'corrected anomaly' (α).
(*a*) Outer planet. Example: *Jupiter, Nabonassar 507 XI 18, 6 a.m.* (cf. XI 3 p. 522)
Given: κ_0 = 290;40°, α = 72;3°.
$\omega = \kappa_0 - 20°$ = 270;40°: $c_5(\omega\ )$ = 0,43 (Table XIII 5).
$\omega > 270°$, so we enter col. 3: c_3(72;3°) = 1;21°.
$\beta = c_3.c_5$ = 1;21 × 0;0,43 ≈ +0;1° (northerly since we took c_3).
Text says that Jupiter occulted δ Cnc, which according to the star catalogue (XXV 5) had a latitude of −0$\frac{1}{6}$°. Thus there is a discrepancy of $\frac{1}{6}$°. Tuckerman (- 240 Sept. 4) gives β ≈ +0;14°. Since δ Cnc was, by modern calculations, almost exactly on the ecliptic at the time of the observation, there could not have been an occultation.

(*b*) Inner planet. Example: Mercury, Nabonassar 486 IV 18, 6 a.m. (cf. IX 7 p. 450)
Given: $\kappa_0 = 129;44°$, $\alpha = 239;15°$.
Table XIII 5, argument α: $c_3 = 1;27°$,
$c_4 = 2;29°$. Since $90° < \kappa_0 < 270°$, we add to the latter $\frac{1}{10}$th of itself: $c'_4 = 2;29° + 0;15° = 2;44°$.
$\kappa' = \kappa_0 + 270° = 39;44°$. $c_5(\kappa') = 45,55$.
$\beta_1 = 1;27° \times 0;45,55 = 1;7°$.
Condition A2 (p. 635) holds, since $\kappa' < 90°$, $90° < \alpha < 270°$, so β_1 is northerly.
$\kappa'' = \kappa_0 + 180° = 309;44°$. $c_5(\kappa'') = 38,11$.
$\beta_2 = c'_4.c_5 = 2;44 \times 0;38,11 = 1;44°$.
Condition A2 (p. 636) holds, since $270° < \kappa'' < 360°$, $\alpha > 180°$, so β_2 is southerly.
$\beta_3 = 0;45° \times c_5(\kappa'').c_5(\kappa'') = 0;18°$. This is southerly.
$\beta = \beta_1 + \beta_2 + \beta_3 = +1;7° - 1;44° - 0;18° = -0;55°$.
Text says Mercury was '3 moons to the north' of δ Cap. In the star catalogue (XXXI 24) this has a latitude of $-2°$; so according to the observation Mercury's latitude should be about $-\frac{1}{2}°$, a discrepancy of about $\frac{1}{2}°$ with the computation. From Tuckerman, for -261 Feb. 12, 6 a.m. Alexandria, I find a latitude of about +0;8°.

Appendix B

Corrections to Heiberg's text

This is a list of all corrections to the Greek text of the standard edition which I have adopted in making the translation (for certain types of corrections omitted see Introduction p. 4). For each item I give the reference in Heiberg's text, the correction (usually the reading of Heiberg followed, after a colon, by the reading I adopt), and the page and note in which I make and, where necessary, justify the correction.

I H16,9 τὰ πλείονα : πλείονα 41 n.30
H23,1 αὐτήν : αὐτόν 44 n.39
H35,18 ἐντεῦθεν : αὐτόθεν 50 n.58
H42,1 λοιπὴ : ἡ λοιπὴ 53 n.62
H48,20 να : νδ 58 n.68
H54,10 δ : γ 59 n.68
H55,43 μα : μδ 59 n.68
H56,15 κζ : κθ 59 n.68
H57,37 νς : νε 59 n.68
H58,13 μα : μδ 59 n.68
H60,17 κς : νς 59 n.68
H65,13 ὑποθεμάτων : ὑποθεματίων 62 n.71
H72,13–15 ὥστε . . . ὑπακουέσθω del. 67 n.80
H75, 2 τὸ σημεῖον : τὰ σημεῖα 68 n.83
H81,29 ια : α 71 n.87
H81,50 κ : α 71 n.87
H83,10 $\overline{\kappa\varepsilon}$: $\overline{\nu\gamma}$ 73 n.89
H83,13 $\overline{\kappa\varepsilon}$: $\overline{\nu\gamma}$ 73 n.89
H86,20 κατὰ δεκαμοιρίαν παράλληλον : κατὰ παράλληλον 28 n.2
H92,8 $\overline{\kappa\gamma}$: $\overline{\kappa\varsigma}$ 77 n.11
H92,11 $\overline{\kappa\gamma}$: $\overline{\kappa\varsigma}$ 77 n.11
H95,18 προεκτιθεμένων : προεκτεθειμένων 79 n.13
H95,22 περιφερείᾳ : περιφερειῶν 79 n.13
H105,13 ∠′γ′ : ∠′ιβ′ 84 n.28
H108,13 $\overline{\lambda\varsigma}$: $\overline{\lambda}$ 85 n.38
H108,20 $\overline{\pi\gamma}$ ιβ′ : $\overline{\pi\gamma}$ $\overline{\iota\beta}$ 85 n.39
H109,9 $\overline{\mu\gamma}$ ∠′ γ′ : $\overline{\mu\gamma}$ ∠′ ι′ 86 n.41
H110,3 $\overline{\mu\gamma}$ $\overline{\delta}$: $\overline{\mu\gamma}$ $\overline{\alpha}$ 86 n.43
H110,6 $\overline{\rho\mu\delta}$: $\overline{\rho\mu}$ δ′ 86 n.45
H111,9 ∠′ γ′ ιβ′ : ∠′ ιβ′ 87 n.52

H111,13 $\overline{\nu\alpha}$ ∠′ ϛ′ : $\overline{\nu\alpha}$ ∠′ 87 n.54
H112,3 $\overline{\lambda}$: $\overline{\alpha}$ 87 n.56
H113,4 ϛ′ : γ′ 88 n.61
H113,5 ιβ′ : ⌈ο 88 n.62
H122,7 $\overline{\mu\alpha}$: $\overline{\mu}$ $\overline{\alpha}$ 93 n.73
H123,11 $\overline{\theta}$: ο 94 n.74
H123,21 ΜΕ : ΜΗ 94 n.75
H138,2 $\overline{\mu\eta}$: $\overline{\mu\eta}$ $\overline{\lambda\beta}$ 99 n.80
H175,7 μθ νη : μθ μη 130 n.108
H181,7 ρ μζ λα λα : ρ μα λα λζ 130 n.108
H183,17 λβ : λβ λ 130 n.108
H186,17 ρλβ ι πθ ν : ρλβ ις πθ μδ 130 n.108
H189,6 δυσμικώτερος seclusi 130 n.110
H196,15 ἀκόλουσθον : ἀκόλουθον 134 n.10
H198,24 ἐφ' ἑαυτοῦ : ὑφ' ἑαυτοῦ 135 n.13
H210,23 να : λ 141 n.29
H210,24 ιβ : να 141 n.29
H210,25 λ : ιβ 141 n.29
H215,38 λε : λϛ 141 n.29
H225,4 ὁμοία seclusi 148 n.39
H225,Fig. Λ addidi 148 n.40
H233,2 σπουδῆς : πάσης σπουδῆς 153 n.45
H239,12 τμῆμα : ἡμικύκλιον 156 n.48
H240,16–17 τῆς ἀνωμαλίας ἐπισκέψεως : τῶν ἀνωμαλιῶν κανονοποιίας 157 n.49
H247,6 $\overline{\beta}$ $\overline{\lambda\delta}$ $\overline{\lambda\varsigma}$: $\overline{\beta}$ $\overline{\lambda\delta}$ 162 n.53
H249,20 $\overline{\beta}$ $\overline{\lambda\delta}$ $\overline{\lambda\varsigma}$: $\overline{\beta}$ $\overline{\lambda\delta}$ 162 n.53
H251,24 πρὸς ἀπογείοις : πρὸς τοῖς ἀπογείοις 165 n.56
H254,5 ἐποιησάμεθα : ποιησόμεθα 166 n.58
H261,14 διάφορον : πλεῖστον διάφορον 171 n.67
H266,5 τῆς σελήνης : τῆς γῆς τουτέστι τοῦ ζῳδιακοῦ διὰ τοῦ κέντρου τῆς σελήνης 173 n.2
H267,4 ταύτας : τὰς αὐτὰς 174 n.3
H269,9 κατὰ τὸ πλάτος : κατὰ πλάτος 175 n.5
H280,5 $\overline{\lambda\alpha}$: $\overline{\lambda}$ 180 n.20
H294,6 ταύτης : καὶ τῆς αὐτῆς 180 n.22
H301,10 σύμφωνος ἀεὶ : σύμφωνος 190 n.28
H317,4–5 ὅμως ὡς μὴ ὑποκειμένου τούτου : ὁμοίως 200 n.42
H317,25 τρίγωνον : ὀρθογώνιον 200 n.43
H318,8 ΒΕΖ : ΒΕΖ ὀρθογώνιον 200 n.44
H319,4 τρίγωνον : ὀρθογώνιον 200 n.43
H319,7 ἐδείχθη $\overline{\rho\kappa}$: $\overline{\rho\kappa}$ 201 n.45
H319,14 τρίγωνον : ὀρθογώνιον 200 n.43
H321,14–15 τοῦ ἐπικύκλου [ἑξήκοντα] ποιεῖ τὸ ἀπ' αὐτῆς $\overline{\gamma\chi}$, ἐὰν τὰ $\overline{\gamma\chi}$: τοῦ ἐπικύκλου τῶν αὐτῶν ἐστιν ξ, ἐὰν τὰ $\overline{\gamma\chi}$ τοῦ τετραγώνου 201 n.46
H332,14 γενομένῃ : γενομένη 208 n.59
H344,5 $\overline{\rho o \varepsilon}$ $\overline{\eta}$: $\overline{\rho o \varepsilon}$ καὶ ἡ 213 n.70

H347,16–17 ἡμίσει καὶ τρίτῳ καὶ δεκάτῳ : ἡμίσει καὶ τρίτῳ καὶ ἡμίσει καὶ τρίτῳ καὶ δεκάτῳ 215 n.75
H353,1–2 τῶν ἐν ἀρχῇ τῆς συντάξεως ὑποδεδειγμένων : τῷ ἐν ἀρχῇ τῆς συντάξεως ἀποδεδειγμένῳ 218 n.3
H353,24–354,1 καὶ διὰ τῆς ἀπεναντίον καὶ παραλλήλου τοῦ κύκλου πλευρᾶς ὥσπερ κεκολλημένος ἀμφοτέραις αὐτῶν ταῖς ἐπιφανείαις: ὥσπερ κεκολλημένος ἀμφοτέραις αὐτῆς ταῖς ἐπιφανείαις 219 n.5
H358,20–21 del. καὶ γράφειν περὶ τὸ Ζ κέντρον τὸν ΔΗ ἔκκεντρον 221 n.8
H360, Fig. corrigenda ut 222 n.9
H363,16 ν′ : να′ 224 n.13
H385,7 ἀποτελουμένης ἀνωμαλίας : ἀποτελουμένης 235 n.28
H395, Fig. corrigenda ut 240 n.33
H404,17–18 del. πρὸς τῇ αὐτῇ γραμμῇ 245 n.41
H416,18–19 transposui post l.8 251 n.50
H417,23 del. πλείστης οὔσης 252 n.55
H431,4 μ̅ ο : μ̅ 260 n.70
H431,13 μ̅ ο : μ̅ 260 n.70
H443,41 μα : κα 264 n.73
H449,16 δι᾽ αὐτοῦ : διὰ τοῦ Η 267 n.81
H451,12–13 del. διὰ τὸ πολὺ μᾶλλον ἐκείνων αὐτὰς μὴ δεδόσθαι 269 n.83
H465,10 ν̅β̅ : ν̅α̅ 277 n.6
H475,2 μ̅ ο : μ̅ 282 n.14
H475,6 ν̅ς̅ ο : ν̅ς̅ 282 n.14
H475,15–17 del. δοθέντος τοῦ κατ᾽ αὐτὴν πλήθους τῶν ἰσημερινῶν ὡρῶν τῆς ἀπὸ τοῦ μεσημβρινοῦ ἀποχῆς 282 n.16
H477,10 δ̅ : δ′ 283 n.19
H485,22 del. καὶ 287 n.31
H490,16 μεγίστης : μέσης 290 n.35
H494,12 μέγιστον : μέσον 292 n.43
H498,8 del. ἀπὸ ο κ̅ε̅ 294 n.47
H501,10 διὰ : καὶ διὰ 295 n.52
H501,18 del. γὰρ 295 n.52
H507,3 del. οὖν 298 n.57
H512,1 φαινομένων : γινομένων 302 n.58
H514,20 γ̅ : γ′ 303 n.62
H519,20 λγ κβ ο : λγ κ β ο 305 n.63
H521,27 μβ : νβ 305 n.63
H521,29 ς : ις 305 n.63
H521,31 μβ : νβ 305 n.63
H537,12–13 del. τῆς ἐκλείψεως ἄνευ 314 n.78
H539,7 ἐκτὸς : ἐντὸς 315 n.82
H544,13 κζ : λδ 318 n.86
H544,23 κδ : κα 318 n.86
Fig. ad finem Vol. I corrigenda ut 316 n.83
II H4,14 τὴν : τῆς 322 n.6
H9,4 λαμβάνει : ἀπολαμβάνει 325 n.32
H11,10 τὸν : τοὺς 326 n.44

H12,12 del. εἰς τὰ ἑπόμενα 327 n.49
H29,7 ι̅β̅ : ιβ′ 336 n.73
H32,1 τ̅ο̅ε̅ : τ̅ο̅θ̅ 337 n.81
H32,18 ἐπέχον : ἀπέχον 338 n.83
H32,19 ι̅β̅ : ιβ′ 338 n.84
H33,20 ἐπέχον : ἀπέχον 338 n.83
H37,2 ταῖς τοῦ : ταῖς τῆς ἐποχῆς ἐπὶ τοῦ 340 n.92
H39,6 ις : ι ς′ 341 n.96
H43,14 κβ ∠′ |ō : κβ ∠′ 343 n.104
H43,15 κγ′ : κ γ′ 343 n.106
H44,19 ἑπόμενος : προηγούμενος 344 n.110
H45,20 πζ : πγ 345 n.111
H46,13 del. ὁμοῦ λ̅α̅ 345 n.114
H47,4 π γ′ : πγ 345 n.112
H47,7 ι γ′ : ιγ 345 n.113
H55,5 ν |ō : νγ ς′ 348 n.123
H58,16 αὐτῶν : αὐτοῦ 350 n.135
H59,3 κδ : κδ ς′ 350 n.133
H64,19 τῶν : τοῦ 353 n.142
H67,19 ις : ι γ′ 354 n.147
H69,13 κς : κγ 354 n.150
H71,18 κς : κγ 355 n.155
H85,18 ι ∠′ : ι 361 n.173
H89,4 ιζ ∠′ : ιζ ς′ 362 n.180
H90,5 ἕκτος : ἐκτὸς 363 n.190
H91,10 κα : κδ 364 n.195
H96,13 δ ∠′ γ′ : ζ δ′ 366 n.212
H96,14 ζ δ′ : δ ∠′ γ′ 366 n.212
H101,6 γ ε′ : γ ς′ 368 n.221
H103,7 γ′ (pr.) : ς′ 369 n.226
H103,8 ∠′ : ς′ 369 n.227
H103,10 β ∠′ : β ∠′ γ′ 369 n.228
H105,7 ς′ : ∠′ 370 n.240
H111,13 κ νο ιη |ō : κ ς′ νο ιη 372 n.7
H111,14 κ ς′ νο ιη : κ νο ιη |ō 372 n.7
H113,7 κε : κθ 373 n.9
H115,18 κγ ∠′ γ′ : κς ∠′ γ′ 375 n.18
H120,10 βορείων : νοτιων 377 n.31
H136,8 αὐτῶν : αὐτοῦ 384 n.76
H147,18 μς ς′ : μς 389 n.100
H149,4 μθ ∠′ δ′ : μθ ∠′ 389 n.101
H161,8 λγ : λ γ′ 395 n.131
H161,12 μγ : μ γ′ 395 n.133
H165,13 α γ′ : λ γ′ 397 n.146
H166,2 αὐτῶν : αὐτῷ 397 n.149
H169,12 ια : ιδ 399 n.157
H172,8 Τοξότου : τόξου 400 n.161

H172,11 Τοξότου : τόξου 400 n.161
H176,18 τῷ νώτῳ : τῶν ὤτων 403 n.172
H176,24 τοῦτο ἠρέμα ὅλον : ὅλον τοῦτο ἠρέμα 403 n.173
H177,13 del. αὐτῇ 403 n.174
H179,4 παρὰ : ὑπὸ 404 n.177
H179,14–15 καὶ τῶν : ἀπὸ τοῦ 404 n.178
H181,5 πλευρὰς : πλευρῶν 405 n.182
H186,13 αὐτοῦ : αὐτῶν 407 n.188
H190,18 ἀνατέλλοντος : ἀνατείλαντος 409 n.195
H190,22 καταδύνοντος : καταδύναντος 409 n.196
H192,19 del. φαινόμενον 410 n.197
H192,20 post μεσουρανήσῃ add. καὶ τὸ ὑπὲρ γῆν τούτου φαινόμενον γίνεται 410 n.197
H194, Fig. corrigenda ut 411 n.200
H198,18 δυνατὸν [εἶναι] : δυνατὸν εἶναι 413 n.204
H200,6 τὸ : τοῦ 414 n.207
H200,7 ΘΖΚ : ΗΘΖΚ 414 n.207
H200,13 κεκλιμένου : ἐγκλινομένου 414 n.208
H203,14 τὸ κατ' αὐτὰς τῶν τῶν ἀστέρων φάσεων τηρήσεις : κατ' αὐτὰς τὰς τῶν φάσεων τηρήσεις 416 n.211
H204,3 ἀπ' αὐτῶν : ἀπὸ 417 n.212
H216,1 $\overline{\zeta\upsilon}$: $\overline{\gamma\upsilon}$ 424 n.25
H216,2 $\overline{\nu\gamma}$: $\overline{\mu\gamma}$ 424 n.26
H219,2 $\overline{\lambda\theta}$: $\overline{\lambda\eta}$ $\overline{\nu\beta}$ $\overline{\lambda}$ 426 n.31
H219,7 $\overline{\nu\beta}$ $\overline{\lambda\eta}$: $\overline{\nu\beta}$ $\overline{\nu\eta}$ 426 n.32
H235,24 ς : νς 426 n.33
H238,3 $\overline{\mu\varepsilon}$: ο $\overline{\mu\varepsilon}$ 426 n.33
H250,17 del. καὶ 442 n.37
H259,4–5 del. ἡ ὑπὸ τῶν ἴσων πλευρῶν 447 n.47
H260,8 post ἀνωμαλίαν add. διαφόρου 448 n.48
H264,18 κθ' : κα' 450 n.58
H264,24 δηλονότι : δὲ 450 n.61
H265,16 del. λ' 451 n.63
H271, Fig. corrigenda ut 454 n.79
H273,19 ιη' : ιθ' 456 n.81
H275,13 κδ' : κα' 456 n.84
H283,4 αὐτοῦ : αὐτῶν 461 n.91
H294,5 $\overline{\iota\eta}$ γ' : $\overline{\iota\eta}$ 467 n.105
H297,5 ιδ' : δ' 469 n.4
H298,14–15 καταλάμπειν : καταλάμψειν 470 n.8
H303,2 ἑκάτερα : ἑκατέρας 472 n.11
H311,4 μέχρι : α' ἔτος 477 n.18
H311,5 ωπδ' : $\overline{\omega\pi\delta}$ ἐστιν ἀπὸ Ναβονασσάρου 477 n.18
H314,22 ἀνωμαλιῶν : ἀνωμαλίας 479 n.20
H318,18 συνοδεύει : συνοδεύσει 481 n.25
H319,8 del. τουτέστιν λειφθεῖσα ὑπ' αὐτῆς 481 n.27
H322,1 διαστάσεως : διαμέτρου στάσεως 484 n.31

H324,8 αὐταὶ : αὗται 486 n.35

H324,22 ΚΛΜ, ΣΤΥ : ΚΛΜ τῶν ΣΤΥ 486 n.37

H329,17 πρὸς τούτῳ : ἐντὸς τούτου 488 n.41

H335,9 $\overline{\xi\theta}$: $\overline{\xi\epsilon}$ 492 n.48

H342,23 ταῦτα : ταὐτὰ 494 n.53

H345,22 ΘΓ : ΘΓΜ 498 n.55

H348,10 del. γ′ 500 n.61

H371, Fig. corrigenda ut 512 n.5

H373, Fig. corrigenda ut 514 n.6

H379,3 post ἑπόμενα add. τοῦ ἀπογείου 518 n.10

H381, Fig. corrigenda ut 519 n.11

H389,2 ἡ ἐκ τοῦ : ἡ ΒΘ ἐκ τοῦ 523 n.18

H396,10 $\overline{\mu\gamma}$: $\overline{\kappa\gamma}$ 527 n.24

H396,13 $\overline{\mu\gamma}$: $\overline{\kappa\gamma}$ 527 n.24

H411,22 $\overline{\iota}$: $\overline{\alpha}$ 537 n.29

H412,1 ὑπόκειται : ὑπέκειτο 537 n.30

H417,13 ὑπόκειται : ὑπέκειτο 540 n.33

H424,6 δ′ : ιδ′ 543 n.37

H425,9 $\overline{\theta}$: $\overline{\iota}$ 543 n.39

H425,14 $\overline{\mu\delta}$: $\overline{\mu\gamma}$ 544 n.39

H428,18 del. πρώτων 545 n.45

H433,4 $\overline{\kappa\varsigma}$: $\overline{\varsigma}$ 547 n.52

H441,49 ιθ : ιε 548 n.55

H442,17 λζ : νζ 548 n.55

H443,34 να : νδ 548 n.55

H443,36 νε : νθ 548 n.55

H443,43 νη : μη 548 n.55

H444,9 κδ : κθ 548 n.55

H460,13 τοῦ ἐκκέντρου : ἐκκέντρου τοῦ 560 n.11

H470,6 τοῦ : τῆς 567 n.28

H470,8 μηδενὸς : μηδὲν 567 n.29

H471,18-19 del. τοιούτων 568 n.34

H471,20 $\overline{\nu}$: $\overline{\nu\eta}$ 568 n.35

H472,5 $\overline{\mu\beta}$: $\overline{\mu\alpha}$ 568 n.36

H474,16 αὐτῶν : αὐταῖς 570 n.41

H475,14 $\overline{\varsigma}$: ς′ 570 n.43

H476,9 αὐτῶν : αὐταῖς 570 n.41

H477,18 αὐτῶν : αὐταῖς 570 n.41

H483,22 $\overline{\nu}\,\overline{\varsigma}$: $\overline{\nu\alpha}$ 575 n.58

H494,20 μεθοδεύομεν : μεθωδεύσαμεν 583 n.82

H497,21 τοῦ ἀπογείου : ἀπὸ τοῦ ἀπογείου 584 n.84

H504,20 del. στίχου 587 n.90

H513,16 del. καὶ 591 n.93

H519,13 $\overline{\nu}$: $\overline{\nu\gamma}$ 595 n.100

H520 del. columnam quartam 596 n.102

H525,23 del. τὸ πλεῖστον 597 n.5

H526,1 del. τῷ πλείστῳ τότε 598 n.6

H537,20 del. τε 602 n.24
H554,11 ΚΛΜ : ΚΑΜ 613 n.36
H590,18 del. μεγίστου 636 n.60
H606,6 κγ α : κγ λ 646 n.83
H606,7 κ η : κ ις 646 n.83

Appendix C

How did Ptolemy derive the mean motions for the five planets?

Our discussion concerns only the mean daily motions in anomaly, since the mean daily motions in longitude are not derived independently: for Venus and Mercury the latter are identical with that of the sun, while for the outer planets they are found by subtracting the mean daily motions in anomaly from the sun's mean daily motion.

The answer to the above question would seem to be provided by those chapters entitled, 'On the correction of the periodic motions [of each planet]', IX 10 (Mercury), X 4 (Venus), X 9 (Mars), XI 3 (Jupiter) and XI 7 (Saturn). In every case Ptolemy determines the position of the planet on the epicycle at one of his own observations, and also at an 'ancient' observation (approximately 400 years earlier). From the (Babylonian) period relations stated in IX 3 he computes how many integer revolutions in anomaly have occurred between the two observations: this plus the increment in degrees derived from the two observations gives the total motion of the planet in anomaly. Division of the latter by the interval in days and fractions of a day between the two observations gives the mean daily motion in anomaly, and Ptolemy explicitly states in every case that this was the basis of the mean daily motion used in the tables (IX 4).

However, if one does the computations implied in the above chapters using Ptolemy's numbers, in no case does one find agreement with the mean daily motions in anomaly which he actually lists,[1] as the following shows.

Ptolemy's mean daily motions in anomaly (IX 3 pp. 424–5)

	♄	0;57,7,43,41,43,40°/d	[1]
	♃	0;54,9,2,46,26,0°/d	[2]
	♂	0;27,41,40,19,20,58°/d	[3]
	♀	0;36,59,25,53,11,28°/d	[4]
	☿	3;6,24,6,59,35,50°/d	[5]
p. 543	♄	travels 35,11,51;27° in 36,57,59;45^{d} → 0;57,7,43,41,*44,18*°/d[2]	[1a]
p. 524	♃	travels 34,31,45;45° in 38,15,32;57,30^{d} → 0;54,9,2,*45,8,48*°/d	[2a]
p. 504	♂	travels 19,13,1;43° in 41,38,1;40^{d} → 0;27,41,40,19,*28,7*°/d[3]	[3a]

[1] Cf. Newton pp. 320–1, 325–7, where the discrepancy is described almost correctly, but implausible consequences drawn.

[2] In these and subsequent computations the last place is rounded on the basis of one more computed place.

[3] Ptolemy gives an increment of '⅔ day', implying 6 a.m. for the first observation and 10 p.m. for the second. If we assume (improbably) that the second was in fact 10;25 p.m. (cf. p. 484 n.32), and

p. 479 ♀	travels 25,35,38;25° in 41,30,52^d → 0;36,59,25,*49,8,51*°/d[4]	[4a]
p. 467 ☿	travels 2,6,52,6;53° in 40,50,13;33,45^d → 3;6,24,6,*58,39,48*°/d[5]	[5a]

The worst of these discrepancies, that for Jupiter,[6] does not produce an error of as much as one minute of arc in 400 years. Hence it is clear that Ptolemy had no motive for 'fudging' here (and also that it is strictly illegitimate to derive a mean motion to the sixth sexagesimal fractional place from observations separated by only 400 years). But, although his observations are essentially in *agreement* with the mean daily motions he uses, the latter cannot be *derived* from them, not at least by the method he states.[7]

An alternative possibility is suggested by the way the derivation of the mean motions is presented in IX 3. There Ptolemy expresses them in the form of 'corrections' to the period relations, e.g. 'for Saturn, 57 returns in anomaly correspond to 59 tropical years plus $1\frac{3}{4}$ days'. These are reduced to degrees and days, e.g. 'Saturn travels (in anomaly) 20520° in 21551;18^d'. It is plausible to suppose that the latter are actually primary, i.e. the corrections 'plus $1\frac{3}{4}$ days' etc. are derived from the equivalences between days and degrees together with the parameter 'one tropical year equals 365;14,48^d'.[8] These equivalences can be derived from the pairs of observations in IX 10 etc., combined with the Babylonian period relations, as follows.

Example: Saturn. From Hipparchus Ptolemy knew the Babylonian period relation, 57 returns in anomaly take place in 59 years, i.e. that the planet travels (57 × 360)° in approximately (59 × 365;14,48)^d. He knew from his pair of observations that it travels 35,11,51;27° in 36,57,59,45^d. From the latter equivalence he could derive a 'correction' to the period of days in the former, by multiplying 36,57,59;45 by (57 × 360) and dividing the result by 35,11,51;27. This produces 5,59,11;17,59,55. . .^{d}, or (rounded to the nearest sixtieth) 21551;18^d, as in IX 3. The corresponding calculations for the other planets are:

♃ 38,15,32;57,30 × (65 × 360) ÷ 34,31,45;45 = 7,12,7;36,42,19. . .^{d} or (rounded) 25927;37^d, as in IX 3.

♂ 41,38,1;40 × (37 × 360) ÷ 19,13,1;43 = 8,0,57;40,45,50. . .^{d} or (rounded) 28857;41^d. Text in IX 3 has 28857;53, emended by me to 28857;43 (cf. n.8).

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the increment actually 16;25^h, this would make the interval 41,38,1;41,2,30^d, leading to 0;27,41,40,*18,46,32*°/d, which is even more discrepant.

[4] But see p. 479 n.21. The interval, which Ptolemy rounds to integer days, should probably be $1\frac{1}{4}$ or $1\frac{1}{2}$ hours less. These corrections lead to daily motions of 0;36,59,25,*51,56,24*° and 0;36,59,25,*52,29,19*°, of which the second is much closer to, but still not identical with, the tabulated daily motion.

[5] Applying the equation of time of −23 mins. to Ptolemy's observation, i.e. taking the increment as 13;7^h, instead of $13\frac{1}{2}$, leads to a daily motion of 3;6,24,*7,3,2*°, which is even more discrepant.

[6] Assuming that we correct the interval for Venus as in n.4.

[7] In case anyone should conjecture that Ptolemy computed the times of the observations more precisely than he states (with e.g. corrections for equation of time), I note that in order to get Ptolemy's mean daily motion accurate to the sixth sexagesimal fractional place directly from the observations, these would have to be recorded to an accuracy of *seconds*, which is totally implausible.

[8] This works well for all planets except Mars (where the text figure, '28857;53^d' is certainly corrupt: I have emended '53' to '43', but '42' would give perfect agreement with the above hypothesis) and Mercury, where '+$1\frac{1}{10}$^{d}' should rather be '+1;3^d.' But, rather than emending to '$1\frac{1}{20}$^{d}' (which is possible), we can regard '$1\frac{1}{10}$^{d}' as simply a small inaccuracy.

♀ 41,30,52 × (5 × 360) ÷ 25,35,38;25 = 48,39;40,5,19. . .^{d} or (rounded) 2919;40^d, as in IX 3.[9]

☿ 40,50,13;33,45 × (145 × 360) ÷ 2,6,52,6;53 = 4,40,2;24,1. . .^{d} or (rounded) 16802;24^d, as in IX 3.

From these 'corrected period relations' the mean daily motions can now be derived:

♄ 20520° in 21551;18^d leads to 0;57,7,43,41,43,39,41. . .°/d, in agreement with [1].

♃ 23400° in 25927;37^d leads to 0;54,9,2,*42,55,52*. . .°/d, in disagreement with [2], and worse than [2a].

♂ 13320° in 28857;41^d leads to 0;27,41,40,*18,39,12*. . .°/d, in disagreement with [3], and worse than [3a].[10]

♀ 1800° in 2919;40^d leads to 0;36,59,25,53,11,27,36. . .°/d, in agreement with [4].[11]

☿ 52200° in 16802;24^d leads to 3;6,24,6,59,35,49,55. . .°/d, in agreement with [5].

Thus, perverse as this procedure may appear, it could theoretically be used to derive Ptolemy's mean motions for Saturn, Venus and Mercury. However, it fails miserably for Jupiter and Mars, which casts doubt on the validity of this explanation in general.

Let us suppose, instead, that Ptolemy found his mean daily motions by some other method. Then the equivalences 'Saturn travels *20520° in 21551;18^d*' etc. can be directly derived by division of 20520 by 0;57,7,43,41,43,40, etc.,[12] and the pairs of observations in IX 10 etc. are simply used as a *check*. E.g. for Saturn Ptolemy found from the observations an increment of 351;27° in 364^y 219$\frac{3}{4}^d$. From the mean motion tables one finds, for the latter interval, 351;26,59°. The corresponding numbers for the other planets are:

♃ 377^y 128^d −1^h	observations 105;45°	tables 105;45,48°
♂ 410^y 231$\frac{2}{3}^d$	observations 61;43°	tables 61;42,55°
♀ 409^y 167^d	observations 338;25°	tables 338;27,48°[13]
☿ 402^y 283^d 13$\frac{1}{2}^h$	observations 246;53°	tables 246;53,28°.

Thus the observations can in every case be regarded as justifying the mean motions used, within the accuracy attainable. On this assumption, Ptolemy had derived his mean motions from some other source, and simply did not bother to

[9] Taking an interval 1$\frac{1}{4}$ or 1$\frac{1}{2}$ hours less (see n.4) makes no difference to the first sexagesimal fractional place.

[10] Taking the sexagesimal fraction of the day as 42,43 or 53 (cf. n.8) produces a progressively smaller mean daily motion and progressively greater disagreement.

[11] It is interesting that this quotient lies almost exactly in the middle between the mean daily motion which Ptolemy gives explicitly (28 in the last sexagesimal place) and that underlying the sections for years and 18-year periods in the mean motion tables (27 in the last sexagesimal place, cf. p. 425 n.29). Is this an indication of incomplete revision?

[12] Mars is still a problem here, since this method also produces 28857;41^d (cf. n.8).

[13] For an interval 1$\frac{1}{2}^h$ less (cf. n.4) one finds from the tables 338;25,30°, in agreement with the result from the observations.

change them on the basis of the observations he quotes (in this he was absolutely justified, since, as we saw above, an interval of 400 years is insufficient to guarantee more than 4 sexagesimal fractional places; he was not of course justified in concealing it from his readers).

This still leaves unexplained the basis of the actual mean motions. One might conjecture that they were derived from observations made over a shorter period (e.g. between Hipparchus and Ptolemy). It is easy to find, by Diophantine analysis, plausible intervals in time and longitude which produce the exact numbers, e.g. for Mars a motion in $274^y\ 189;16^d$ of 128 revolutions plus 169;32° leads to a mean daily motion of 0;27,41,40,19,20,57,59°/d. But in the absence of any evidence for such observations by Hipparchus this remains mere arithmetical juggling, and we must admit that the origin of these numbers, at least for Jupiter and Mars, and probably for all the planets, remains unknown.[14]

[14] An alternative conjecture is that the mean motions were indeed derived from the quoted observations, but by applying a 'correction' to an earlier (?Hipparchan) mean motion, in the same way as the mean motion in lunar anomaly was corrected in IV 7 (and in lunar latitude in the Canobic Inscription). But since no such mean motion is mentioned by Ptolemy, the details would be irrecoverable.

Bibliography

Aaboe, A., 'On the Babylonian Origin of Some Hipparchian Parameters'. Centaurus 4, 1955, 122–125.

ACT: Astronomical Cuneiform Texts, ed. O. Neugebauer. 3 vols. Princeton and London, n.d. [1955].

Allinson, Francis G., Lucian Satirist and Artist. New York, 1926.

Apollonius, *Conics*: Apollonii Pergaei quae Graece exstant ed. I. L. Heiberg. 2 vols. Leipzig (Teubner), 1891, 1893.

Aratus: Arati Phaenomena rec. Ernestus Maass. Berlin, 1893.

Archimedes, Opera Omnia ed. J. L. Heiberg. Second edn. 3 vols. Leipzig (Teubner), 1910–1915.

Archimedes (Heath): The Works of Archimedes edited in modern notation with introductory chapters by T. L. Heath, with a supplement, The Method of Archimedes recently discovered by Heiberg. New York, n.d. (Dover), reprint of editions Cambridge 1897, 1912.

Aristarchus: see Heath, T. L.

Aristotle's Metaphysics. A revised text with introduction and commentary by W. D. Ross. 2 vols. Oxford, 1924.

Aristotle, Meteorologica, with an English translation by H. D. P. Lee. Second edn. London and Cambridge, Mass. (Loeb Classical Library), 1962.

Athenaeus: Athenaei Naucratitae Dipnosophistarum Libri XV rec. G. Kaibel. 3 vols. Leipzig (Teubner), 1887–1890.

Autolycus de Pitane, ed. Joseph Mogenet. (Université de Louvain, Recueil de travaux d'histoire et de philologie, 3ᵉ sér. fasc. 37). Louvain, 1950.

Avienius: Rufi Festi Avieni Carmina rec. Alfred Holder. Innsbruck, 1887.

Bayer: Ioannis Bayeri Vranometria, omnium asterismorum continens schemata. Augustae Vindelicorum, 1603.

Bečvář, Antonín, Atlas of the Heavens. Atlas Coeli 1950.0. Prague and Cambridge, Mass., 1958. [Also known as the 'Skalnate Pleso Atlas'].

Berger, Hugo, Die geographischen Fragmente des Eratosthenes. Leipzig, 1880.

Böckh, August, Ueber die vierjährigen Sonnenkreise der Alten, vorzüglich den Eudoxischen. Berlin, 1863.

Boll, Franz, 'Studien über Claudius Ptolemäus'. Jahrbücher für Classische Philologie, Supplementband 21, 1894, 51–244.

Boll, F., and Gundel, W., 'Sternbilder, Sternglaube und Sternsymbolik bei Griechen und Römern'. Ausführliches Lexikon der griechischen und römischen Mythologie ed. W. H. Roscher, Bd. 6. Leipzig and Berlin, 1924–37, cols. 867–1071.

Bouché-Leclercq, A., L'Astrologie Grecque. Paris, 1899.

Britton [1]: John Phillips Britton, On the Quality of Solar and Lunar Observations and Parameters in Ptolemy's *Almagest*. Ph.D. Thesis, Yale, 1967.

Britton [2]: John P. Britton, 'Ptolemy's Determination of the Obliquity of the Ecliptic'. Centaurus 14, 1969, 29-41.

BSC: Yale University Observatory, Catalogue of Bright Stars. Third Revised Edition by Dorrit Hoffleit. New Haven, 1964.

Burnham: Robert Burnham Jr., Burnham's Celestial Handbook. 3 vols. New York, 1978.

Calcidius: Timaeus a Calcidio translatus commentarioque instructus, ed. J. H. Waszink. London and Leiden, 1962.

Callimachus, Aetia-Iambi-Lyric Poems, ed. C. A. Trypanis. London and Cambridge, Mass. (Loeb Classical Library), 1958.

Campanus: Campanus of Novara and medieval planetary Theory, *Theorica Planetarum*, ed. Francis S. Benjamin Jr. and G. J. Toomer. Madison, Wisconsin, 1971.

CCAG: Catalogus Codicum Astrologorum Graecorum [I], Codices Florentinos descr. A. Olivieri. Bruxellis, 1898.

Cumont, Franz, 'Les noms des planètes et l'astrolatrie chez les Grecs'. L'Antiquité Classique 4, 1935, 5-43.

Daremberg-Saglio: Dictionnaire des Antiquités Grecques et Romaines, sous la direction de Ch. Daremberg et Edm. Saglio. 5 parts. Paris, 1875-1919.

Delambre, J. B. J., Histoire de l'astronomie ancienne. 2 vols. Paris, 1817.

Diels-Kranz: Die Fragmente der Vorsokratiker, von Hermann Diels. Zehnte Auflage herausgegeben von Walther Kranz. 3 vols. Berlin, 1961.

Dindorf, *Thesaurus*: Thesaurus Graecae Linguae ab Henrico Stephano constructus, tertio ed. Carolus Benedictus Hase, Guilielmus Dindorfius et Ludovicus Dindorfius. 8 vols. Paris, 1831-51.

Dio Cassius: Cassii Dionis Cocceiani Historiarum Romanarum quae supersunt ed. U. P. Boissevain. 5 vols. repr. Berlin, 1955.

Epicurus, *Letter to Pytheas*: Epicurus, Epistolae Tres et Ratae Sententiae ed. P. von der Muehll. Leipzig (Teubner), 1922.

Eratosthenes, *Geography*: see Berger, Hugo.

Eratosthenis Catasterismorum Reliquiae ed. C. Robert. Berlin, 1878.

Euclid: Euclidis Elementa ed. I. L. Heiberg. (Euclidis Opera Omnia vols. 1-5). Leipzig (Teubner), 1883-8.

Euclid (Heath): The Thirteen Books of Euclid's Elements, translated by Sir Thomas L. Heath. Second edn. 3 vols. Cambridge, 1926.

Euclid, *Data*: Euclidis Data cum commentariis Marini et scholiis antiquis ed. Henricus Menge. (Euclidis Opera Omnia vol. 6). Leipzig (Teubner), 1896.

Euclid, *Phaenomena*: Euclidis Phaenomena et Scripta Musica ed. Henricus Menge. (Euclidis Opera Omnia vol. 8). Leipzig (Teubner), 1916.

Eudoxi Ars Astronomica qualis in charta Aegyptiaca superest ed. Fr. Blass. Kiel [University Programme], 1887.

Fotheringham [1]: J. K. Fotheringham, 'The Secular Acceleration of the Moon's Mean Motion as Determined from the Occultations in the *Almagest*', Monthly Notices of the Royal Astronomical Society 75, 1915, 377-94.

Fotheringham [2]: J. K. Fotheringham, 'The Secular Acceleration of the Moon's Mean Motion as Determined from the Occultations in the *Almagest* (a Correction)'. Monthly Notices of the Royal Astronomical Society 83, 1923, 370–3.

Fotheringham [3] J. K. Fotheringham, 'Note on the Secular Accelerations of the Sun and Moon as determined from the Ancient Lunar and Solar Eclipses, Occultations, and Equinox Observations'. Monthly Notices of the Royal Astronomical Society 80, 1920, 578–81.

Fraser [1] P. M. Fraser, Ptolemaic Alexandria. 3 vols. Oxford, 1972.

Fraser [2] P. M. Fraser, 'Bibliography: Greco-Roman Egypt.' Journal of Egyptian Archaeology 47, 1961, 139–49.

Galen: Claudii Galeni Pergameni Scripta Minora Vol. II ex recognitione Iwani Mueller. Leipzig (Teubner), 1891.

Galen, 'On Seven-Month Children': R. Walzer, Galens Schrift 'Ueber die Siebenmonatskinder'. Rivista degli Studi Orientali 15, 1935, 323–57.

GAS VI: Fuat Sezgin, Geschichte des Arabischen Schrifttums, Band VI, Astronomie bis ca. 430 H. Leiden, 1978.

Germanici Caesaris Aratea cum Scholiis ed. Alfred Breysig. Berlin, 1867.

Geminus, *Eisagoge*: Gemini Elementa Astronomiae ed. C. Manitius. Leipzig (Teubner), 1898.

Ginzel, F. K., Handbuch der mathematischen und technischen Chronologie. 3 vols. Leipzig, 1906–14.

Glowatzki, Ernst and Göttsche, Helmut, Die Sehnentafel des Klaudios Ptolemaios. Nach den historischen Formelplänen neuberechnet. München, 1976.

Gundel, W., article 'Kyknos 9'. RE 11.2, 1922, cols. 2442–51.

Halma [N., ed. and tr.], Composition Mathématique de Claude Ptolémée. 2 vols. Paris, 1813, 1816.

HAMA: O. Neugebauer, A History of Ancient Mathematical Astronomy. 3 vols. Berlin-Heidelberg-New York, 1975.

Hartner, Willy, 'The Mercury Horoscope of Marcantonio Michiel of Venice'. Vistas in Astronomy, ed. Arthur Beer, vol. 1, London and New York, 1955, 84–138.

Haskins, C. H., Studies in the History of Mediaeval Science. Cambridge, Mass., 1924, repr. New York, 1960.

Heath, T. L., Aristarchus of Samos, the Ancient Copernicus. Oxford, 1913.

Heath, HGM: Sir Thomas Heath, A History of Greek Mathematics. 2 vols. Oxford, 1921.

Heath. See also Euclid.

Heiberg, J. L. See Apollonius, Euclid, Ptolemy, Theodosius.

Hennig, Richard, Terrae Incognitae I, Altertum bis Ptolemäus. Leiden, 1936.

Hephaestionis Thebani Apotelesmaticorum Libri Tres ed. David Pingree. Vol. I. Leipzig (Teubner), 1973.

Heron, *Belopoeica*: see Marsden, Greek and Roman Artillery.

Heron, *Dioptra*: Herons von Alexandria Vermessungslehre and Dioptra, ed. Hermann Schöne. (Heronis Alexandrini Opera quae supersunt omnia, Vol. III). Leipzig (Teubner), 1903.

Heron, *Pneumatica*: Heronis Alexandrini Opera quae supersunt omnia, Vol. I, Pneumatica et Automata rec. Wilhelm Schmidt. Leipzig (Teubner), 1899.

Hesiod, *Works and Days*: Hésiode, Texte établi et traduit par Paul Mazon. Paris (Collection Budé), 1951.

Hipparchus, *Comm. in Arat.*: Hipparchi in Arati et Eudoxi Phaenomena Commentariorum Libri Tres rec. Carolus Manitius. Leipzig (Teubner), 1894.

Homer, Hymns: The Homeric Hymns. ed. T. W. Allen, W. R. Halliday and E. E. Sikes. Second edn. Oxford, 1936.

Honigmann, Ernst, Die sieben Klimata und die ΠΟΛΕΙΣ ΕΠΙΣΗΜΟΙ. Heidelberg, 1929.

Hultsch, 'Sehnentafeln': F. Hultsch, 'Die Sehnentafeln der griechischen Astronomen'. Das Weltall 2, 1901/2, 49–55.

Ibn aṣ-Ṣalāḥ, Zur Kritik der Koordinatenüberlieferung im Sternkatalog des *Almagest*, ed. Paul Kunitzsch. (Abh. Ak. Wiss. Göttingen, Phil.-hist. Kl., 3F, Nr.94). Göttingen, 1975.

Ideler, Ludwig, Historische Untersuchungen über die astronomischen Beobachtungen der Alten. Berlin, 1806.

Kiepert, H. and R., Formae Orbis Antiquae. Berlin, 1893–1914.

King, L. W., Babylonian Boundary-Stones and Memorial-Tablets in the British Museum. With an Atlas of Plates. London, 1912.

Kühner-Gerth: Ausführliche Grammatik der Griechischen Sprache von Dr. Raphael Kühner. Zweiter Teil. Satzlehre. 3[e] Auflage von Dr. Bernhard Gerth. 2 vols. Hannover and Leipzig, 1898, 1904.

Kugler, F. X., Die Babylonische Mondrechnung. Freiburg im Breisgau, 1900.

Kugler, SSB: F. X. Kugler, Sternkunde und Sterndienst in Babel. 2 vols. Münster, 1907–24.

Kunitzsch, Paul, Der Almagest. Die Syntaxis Mathematica des Claudius Ptolemäus in arabisch-lateinischer Überlieferung. Wiesbaden, 1974.

Lewis, Naphtali, Papyrus in Classical Antiquity. Oxford, 1974.

LSJ: A Greek-English Lexicon compiled by Henry George Liddell and Robert Scott. A New Edition Revised . . . by Henry Stuart Jones. 2 vols. Oxford, 1940. Supplement, edited by E. A. Barber. Oxford, 1968.

Maass, E. (ed.), Commentariorum in Aratum Reliquiae. Berlin, 1898.

Manitius, Karl, 'Fixsternbeobachtungen des Altertums'. Das Weltall 5, 1904/5, 14–18, 23–6.

Manitius [translation]: Ptolemäus, Handbuch der Astronomie. Deutsche Übersetzung von K. Manitius. 2 vols. Leipzig, 1912, 1913. Second edn., revised by O. Neugebauer. Leipzig, 1963.

Marsden, E. W., Greek and Roman Artillery. Technical Treatises. Oxford, 1971.

Menelaus, *Sphaerica*: Max Krause, Die Sphärik von Menelaus aus Alexandrien in der Verbesserung von Abū Naṣr Manṣūr b. ʿAlī b. ʿIrāq. (Abh. Ges. Wiss. zu Göttingen, Phil.-hist. Kl., 3F., 17). Berlin, 1936.

Mercati, Iohannes, and Franchi de' Cavalieri, Pius, Codices Vaticani Graeci, Tomus I. Roma, 1923.

MGH: Monumenta Germaniae Historica, Auctorum Antiquissimorum Tomus XIII, Chronica Minora Saec. IV.V.VI.VII ed. Th. Mommsen, Vol. III. Berlin 1898.

Neugebauer [1]: O. Neugebauer, 'Untersuchungen zur antiken Astronomie V. Der Halleysche "Saros" und andere Ergänzungen zu UAA III'. Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik B 4, 1938, 407–11.

Neugebauer [2]: O. Neugebauer, 'On Some Aspects of Early Greek Astronomy'. Proceedings of the American Philosophical Society 116 no. 3, June 1972, 243–51.

Neugebauer [3]: O. Neugebauer, 'Über eine Methode zur Distanzbestimmung Alexandria-Rom bei Heron'. Det Kgl. Danske Videnskabernes Selskab, Hist.-fil. Medd. XXVI 2, 1938.

Neugebauer, O. and Schmidt, O., 'Hindu Astronomy at Newminster in 1428'. Annals of Science 8, 1952, 221–8.

Neugebauer, O. and Van Hoesen, H. B., Greek Horoscopes. (Memoirs of the American Philosophical Society, Vol. 48). Philadelphia, 1959.

Neugebauer, O. See also HAMA.

Neugebauer, P. V., Spezieller Kanon der Mondfinsternisse für Vorderasien und Ägypten von 3450 bis 1 v. Chr. Astronomische Abhandlungen. (Ergänzungshefte zu den Astronomischen Nachrichten Bd. 9 Nr. 2). Kiel, 1934.

Neugebauer, P. V., Astronomische Chronologie. 2 vols. Berlin and Leipzig, 1929.

Newton, Robert R., The Crime of Claudius Ptolemy. Baltimore and London, 1977.

Op. Min. See Ptolemy, *Opera Minora.*

Oppolzer, Th. v., Canon der Finsternisse (Ak. d. Wiss., Wien, Denkschriften, LII). Wien, 1887. English translation by Owen Gingerich as Canon of Eclipses. New York, 1962.

The Oxford English Dictionary. 13 vols. Oxford, 1933.

Pappus, Commentary on the Almagest: see Rome [1].

Pappus, *Synagoge*: Pappi Alexandrini Collectionis quae supersunt, ed. F. Hultsch. 3 vols. Berlin, 1875–8.

Parker-Dubberstein: Richard A. Parker and Waldo H. Dubberstein, Babylonian Chronology 626 B.C.–A.D. 75 (Brown University Studies XIX). Providence, 1956.

Pedersen: Olaf Pedersen, A Survey of the Almagest. (Acta Historica Scientiarum Naturalium et Medicinalium Vol. 30). Odense University Press, 1974.

Peters, Christian Heinrich Friedrich, and Knobel, Edward Ball, Ptolemy's Catalogue of Stars. A Revision of the Almagest. Washington, 1915.

Petersen, Viggo M., and Schmidt, Olaf, 'The Determination of the Longitude of the Apogee of the Orbit of the Sun according to Hipparchus and Ptolemy'. Centaurus 12, 1967, 73–96.

Pfeiffer, Erwin, Studien zum antiken Sternglauben. (ΣΤΟΙΧΕΙΑ II). Leipzig and Berlin, 1916.

P-K: see Peters and Knobel.

Pliny, NH: Pliné l'Ancien, Histoire naturelle, Livre II, texte établi, traduit et commenté par Jean Beaujeu. Paris (Collection Budé), 1950.

Pollux: Pollucis Onomasticon ed. E. Bethe. 3 vols. Leipzig (Teubner), 1900–37.

Porphyry, *Comm. on Harmonica*: Porphyrius, Εἰς τὰ ἁρμονικὰ Πτολεμαίου ὑπόμνημα ed. Ingemar Düring (Göteborgs Högskolas Årsskrift XXXVIII 1932:2).

Posidonius, I. The Fragments, ed. L. Edelstein and I. G. Kidd. Cambridge, 1972.

P. Oxy.: The Oxyrhynchus Papyri. Part I, ed. B. P. Grenfell and A. S. Hunt. London, *1898*. Part XIX, ed. E. Lobel, E. P. Wegener, C. H. Roberts and H. I. Bell. London, 1948.

P. Ryl.: Catalogue of the Greek Papyri in the John Rylands Library Manchester. Vol. I ed. Arthur S. Hunt. Manchester and London, 1911.

Price, Derek J., 'Precision Instruments to 1500', with a section on Hero's instruments by A. G. Drachmann. A History of Technology, ed. Charles Singer et al., Vol. III, Oxford, 1957, 582–619.

Pritchard, James B. (ed.), Ancient Near Eastern Texts Relating to the Old Testament. Third edn. Princeton, 1969.

Proclus, Hypotyposis Astronomicarum Positionum, ed. C. Manitius. Leipzig (Teubner), 1909.

Pseudo-Eratosthenes: see Eratosthenis Catasterismorum Reliquiae.

Ptolemy, Almagest: Claudii Ptolemaei Opera quae exstant omnia. Vol. I, Syntaxis Mathematica, ed. J. L. Heiberg. 2 vols. Leipzig (Teubner), 1898, 1903.

Ptolemy, Almagest. See also Halma, Manitius, Taliaferro.

Ptolemy, *Apotelesmatica*: Claudii Ptolemaei Opera quae exstant omnia. Vol. III, ΑΠΟΤΕΛΕΣΜΑΤΙΚΑ ed. F. Boll et Ae. Boer. Leipzig (Teubner), 1957.

Ptolemy, *Geography*: Claudii Ptolemaei Geographia ed. C. F. A. Nobbe. 2 vols. Leipzig, 1843, 1845.

Ptolemy, Handy Tables: N. Halma (ed.), Tables manuelles astronomiques de Ptolémée et de Théon. 3 parts. Paris, 1822–5.

Ptolemy, *Op(era) Min(ora)*: Claudii Ptolemaei Opera quae exstant omnia. Vol. II, Opera Astronomica Minora, ed. J. L. Heiberg. Leipzig (Teubner), 1907.

Ptolemy, *Optics*: L'Optique de Claude Ptolémée dans la version latine d'après l'arabe de l'émir Eugène de Sicile, ed. Albert Lejeune. (Université de Louvain, Recueil de travaux d'histoire et de philologie, 4e série, fasc. 8). Louvain, 1956.

Ptolemy, *Phaseis*: in *Opera Minora*, 3–67.

Ptolemy, *Planetary Hypotheses*: The Arabic Version of Ptolemy's *Planetary Hypotheses*, ed. Bernard R. Goldstein. Transactions of the American Philosophical Society, N.S. Vol. 57.4. Philadelphia, 1967.

R-E: Paulys Real-Encyclopädie der classischen Altertumswissenschaft, Neue Bearbeitung . . . von G. Wissowa, W. Kroll [and others]. Stuttgart, 1894–

Regiomontanus, Epytoma: Epytoma Ioannis De monte regio In almagestum ptolomei. Venice, 1496.

Rehm, A., 'Hipparchos 18', R-E VIII.2, 1913, cols. 1666-81.

Rehm, A., Griechische Windrosen, (SB Königl. Bayer. Ak. d. Wiss., Philos.-philol. u. hist. Kl. 1916.3).

Richter, Gisela M. A., A Handbook of Greek Art. Third edn. London, 1963.

Riddell, R. C., 'The Latitudes of Venus and Mercury in the Almagest'. Archive for History of Exact Sciences 19, 1978, 95-111.

Robert, Louis, A travers l'Asie Mineure. (Bibliothèque des Ecoles Françaises d'Athènes et de Rome, 239). Paris, 1980.

Rome [1]: A. Rome (ed.), Commentaires de Pappus et de Théon d'Alexandrie sur l'Almageste.

Tome I. Pappus d'Alexandrie, Commentaire sur les livres 5 et 6 de l'Almageste. (Studi e Testi 54). Roma, 1931.

Tome II. Théon d'Alexandrie, Commentaire sur les livres 1 et 2 de l'Almageste. (Studi e Testi 72). Città del Vaticano, 1936.

Tome III. Théon d'Alexandrie, Commentaire sur les livres 3 et 4 de l'Almageste. (Studi e Testi 106). Città del Vaticano, 1943.

Rome [2]: A. Rome, 'Glanures dans l'idylle 15 de Théocrite'. Académie royale de Belgique, Bulletin de la Classe des Lettres, 5^e sér. T. 37, 1951, 260-7.

Rome [3]: A. Rome, 'Les Observations d'Équinoxes de Ptolémée. Ptolémée et le mouvement de l'apogée solaire'. Ciel et Terre 59, 1943, 1-15.

Rome [4]: A. Rome, 'L'Astrolabe et le Météoroscope d'après le commentaire de Pappus sur le 5^e livre de l'Almageste'. Annales de la Société Scientifique de Bruxelles 47, 1927, Deuxième partie, Mémoires, 77-102.

Rome [5]: A. Rome, 'Les observations d'équinoxes et de solstices dans le chapitre 1 du livre 3 du Commentaire sur l'Almageste par Théon d'Alexandrie'. [I] Annales de la Société Scientifique de Bruxelles 57, sér. I, 1937, 213-36. [II] ibid. 58, sér. I, 1938, 6-26.

Rome [6]: A. Rome, 'The Calculation of an Eclipse of the Sun according to Theon of Alexandria'. Proceedings of the International Congress of Mathematicians, Cambridge, Mass., U.S.A., Aug. 30-Sept. 6, 1950. I, 209-19.

Sachs [1]: A. J. Sachs, 'Babylonian observational astronomy'. Philosophical Transactions of the Royal Society, London, A.276, 1974, 43-50.

Sach [2]: A. J. Sachs, 'A classification of the Babylonian astronomical tablets of the Seleucid period'. Journal of Cuneiform Studies 2, 1948, 271-90.

Samuel, Alan E., Greek and Roman Chronology. Calendars and Years in Classical Antiquity. (Handbuch der Altertumswissenschaft, Abt. I, Teil 7). München, 1972.

Sattler, Peter, Studien aus dem Gebiet der Alten Geschichte. Wiesbaden, 1962.

Sawyer, Frederick W., 'On Ptolemy's Determination of the Apsidal Line for Venus'. Appendix to: Bernard R. Goldstein, 'Remarks on Ptolemy's Equant Model in Islamic Astronomy'. ΠΡΙΣΜΑΤΑ (Festschrift für Willy Hartner), ed. Y. Maeyama and W. G. Saltzer. Wiesbaden, 1977, 169-81.

Schjellerup, C. F. C., 'Recherches sur l'astronomie des anciens III. Sur les conjonctions d'étoiles avec la lune rapportées par Ptolémée'. Copernicus 1, 1881, 223-36.

Schlachter, Alois, Der Globus, seine Entstehung und Verwendung in der Antike. (ΣΤΟΙΧΕΙΑ VIII). Leipzig and Berlin, 1927.

Schmidt, Olaf, 'Bestemmelsen af Epoken for Maanens Middelbevaegelse i. Bredde hos Hipparch og Ptolemaeus'. Matematisk Tidsskrift B 1937, 27-32.

Schram, Robert, Kalendariographische und Chronologische Tafeln. Leipzig, 1908.

See, T. J. J., 'Historical Researches Indicating a Change in the Color of Sirius, between the Epochs of Ptolemy, 138, and of Al Sûfî, 980 A.D.' Astronomische Nachrichten 229, 1927, cols. 245-72.

Seidl, Ursula, 'Die babylonischen Kudurru-Reliefs'. Deutsches Archäologisches Institut, Baghdader Mitteilungen 4, 1968, 7-220.

Strabo: The Geography of Strabo, with an English translation by Horace Leonard Jones. 8 vols. London and Cambridge, Mass. (Loeb Classical Library), 1917-32.

Swerdlow, Noel, 'Hipparchus on the Distance of the Sun'. Centaurus 14, 1969, 287-305.

Swerdlow, Noel M., 'The origin of Ptolemaic planetary theory'. Forthcoming.

Swerdlow, N., and Neugebauer, O., Mathematical Astronomy in Copernicus' *De Revolutionibus*. (Studies in the History of Mathematics and Physical Sciences). Forthcoming.

Taliaferro, R. Catesby. The Almagest by Ptolemy. *In* Great Books of the Western World, Chicago (Encyclopaedia Britannica), 1952, Vol. 16.

Theodosius, *Sphaerica*: Theodosius Tripolites, Sphaerica, von J. L. Heiberg. (Abh. der Ges. d. Wiss. zu Göttingen, Phil-hist. Kl., N.F. XIX.3). Berlin, 1927.

Theon of Alexandria, Commentary on the Almagest: see Rome [1].

Theon of Smyrna: Theonis Smyrnaei Expositio rerum mathematicarum ad legendum Platonem utilium, ed. E. Hiller. Leipzig (Teubner), 1878.

Theophrastus, *Historia Plantarum*: Theophrastus, Enquiry into Plants, with an English translation by Sir Arthur Hort. 2 vols. London and Cambridge, Mass. (Loeb Classical Library), 1916.

Thiele, Georg, Antike Himmelsbilder. Berlin, 1898.

Tihon, Anne, 'Le calcul de l'éclipse de Soleil du 16 juin 364 p.C. et le "Petit Commentaire" de Théon'. Bulletin de l'Institut Belge de Rome, fasc. 46-7, 1976-7, 35-79.

Toomer [1]: G. J. Toomer, 'The Mathematician Zenodorus'. Greek, Roman and Byzantine Studies 13, 1972, 177-92.

Toomer [2]: G. J. Toomer, 'The Chord Table of Hipparchus and the Early History of Greek Trigonometry'. Centaurus 18, 1973, 6-28.

Toomer [3]: G. J. Toomer, review of Olaf Pedersen, A Survey of the Almagest, Archives Internationales d'Histoire des Sciences 27, 1977, 137-50.

Toomer [4]: G. J. Toomer, review of E. Glowatzki and H. Göttsche, Die Sehnentafel des Klaudios Ptolemaios, Centaurus 21, 1977, 321-3.

Toomer [5]: article PTOLEMY. Dictionary of Scientific Biography XI, 1975, 186-206.

Toomer [6]: article HIPPARCHUS. Dictionary of Scientific Biography XV, 1978, 207-24.

Toomer [7]: article METON. Dictionary of Scientific Biography IX, 1974, 337-40.

Toomer [8]: G. J. Toomer, 'The Size of the Lunar Epicycle According to Hipparchus'. Centaurus 12, 1967, 145-50.

Toomer [9]: G. J. Toomer, 'Hipparchus on the Distances of the Sun and Moon'. Archive for History of Exact Sciences 14, 1974, 126-42.

Toomer [10]: G. J. Toomer, 'A Survey of the Toledan Tables'. Osiris 15, 1968, 5-174.

Toomer [11]: G. J. Toomer, 'Hipparchus' Empirical Basis for His Lunar Mean Motions'. Centaurus 24, 1981, 97-109.

Tuckerman: Bryant Tuckerman, Planetary, Lunar and Solar Positions at Five-day and Ten-day Intervals. [Vol. I,] 601 B.C. to A.D. 1 (Memoirs of the American Philosophical Society, Vol. 56), Philadelphia, 1962. [Vol. II,] A.D. 2 to A.D. 1649 (Memoirs of the American Philosophical Society, Vol. 59), Philadelphia, 1964.

van der Waerden, Bartel Leendert, 'Ewige Tafeln'. Arithmos-Arrythmos, Skizzen aus der Wissenschaftsgeschichte, Festschrift für Joachim Fleckenstein zum 65. Geburtstag, ed. K. Figala and E. H. Berninger, München, 1979, 285-93.

Vettius Valens: Vettii Valentis Anthologiarum Libri ed. Guilelmus Kroll. Berlin, 1908.

Vogt, Heinrich, 'Versuch einer Wiederherstellung von Hipparchs Fixsternverzeichnis'. Astronomische Nachrichten 224, 1925, cols. 17-54.

Wilson, Curtis, 'The Inner Planets and the Keplerian Revolution'. Centaurus 17, 1973, 205-48.

Xenophon, Memorabilia: Xenophontis Commentarii rec. Carolus Hude. Leipzig (Teubner), 1934.